Downlink Cellular Interference Alignment

Paul Ferrand, Jean-Marie Gorce

To cite this version:

Paul Ferrand, Jean-Marie Gorce. Downlink Cellular Interference Alignment. [Research Report] RR-8543, INRIA. 2014. hal-00996728

HAL Id: hal-00996728

https://hal.inria.fr/hal-00996728

Submitted on 26 May 2014

Downlink Cellular Interference Alignment

Paul Ferrand, Jean-Marie Gorce

RESEARCH REPORT

N° 8543

Mai 2014

Project-Team Socrate

ISSN 0249-6399   ISRN INRIA/RR--8543--FR+ENG

Downlink Cellular Interference Alignment

Paul Ferrand, Jean-Marie Gorce

Project-Team Socrate

Research Report n° 8543 — Mai 2014 — 31 pages

Abstract: Cellular networks have been notoriously interference-limited systems in dense urban areas, where base stations are deployed in close proximity to one-another. Recently, a signal processing method called Interference Alignment has emerged, making use of the increasing signal dimensions available in the system through multiple-input multiple output (MIMO) and Orthogonal Frequency Division Multiplexing (OFDM) technologies. In this report, we review the state of the art of interference alignment since its foundation, and we detail algorithms and baseline comparisons to make when applying interference alignment schemes to downlink cellular networks. We also propose a number of research directions of interest which are not yet answered in the current literature.

Key-words: interference alignment, signal processing, cellular networks

RESEARCH CENTRE
GRENOBLE – RHÔNE-ALPES

Inovallée
655 avenue de l'Europe Montbonnot
38334 Saint Ismier Cedex

Alignement d'interférence dans les réseaux cellulaires en voie descendante

Résumé : Les réseaux cellulaires ont été l'exemple typique de réseaux dont les performances sont limités par les interférences, particulièrement dans les régions urbaines. Récemment, une nouvelle technique de traitement du signal appelée "alignement d'interférences" a été dévelopée, et permet d'utiliser les dimensions du signal reçu à travers les technologies MIMO (multiple input multiple output) et OFDM (orthogonal frequency division multiplexing) pour annuler tout ou partie de l'interférence reçue par les mobiles. Dans ce rapport, nous évaluons la littérature liée à l'alignement d'interférence et nous détaillons les algorithmes existants et leur application aux réseaux cellulaires en voie descendante. Nous proposons ensuite un ensemble de directions de recherche d'intérêt par rapport à l'état de l'art actuel.

Mots-clés : alignement d'interférences, traitement du signal, réseaux cellulaires

List of acronyms

MIMO multiple-input multiple output

d.o.f. degrees of freedom

SNR signal-to-noise ratio

SINR signal-to-interference-plus-noise ratio

w.l.o.g. without loss of generality

QoS quality of service

SC small cell

BS base station

IA Interference Alignment

CSI channel side information

w.r.t. with respect to

UE user equipment

OFDM Orthogonal Frequency Division Multiplexing

BAU Business-as-usual

MMSE minimum mean-square error

OFDMA Orthogonal Frequency Division Multiple Access

CoMP coordinated multi-point

MSE Mean-square error

FFR fractional frequency reuse

1 Introduction

Interference management techniques are a recurrent focus in many communication setups. In cellular systems, interference from neighboring base stations are still one of the foremost constraint on the deployment of the network, and will create outages at the cell edges as well as the need for complex handovers. Such trend can also be seen in dense urban wireless local area networks, where an apartment building will typically have dozens of networks competing over the same bandwith. A classical way to approach these issues are through medium access control techniques, which in turn severely reduce the performance of each individual nodes system to ensure cohabitation. Interference management is thus critical in most modern communication networks.

Interference management for cellular network has been first and foremost implemented through smart reuse of the resources allocated to the network as a whole, mostly through so-called Frequency Division Multiple Access (FDMA) techniques. Common schemes include *reuse-n* schemes where neighboring cells do not interference on each over resources, as well as fractional frequency reuse (FFR) schemes [1]. More recently, making use of the increase signal space available through both MIMO systems as well as OFDM techniques, a new concept has emerged. Interference Alignment (IA) [2] aims at using these signaling dimensions so that transmitters cooperative design their signals in order for interference to overlap at the receivers. Somewhat surprisingly, as long as the number of dimension increases with the network size, the authors of [2] showed that the sum-throughput of the network can grow linearly with the network size also.

Our goal in this report is to study the opportunity of using interference alignment as an advanced communication technique in the Greentouch arsenal. Most research on IA focused on the performance gain in idealized scenarios. We aim at focusing on the potential energy gain of this approach, as outlined in [3], as well as the practicability of the implementation on current hardware. While [3] detailed interference cancellation as a tool and the impact it has on network performance, the current report is aimed at describing the existing IA algorithms available in the state of the art, as well as our planned iterations and research objectives on this front. We introduce advances from the literature up to the time of writing, and subsequently develop a global model encompassing the different scenarii encountered in applying IA to cellular networks. We then focus on 2 specific scenarii of interest and study their particularities, as well as the different algorithms available to achieve IA performance gains in practice. The last section of the report is devoted to the extension and specialization of the schemes we plan to develop, as well as their implementation both in simulation and on software-radio hardware.

1.1 Notations

In the sequel, we denote vectors using a lowercase boldface notation, e.g. $\mathbf{x}$, matrices using an uppercase boldface notation, e.g. $\mathbf{C}$ and scalar in a normal typeface. Unless specified explicitely, all scalars, vectors and matrices are complex-valued. The set of complex-valued matrices with M rows and N columns is denoted $\mathbb{M}(M, N)$. The trace of a matrix $\mathrm{tr}(\cdot)$ is the sum of diagonal elements of the matrix. The determinant of a matrix is denoted as $|\cdot|$. The superscript T indicates the transpose of a matrix or a vector, whereas the superscript $\dagger$ indicates the hermitian – the conjugate transpose – of a matrix. The operator $\nu_d[\cdot]$ refers to the eigenvector of the d^{th} lowest eigenvalue of a matrix. Unless specified, the norm operator $\|\cdot\|$ refers to the euclidean norm when applied to vectors, and the Frobenius norm when applied to matrices. We denote $[\mathbf{A}]^i$ the i^{th} column of the matrix $\mathbf{A}$, and $[\mathbf{A}]_i$ the i^{th} line of the matrix $\mathbf{A}$. Finally, $\mathbf{null}(\cdot)$ denotes the space orthogonal to the specified vectors or the columns of the specified matrix, whereas **span**$(\cdot)$ refers to the subspace spanned by the specified vectors of the columns of the specified matrix.

1.2 Mathematical preamble

A large body of results on interference channels focus on the notion of degrees of freedom (d.o.f.), which relate to the number of elementary information streams that may flow in the network with sufficiently large signal-to-noise ratio (SNR) at the receivers. We introduce this performance metric through its basic definition on MIMO channels under an additive Gaussian model, with M antennas at the transmitter and N antennas at the receiver. The output of this channel, $\mathbf{y} \in \mathbb{C}^N$ is linked to the input symbols $\mathbf{x} \in \mathbb{C}^M$ by the relation:

$$\mathbf{y} = \mathbf{G}\mathbf{x} + \mathbf{x} \tag{1}$$

The matrix G is composed of $M \times N$ complex channel gains. We can assume without loss of generality (w.l.o.g.) that $\mathbb{E}\left[\mathbf{x}\mathbf{x}^\dagger\right] = \mathbf{I}_N$, where $\mathbf{I}_N$ is an identity matrix of size $N \times N$ [4, Rem.9.1]. Furthermore, we assume that the total power used by the symbol $\mathbf{x}$ is bounded by P, i.e. $\mathrm{tr}(\mathbb{E}\left[\mathbf{x}\mathbf{x}^\dagger\right]) \leq P$. With conditions on the matrix $\mathbf{G}$, [5] showed that using the singular decomposition of $\mathbf{G} = \mathbf{U}\boldsymbol{\Delta}\mathbf{V}^\dagger$, the capacity of this channel may be written:

$$C = \max_{\mathbf{K}:\mathrm{tr}(\mathbf{K}) \leq P} \log |\mathbf{I}_d + \boldsymbol{\Delta}\mathbf{K}\boldsymbol{\Delta}| \tag{2}$$

We define $d = \min(M, N)$ and $K = \mathbf{V}^\dagger \mathbb{E}\left[\mathbf{x}\mathbf{x}^\dagger\right] \mathbf{V}$ is the transformed covariance matrix of the input symbols. Since $\mathbf{V}$ is unitary by definition, the trace constraint on the power of $\mathbf{x}$ transfers to a trace constraint on $\mathbf{K}$. Both $\mathbf{I}_d$ and $\boldsymbol{\Delta}$ are diagonal matrices. By Hadamard's inequality, we know that $|\mathbf{A}| \leq \prod_i A_{ii}$ where A_{ii} are the diagonal entries of $\mathbf{A}$. Thus:

$$|\mathbf{I}_d + \boldsymbol{\Delta}\mathbf{K}\boldsymbol{\Delta}| \leq \prod_{i=1}^{d} (1 + \Delta_{ii} K_{ii} \Delta_{ii}) \tag{3}$$

with equality iff $\mathbf{K}$ is diagonal. This result implies that at the optimal, the Gaussian MIMO channel reduces to a product Gaussian channel, i.e. d parallel Gaussian channels. The optimal covariance matrix $\mathbf{K}$ may be determined through water-filling [4, Sec.3.2.3]. Let's denote $\tilde{\mathbf{K}}$ the optimal transformed covariance matrix $\mathbf{K}$ such that the capacity C in (2) is maximized. We can write:

$$C = \sum_{i=1}^{d} \log\left(1 + \Delta_{ii}^2 \tilde{K}_{ii}\right) \tag{4}$$

If we assume $\Delta_{ii} \neq 0$ for all i, when the power constraint P increases, the water level is deep and allocating equal amounts of powers to all the antennas becomes optimal asymptotically as $P \to \infty$. Thus, we can rewrite:

$$C = d \log(P) + o(\log(P)) \tag{5}$$

The number d represents the number of spatial dimensions available to the transmitter, as modified by the channel G, and is termed as *degrees of freedom* as to represent the available *parallel* channels available for signalization. Traditional orthogonalization will also increase the number of d.o.f.. For example, using frequency division will provide additional d.o.f., as frequency-divided channels are also traditionnaly modeled by parallel Gaussian channels.

2 State of the art

We focus here on the literature specific to the actual interference alignment techniques as they have been developped in the recent years. A previous report [3] describes the modeling of cellular networks, as well as the the effect of interference on cellular networks. The potential gains in terms of signal-to-interference-plus-noise ratio (SINR) geometry of interference alignment are developped in this report.

2.1 Interference alignment

In the current literature, IA refers to a large range of signal processing techniques that aim at using the *dimensionality* of signals transferred over the network as to reduce the space spanned by unwanted signals at the receiver, thereby reducing or completely cancelling interference. One of the key results from the application of IA ideas to networks is that, under specific conditions, dense and high-power wireless networks are not fundamentally interference limited. As an example, under idealized assumptions, using IA in the setting of an *interference channel* formed by K transmitter-receiver pairs interfering between one another allows *each* pair to achieve a data rate equal to half of his interference-free channel, *regardless of* K [2]. The theory developped on the early work of [6, 7] on the *X channel* and *interference channel* with 2 transmitter-receiver pairs.

Regardless of the setting, the idea behind interference alignment is as follows, and traces back to the so-called *index coding* problem of source coding (links between IA and the index coding problem are treated in [8] and references therein). The idea originates from linear algebra. Consider the system of equations [9]:

$$y_1 = 3x_1 + 2x_2 + 3x_3 + x_4 + 5x_5 \tag{6}$$

$$y_2 = 2x_1 + 4x_2 + x_3 - 3x_4 + 5x_5 \tag{7}$$

$$y_3 = 4x_1 + 3x_2 + 5x_3 + 2x_4 + 8x_5 \tag{8}$$

In a general setting, by observing y_1, y_2 and y_3, one cannot recover the values of the five unknowns. Solving for all unknowns would require *at least* five observations. Nevertheless, if we are only interested in the value of x_1, it turns out that in this setup it is possible to recover its value because the *interfering space* (the values $(x_2, \dots, x_5)$) only span a 2 dimensional vector space. Consider the projection of $\mathbf{y} = (y_1, y_2, y_3)^T$ onto the vector $\mathbf{u} = (17, -1, -10)^T$:

$$\mathbf{u}^T \mathbf{y} = 9x_1 \tag{9}$$

Through a simple linear projection of the observed vector, x_1 can thus be recovered free from interference even though the original space spanned by the signal was $5 > 3$. This general principle forms the basis of interference alignment, although most of the difficulty lies in the specific design of transmit symbols and decoding algorithms allowing the system to *effectively* consolidate interference at each receiver in a small subspace. One of the key condition for this technique to work stems from the fact that, in realistic wireless channels, each receiver will observe different combination of the symbols sent because of the random and independent effects of the channel. Since each receiver observes a different signal, it is possible to carefully design the transmitted signal to ensure that interference may be constrained in a subspace of the received signal.

