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**INVENTORY AND PRICING IN GLOBAL OPERATIONS:  
LEARNING FROM OBSERVED DEMANDS**

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# **Inventory and Pricing in Global Operations: Learning From Observed Demand**

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## Abstract

We develop a two-period model applicable to global sourcing by considering a firm that operates in two markets: one is located in the U.S. and the second is in a country having a selling season that does not overlap with the U.S. selling season. Demand for each market depends linearly on the selling price and includes an unknown scale parameter. We assume that the firm learns from sales in the first market to assist decision making in the second. We also assume a single procurement opportunity, but allow the firm to ship leftovers from the first market to the second if doing so is profitable. Our results include the characterization of the optimal recourse policy, which represents the firm's decisions made at the beginning of the second selling season after it observes both sales in the first market and a realized value of the foreign exchange rate. Additionally, we provide a sufficient condition for reducing the optimization problem to a maximization over a single variable that we interpret as the safety stock for the first market. Further, we provide evidence that the sufficient condition is a rather mild one, likely to be satisfied in practical applications. We also establish a lower bound on the optimal value of the first-market safety stock, thereby truncating the search region of the last decision variable. This lower bound represents the optimal safety stock for the first market if that decision were made myopically, without regard to its effect on the profit associated with the second market.

## 1.0 Introduction

Consider a single, monopolistic firm that operates in two separate markets and desires to establish the optimal quantity to procure and selling price to set for its product in each market. The firm offers the same product for sale in both markets, but the selling seasons are non-overlapping. Consequently, the firm is provided the opportunity to transfer some or all of the leftovers remaining from the first market to the second market for possible sale in the second selling season. In addition, the respective market demand functions -- each of which are price dependent and include a scale parameter that is unknown at the start of the first selling season -- are correlated perfectly due to homogenous customer preferences. As a result, the firm can revise its characterization of the unknown demand parameter applicable to the second market by observing sales in the first market. In particular, if there are leftovers in the first market, then the firm's sales and demand are equivalent; thus, the firm deduces the value of the unknown parameter, thereby eliminating the associated uncertainty in the demand function for the second market. If, however, there are no leftovers in the first market, the firm cannot deduce the value of the unknown parameter. Consequently, that parameter continues to contribute uncertainty to the demand function for the second market, although the firm can update its characterization of that uncertainty based on information obtained from the first market.

We assume that the firm commits to its procurement quantities for both markets concurrently, at the beginning of the first selling season. We do not require that the firm receive both its procurement quantities at the beginning of the first selling season, only that it establish at that time a contractual arrangement governing the specified amount to be delivered at the start of each selling season. One motivation for this restriction is the desire to establish a modeling framework for the firm that negotiates a cost discount schedule by bringing larger procurement quantities to the bargaining table. Perhaps a more appropriate motivation, though, is that this particular procedure is common in the fashion goods and related industries for which often there exists only one procurement opportunity.

A key implication of having only one procurement opportunity is that the decision of a procurement quantity for the second market is made with limited information. Since the firm learns from its operations in the first market, it is likely that it would choose a procurement quantity differently if it were to make its decision after rather than before the first selling season. As recourse, though, the firm has two options. First, it can affect demand in the second market by changing its selling price. And second, if at the beginning of the second selling season the firm reaches the conclusion that the amount of stock procured for the second market is less than it desires, it can supplement its supply by transferring a portion of the leftovers remaining from the first market. However, this option might provide only limited recourse since the firm is constrained by the number of leftovers remaining from the first market. If no leftovers remain from the first market, then the firm's resulting management situation represents an application of a yield-management problem. This is because, in such a case, although the firm's capacity is fixed, it can affect the trade-off between expected leftovers and expected shortages by altering market demand through its selling price. (Weatherford and Bodily (1992) provide a general classification scheme for yield-management problems; the one described here is similar to an airline or hotel booking problem in which there is only one price class and demand is uncertain.)

In order to keep general the formulation of this model, we assume that the two markets are separated by international boundaries, thereby establishing global sourcing as the primary application. Non-international applications -- e.g., a scenario in which a firm operating in a single country uses an initial test market before offering a new product for sale in a second market -- are obtained by fixing the foreign exchange rate at one.

The management situation described above is an extension to a problem formulated by Kouvelis and Gutierrez (1992), who developed an optimal (profit-maximizing) strategy for an international firm that sells a fashion good in two, non-overlapping markets. In their model, Kouvelis and Gutierrez incorporate foreign exchange risk and provide the alternative of transferring to the second market some portion of the leftovers remaining from the first market; but, they assume that the selling price is given (for each market) and that the firm is

provided a second opportunity to procure (at the beginning of the second market). They also assume that the firm does not learn; that is, the firm does not revise after the first selling season the uncertainty associated with the second market.

Other relevant papers appearing either in the operations management or the economics literature address only aspects of the model considered here. For example, Krouse and Senchack (1977); Harpaz, Lee, and Winkler (1982); Braden and Freimer (1991); and Nahmias (1993) incorporate the idea of learning from censored information in their inventory models, but they assume that the firm's selling price is given rather than incorporate it as a decision variable. On the other hand, Grossman, Kihlstrom, and Mirman (1977); Lazear (1986); Balvers and Cosimano (1990); Trefler (1993); and Braden and Oren (1994) develop economic models for learning the demand curve, but they assume that the firm's stocking quantity is given rather than incorporate it as a decision variable. Whitin (1955); Mills (1959 and 1960); Karlin and Carr (1962); Ernst (1970); Zabel (1970 and 1972); Thowsen (1975); Young (1978); and Polatoglu (1991) study management situations in which the firm's stocking quantity and selling price are joint decision variables, but these models do not include learning.

## 2.0 Notation and Assumptions

Define markets 1 and 2 as the domestic and foreign markets, respectively. Correspondingly, associate selling season  $i$  with market  $i$ , for  $i = 1, 2$ . Then, let  $p_i$  denote the selling price for market  $i$ . In addition, let  $q$  and  $I$  (no subscripts) denote the quantities procured for markets 1 and 2, respectively. (Recall that  $I$ , the quantity procured for market 2, is ordered at the beginning of the first selling season; consequently, it serves as an initial stock of inventory already on-hand prior to the time decisions are made for market 2). In addition, designate  $x$  as the number of leftovers from market 1 transferred to market 2 ( $x \leq \text{market 1 leftovers}$ ).

Let  $c$  denote the cost of procuring a unit and let  $\tau$  represent the cost of transferring a unit from market 1 to market 2. We assume no penalty cost for a lost sale other than forfeited profit. That is, we let the per-unit goodwill cost of a shortage equal zero. We justify this assumption in

two ways. First, each of the two markets has only one selling season. Consequently, goodwill cost has little meaning. Second, the firm is a price-setter. This means that the firm affects demand by changing its selling price. Therefore, the lost goodwill resulting in an unsatisfied demand can be thought of within the context of the parameters of the demand function. Note, however, a per-unit cost of goodwill can be included explicitly in the model without changing the structure of the results. Similarly, we assume that the cost of discarding a unit is zero and that there is no salvage value. Again, either a per-unit discarding cost or a per-unit salvage value can be included without difficulty.

Next, consider the demand structure of the two markets. Without loss of generality, we assume an identical form for the two demand functions (recall that earlier, in order to include learning in the modeling framework, we specified that the two demand functions are correlated perfectly). Specifically, let demand be a decreasing, linear function of  $p$  and include an initially unknown scale parameter as an additive term:  $D(p) = y(p) + \varepsilon$ , where  $y(p) \equiv a - bp$  and both  $a$  and  $b$  are known, but  $\varepsilon$  is not. Correspondingly, assign  $F(\bullet)$  as a subjective cumulative distribution function to characterize the firm's initial beliefs regarding  $\varepsilon$  and define  $f(\bullet) = dF(\bullet)$  as the probability density function. Also, define  $h(\bullet) = f(\bullet)/[1-F(\bullet)]$  as  $\varepsilon$ 's hazard rate; let  $E[\varepsilon]$  denote the expected value of  $\varepsilon$  and assume  $-a \leq A \leq \varepsilon \leq B$ . Further, assume that  $F(\bullet)$  is such that  $h(\bullet)$  is a non-decreasing function. Examples of distributions having a non-decreasing hazard rate include the uniform, normal, and many gamma distributions (refer to Barlow and Proschan (1975) for a more extensive list).

We choose the firm's domestic currency (e.g., U.S. dollars) as the dimension for all monetary-related parameters and variables, including  $p_2$ . But, since market demand's dependency on price is a local phenomenon, we convert  $p_2$  to the applicable local currency by multiplying it by the foreign exchange rate,  $\eta$ , which, we assume, is a random variable. As a result, we express the demand function applicable to the second market more appropriately as  $D(p|\eta) = y(p|\eta) + \varepsilon$ , where  $y(p|\eta) = a - b\eta p$ ,  $p$  is in the domestic currency, and  $0 < \alpha \leq \eta \leq \beta$ .