As a communication technique, IA shows the most gains in the high SINR regime, when the transmission power grows, although one can characterize the gains at any SNR in specific cases [10]. As such theoretical work on IA focuses on the d.o.f. achievable in specific communication models. The original work on IA centered on the MIMO X channel [7] and the MIMO interference channel [6], both with 2 transmitter-receiver pairs. Cadambe and Jafar developped an IA scheme allowing to achieve the outer-bound on the d.o.f. in the K-users MIMO interference channel in [11, 2], as well as the general MIMO X channel in [12]. These results have been extended by Karmakar *et al.* in [13], who characterized the capacity of the MIMO interference channel within 1 bit over the whole SINR range[1]. Grokop *et al.* showed in [14] that the achievability of the d.o.f. region in the case of the K-users interference channel requires the dimension of the signal space to grow exponentially. In practice, in order to be able to solve the linear systems as presented before and effectively constrain interference in a subspace of the received signal space, one has to increase the number of antennas available on the transmitters and receivers. When this is not possible, additional dimensions can be obtained by signaling over different frequency bands or transmission slots, provided that significant diversity is obtained from these *channel extensions*. As an example, one can simply use the fact that if the effect of the channel is modeled through a symmetric probability distribution, then there exists states over time that will naturally cancel each other. A transmitter that has knowledge of these states can use this particularity to design its transmitted symbols to achieve IA, provided that it can transmit over an infinite period of time [15]. Not using channel extensions can in many cases limit the achievable d.o.f., since the outer-bounds on the d.o.f. region are fractional for most channels [16]. In generic random channels, conditions for the solvability of the linear systems for IA were studied in [17], and linked to the solvability of a system of polynomial equations. Yetis *et al.* [17] showed in particular that so-called *proper* systems where the number of equations does not exceed the number of variables are likely to be feasible in the K-user interference channel. Nevertheless, as the number of users grows, the complexity of computing a solution to the IA problems becomes untractable ; in the case of the interference channel, Razaviyayn *et al.* showed that the problem is indeed NP-hard when receivers have more that 3 antennas [18]. Ning *et al.* proved feasibility results for IA solutions when frequency diversity is available in the systems, thereby giving a specific block-matrix form to the channel matrix. Further theoretical results on IA may be found in [9] and references therein.

2.2 Effect of imperfect and delayed state information at the transmitter

In most theoretical settings, achieving the gains of IA requires perfect knowledge of the channels coefficients between the transmitter and receivers, in order to properly design the transmitted symbols and decoding algorithms. In a practical setting however, especially when implementing IA, channel side information (CSI) may not be available at the transmitter and/or receiver, or may be delayed or degraded. This issue is a key focus of contemporary research on applying IA in wireless communications networks [19], as imperfect CSI will lead the interference subspace to *leak* into the signal subspace. A large body of work already showed that without CSI at the transmitter, the achievable d.o.f. region collapses in a number of settings related to cellular communications, especially broadcast [20] and the 2-user interference channel [21]. Vase and Varanasi characterized the achievable d.o.f. region for the MIMO broadcast and interference channel, and further identify the cases and hypotheses under which the region does in fact *not* collapse without transmitter CSI. Nevertheless, Maddah-Ali and Tse showed in [22] and [23] that in some cases delayed CSI can actually provide a gain in d.o.f. even when the delay is too large to ensure correct channel tracking by the transmitter. Vaze and Varanasi studied the cases where delayed CSI is indeed useful in MIMO broadcast and interference channels in [24], as well as their d.o.f. region without transmitter CSI [25]. Jafar also showed that under specific conditions on the

---

[1] Recall that d.o.f. characterization focus only the asymptotic high SINR range.

wireless channel behavior, it was possible to achieve IA gains without any transmitter CSI [26]. The key idea in that case is to use the channel temporal correlation statistics, in particular the fact that only part of the channel may vary over time, thereby allowing the receiver to effectively cancel interference by tracking the changes and substracting the received signals. Similar results have been presented in [27] by using antenna switching techniques.

In the most common case of imperfect or noisy CSI, one has to account for the leaked interference in the signal space due to incorrect information about the channel coefficient. El Ayach *et al.* studied an algorithm effectively tracking the channel and minimizing the interference leakage and noise on the channel predictions simultaneously in [28]. The authors extended their algorithm in [29] and showed that by projecting the required CSI onto the Grassmanian manifold, one can effectively reduce the feedback overhead by exploiting both the temporal correlation of the channel and the Grassmanian structure. A Grassmanian gradient descent algorithm was also described in [30], showing similar benefits. Krishnamachari and Varanasi bounded the needed feedback transmission rate required to achieve the d.o.f. region of the MIMO interference channel [31], as well as the degradation in d.o.f. stemming from insufficient feedback. Rezaee and Guillaud studied a similar problem and derived the optimal feedback scaling with respect to (w.r.t.) the transmission power of the source and showed that without scaling, the capacity of the system effectively plateaus [32, 33].

2.3 Numerical algorithms for IA

Linear interference alignment solutions in practical systems are at first sight very viable candidates for improving the performance of the interference-limited cellular networks. In effect, the schemes to achieve interference alignment only act on the precoding and decoding matrices and thus should readily adapt to the MIMO-OFDM setups used in 4^{th} generation cellular networks and envisioned for the next generation. Nonetheless, as partially discussed in the preceding section, some challenges may be identified before effectively achieving the theoretical gains of IA[19]:

- The dimensionality required to achieve IA grows faster than exponentially in the number of users, whereas practical systems will be limited in both frequency and antenna diversity.
- IA shows most of its gains in the high-SINR regime, in interference limited systems with a low number of interferers.
- Computation of the precoding and decoding matrices is heavily reliant on CSI at both transmitters and receivers. The overhead of CSI acquisition must be minimized and traded off for the performance of the IA scheme.
- In some schemes, the synchronization and organization of the network will be paramount in achieving the gains projected by IA. Centralized control risk increasing the load on the backhaul network and risk missing delay constraints due to the transfer of CSI and computation time of IA solutions.

The practical computation of precoding and decoding matrices in interference channels has been treated in a number of paper. It has been shown that exactly computing the matrices for the interference channel with more than 3 antennas receivers is indeed a NP-hard problem [18]. As such, most approaches will use heuristics to derive adequate IA solutions targeted at a specific IA scheme. We focus here on distributed approaches. The *Interference plus noise leakage (INL)* algorithm presented in [34] aims at finding the smallest interference subspace and the associated precoders and decoders, without considering the resultant SINR. The most common algorithm aims at maximizing the SINR while reducing the interference space in the manner of [34], and is thus called *Max-SINR* [35]. A similar approach has been taken by Kim and Torlak [36] to improve the precoders found by the original algorithm proposed by Cadambe and Jafar in [2]. An extension of the *Max-SINR* algorithm relieving the hypothesis of reciprocal channels has been introduced in [37], and Schmidt *et al.* introduced a generalized version using Mean-square error (MSE) terms rather than SINR terms, with weights on the objective function [38], thereby allowing for quality of service (QoS) constraints. The case where all nodes do not participate in the IA scheme has been treated in [39]. Peters and Heath also prove the convergence of the *Max-SINR* algorithm under a colored gaussian noise hypothesis in [39]. We detail these algorithms in section 3.1.

These algorithms have been implemented on hardware platforms. The team of Pr. Heath developed an IA testbed formed by 3 transmitter-receiver pairs with 2 antennas each, and successfully implemented an IA scheme using an INL algorithm tuned for this setup (see e.g. [28, 19] and references therein). The results highlight the complexity of the computation and the necessary tradeoffs to be made for practical implementation of IA. Similar results have been found at UT Vienna by Mayer *et al.* in a realistic outdoor testbed. They highlight precisely the processing and delay incurred by the necessary estimation and feedback of CSI to the base stations [40].

3 Interference alignment in downlink cellular networks

As shown above IA drove strong interest in the research community for different settings far beyond the initial K-user intereference channel. The recent review [19] highlights the different technical challenges to be solved before envisioning a practical application among which implementing accurate feedback loops is probably the most important challenge. But beyond the practical implementation of IA solutions in a network, the actual model of the network tends to be complex and involve a large number of hypothesis relative to the number of base stations (BSs), user equipments (UEs), network geometry and antenna/frequency diversity. These assumptions, or lack thereof, are needed and will play a significant role in the design of IA schemes. These IA schemes are in return heavily tuned to the specific hypotheses made and may not adapt to all cellular configurations, thereby justifying the need to develop a scheme adapted to the Greentouch cellular model. A downlink cellular network is basically an *interfering broadcast channel*, where BSs will transmit towards a number of users and interfere with each other. We state below the general formulation for the downlink interferering broadcast channel, and subsquently review the literature related to the problem based on the hypotheses and simplifications made.

We consider a set of B BSs indexed in $\{1, \dots, B\}$ and a set of U UEs indexed in $\{1, \dots, U\}$. As a simplification, all BSs will be equipped with M antennas and all UEs with N antennas. We only focus on the downlink problem. Each user is linked to a specific BS, and the set of all users linked to BS b is denoted by $\mathcal{U}_b$. The communications take place over a bandwith W separated into F orthogonal sub-bands of equal size W/F. For a sub-band f, with $1 \leq f \leq F$, the power allocated by a BS b, with $1 \leq b \leq B$ is denoted by $p_b^{(f)}$, and the global power used by the BS is constrained to P. In practical systems, usually, a *scheduling* operation will determine the sub-bands a specific user will transmit on. We do not consider scheduling explicitely in the model ; in effect, scheduling is either done before applying the IA techniques described here or implicitely managed in the IA schemes, in which case all users share the whole frequency space. As multiple BSs compete over the same resource set, a user will experience two kind of interferences. The first kind of interference comes from BSs other than the UE's and is termed *out-of-cell interference* or *inter-cell interference*. A base station may also have to serve multiple users simultaneously, thereby sending more than one stream of information in a single signal. This leads to *intra-cell interference* due to the superposition of information streams, which can be cancelled through beamforming techniques or managed through iterative decoding and/or dirty paper coding [4, 41]. We model the signals received by the UE u from the BS b using a static pathloss $L_{b,u}$ dependent on the distance $d(b,u)$ between the BS and the UE, a large-scale random component $S_{b,u}$ which follows the Greentouch shadowing model. For simplicity of presentation, we consider the aggregate large-scale effects as a single variable:

$$\gamma_{b,u}^{(f)} = L_{b,u} S_{b,u} p_b^{(f)} \tag{10}$$

We further denote $\mathbf{\Gamma}_{b,u}^{(f)} = \sqrt{\gamma_{b,u}^{(f)}} \mathbf{I}_N$, and the pathloss matrix from BS b to user u is constructed as $\mathbf{\Gamma}_{b,u} = \mathbf{diag}\left(\mathbf{\Gamma}_{b,u}^{(1)}, \dots, \mathbf{\Gamma}_{b,u}^{(F)}\right)$. All matrices and scalars are supposed formed of complex coefficients. Additional fading per sub-band is captured through a random matrix $\mathbf{H}_{b,u}^{(f)} \in \mathbb{M}(N, M)$. The fading matrix over all sub-bands is denoted $\mathbf{H}_{b,u} = \mathbf{diag}\left(\mathbf{H}_{b,u}^{(1)}, \dots, \mathbf{H}_{b,u}^{(F)}\right)$, and so we have $\mathbf{H}_{b,u} \in \mathbb{M}(F \times N, F \times M)$. Each BS send a space-frequency codeword $\mathbf{x}_b = \left[\left(\mathbf{x}_b^{(1)}\right)^\dagger, \dots, \left(\mathbf{x}_b^{(F)}\right)^\dagger\right]^\dagger$ such that each for any $1 \leq f \leq F$, the codeword $\mathbf{x}_b^{(f)}$ has size $M \times 1$. Without any precoding or decoding, and assuming that all BSs transmit on the whole bandwith, the signal received by a specific user u can be written as:

$$\mathbf{y}_u = \sum_{b=1}^{B} \mathbf{\Gamma}_{b,u} \mathbf{H}_{b,u} \mathbf{x}_b + \mathbf{z}_u \qquad \mathbf{z}_u \sim \mathcal{CN}(0, \sigma) \tag{11}$$

We model the precoding and decoding operation over the whole available signal dimensions in frequency and space. A BS aims at sending a $d_u \geq 1$ symbols towards each of its users $u \in \mathcal{U}_b$. The symbols are ordered as $\{s_{u,i}\}_{1 \leq i \leq d_u}$ and may be stacked into a vector $\mathbf{s}_u$. To send a symbol, the BS uses a precoding matrix $\mathbf{C}_u \in \mathbb{M}(F \times M, d_u)$, where $[\mathbf{C}_u]^i$, the i^{th} column of the precoding matrix, is the *precoding vector* associated with the symbol $s_{u,i}$. Upon receiving its signal, the user u can apply a linear operation on the signal received modeled through a matrix $\mathbf{D}_u \in \mathbb{M}(d_u, F \times N)$ to decode its symbols, such that:

$$\tilde{\mathbf{y}}_u = \mathbf{D}_u \mathbf{y}_u = \mathbf{D}_u \sum_{b=1}^{B} \sum_{v \in \mathcal{U}_b} \mathbf{\Gamma}_{b,u} \mathbf{H}_{b,u} \mathbf{C}_v \mathbf{s}_v + \mathbf{D}_u \mathbf{z}_u \tag{12}$$

For the purpose of describing iterative algorithms when necessary, we will denote the "reversed", uplink network parameters (from the UEs to the BSs) using left-arrow stacked over the letters. In essence, $\overleftarrow{\mathbf{H}}_{b,u}$ is the channel matrix from user u to BS b, $\overleftarrow{\mathbf{C}}_u$ is the precoder used by UE u in the uplink, and $\overleftarrow{\mathbf{D}}_b$ is the decoder used by the BS b for uplink transmissions. In the uplink, a user sends a signal $\overleftarrow{\mathbf{x}}_u = \overleftarrow{\mathbf{C}}_u \overleftarrow{\mathbf{s}}_u$ and a BS receives $\overleftarrow{\mathbf{y}}_b$ as follows:

$$\overleftarrow{\mathbf{D}}_b \overleftarrow{\mathbf{y}}_b = \overleftarrow{\mathbf{D}}_b \sum_{u=1}^{U} \mathbf{\Gamma}_{b,u} \overleftarrow{\mathbf{H}}_{b,u} \overleftarrow{\mathbf{C}}_u \overleftarrow{\mathbf{s}}_u + \overleftarrow{\mathbf{D}}_b \overleftarrow{\mathbf{z}}_b \tag{13}$$

Most uplink analysis in this and subsequent reports will nevertheless be restricted to channel estimation and feedback rather than actual data transmissions. As such, symbols sent by the users are expected to be quantized as a feedback rate – in which case we are only interested in the SINR of the streams – or known by the BS and used for estimation. The actual content of the symbol vector $\overleftarrow{\mathbf{s}_u}$ is thus either fixed and known, or will match the downlink description of the streams from the BS towards the UE.

In the downlink, to evaluate the sum-rate achievable by a user u, we will have to study the rate of each stream captured through the symbols $s_{u,i}$. Assuming, w.l.o.g., that the user u is linked to the BS 1, the received signal can thus be globally decomposed as:

$$\tilde{\mathbf{y}}_u = \overbrace{\mathbf{D}_u \boldsymbol{\Gamma}_{1,u} \mathbf{H}_{1,u} \mathbf{C}_u \mathbf{s}_u}^{\text{Desired signal}} + \overbrace{\sum_{\substack{v \in \mathcal{U}_1 \\ v \neq u}} \mathbf{D}_u \boldsymbol{\Gamma}_{1,u} \mathbf{H}_{1,u} \mathbf{C}_v \mathbf{s}_v}^{\text{Intra-cell interference}} + \overbrace{\sum_{b=2}^{B} \sum_{v \in \mathcal{U}_b} \mathbf{D}_u \boldsymbol{\Gamma}_{b,u} \mathbf{H}_{b,u} \mathbf{C}_v \mathbf{s}_v}^{\text{Inter-cell interference}} + \mathbf{D}_u \mathbf{z}_u \tag{14}$$

More compactly:

$$\tilde{\mathbf{y}}_u = \mathbf{D}_u \boldsymbol{\Gamma}_{1,u} \mathbf{H}_{1,u} \left(\mathbf{C}_u \mathbf{s}_u + \sum_{\substack{v \in \mathcal{U}_1 \\ v \neq u}} \mathbf{C}_v \mathbf{s}_v \right) + \mathbf{D}_u \sum_{b=2}^{B} \sum_{v \in \mathcal{U}_b} \boldsymbol{\Gamma}_{b,u} \mathbf{H}_{b,u} \mathbf{C}_v \mathbf{s}_v + \mathbf{D}_u \mathbf{z}_u \tag{15}$$

Since the matrix $\boldsymbol{\Gamma}_{b,u}$ is diagonal for any b and u, necessary conditions on the achievability of inter-cell IA would include, for a given user u:

$$\mathbf{D}_u \mathbf{H}_{b,u} \mathbf{C}_v = \mathbf{0}_{\mathbb{M}} \quad \forall (b,v) \in \{(b,v) \,|\, v \in \{1, \ldots, U\},\, v \neq u,\, v \in \mathcal{U}_b\} \tag{16a}$$

$$\operatorname{rank}\left(\mathbf{D}_u \mathbf{H}_{1,u} \mathbf{C}_u\right) = d_u \tag{16b}$$

These conditions ensure that the inter-cell interference is nulled for any signal sent by the interfering BSs, while still preserving the necessary signal space to achieve a multiplexing gain d_u at high SINR. The intra-cell interference may then be treated in a complementary manner, either through interference cancellation or dirty paper coding. However, in such a complex formulation, these conditions are hard to evaluate and translate into actual effective constructions of precoders and decoders achieving IA. Consequently, most of the literature considers a subset of the general problem and develop schemes adapted to the model studied. We review theses simplifications w.r.t. the goal of deriving a practical IA scheme for the Greentouch network model.

3.1 Interference alignment in the K-user interference channel

A classical MIMO based IA method for cellular networks is a direct application of the K-users IA scheme introduced in section 2.1 – subsequent details may be found in [9]. For the sake of clarity, the model comprises a set of K transmitter-receiver pairs which interfer each other, as represented on Fig.1. The application of this model in the context of cellular networks has been proposed by [42], and extended in [43]. The authors of [44, 45] study the form of IA solutions in partially connected interference channels, where a subset of cross-channels between BSs and UEs are active at a point in time. They provide an algorithm checking the feasibility of the IA problem, as well as optimizing the d.o.f. achieved by each users. This approach, however presented, will require the network to form "pairs" of BSs and UEs designated to interact over the same set of resources in space, frequency and time, and only a single user from a specific BS will be allocated the resource. This leads to a specialization of the interfering broadcast channel to an interference channel:

- Each user is limited to a single frequency sub-band
- Each frequency sub-band is allocated to exactly one user

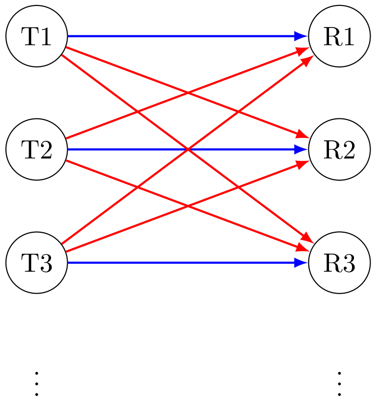

Figure 1: K-users interference channel. The first 3 transmitter-receiver pairs are represented on this figure. The blue lines indicate the desired links, whereas the red lines indicates the interfering links.

As such, interference inside a single frequency sub-band will only come in the form of inter-cell interference from external BSs, and the user *scheduling* is done before applying IA techniques. Therefore, in the preceding model, we can simplify the study to a single sub-band and generalize for all users of a BS on each sub-band, leading to $F = 1$ and $B = U = K$. Under these hypotheses, the conditions (16) can be specialized, and a d.o.f. distribution for all users $(d_1, \ldots, d_U)$ is achievable iif there exists $\mathbf{C}_b$ and $\mathbf{D}_k$ precoding and decoding matrices s.t. for any $k \in \{1, \ldots, U\}$:

$$\mathbf{D}_k \mathbf{H}_{b,k} \mathbf{C}_b = 0 \qquad \forall b \neq k \tag{17a}$$

$$\text{rank}\left(\mathbf{D}_k \mathbf{H}_{k,k} \mathbf{C}_k\right) = d_k \tag{17b}$$

It has been proved ([2, 17, 9] and references therein) that the existence of this scheme depends on the dimensions of the problem $(K, M, n, (d_1, \ldots, d_U))$ and not on the specific channel realizations, if the channels are assumed drawn from a continuous distribution. Conditions (17) suppose a fixed d.o.f. distribution, but the "d.o.f. achievable region" can be determined for the K-users interference channel and is given by [46]:

$$\mathcal{D} = \{(d_1, \cdots, d_U) \mid d_i + d_j \leq N \quad \forall (i,j) \quad i \neq j\} \tag{18}$$

which means that in a fair approach, the receiver dimension N is divided into two equal parts ; one for interference and one for the useful signal. When all $d_i = 1$ and without channel extensions, Tresch *et al.* provide the following limit [47]:

$$N + M - 1 \geq K \tag{19}$$

For instance, using this scheme with 2×2 MIMO systems allows to manage only 3 users simultaneously. Therefore, the authors of [47] proposed to form clusters of BSs, which cooperate to serve their mobiles using the IA scheme (Fig.2). They observed a good capacity improvement but only in the center of the clusters which is normal since border mobiles still suffer from inter-cluster interferences. Less intuitivele, even mobiles near the center of the cluster – and thus far from the 3 cooperating BSs – gain relatively little compared to an optimal MIMO transmission associated with a 1/3 frequency reuse scheme. One could overcome this issue by forming dynamic clusters depending on the position of the mobiles but the coordination would become cumbersome, and even in this case a proper frequency reuse scheme could remain more efficient. Furthermore, a global knowledge of CSI is required to form the clusters. This scheme has been generalized and extended in [43] to arbitrary clusters over each frequency sub-band, thus still requiring a centralized knowledge and decision system. Nonetheless, the authors develop an achievable IA scheme for a dynamically constructed cluster of K BSs and d streams per user, verifying:

$$M + N - (K+2)d \geq 0 \qquad d \leq \min(M, N) \qquad d \leq \max(M, N) \tag{20}$$

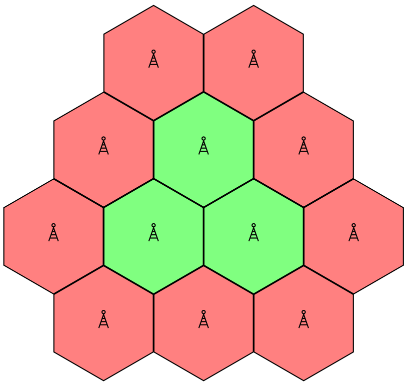

Figure 2: Clustering of base stations in a group of 3. Red cells indicate out-of-cluster interferers. This example in a 2 × 2 MIMO configuration allows each cluster of BSs to serve 3 users using interference alignment using the technique developed in [42].

A similar approach has been proposed and evaluated on actual measurements in [28], and another scheme is evaluated by simulation in [48]. The results show again a questionable gain of interference alignment compared to other techniques. However, as mentioned above, IA has been performed independently on each frequency in the OFDM frequency spectrum after each mobile has been allocated a given sub-band. In [48], two options are proposed: the first makes clusters with the 3 sectorial antennas of a same site while the second makes clusters with the 3 sectorial antennas of adjacent cells pointing in the same direction. In such situation, we have for each carrier, a set of three pairs TX-RX which aim at performing an interference alignment. The former results however suffer from two main limitations. First, the working space is mostly limited to 2 × 2 MIMO space which drastically reduces the available signal space for performing IA. As shown before, at most 3 users can be managed through this setup and each will have to sacrifice half of its signaling space to achieve IA. So without coordination we have a potential of 6 d.o.f. (2 per user) but suffering from strong interference, while with IA, we have only 3 d.o.f. with no interference (except from outside the cluster). Thus the d.o.f. lost through IA should be compensated by a strong SINR gain, which is revealed to be unsufficient in the related experiments.

The second limitation comes from the need of perfect and centralized CSI in the cluster to align interference. It has been also shown that reducing the CSI further drastically reduces the performance of the system [48]. The only option thus relies on increasing the signal space, i.e. by equipping all BSs and mobiles with more antennas, but expecting more than 4 antennas on mobiles is right now utopic. It can be also increased by considering not only the MIMO space but also the frequency space if the carriers suffer from independent fading states as suggested in [49]. However, any approach in this research direction is likely to require centralized coordination and CSI, whereas our focus in this work is rather on distributed coordination.