Let  $\Phi(\bullet)$  and  $\phi(\bullet)$  correspond to the cumulative distribution function and probability density function, respectively, associated with  $\eta$ .

Note that we reach this same interpretation of the demand function for the second market if we assume instead that  $p_2$  is set in the foreign (local) currency. To demonstrate, let  $p_F$  denote the selling price for the second market, set in the foreign currency. Then the (local) demand function is  $D_F(p_F) = a - bp_F + \varepsilon$ . Correspondingly, the revenue generated from sales in the foreign market is  $R_F = p_F \min\{a - bp_F + \varepsilon, q_F\}$ , where  $q_F$  represents the stocking quantity for the foreign market and  $R_F$  is in the foreign currency. This revenue ( $R_F$ ) is converted into the domestic currency by multiplying it by  $1/\eta$ , the foreign exchange rate for changing foreign currency to domestic currency. Thus, the revenue generated from sales in the foreign market, converted to the domestic currency, is  $R_D = (1/\eta)R_F = (1/\eta)p_F \min\{a - bp_F + \varepsilon, q_F\}$ . However, by defining  $p_D = (1/\eta)p_F$  as the selling price for the second market, converted into the domestic currency, we obtain  $R_D = p_D \min\{a - bp_D + \varepsilon, q_F\}$ , which leads to the same interpretation of the demand function applicable to the second market as above.

Notice the following modeling difference between the parameters  $\varepsilon$  and  $\eta$ . On the one hand, the value of  $\varepsilon$  is fixed, having associated with it at the beginning of the first selling season a probability distribution only because its value is unknown to the firm at that time. However, since the firm learns in the first market, it begins the second selling season better informed with respect to  $\varepsilon$ : either  $\varepsilon$  is known (if demand is observed) or the range of possible values of  $\varepsilon$  is truncated (if sales are observed). On the other hand, the value of  $\eta$  is random, assuming realized values at different points in time. However, like Kouvelis and Gutierrez, we assume that  $\eta$  is observed at the start of the second selling season and remains constant for the duration of the season. This particular assumption is convenient because it means that the firm is aware fully of the foreign exchange rate at the time that it sets its selling price for the second market. This assumption also is realistic because a firm operating in an environment such as that described here can purchase (through financial markets) a foreign exchange rate option, thereby allowing it



to lock in at a specific rate at any point in time. Let  $v$  denote the realized value of  $\eta$  observed at the beginning of the second selling season.

A notable benefit of modeling foreign exchange risk in the manner described above is that the uncertainty in foreign exchange is absorbed into demand, thereby providing greater modeling flexibility. In fact, one of the results of this paper is that no assumptions regarding the specific form of  $\Phi(\bullet)$  are required for analytical tractability, which is a characteristic differentiating the international model developed here from related models of global manufacturing distribution networks (see Huchzermeier (1991)).

Next, we introduce the transformation of variables,  $z \equiv q - y(p_1)$ , in order to simplify the mathematical analysis. The definition of  $z$  provides a convenient substitution for  $q$  because it yields a compact formula for calculating leftovers or shortages for the first market. In particular, if  $z > \varepsilon$  (i.e., if  $q = y(p_1) + z > D(p_1) = y(p_1) + \varepsilon$ ), then  $z - \varepsilon$  denotes the leftovers in market 1; if  $z \leq \varepsilon$  (i.e., if  $q \leq D(p_1)$ ), then  $\varepsilon - z$  denotes the shortages in market 1. Also, since safety stock is defined as the difference between stocking quantity and expected demand, we interpret the decision variable,  $z$ , as a surrogate for the market 1 safety stock:  $ss_1 \equiv q - E[D(p_1)] = q - (y(p_1) + E[\varepsilon]) = z - E[\varepsilon]$ . In other words, the deviation of  $z$  from the mean of the unknown demand parameter represents the firm's safety stock for market 1. Consequently, if  $E[\varepsilon] = 0$ , then  $z$  is equivalent to  $ss_1$ . A similar transformation of variables for market 2, although possible, is not convenient because  $I$  and  $p_2$  are not determined simultaneously.