3.2 Interference alignment in interfering broadcast channels

By relaxing the strict BS-UE pairing of the preceding section, the system reverts to a downlink broadcast interference channel and capture the expected situation of next generation networks. This relaxation intuitively holds great potential ; in effect, we are trading the complexity of intra-cell interference management for the gain of pooling and dynamically scheduling the resources to the UEs. Nevertheless, the complexity of the model leads to various simplifications and hypotheses to facilitate its analysis in the literature. In [50], the authors present a low-feedback scheme for a 3-cells honeycomb setup, where *fractional frequency reuse* is used by the base station to reduce interference on the cell-edge users. Their work presents an IA scheme designed to cancel the *dominant interferers* on a subset of the UEs, whose signal is strong enough for the cross-channel to be reliably estimated. Each UE able to implement this scheme get 1 d.o.f. free from interference. Da and Zhang present a simple scheme for OFDM downlink cellular networks using IA over different sub-bands, and develop an algorithm to optimally distribute the frequency and power resources to individual users [51]. They suppose a fixed scheduling of OFDM sub-bands for the users as to extract maximal frequency diversity, and show that the resulting optimization problem is quasi-concave in the power allocation. Suh *et al.* develop in [49] a scheme for the downlink celullar networks, extended from [52] which treated uplink channels. Their solution allows users in a cell to cancel their dominant interferer as well as the intra-cell interference for users in the same sub-band, achieving 1 d.o.f., without any communication between the BSs. The IA scheme further implements a novel approach which gives a tradeoff between the power gain of matched filtering at low-SNR and interference alignment gains at high SNR, thus being efficient for a large range of practical cases in cellular networks. An opportunistic IA scheme is presented in [53]. The implementation used pre-formed transmit beamformers, and allows users to use their local CSI to compute a sub-optimal IA solution but in a completely independent manner, using a *Max-SINR* algorithm. Hwang derived the conditions for the existence of IA solutions in downlink cellular networks [54], with the particularity that all interferers at a UE are aligned in a rank-1 vector space. Shin *et al.* show in [55] that a closed form solution to the interference alignment problem exists in a simple a cellular setup with 2 BSs and 2 UEs. The solution, when applicable, requires to solve a large matrix eigenproblem, whose computation becomes very expensive as the size of the network grows. Their work is extended in [56] to larger heterogeneous networks, where solutions for the IA problem are cast as generalized eigenvalue problems. In parallel, Tang and Lambotharan [41] generalized the grouping method of [55] to more general downlink cellular broadcast problem. The algorithm complexity to obtain the IA solutions are evaluated, as well as necessary conditions for the number of transmit and receive antennas for the problem to be cast as an eigenvalue problem.

As an example on how to derive an IA solution for the interfering broadcast channel, we detail the scheme of [49], which assumes the following w.r.t. the model described in section 3:

- Each user is allocated 1 stream: $d_u = 1 \quad \forall u \in \{1, \ldots, U\}$. This causes the symbol sent to user u to be a scalar s_u, and precoding/decoding matrices to become the vectors $\mathbf{c}_u$ and $\mathbf{d}_u$.
- The BSs and the UEs have the same number of antennas: $M = N$
- The number of users U served by a BS through IA is limited to $N - 1$

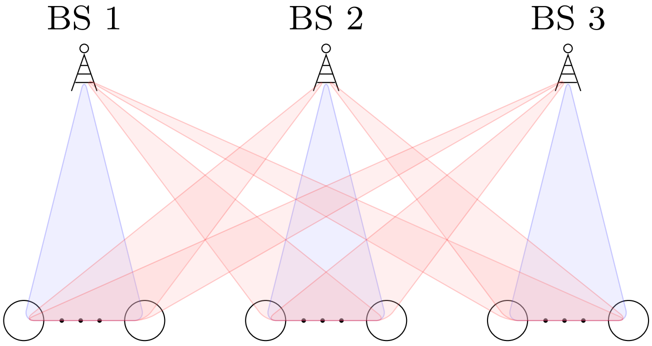

Figure 3: General downlink interference broadcast channel, with 3 BSs and an arbitrary number of UEs.

Some of these assumptions may be relaxed, although they lead to potentially suboptimal generalization of the IA scheme. Furthermore, while Suh *et al.* do indicate the possibility of using multiple frequency sub-bands, the scheduling has to be done beforehand, as was the case with the K-users interference channel schemes presented above. Another particularity of the scheme of [49] is that the *strongest* interferer is removed. When using frequency-based signal dimensions however, as seen from (10), such definition becomes arbitrary. We suppose an additional precoder $\mathbf{P} \in \mathbb{M}(N, N-1)$, shared by all BSs. The received signal, considering precoders and decoders, is thus written as follows using the assumptions of [49]:

$$\tilde{y}_u = \mathbf{d}_u \mathbf{\Gamma}_{1,u} \mathbf{H}_{1,u} \mathbf{P} \left(\mathbf{c}_u s_u + \sum_{\substack{v \in \mathcal{U}_1 \\ v \neq u}} \mathbf{c}_v s_v \right) + \mathbf{d}_u \sum_{b \in \{2,\dots,B\}} \sum_{v \in \mathcal{U}_b} \mathbf{\Gamma}_{b,u} \mathbf{H}_{b,u} \mathbf{P} \mathbf{c}_v s_v + \mathbf{d}_u \mathbf{z}_u \tag{21}$$

Each BS will have $N-1$ precoders of size $N-1 \times 1$. Using pilot signals, a user u can estimate the interference from neighboring BSs. Assuming we want to cancel interference from BS β, with $\beta \neq 1$, the user can always find a null vector $\mathbf{d}_u$ such that $\mathbf{d}_u \mathbf{H}_{\beta,u} \mathbf{P} = 0$ since $\mathbf{H}_{\beta,u} \mathbf{P} \in \mathbb{M}(N, N-1)$. The user then feeds back its own specific signal space $\mathbf{d}_u \mathbf{H}_{1,u} \mathbf{P}$ to its BS, which can construct the zero forcing precoders $\mathbf{c}_u$ for all its users such that intra-cell interference is cancelled. Such a precoder also exists since there is exactly $N-1$ dimensions available. The received signal is thus:

$$\tilde{y}_u = \mathbf{d}_u \mathbf{\Gamma}_{1,u} \mathbf{H}_{1,u} \mathbf{P} \mathbf{c}_u s_u + \overbrace{\mathbf{d}_u \sum_{\substack{b \in \{2,\dots,B\} \\ b \neq \beta}} \sum_{v \in \mathcal{U}_b} \mathbf{\Gamma}_{b,u} \mathbf{H}_{b,u} \mathbf{P} \mathbf{c}_v s_v}^{\text{Residual out-of-cell interference}} + \mathbf{d}_u \mathbf{z}_u \tag{22}$$

The approach of Suh *et al.* basically allows all intra-cell interferers to span the space of the the cancelled out-of-cell interferer, thereby only leaving for a given user u a single stream with a residual interference term. The authors note that the performance of this approach is dampened by the residual interference term, which as seen in [3] may be large. They also propose an alternative approach. Rather than aiming at completely removing the interference, the user will design a decoder adapter to the interference space. Consider the covariance matrix of interference and noise at a user u:

$$\mathbf{\Phi}_u = (1 + \mathsf{INR}_{\mathrm{rem}})\mathbf{I} + \frac{\mathsf{SNR}}{K} \sum_{v \in \mathcal{U}_\beta} \mathbf{\Gamma}_{\beta,u}\mathbf{H}_{\beta,u}\mathbf{P}\mathbf{c}_v\mathbf{c}_v^\dagger\mathbf{P}^\dagger\mathbf{H}_{\beta,u}^\dagger\mathbf{\Gamma}_{\beta,u}^\dagger \tag{23}$$

The $\mathsf{INR}_{\mathrm{rem}}$ term is the aggregate residual interference term, which is supposed white Gaussian. The users have no knowledge of the zero-forcing precoders $\mathbf{c}_v$ used by BS β for its users – it changes at each channel use. Since $\mathbf{c}_v$ is dependent on $\mathbf{P}$, the global precoder, carefully designing $\mathbf{P}$ allows to compute a good approximation of $\mathbb{E}\left[\mathbf{c}_v\mathbf{c}_v^\dagger\right]$ and thus use the expected value over the whole user precoders $\mathbf{c}_v$ of $\mathbf{\Phi}_u$ as a decoder. This approach is akin to a minimum mean-square error (MMSE) receiver in a point-to-point MIMO channel, and thus captures some beamforming gain at low SINR, thereby outperforming the IA receiver presented above in this region. Furthermore, the authors of [49] also show that through an empirical design of the global precoder $\mathbf{P}$, the MMSE-like receiver can also outperform IA at high SINR, making this approach a strong candidate for practical implementation. Some limitations are present in the scheme, the main one being that there is no simple and optimal extension for asymmetric antenna configurations.

In [57], a slightly different approach is used. Assuming that $N \geq M$, e.g. UEs have more antennas than BSs, every interfering channel is invertible almost surely. When $N < M$, frequency extensions and scheduling is proposed by the authors to attain the requirements. The authors only consider a simple BSs geometry with 2 BSs, but the scheme can be readily extended to a more general case and assume that only the strongest interferer is removed. All users agree on a decoding vector $\mathbf{v}_{\mathrm{ref}}$ of size M and using the pseudo-inverse of the strongest interfering BS β, the received symbol for user u can be written:

$$\tilde{y}_u = \mathbf{v}_{\mathrm{ref}}^\dagger\mathbf{H}_{\beta,u}^+\mathbf{\Gamma}_{1,u}\mathbf{H}_{1,u}\mathbf{x}_1 + \mathbf{v}_{\mathrm{ref}}^\dagger\mathbf{\Gamma}_{\beta,u}\mathbf{x}_\beta + \mathbf{v}_{\mathrm{ref}}^\dagger\mathbf{H}_{\beta,u}^+ \sum_{\substack{b \in \{2,\ldots,B\} \\ b \neq \beta}} \mathbf{\Gamma}_{b,u}\mathbf{H}_{b,u}\mathbf{x}_b + \mathbf{v}_{\mathrm{ref}}^\dagger\mathbf{H}_{\beta,u}^+\mathbf{z}_u \tag{24}$$

The decoder used here is $\mathbf{d}_u = \mathbf{v}_{\mathrm{ref}}^\dagger\mathbf{H}_{\beta,u}^+$ where $\mathbf{H}_{\beta,u}^+ = (\mathbf{H}_{\beta,u}^\dagger\mathbf{H}_{\beta,u})^{-1}\mathbf{H}_{\beta,u}^\dagger$. As long as all the $\mathbf{x}_b$ are constructed in the null-space of $\mathbf{v}_{\mathrm{ref}}$ (using for example zero-forcing beamforming [57]), the term $\mathbf{v}_{\mathrm{ref}}^\dagger\mathbf{\Gamma}_{\beta,u}\mathbf{x}_\beta$ will cancel. To form its beamforming matrix, the BS will need to gather knowledge of the receive vector $\mathbf{v}_{\mathrm{ref}}$, as well as the equivalent channels towards the users $\mathbf{v}_{\mathrm{ref}}^\dagger\mathbf{H}_{\beta,u}^\dagger\mathbf{H}_{1,u}$, to apply intra-cell cancellation methods. This approach is thus equivalent to the basic form of downlink IA of Suh *et al.* [49], although the "common signal space reducing parameter" is different. In [57], the users agree on a common $\mathbf{v}_{\mathrm{ref}}$ and basically agree that they constrain themselve to a rank-1 subspace, whereas in [49], the BSs agree on a common precoder $\mathbf{P}$ and reduce their signal space. Therefore, the approach of [49] seems more amenable to extensions for users to decode multiple streams, or globally adapt the signal space at the BSs. The scheme of [57] is evaluated in a practical setup in [58] and exhibits a significant gain compared to a MIMO transmission. However, the simulations use MMSE receivers rather than zero-forcing receivers. This lets us assume that it is probably more efficient to use the IA concept to manage the subspaces used for signalization at the BSs, but have users employ MMSE-like receivers as suggested in [49]. The superiority of MMSE receivers has also been noted in [48].

Both approaches offers much more freedom in the use of IA than the K-users IA approach of the preceding section. It is worth noting that these are not in the strict sense IA schemes because there is not a real alignment of interference from several BSs. However, they both integrate the interference free subspace concept of IA. Each BS transmit inside a restricted subspace to preserve a given free subspace for mobiles of neighborin BSs. This subspace doesn't need to be specific because the exact span of this subspace is receiver-dependent. This approach can be compared to fractional frequency reuse schemes [1], where an amount of resources is dedicated to edge users. However, in the downlink IA schemes presented here, this interference free subspace is different for each mobile, rather than being chosen by the BS. For instance, consider the case with two cells A and B. If BS A works with a reduced signal space of 1 unit, this is sufficient to create a free dimension at each receiver of BS B. Because the channel randomizes the orientation of this free dimension, each mobile of BS B feeds a different equivalent channel to their BS. When all these mobiles have send their feedback, it can attribute a precoding vector to each mobile, with orthogonalization techniques such as zero-forcing precoding, to avoid intra-cell interference.

3.3 OFDMA based Interference Alignment

Interference alignment has been proposed initially for the context of MIMO communications. But as noted from the earliest studies of interference alignment, it is however possible to exploit *channel extensions* in frequency or time [2]. In fact, as many upper bounds on achievable d.o.f. are rational, they require the use of such extensions ; using only space dimensions would result in integer d.o.f.. Although MIMO and frequency spaces may be combined to increase the total dimensions available for IA, we focus mainly on pure frequency space approaches in this section. Using frequency space is relevant for LTE like systems based on Orthogonal Frequency Division Multiple Access (OFDMA) since the frequency dimensions are much larger than the space dimensions (receivers are typically limited to 2 up to 4 antennas). Of course, the actual independent dimensions are much lower than the number of carriers due to channel correlation between adjacent sub-carriers, but at least 10 independent carriers over the spectrum may be expected in non line-of-sight conditions.

Basically, the problem formulation is similar to the MIMO based IA except that the channel matrices are diagonal. Let consider a setup similar to Fig.3 where the channel should now be understood as frequency channels. If we consider a 3-user interference channel, it is possible to show that no IA solution exists for each user to attain 1 d.o.f. while globally using only 2 frequency sub-bands. This is due to the fact that diagonal matrices reduce the number of independent quadratic equations in the IA conditions (16), thereby only allowing trivial solutions to the IA problem [17]. This issue also arises in practice in the numerical evaluation presented in [51], which is presented thereafter, where the authors note a small but non-zero residual interference term after applying the iterative *Max-SINR* algorithm [35]. On the other hand, using 4 frequency sub-bands however allows each user to achieve 2 d.o.f.

The authors of [51] present a basic scheme aiming at implementing IA over a subset of the frequency sub-bands of multiple BSs. The hypotheses made by the authors are as follows:

- Each user is allocated a pair of frequency sub-bands, spaced $W/2$ apart from each other. The total number of users per-cell is thus constrained to $F/2$
- For each pair of frequency sub-bands, 3 BSs cooperate to form a classical 3-user interference channel, and aim at cancelling interference for their own user.
- Iterative computation of the interference alignment solutions to (16) is performed using the algorithm in [35].
- A power allocation per BS is performed as to maximize the sum-rate of each cell, taking the IA scheme into account. This means that scheduling of the users onto the frequency sub-band pairs is done before the power allocation.

As mentioned, no exact IA solution exist for this setup, and residual interference will remain after the iterative computation of IA solutions. The authors then propose to extend the scheme using hybrid approaches similar to a fractional frequency reuse, where a subset of the frequency sub-bands is reserved for cell-edge users.

In [59], the authors propose an experimental study of practical IA in the frequency domain, in an approach similar to that of [51] but considering a larger number of frequency sub-bands for each user. As was the case for the scheme of [51], this is equivalent to a K-user interference channel where channel matrices are diagonal, and thus has limitations similar to the approaches presented in section 3.1. The authors evaluate different algorithms for numerical computation of the IA solutions, which shows clear gains from the *Max-SINR* and *Max-Sum rate* approaches.

In [60] proposes an iterative approach that resembles to the scheme proposed by Tse but with a complete feedback between BS so that the BS can also optimize their transmission codebooks. A central BS receives all channels and optimizes the coders by removing outcell interference. They show that the system performs almost as well as a block diagonalization approach but where a complete synchronization and data exchange is required. A similar approach is taken in [61], where the IA conditions are solved globally over the space and frequency domain jointly, in a numerical manner, and where all interference is aimed at being cancelled. The authors conjecture that for d streams per user, K users per cell and symmetric antenna configurations on BSs and UEs, a necessary condition for the feasibility of IA in the MIMO-OFDM interfering broadcast channel is:

$$(BK + 2)d \leq F \times N \tag{25}$$

which clearly shows that for a fixed number of resources $F \times N$, increasing B reduces linearly either d or K, *if we aim at removing every interferer.* This highlights the main drawback of IA: if we want to align simultaneously all interference from all BSs, the number of constraints become too complex and the number of users served diminishes drastically. However, we will show in the next section that focusing on removing the first interferers provides substantial gains, and may even lead the network out of the interference-limited regime in some cases.

4 Implementation scenarii and algorithms

As described above, several directions are possible for using IA inside the Greentouch framework. First, to the best of our knowledge, most studies already published evaluated the gain of IA in terms of sum-rate or SINR improvement but never estimated the potential energy gain of this approach, especially when coupling IA with global power reduction as hinted in [3]. We identified 2 main scenarii relevant for Greentouch where IA techniques may be exploited.

4.1 Inter-cell cooperative IA

These schemes use the ideas presented in section 3.1, and aim at forming clusters of BSs where a select number of UE-BS pairs apply interference alignment jointly. As seen from [3], removing up to 2 interferers is conjectured to provide the best complexity/performance tradeoff. As such, we can readily imagine 3 BSs grouping their respective facing sectors to achieve IA on 3 users, as shown on Fig.4.

This setup is directly based on the K-user interference channel. We suppose here that scheduling is done beforehand and that users do not share their allocated subcarriers. From [42] we know that most gains are expected at the center of the cell clusters, which lets us bias the scheduler in order to have cell-edge users on a select group of sub-carriers that is common to all BSs. If need be, beyond the space dimensions available inside a subcarrier, multiple groups of sub-carriers can be formed and scheduled to add frequency dimensions for the IA problem. As an example,

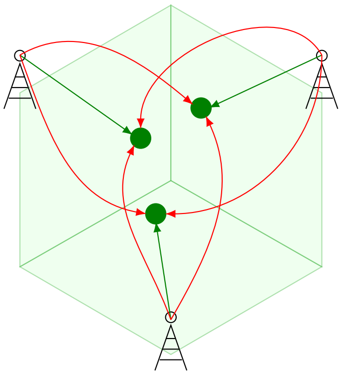

Figure 4: Sector groups for inter-cell cooperative IA. Three BSs coordinate their precoders to achieve IA on 3 users distributed over the 3 sectors.

attaining the theoretical upper bound of 3/2 in the 3-user interference channel requires multiple frequency extensions [2]. Multiple points can be made for and against this approach:

- This scheme provides an interesting baseline. It is similar to coordinated multi-point (CoMP) while requiring less backhaul communications between the base stations. In short, only the precoders have to be jointly designed by the BSs of the cluster based on the global CSI ; the signals and symbols actually sent are irrelevant in IA.

- As far as experimental validation goes, the basic 3-user interference channel has already been implemented and studied on various platforms [28, 40] as well as extensive simulations [58, 48]. While the baseline is interesting in our study, it provides little breakthrough compared to the state of the art. Nonetheless, few of these approaches considered the impact of using frequency-space signal dimensions simultaneously, and none were focused on decreasing the global energy consumption of the network.

- Basic Business-as-usual (BAU) systems with $d = 1$,$F = 1$, $M = 2$, $N = 2$ and 3 BS-UE pairs accepts closed form expressions for the IA problem [2]. For other setups, iterative algorithms like the one proposed in [35] converges almost surely.

- This approach is held back by the need of centralized cooperation between the BSs, or multiple iterations and channel estimations when considering distributed numerical optimizations. Therefore, while the performance gain may prove interesting, the overall cost of the solution will require extensive system-level simulations, as to compute the global impact of backhauling, delay and imperfect knowledge of the channel coefficients on the performance of the whole system.

The state-of-the-art algorithms available for this setup at the moment can be split up in three categories, all stemming from a similar analysis yet differing in their optimization objectives, as discussed in [39]. Let us consider a K-user interference channel, with an arbitrary yet fixed space, time and frequency resource set available for the transmitter-receiver pairs. This resource set generates channel matrices between the nodes that will be either full, diagonal or block-diagonal depending on the combination of resources considered. The authors of [35] present both an interference space minimization algorithm, as well as the well-used Max-SINR algorithm. Both presentations are based on some key hypotheses:

- The precoding and decoding matrices $\mathbf{C}_b$ and $\mathbf{D}_u$ have orthonormal column vectors. Verifying the IA conditions (16) only requires orthogonal column vectors, but the orthonormality conditions allows for a clearer treatment of the power allocation described below. Essentially, the precoding does not change the power distribution between the symbols and the actual signals sent.
- The transmitted symbols $\mathbf{s}_b$ are chosen from a Gaussian codebook with variance $P/D_b \cdot \mathbf{I}$. Since each transmitter is paired with a single receiver, this also implies $D_b = d_u$ is the theoretical d.o.f. achieved by the BS-UE pair if interference is effectively removed. The equal power allocation between the source symbol is optimal at high SINR [62], and as such does not impact the achievable d.o.f.. In practice however, an unequal power allocation between the streams may provide substantial beamforming gains.
- The channels are supposed reciprocal, i.e. $\mathbf{H}_{b,u} = \overleftarrow{\mathbf{H}}_{b,u}^{\dagger}$. As the algorithms are iterative in nature, this assumption has a profound impact on the algorithm and the form of the solutions. In particular, from the IA conditions (16), we have that achieving IA in both directions requires:

$$\begin{aligned}
\mathbf{D}_u^{\dagger}\mathbf{H}_{b,u}\mathbf{C}_u &= \mathbf{0}_{\mathbb{M}} && \forall (b,u) \in \{(b,u)\,|\,1 \leq b \leq K,\, 1 \leq u \leq K,\, b \neq u\} && \text{(26a)}\\
\operatorname{rank}\left(\mathbf{D}_u^{\dagger}\mathbf{H}_{b,u}\mathbf{C}_b\right) &= d_u && \forall (b,u) \in \{(b,u)\,|\,1 \leq b \leq K,\, 1 \leq u \leq K,\, b = u\} && \text{(26b)}\\
\overleftarrow{\mathbf{D}}_u^{\dagger}\overleftarrow{\mathbf{H}}_{b,u}\overleftarrow{\mathbf{C}}_u &= \mathbf{0}_{\mathbb{M}} && \forall (b,u) \in \{(b,u)\,|\,1 \leq b \leq K,\, 1 \leq u \leq K,\, b \neq u\} && \text{(26c)}\\
\operatorname{rank}\left(\overleftarrow{\mathbf{D}}_u^{\dagger}\overleftarrow{\mathbf{H}}_{b,u}\overleftarrow{\mathbf{C}}_u\right) &= d_u && \forall (b,u) \in \{(b,u)\,|\,1 \leq b \leq K,\, 1 \leq u \leq K,\, b = u\} && \text{(26d)}
\end{aligned}$$

  Since $\mathbf{H}_{b,u} = \overleftarrow{\mathbf{H}}_{b,u}^{\dagger}$, choosing $\mathbf{C}_u = \overleftarrow{\mathbf{D}}_u$ and $\overleftarrow{\mathbf{C}}_u = \mathbf{D}_u$ ensures that the IA conditions are satisfied for the downlink if they're satisfied for the uplink, and conversely.

The first algorithm of [35] aims at minimizing the *interference leakage* at all receiver. The interference leakage for a user u is defined as follows:

$$\mathcal{I}_{\mathrm{IL}}(u) = \operatorname{tr}\left(\mathbf{D}_u^{\dagger}\mathbf{Q}_u\mathbf{D}_u\right) \qquad \mathbf{Q}_u = \sum_{\substack{1 \leq b \leq K \\ b \neq u}} \frac{P}{d_u}\mathbf{\Gamma}_{b,u}\mathbf{H}_{b,u}\mathbf{C}_u\mathbf{C}_u^{\dagger}\mathbf{H}_{b,u}^{\dagger}\mathbf{\Gamma}_{b,u}^{\dagger} \tag{27}$$

In this expression, $\mathbf{Q}_u$ is the interference covariance term. We consider that $\mathbf{C}_b$ has been chosen for all $1 \leq b \leq K$, and the user thus has to choose $\mathbf{D}_u$ such that $\mathbf{L}_u$ is minimized – and $\mathbf{D}_u$ has orthonormal columns. The partial problem thus writes, for each user u:

$$\begin{aligned}
\min_{\mathbf{D}_u}.\quad & \mathcal{I}_{\mathrm{IL}}(u) = \operatorname{tr}\left(\mathbf{D}_u^{\dagger}\mathbf{Q}_u\mathbf{D}_u\right)\\
\text{s.t.}\quad & \mathbf{D}_u^{\dagger}\mathbf{D}_u = \mathbf{I}
\end{aligned} \tag{28}$$

A solution to the problem is to take $\mathbf{D}_u$ as the space spanned by the d_u smallest eigenvalues of $\mathbf{Q}_u$. Using the notation $\nu_i[\cdot]$ as the eigenvector corresponding to the i^{th} smallest eigenvalue of a matrix, the columns of $\mathbf{D}_u$ as thus chosen as:

$$[\mathbf{D}_u]^d = \nu_d\left[\mathbf{Q}_u\right] \qquad 1 \leq d \leq d_u = D_b \tag{29}$$

The user thus sets its receive filter as $\mathbf{D}_u^{\dagger}$, and subsequently uses it as a precoder in a feedback transmission $\overleftarrow{\mathbf{C}}_u = \mathbf{D}_u$. In a similar manner, the BSs compute their optimal receive filters for the feedback transmission, and uses them as precoder in the next iteration. It can be shown[35] that the leakage interference term is decreasing at each step, and bounded below, which means that the algorithm converges towards a local optimum. This approach can be extended to include a potentially colored noise or uncorrelated interference term in the objective, another approach termed *interference plus noise leakage minimization* [28].