Finally, we define  $V(I, z, u, v)$ ,  $W(I, z, v)$ , and  $\Pi(z, p_1, I)$  as follows:

- $V(I, z, u, v)$  = expected profit associated with market 2 when an optimal policy is followed, given that at the beginning of the second selling season  $v$  is the foreign exchange rate,  $u$  denotes the revealed value of  $\varepsilon$  (i.e.,  $\varepsilon = u$ ),  $z - u$  is the number of leftovers remaining from market 1, and  $I$  is the initial inventory on-hand.
- $W(I, u, v)$  = expected profit associated with market 2 when an optimal policy is followed, given that at the beginning of the second selling season  $v$  is the foreign

exchange rate,  $u$  denotes the lower bound for  $\varepsilon$  (i.e.,  $\varepsilon \geq u$ ), there are no leftovers from market 1, and  $I$  is the initial inventory on-hand.

$\Pi(z, p_1, I) =$  total expected profit for the two markets combined, computed at the beginning of the first selling season, given that  $z$ ,  $p_1$ , and  $I$  are the decisions made at that time and an optimal recourse policy is followed in the second selling season.

Notice the difference between  $V(I, z, u, v)$  and  $W(I, u, v)$ : the former corresponds to the case in which leftovers remain from market 1 and consequently, the demand function for market 2 is deterministic (because  $\varepsilon$  is known as of the beginning of the second selling season); the latter corresponds to the complimentary case in which no leftovers remain from market 1 and consequently, the demand function for market 2 contains uncertainty (because  $\varepsilon$  still is not known as of the beginning of the second selling season).

### 3.0 Formulation of the Firm's Objective Function

The firm's objective is to choose  $z$ ,  $p_1$ , and  $I$  such that  $\Pi(z, p_1, I)$  is maximized, where  $\Pi(z, p_1, I)$  accounts for the fact that the firm chooses optimally its recourse decisions for market 2 (the recourse decisions are  $p_2$  and  $x$ , which represent the decisions made prior to the second selling season, but after the firm observes both sales from market 1 and a value for  $\eta$ ). To develop an expression for  $\Pi(z, p_1, I)$ , we consider the sequence of events relevant to its formulation and the corresponding profit implications associated with each event. Note that, in developing  $\Pi(z, p_1, I)$ , we do not discount revenues and costs associated with the second selling season. This is an arbitrary decision made only as an attempt to provide a less cumbersome presentation.

At the beginning of the first selling season, the firm chooses  $p_1$ , thereby establishing the demand for its product that corresponds to market 1:  $D(p_1) = y(p_1) + \varepsilon$ , where, recall,  $\varepsilon$  is unknown. In addition, the firm procures  $q = y(p_1) + z$  units to stock for possible sale in market 1 and  $I$  units to stock for possible sale in market 2 (the  $I$  units, however, are not received by the

firm until the beginning of the second selling season; hence, the firm incurs no holding costs associated with  $I$ ). The total procurement cost is  $c(y(p_I) + z + I)$ .

Then, if  $\varepsilon < z$  (i.e., if  $D(p_I) = y(p_I) + \varepsilon < q = y(p_I) + z$ ), the firm satisfies each unit demanded in market  $I$  -- thereby generating a sales revenue of  $p_I D(p_I) = p_I(y(p_I) + \varepsilon)$  -- and has  $z - \varepsilon$  leftovers. In addition, given that the firm acts optimally when choosing its recourse decisions, the contribution to profit associated with the second selling season is  $V(I, z, \varepsilon, v)$  since, at the beginning of the second selling season, the firm knows the value of  $\varepsilon$  ( $\varepsilon = D(p_I) - y(p_I)$ , where  $D(p_I)$  is known because it is equivalent to market  $I$  sales).

If, instead,  $\varepsilon \geq z$  (i.e., if  $D(p_I) \geq q$ ), the firm sells in market  $I$  its complete stock of  $q = y(p_I) + z$  units -- thereby generating a sales revenue of  $p_I(y(p_I) + z)$  -- and has no leftovers. In addition, given that the firm acts optimally when choosing its recourse decisions, the contribution to profit associated with the second selling season is  $W(I, z, v)$  since, at the beginning of the second selling season, the firm knows only that  $z$  is the lower bound for  $\varepsilon$ .