By noting that the objective of the interference leakage minimization does not take the direct received power into account, the authors of [35] proposed another algorithm – also in a distributed form. The goal is there to maximize the SINR of the user for each stream. Considering a stream $1 \leq d \leq d_u$ at user u, the SINR of the stream is:

$$\mathsf{SINR}_{u,d} = \frac{P}{d_u} \cdot \frac{[\mathbf{D}_u^\dagger]_d \left(\mathbf{\Gamma}_{u,u}\mathbf{H}_{u,u}[\mathbf{C}_u]^d[\mathbf{C}_u^\dagger]_d\mathbf{H}_{u,u}^\dagger\mathbf{\Gamma}_{u,u}^\dagger\right)[\mathbf{D}_u]^d}{[\mathbf{D}_u^\dagger]_d\mathbf{B}_{u,d}[\mathbf{D}_u]^d} \tag{30}$$

where $\mathbf{B}_{u,d}$ is the interference plus noise covariance matrix:

$$\mathbf{B}_{u,d} = N \cdot \mathbf{I} + \overbrace{\sum_{\substack{1 \leq b \leq K \\ b \neq u}} \frac{P}{d_b}\mathbf{\Gamma}_{b,u}\mathbf{H}_{b,u}\mathbf{C}_b\mathbf{C}_b^\dagger\mathbf{H}_{b,u}^\dagger\mathbf{\Gamma}_{b,u}^\dagger}^{\textbf{Out-of-cell interference}} + \overbrace{\frac{P}{d_u}\sum_{\substack{1 \leq l \leq d_u \\ l \neq d}} \mathbf{\Gamma}_{u,u}\mathbf{H}_{u,u}[\mathbf{C}_u]^l[\mathbf{C}_u^\dagger]_l\mathbf{H}_{u,u}^\dagger\mathbf{\Gamma}_{u,u}^\dagger}^{\textbf{Cross-stream interference}} \tag{31}$$

The optimal beamforming vector for the d^{th} stream of user u is the MMSE solution:

$$[\mathbf{D}_u]^d = \frac{\mathbf{B}_{u,d}^{-1}\mathbf{H}_{u,u}[\mathbf{C}_u]^d}{\|\mathbf{B}_{u,d}^{-1}\mathbf{H}_{u,u}[\mathbf{C}_u]^d\|} \tag{32}$$

As before, the solution may be iterated using channel estimations at the nodes and transmitting on the feedback link through the channel reciprocity property. We can note however that while this solution provides the gains of IA, it does not aim at aligning interference, but rather computes the optimal receivers – albeit without successive cancellation between streams [62]. At high SINR, we expect the *Max-SINR* algorithm to perform as well as the interference leakage minimization, and be more adaptative in other SINR regions. The partial problem is written, for the decoder side, as:

$$\begin{aligned} \max_{\mathbf{D}_u}. \quad & \mathcal{I}_{\text{SINR}}(u) = \sum_{1 \leq d \leq d_u} \mathsf{SINR}_{u,d} \\ \text{s.t.} \quad & \mathbf{D}_u^\dagger\mathbf{D}_u = \mathbf{I} \end{aligned} \tag{33}$$

The last family of optimization techniques aims at minimizing the MSE of the estimated symbols, and has been presented in [38, 39]. The objective is there to minimize the MSE of the estimated symbols at the receivers globally:

$$\mathcal{I}_{\text{MSE}} = \sum_{u=1}^{K} \mathbb{E}\left\|\mathbf{D}_u^\dagger\mathbf{y}_u - \mathbf{s}_u\right\|_2^2 \tag{34}$$

$$= \sum_{u=1}^{K} \mathbb{E}\left\|\mathbf{D}_u^\dagger\left(\mathbf{\Gamma}_{u,u}\mathbf{H}_{u,u}\mathbf{C}_u\mathbf{s}_u + \underbrace{\sum_{\substack{1 \leq b \leq K \\ b \neq u}} \mathbf{\Gamma}_{b,u}\mathbf{H}_{b,u}\mathbf{C}_b\mathbf{s}_b + \mathbf{z}_u}_{\textbf{Interference signals}}\right) - \mathbf{s}_u\right\|^2 \tag{35}$$

In [38, 39], precoders and decoders are not subjected to an orthonormality constraint, symbols sent are normalized, $\mathbb{E}\left[\mathbf{s}_u\mathbf{s}_u^\dagger\right] = \mathbf{I}$, and [39] treats the general case where the noise is non-white, i.e. $\mathbf{Z}_u = \mathbb{E}\left[\mathbf{z}_u\mathbf{z}_u^\dagger\right]$. The power constraint is thus relegated to the precoders $\mathbf{C}_u$. Nonetheless, for any $\mathbf{C}_u$, there exists some diagonal matrix $\mathbf{\Psi}_u$ and normalized $\tilde{\mathbf{C}}_u$ such that $\mathbf{C}_u = \tilde{\mathbf{C}}_u\mathbf{\Psi}_u$. Therefore, we can freely consider the power constraint enforced either on the precoders $\mathbf{C}_u$, or on the symbols $\mathbf{s}_u$ through the relation $\mathbf{C}_u\mathbf{s}_u = \tilde{\mathbf{C}}_u\mathbf{\Psi}_u\mathbf{s}_u$. In lieu of the orthonormality constraint, the authors of these algorithms use the relaxed power constraint $\|\mathbf{C}_b\|_F^2 = \operatorname{tr}(\mathbf{C}_b\mathbf{C}_b^\dagger) \leq P$, for all $b \in \{1, \ldots, K\}$. For the whole network, this thus writes:

$$\begin{aligned} &\underset{(\mathbf{D}_u)_{1\leq u\leq U}}{\text{min.}} && \mathcal{I}_{\text{MSE}} \\ &\text{s.t.} && \|\mathbf{C}_b\|_F^2 \leq P \end{aligned} \tag{36}$$

Using these hypotheses, at each iteration, for fixed receiving filters $\mathbf{D}_u$, $1 \leq u \leq K$, the optimal precoders are:

$$\mathbf{C}_b = \left(\lambda_b\mathbf{I} + \sum_{u=1}^{K}\mathbf{\Gamma}_{b,u}^\dagger\mathbf{H}_{b,u}^\dagger\mathbf{D}_u\mathbf{D}_u^\dagger\mathbf{H}_{b,u}\mathbf{\Gamma}_{b,u}\right)^{-1}\mathbf{\Gamma}_{b,b}^\dagger\mathbf{H}_{b,b}^\dagger\mathbf{D}_b \tag{37}$$

The scalar coefficient λ_b is a Lagrangian multiplier chosen such that the power constraint $\operatorname{tr}(\mathbf{C}_b\mathbf{C}_b^\dagger) \leq P$ is met. No closed-form is available for its expression, although $\mathbf{C}_b(\lambda_b)$ is decreasing in λ_b and thus Newton iterations yield the desired value efficiently. Similarly, at each step, for fixed transmit precoders $\mathbf{C}_b$, $1 \leq b \leq K$, the optimal decoders are:

$$\mathbf{D}_u = \left(\sum_{b=1}^{K}\mathbf{\Gamma}_{b,u}\mathbf{H}_{b,u}\mathbf{C}_u\mathbf{C}_u^\dagger\mathbf{H}_u^\dagger\mathbf{\Gamma}_{b,u}^\dagger + \mathbf{Z}_u\right)^{-1}\mathbf{\Gamma}_{u,u}\mathbf{H}_{u,u}\mathbf{C}_u \tag{38}$$

At each step, the solutions presented here verify the Karush-Kuhn-Tucker conditions, and each iteration reduces the global MSE and thus converges towards a stationary point of the original problem [39]. The minimum MSE optimization objective can be linked to the minimum interference leakage objective, by considering the MSE between the processed received symbols on the direct link $\mathbf{D}_u^\dagger\mathbf{\Gamma}_{b,u}\mathbf{H}_{b,u}\mathbf{C}_u\mathbf{s}_u$ rather than $\mathbf{s}_u$ [39]. Furthermore, it is related to the max-SINR algorithm in the reduced treatment of [38] considering only 1 stream per user. An extension of the algorithm with weights on the MSE of each BS-UE pair has been treated in [63], and linked to a weighted sum-rate maximization objective, thereby providing treatment of QoS constraints.

4.2 Decentralized IA scenario

We presented two approaches for decentralized IA in cellular networks in section 3.2. Compared to the K-user interference channel, in the interfering broadcast channel model, one has to take *intra-cell interference* into account. Streams of information from the BS have different destinations, and since the BS multiplexes these streams in a single symbol, they interferer between one another. This is a classical multi-user problem known in the broadcast channel litterature, as well as in multi-user MIMO techniques. The approaches of [49] and [57] that we presented share common characteristics, although the actual implementation between them differs slightly. These similarities are listed below, and Fig.5 presents a graphical view of the functioning of both schemes:

- Through additional precoding or decoding, the signal space used by the BSs is reduced in order to allow users to find a receiving subspace orthogonal to the interference.

- The intra-cell interference is aimed at spanning the out-of-cell interference space on the user side. This is the part where IA is actually implemented ; users will choose some receiving subspace where interference is cancelled or diminished, and ask their BS to project intra-cell interference onto that space, through zero-forcing beamforming.

- Beyond initial parameters, no sharing of information, nor iterations, are necessary to achieve the gains promised by the schemes. UEs only need to estimate the channel of their strongest interferer, compute their receive filters based on this knowledge, and transmit their *equivalent* received space to their BS.

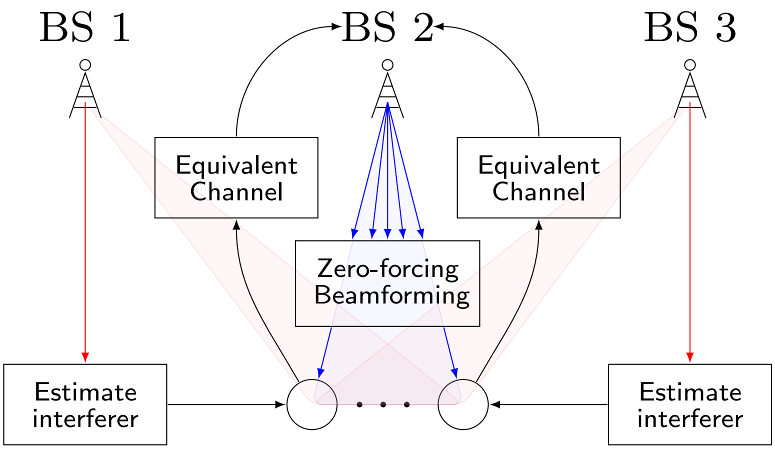

Figure 5: Decentralized IA for interfering broadcast channels. The scheme incorporates channel estimation, computation of the receive filter and zero-forcing from the BS.

Both schemes, in their actual shape, suffer from some drawbacks. They only consider singular stream allocations to users, and a generalization isn't clear. We discuss some possibilities in the next section. Furthermore, they only consider removing 1 interferer only. The scheme of [49] hints at the possibility of adapting the MMSE decoder to more than one interferer, but the performance of such a change has not been evaluated yet. If the 2 strongest interferers are close to colinear, then the gains of such an approach are likely consequent. The number of available dimensions and the overlap between the interferers will also likely play an important role.

A different approach has been presented in [55] and extended by Tang and Lambotharan in [41]. The goal is here to align all inter-cell and intra-cell interferers onto the same subspace. Consider for example network with 2 BSs and 2 UEs per BS. In essence in this network, a BS has to consider 4 receiving subspaces and send its information towards its users, while managing interference onto the other users. This problem can be simplified by *grouping* the users in a cell in such a way that the interference seen from a BS spans the same subspace *on both users.* Let the users of BS 1 be indexed by 1 and 2. Upon estimating the interfering channel, the condition for the subspaces to overlap is thus:

$$\mathbf{span}\{\mathbf{H}_{2,1}^{\dagger}\mathbf{D}_1\} = \mathbf{span}\{\mathbf{H}_{2,2}^{\dagger}\mathbf{D}_2\} = \mathbf{G}_1 \tag{39}$$

The variables of interest are the decoding matrices $\mathbf{D}_1$ and $\mathbf{D}_2$. This can be solved through the matrix equation [55]:

$$\begin{pmatrix} \mathbf{I} & -\mathbf{H}_{2,1}^{\dagger} & \mathbf{0} \\ \mathbf{I} & \mathbf{0} & -\mathbf{H}_{2,2}^{\dagger} \end{pmatrix} \begin{pmatrix} \mathbf{G}_1 \\ \mathbf{D}_1 \\ \mathbf{D}_2 \end{pmatrix} = \mathbf{0} \tag{40}$$

If a solution verifying (39) can be found, both users in the group can be considered as one for the interfering BS. Hence, if BS 1 can use the fact that its precoder columns need to lie in the proper nullspaces of the interference terms. For example, if we consider $d = 1$ stream per user, the precoders $\mathbf{c}_1$ and $\mathbf{c}_2$ verify:

$$\mathbf{c}_1 \subset \mathbf{null}\{[\mathbf{G}_1 \quad \mathbf{H}_{1,2}^{\dagger}\mathbf{D}_1]^{\dagger}\} \tag{41}$$

$$\mathbf{c}_2 \subset \mathbf{null}\{[\mathbf{G}_1 \quad \mathbf{H}_{1,1}^{\dagger}\mathbf{D}_2]^{\dagger}\} \tag{42}$$

The symbol streams thus are orthogonals to the interference subspace for each user. Extensions for multiple streams and more BSs is treated by [41]. For K users per BS, B BSs and d streams per users, the conditions for achievability of this scheme on the number of transmit and receive antennas are:

$$\begin{aligned} M &\geq d(1 + K(L-1)) \\ N &\geq d(1 + (K-1)(L-1)) \end{aligned}$$

It is yet unclear whether using frequency diversity rather than antennas leads to harsher conditions on the achievability of the scheme.

Shi *et al.* [64] develop an extension of the *Min-MSE* algorithm applicable to interfering broadcast channels in a distributed fashion. As such, this is an interesting baseline for performance comparisons of the more refined IA schemes presented above. In effect, the iterative algorithm of [64] does not aim at aligning interference, but rather at designing precoders and receive filters maximizing some utility function of the different information streams in the network through a weighted MMSE approach. The general optimization problem the authors are solving is represented in the following way. Let $\mathbf{E}_u$ be the MSE variance at user u, and assume user u is linked to BS β:

$$\mathbf{E}_u = (\mathbf{I} - \mathbf{D}_u^{\dagger}\mathbf{H}_{\beta,u}\mathbf{C}_u)(\mathbf{I} - \mathbf{D}_u^{\dagger}\mathbf{H}_{\beta,u}\mathbf{C}_u)^{\dagger} + \sum_{1 \leq b \leq B} \sum_{\substack{v \in \mathcal{U}_b \\ v \neq u}} \mathbf{D}_u^{\dagger}\mathbf{H}_{b,u}\mathbf{C}_v\mathbf{C}_v^{\dagger}\mathbf{H}_{b,u}^{\dagger}\mathbf{D}_u + \sigma_u^2\mathbf{D}_u^{\dagger}\mathbf{D}_u$$

The problem in its most basic form is thus written:

$$\begin{aligned} \underset{(\mathbf{D}_u,\mathbf{C}_u)_{1\leq u\leq U}}{\text{minimize}} \quad & \mathcal{I}_{\text{MSE}} = \sum_{1\leq u\leq U} \operatorname{tr}(\mathbf{E}_u) \\ \text{subject to} \quad & \|\mathbf{C}_u\|_F^2 \leq P \qquad 1 \leq u \leq U \end{aligned} \tag{43}$$

The constraint on the precoder's norm may be written as a trace constraint. Shi *et al.* show that many kind of more complex problems may be derived from the MMSE minimization problem with the addition of a variable weight matrix. In particular, they show that maximizing the weighted sum-rate $\sum_u \alpha_u R_u$ of all nodes in the network is equivalent – in the sense that it has the same *global* optimum – to the following problem:

$$\begin{aligned} \underset{(\mathbf{W}_u,\mathbf{D}_u,\mathbf{C}_u)_{1\leq u\leq U}}{\text{minimize}} \quad & \sum_{1\leq u\leq U} \alpha_u \left(\operatorname{tr}(\mathbf{W}_u\mathbf{E}_u) - \log|\mathbf{W}_u|\right) \\ \text{subject to} \quad & \|\mathbf{C}_u\|_F^2 \leq P \qquad 1 \leq u \leq U \end{aligned} \tag{44}$$

By noting that the problem is convex in $\mathbf{W}_u$, $\mathbf{D}_u$ and $\mathbf{C}_u$ separately, the authors propose an iterative block descent algorithm which reaches a stationary and locally optimal point provably. This algorithm is amenable to distributed implementation, and thus is an interesting baseline to compare the performance of IA schemes to more traditional distributed beamforming schemes. The implementation of the distributed scheme is straightforward, and in practice similar to the *Max-SINR* algorithm or the MSE minimization of the preceding section.

4.3 Extensions and complementary scenarii

The decentralized IA scheme is well adapted to integration with small cells. In effect, since the base stations reduce their own signal space in order to allow users to compute interference-reducing receive filters, a small cell (SC) can proceed in a similar manner and be effectively treated by the algorithm as a macro BS. Some alternative approaches can also be envisionned for SCs. In [65, 66], the authors propose schemes termed *cognitive* IA, where the SCs users estimate the channels from the macro BSs and use an orthogonal space for all their transmissions, thereby allowing the main communication backbone to communicate transparently. In effect, all the computation is delegated to the SCs.

Such an extension may be envisionned with the decentralized approach of [49, 57]. By reducing their used signal space, the BSs are leaving part of the global space free for small cells to use. Since the most limiting aspect of performances for users in a SC is the interference from the neighboring BS, the SC access point may use orthogonalization techniques to send its information in a space that is orthogonal to the interference space at the receiver. In effect, this approach is akin to a *Fractional Frequency Reuse* scheme [1], where the freed signal dimensions are much more dynamic than traditional FFR schemes ; each user will see a different free subspace and thus the BS has much more latitude in the way it can handle its transmissions.

Describing the previous scenarii, we focused on distributed optimization of the decoders and precoders. As seen in our literature review, and pointed out in [19], much of the focus of practical interference alignment lies in the handling and quality of the feedback from the users to their BS. We expect to work in the immediate future on several aspects of this particular roadblock for IA:

- Although the complete *interference cancellation* of IA is attractive at first, in practice numerous works showed that MMSE receivers actually performed better. For any of the scenarii presented above, [49, 48, 35] used MMSE receivers and achieved higher rates than “pure” interference alignment. MMSE receivers are also more robust to channel estimation errors, and are easier to handle when trying to model uncertainty on the state of the different channels. While the algorithms we plan to develop root themselves in IA techniques, we plan to use MMSE decoders as receive filters.

- A key point of the feedback quality can be linked to subspace tracking methods derived from statistics and estimation. In effect, in order to construct its precoders, the BS will want to track the *minor* subspace of interference – complementary to the principal subspace – and use this subspace for transmission. Iterative subspace tracking algorithms have been studied extensively in the late 1990s, such as the well known *Projection Approximation Subspace Tracking* (PAST) [67], which may adapt readily to our setup. More recently, these approaches have been adapted to sparse or extremely noisy data using results from the compressive sensing literature (see [68] and references therein).

- Beyond the successful tracking of subspaces, the goal is ultimately to have a stable network where variations of resource allocations and, in our case, precoding and decoding matrices are thoroughly controlled to avoid transient unstable states. Such stability over time may be obtained through control theory, as demonstrated in [69]. In practice, the resource allocation changes are constrained to deviate smoothly from the present to the target state, rather than abruptly. Therefore, the system is in effect pursuing its target and not reach it, making the whole network evolution slower and thus predictable. This method may be extended in our case, using the minor subspaces and interference power tracked by the BSs for their respective users.

Finally, a complementary goal to this work would be to better predict the gains of IA in practical cases. This is needed for example in the first scenario ; if a BS, or a group of BSs, are able to assess that a group of their users would benefit from using interference alignment, they may trigger an opportunistic cooperation towards this aim. In order to take this decision however, one clearly needs a tractable way to compute the IA gains without actually implementing the scheme. An interesting approach on this subject has been presented in [44] based on the ergodic capacity of MIMO channels.

5 Conclusion

In a previous report, we identified that much of the expected gain of IA techniques will come through the removal of the first, or the first two, strongest interferers. Throughout this report, we described two IA schemes with some variations that provide a base to achieve this goal. The first scheme stems from the more classical IA model, the K-user interference channel. In this model, a cluster of K BSs chooses to dedicate part of their resources to exactly K UEs, and apply classical IA to make interfering subspaces overlap on the UEs. The second scheme is oriented towards interfering broadcast channels, which closely model the behavior of actual cellular systems in the downlink direction. In this case, at least 2 approaches in the literature aim at removing the strongest interferer while ensuring a rank-1 subspace with reduced interference for each user in the cell, provided that enough dimensions are available for signaling.

We plan to iterate on these results, on several key points described in the last section of the report. First, most practical implementations and simulations consider only the spatial dimensions – i.e. antennas – and the frequency dimensions as channel extensions rather than actual dimensions. In our experiments, we plan to not only focus on energy gains, but also on the use of mainly frequency dimensions to see if IA shows enough gains in this setup. If indeed it does, the complexity of adding IA in existing systems is relatively low, therefore making IA a good candidate for implementation in future generation wireless networks. In particular, using IA over the frequency space, we expect to provide gains over the full-reuse or FFR schemes currently implemented in cellular networks.

References

[1] Abdelbaset S. Hamza, Shady S. Khalifa, Haitham S. Hamza, and Khaled Elsayed. A survey on inter-cell interference coordination techniques in OFDMA-based cellular networks. *IEEE Communications Surveys and Tutorials*, 15(4):1642–1670, Aug 2013. doi: 10.1109/SURV.2013.013013.00028.

[2] V. R. Cadambe and S. A. Jafar. Interference alignment and degrees of freedom of the k-user interference channel. *IEEE Transactions on Information Theory*, 54(8):3425–3441, Aug 2008. doi: 10.1109/TIT.2008.926344.

[3] J.-M. Gorce and P. Ferrand. Interference modeling and cellular network geometry. Technical report, Inria, 2014.

[4] A. El Gamal and Y.-H. Kim. *Network Information Theory.* Cambridge University Press, 2011.

[5] E. Telatar. Capacity of multi-antenna gaussian channels. *European Transactions on Telecommunications*, 10(6):585–595, 1999.

[6] S. A. Jafar and M. J. Fakhereddin. Degrees of freedom for the MIMO interference channel. *IEEE Transactions on Information Theory*, 53(7):2637–2642, 2007. doi: 10.1109/TIT.2007.899557.

[7] Syed A. Jafar and Shlomo Shamai (Shitz). Degrees of freedom region of the MIMO x channel. *IEEE Transactions on Information Theory*, 54(1):151–170, Jan 2008. doi: 10.1109/TIT.2007.911262.

[8] Syed Jafar. Topological interference management through index coding. *IEEE Transactions on Information Theory*, 60:529–568, 2014. doi: 10.1109/TIT.2013.2285151.

[9] S. A. Jafar. Interference alignment – a new look at signal dimensions in a communication network. *Foundation and Trends in Commun. and Inf. Theory*, 7(1):1–134, 2011. doi: 10.1561/0100000047.

[10] A. Özgür and D. N. C. Tse. Achieving linear scaling with interference alignment. In *IEEE International Symposium on Information Theory (ISIT)*, 2009.

[11] V. R. Cadambe and S. A. Jafar. Degrees of freedom of wireless networks - what a difference delay makes. In *Asilomar Conference on Signals, Systems and Computers*, pages 133–137. Institute of Electrical and Electronics Engineers, Nov 2007. ISBN 978-1-4244-2109-1. doi: 10.1109/ACSSC.2007.4487180.

[12] V. R. R. Cadambe and S. A. Jafar. Interference alignment and the degrees of freedom of wireless x networks. *IEEE Transactions on Information Theory*, 55(9):3893–3908, 2009. doi: 10.1109/TIT.2009.2025541.

[13] Sanjay Karmakar and Mahesh K. Varanasi. The capacity region of the MIMO interference channel and its reciprocity to within a constant gap. *IEEE Transactions on Information Theory*, 59(8):4781–4797, Aug 2013. doi: 10.1109/TIT.2013.2256231.

[14] L. H. Grokop, D. N. C. Tse, and R. D. Yates. Interference alignment for line-of-sight channels. *IEEE Transactions on Information Theory*, 57(9):5820–5839, Sep 2011. doi: 10.1109/TIT.2011.2161914.

[15] B. Nazer, M. Gatspar, S. A. Jafar, and S. Vishwanath. Ergodic interference alignment. In *IEEE International Symposium on Information Theory (ISIT)*, 2009.

[16] Meisam Razaviyayn, Gennady Lyubeznik, and Zhi-Quan Luo. On the degrees of freedom achievable through interference alignment in a MIMO interference channel. *IEEE Transactions on Signal Processing*, 60(2):812–821, Feb 2012. doi: 10.1109/TSP.2011.2173683.

[17] C M Yetis, Tiangao Gou, S A Jafar, and A H Kayran. On feasibility of interference alignment in MIMO interference networks. *IEEE Transactions on Signal Processing*, 58(9):4771–4782, Sep 2010. doi: 10.1109/TSP.2010.2050480.

[18] Meisam Razaviyayn, Maziar Sanjabi, and Zhi-Quan Luo. Linear transceiver design for

interference alignment: Complexity and computation. *IEEE Transactions on Information Theory*, 58(5):2896–2910, May 2012. doi: 10.1109/TIT.2012.2184909.

[19] O. El Ayach, S. W. Peters, and R. W. Heath. The practical challenges of interference alignment. *IEEE Wireless Communications Magazine*, 20(1):35–42, Feb 2013. doi: 10.1109/MWC.2013.6472197.

[20] S.A. Jafar and A.J. Goldsmith. Isotropic fading vector broadcast channels: The scalar upper bound and loss in degrees of freedom. *IEEE Transactions on Information Theory*, 51(3): 848–857, Mar 2005. doi: 10.1109/TIT.2004.842621.

[21] A. Host-Madsen. Capacity bounds for cooperative diversity. *IEEE Transactions on Information Theory*, 52(4):1522–1544, Apr 2006. doi: 10.1109/TIT.2006.871576.

[22] M. A. Maddah-Ali and D. N. C. Tse. On the degrees of freedom of miso broadcast channels with delayed feedback. Technical report, University of California at Berkeley, 2010.

[23] Mohammad Ali Maddah-Ali and David Tse. Completely stale transmitter channel state information is still very useful. *IEEE Transactions on Information Theory*, 58(7):4418–4431, Jul 2012. doi: 10.1109/TIT.2012.2193116.

[24] Chinmay S. Vaze and Mahesh K. Varanasi. The degrees of freedom region and interference alignment for the MIMO interference channel with delayed CSIT. *IEEE Transactions on Information Theory*, 58(7):4396–4417, Jul 2012. doi: 10.1109/TIT.2012.2194270.

[25] Chinmay S. Vaze and Mahesh K. Varanasi. The degree-of-freedom regions of MIMO broadcast, interference, and cognitive radio channels with no CSIT. *IEEE Transactions on Information Theory*, 58(8):5354–5374, Aug 2012. doi: 10.1109/TIT.2012.2201349.

[26] Syed A. Jafar. Blind interference alignment. 6(3):216–227, Jun 2012. doi: 10.1109/JSTSP.2012.2187877.

[27] T. Gou, C. Wang, and S. A. Jafar. Aiming perfectly in the dark-blind interference alignment through staggered antenna switching. *IEEE Transactions on Signal Processing*, 59(6):2734–2744, 2011. doi: 10.1109/TSP.2011.2129517.

[28] O. El Ayach, S. W. Peters, and R. W. Heath. The feasibility of interference alignment over measured MIMO-OFDM channels. *IEEE Transactions on Vehicular Technology*, 59 (9):4309–4321, Nov 2010. doi: 10.1109/TVT.2010.2082005.

[29] O. El Ayach and R. W. Heath. Grassmannian differential limited feedback for interference alignment. *IEEE Transactions on Signal Processing*, 60(12):6481–6494, Dec 2012. doi: 10.1109/TSP.2012.2218238.

[30] Mohsen Rezaee and Maxime Guillaud. Interference channel sum rate optimization on the grassmann manifold. In *International Symposium on Wireless Communication Systems (ISWCS)*, pages 571–575. Institute of Electrical and Electronics Engineers, Aug 2012. ISBN 978-1-4673-0762-8. doi: 10.1109/ISWCS.2012.6328432.

[31] Rajesh Krishnamachari and Mahesh Varanasi. Interference alignment under limited feedback for MIMO interference channels. *IEEE Transactions on Signal Processing*, 61(15):3908–3917, Jul 2013. doi: 10.1109/TSP.2013.2264055.

[32] Mohsen Rezaee and Maxime Guillaud. Limited feedback for interference alignment in the k-user MIMO interference channel. In *IEEE Information Theory Workshop (ITW)*, pages 667–671. Institute of Electrical and Electronics Engineers, Sep 2012. ISBN 978-1-4673-0223-

4. doi: 10.1109/ITW.2012.6404762.

[33] Mohsen Rezaee, Maxime Guillaud, and Fredrik Lindqvist. CSIT sharing over finite capacity backhaul for spatial interference alignment. In *IEEE International Symposium on Information Theory (ISIT)*, pages 569–573. Institute of Electrical and Electronics Engineers, July 2013. ISBN 978-1-4799-0446-4. doi: 10.1109/ISIT.2013.6620290.

[34] Steven W. Peters and Robert W. Heath. Interference alignment via alternating minimization. In *IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, pages 2445–2448. Institute of Electrical and Electronics Engineers, Apr 2009. ISBN 978-1-4244-2353-8. doi: 10.1109/ICASSP.2009.4960116.

[35] K. Gomadam, V. R. Cadambe, and S. A. Jafar. A distributed numerical approach to interference alignment and applications to wireless interference networks. *IEEE Transactions on Information Theory*, 57(6):3309–3322, Jun 2011. doi: 10.1109/TIT.2011.2142270.

[36] D. Kim and M. Torlak. Optimization of interference alignment beamforming vectors. *IEEE Journal on Selected Areas in Communications*, 28(9):1425–1434, 2010. doi: 10.1109/JSAC.2010.101204.

[37] Bo Niu and Alexander M. Haimovich. Interference subspace tracking for network interference alignment in cellular systems. In *IEEE Global Telecommunications Conference (GLOBECOM)*, pages 1–5. Institute of Electrical and Electronics Engineers, Nov 2009. ISBN 978-1-4244-4148-8. doi: 10.1109/GLOCOM.2009.5426236. URL `http://dx.doi.org/10.1109/GLOCOM.2009.5426236`.

[38] David A. Schmidt, Changxin Shi, Randall A. Berry, Michael L. Honig, and Wolfgang Utschick. Minimum mean squared error interference alignment. In *Asilomar Conference on Signals, Systems and Computers*, pages 1106–1110. Institute of Electrical and Electronics Engineers, Nov 2009. ISBN 978-1-4244-5825-7. doi: 10.1109/ACSSC.2009.5470055.

[39] S. W. Peters and R. W. Heath. Cooperative algorithms for MIMO interference channels. *IEEE Transactions on Vehicular Technology*, 60(1):206–218, Jan 2011. doi: 10.1109/TVT.2010.2085459.

[40] Martin Mayer, Gerald Artner, Gabor Hannak, Martin Lerch, and Maxime Guillaud. Measurement based evaluation of interference alignment on the vienna mimo testbed. In *International Symposium on Wireless Communication Systems (ISWCS)*, 2013.

[41] Jie Tang and Sangarapillai Lambotharan. Interference alignment techniques for MIMO multi-cell interfering broadcast channels. *IEEE Transactions on Communications*, 61(1): 164–175, Jan 2013. doi: 10.1109/TCOMM.2012.100912.110644. URL `http://dx.doi.org/10.1109/TCOMM.2012.100912.110644`.

[42] R. Tresch and M. Guillaud. Clustered interference alignment in large cellular networks. In *IEEE International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC)*, pages 1024–1028, Sept 2009. doi: 10.1109/PIMRC.2009.5450107.

[43] Manfred Westreicher and Maxime Guillaud. Interference alignment over partially connected interference networks: Application to the cellular case. In *IEEE Wireless Communications and Networking Conference (WCNC)*, pages 647–651, 2012. ISBN 978-1-4673-0437-5. doi: 10.1109/WCNC.2012.6214448. URL `http://dx.doi.org/10.1109/WCNC.2012.6214448`.

[44] M. Guillaud and D. Gesbert. Interference alignment in the partially connected K-user MIMO interference channel. In *European Signal Processing Conference (EUSIPCO)*, Barcelona,

SPAIN, 08 2011.

[45] Liangzhong Ruan, V. K. N. Lau, and Xiongbin Rao. Interference alignment for partially connected MIMO cellular networks. *IEEE Transactions on Signal Processing*, 60(7):3692–3701, Jul 2012. doi: 10.1109/TSP.2012.2192432. URL `http://dx.doi.org/10.1109/TSP.2012.2192432`.

[46] Huarui Yin. Comments on degrees of freedom region for k-user interference channel with m antennas. *arXiv preprint arXiv:1011.3812*, 2010.

[47] Roland Tresch, Maxime Guillaud, and Erwin Riegler. On the achievability of interference alignment in the k-user constant mimo interference channel. In *IEEE Workshop on Statistical Signal Processing*, pages 277–280. IEEE, 2009.

[48] D. Aziz, F. Boccardi, and A. Weber. System-level performance study of interference alignment in cellular systems with base-station coordination. In *IEEE International Symposium on Personal Indoor and Mobile Radio Communications (PIMRC)*, pages 1155–1160. IEEE, 2012.

[49] C. Suh, M. Ho, and D. N. C. Tse. Downlink interference alignment. *IEEE Transactions on Communications*, 59(9):2616–2626, 2011. doi: 10.1109/TCOMM.2011.070511.100313.

[50] Vinoth Nagarajan and Bhaskar Ramamurthi. Distributed cooperative precoder selection for interference alignment. *IEEE Transactions on Vehicular Technology*, 59(9):4368–4376, Nov 2010. doi: 10.1109/TVT.2010.2068569. URL `http://dx.doi.org/10.1109/TVT.2010.2068569`.

[51] B. Da and R. Zhang. Exploiting interference alignment in multi-cell cooperative OFDMA resource allocation. In *IEEE Global Telecommunications Conference*, pages 1–5. Institute of Electrical and Electronics Engineers, Dec 2011. ISBN 978-1-4244-9268-8. doi: 10.1109/GLOCOM.2011.6133870.

[52] C. Suh and D. N. C. Tse. Interference alignment for cellular networks. In *Allerton Conference on Communications, Control and Computing (Allerton)*, 2008.

[53] Jubin Jose, Sundar Subramanian, Xinzhou Wu, and Junyi Li. Opportunistic interference alignment in cellular downlink. In *Allerton Conference on Communication, Control, and Computing (Allerton)*, pages 1529–1534. Institute of Electrical and Electronics Engineers, Oct 2012. ISBN 978-1-4673-4539-2. doi: 10.1109/Allerton.2012.6483401. URL `http://dx.doi.org/10.1109/Allerton.2012.6483401`.

[54] Duckdong Hwang. Interference alignment for the multi-cell multiuser interference channel. *IEEE Communications Letters*, 16(6):831–833, Jun 2012. doi: 10.1109/LCOMM.2012.041112.120118. URL `http://dx.doi.org/10.1109/LCOMM.2012.041112.120118`.

[55] Wonjae Shin, Namyoon Lee, Jong-Bu Lim, Changyong Shin, and Kyunghun Jang. On the design of interference alignment scheme for two-cell MIMO interfering broadcast channels. *IEEE Transactions on Wireless Communications*, 10(2):437–442, Feb 2011. doi: 10.1109/TWC.2011.120810.101097. URL `http://dx.doi.org/10.1109/TWC.2011.120810.101097`.

[56] Wonjae Shin, Wonjong Noh, Kyunghun Jang, and Hyun-Ho Choi. Hierarchical interference alignment for downlink heterogeneous networks. *IEEE Transactions on Wireless Communications*, 11(12):4549–4559, Dec 2012. doi: 10.1109/TWC.2012.101912.120421.

[57] A. Bayesteh, A. Mobasher, and Y. Jia. Downlink multi-user interference alignment in two-

cell scenario. In *Canadian Workshop on Information Theory (CWIT)*, 2011.

[58] Harri Niemelainen, Helka-Liina Maattanen, and Mikko Valkama. System-level studies for multi-user interference alignment in a homogeneous network. In *IEEE Vehicular Technology Conference (VTC Fall)*, pages 1–6. IEEE, 2013.

[59] R. Brandt, H. Asplund, and M. Bengtsson. Interference alignment in frequency - a measurement based performance analysis. In *International Workshop on Systems, Signals and Image Processing (IWSSIP)*, pages 227–230, April 2012.

[60] Jan Schreck and Gerhard Wunder. Iterative interference alignment for cellular systems with user selection. In *IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, pages 2833–2836. IEEE, 2012.

[61] V. Kafedziski and T. Javornik. Frequency-space interference alignment in multi-cell MIMO OFDM downlink systems. In *IEEE Vehicular Technology Conference (VTC Spring)*. IEEE, 2013. ISBN http://id.crossref.org/isbn/978-1-4673-6337-2. doi: 10.1109/vtcspring.2013.6692572. URL `http://dx.doi.org/10.1109/VTCSpring.2013.6692572`.

[62] D. N. C. Tse and P. Vishwanath. *Fundamentals of Wireless Communications*. Cambridge University Press, 2005.

[63] F. Negro, S.P. Shenoy, I. Ghauri, and D.T.M. Slock. On the mimo interference channel. In *Information Theory and Applications Workshop (ITA)*, 2010.

[64] Q. Shi, M. Razaviyayn, Z.-Q. Luo, and C. He. An iteratively weighted MMSE approach to distributed sum-utility maximization for a MIMO interfering broadcast channel. *IEEE Transactions on Signal Processing*, 59(9):4331–4340, Sep 2011. doi: 10.1109/TSP.2011.2147784.

[65] Marco Maso, Leonardo S Cardoso, Ejder Baştuğ, Nguyen Linh-Trung, Mérouane Debbah, Ozgur Ozdemir, et al. On the practical implementation of vfdm-based opportunistic systems: issues and challenges. *REV Journal on Electronics and Communications*, 2(1-2), 2012.

[66] Marco Maso, Merouane Debbah, and Lorenzo Vangelista. A distributed approach to interference alignment in OFDM-based two-tiered networks. *IEEE Transactions on Vehicular Technology*, 62(5):1935–1949, Jun 2013. doi: 10.1109/TVT.2013.2245516. URL `http://dx.doi.org/10.1109/TVT.2013.2245516`.

[67] Bin Yang. Projection approximation subspace tracking. *IEEE Transactions on Signal Processing*, 43(1):95–107, 1995. ISSN 1053-587X. doi: 10.1109/78.365290. URL `http://dx.doi.org/10.1109/78.365290`.

[68] Jun He, Laura Balzano, and John C.S. Lui. Online robust subspace tracking from partial information. http://arxiv.org/abs/1109.3827v2, 2011.

[69] Virgile Garcia, Nikolai Lebedev, and Jean-Marie Gorce. Model predictive control for smooth distributed power adaptation. In *IEEE Wireless Communications and Networking Conference (WCNC)*, 2012.

RESEARCH CENTRE
GRENOBLE – RHÔNE-ALPES

Inovallée
655 avenue de l’Europe Montbonnot
38334 Saint Ismier Cedex

Publisher
Inria
Domaine de Voluceau - Rocquencourt
BP 105 - 78153 Le Chesnay Cedex
inria.fr

ISSN 0249-6399