In summary, then, the firm's total, two-market, conditional profit,  $P(z, p_I, I)$ , depends on the relationship between  $\varepsilon$  and  $z$  as well as the value of  $\eta$  observed at the beginning of the second selling season:

$$P(z, p_I, I) = -c(y(p_I) + z + I) + \begin{cases} p_I(y(p_I) + \varepsilon) + V(I, z, \varepsilon, v) & \text{if } \varepsilon < z \text{ and } \eta = v \\ p_I(y(p_I) + z) + W(I, z, v) & \text{if } \varepsilon \geq z \text{ and } \eta = v \end{cases}$$

Therefore, the total expected profit is:

$$\Pi(z, p_I, I) \equiv E[P(z, p_I, I)] = \Psi(p_I) - L(z, p_I) + \pi(z, I) \quad (1)$$

where

$$\Psi(p_I) \equiv \int_A^B (p_I - c)(y(p_I) + u) f(u) du = (p_I - c)(y(p_I) + E[\varepsilon]) \quad (2)$$

$$L(z, p_I) \equiv \int_A^z c(z - u) f(u) du + \int_z^B (p_I - c)(u - z) f(u) du \equiv c\Lambda(z) + (p_I - c)\Theta(z) \quad (3)$$

and

$$\begin{aligned}
\pi(z, I) &\equiv -cI + \int_{\alpha}^{\beta} \left[ \int_A^z V(I, z, u, v) f(u) du \right] \phi(v) dv + \int_{\alpha}^{\beta} \left[ \int_z^B W(I, z, v) f(u) du \right] \phi(v) dv \\
&= -cI + \int_{\alpha}^{\beta} \left[ \int_A^z V(I, z, u, v) f(u) du + [1 - F(z)] W(I, z, v) \right] \phi(v) dv
\end{aligned} \tag{4}$$

Equation (2) represents the single-period riskless profit function (Mills, 1959). That is, the single-period profit for a given price in a certainty-equivalent problem in which  $\varepsilon$  is replaced by its expected value. Equation (3) often is referred to as the single-period expected loss function (Silver and Peterson, 1985), which assesses an overage cost ( $c$ ) for each unit leftover when  $z$  is chosen too large relative to the actual value of  $\varepsilon$  and an underage cost ( $p_I - c$ ) for each unit shortage when  $z$  is chosen too small ( $\Lambda(z) \equiv \int_A^z (z-u)f(u) du$  and  $\Theta(z) \equiv \int_z^B (u-z)f(u) du$  denote the expected number of leftovers and shortages, respectively). Equation (4) can be interpreted as the expected profit associated with market 2 given that the firm chooses  $z$  and  $I$  at the beginning of the first selling season and then follows an optimal recourse policy after additional information is provided at the beginning of the second selling season. Thus, (1) can be interpreted as the sum of the expected profits associated with market 1 and market 2, where the expected profit associated with market 1 represents the profit that would occur in the absence of uncertainty, less the expected loss due to uncertainty.

Correspondingly, the firm's objective at the beginning of the first selling season can be written compactly as:

$$\begin{aligned}
&\underset{z, p_I, I}{\text{maximize}} \Pi(z, p_I, I)
\end{aligned} \tag{5}$$

Notice from (4) that the profit associated with market 2 does not depend on  $p_I$ . Consequently, we can reduce (5) to a function of the two variables,  $z$  and  $I$ , by expressing  $p_I^*$ , the optimal selling price for the first market, in terms of  $z$  as we would if market 2 did not exist.

*Lemma 1. The optimal selling price for market 1 can be expressed uniquely as a function of  $z$ :*

$$p_1^* \equiv p(z) = \frac{a + bc + E[\varepsilon] - \Theta(z)}{2b}$$

Proof: From (1)-(4),  $\Pi(z, p_1, I)$  is concave in  $p_1$  for given values of  $z$  and  $I$ , thereby implying that  $p_1^*$  satisfies  $\partial \Pi(z, p_1, I) / \partial p_1 = 0$ :

$$\frac{\partial \Pi(z, p_1, I)}{\partial p_1} = \frac{d\Psi(p_1)}{dp_1} - \frac{\partial L(z, p_1)}{\partial p_1} = a - bp_1 + E[\varepsilon] - bp_1 + bc - \Theta(z)$$

and

$$\frac{\partial^2 \Pi(z, p_1, I)}{\partial p_1^2} = -2b < 0$$

$$\therefore p_1^* = \frac{a + bc + E[\varepsilon] - \Theta(z)}{2b}$$

□

Since the expected number of shortages,  $\Theta(z)$ , is non-increasing in  $z$ ,  $p(z)$  is non-decreasing in  $z$ .

Substituting  $p_1^* = p(z)$  into (5) reduces the firm's optimization problem to a maximization over only  $z$  and  $I$ :

$$\underset{z, I}{\text{maximize}} \quad \Pi(z, p(z), I) \tag{6}$$

Even given this reduced form of the firm's objective function, the determination of  $z^*$  and  $I^*$  -- the optimal values of  $z$  and  $I$ , respectively -- is not straightforward because the computation depends on the structure of the firm's optimal recourse policy associated with market 2. That is, the solution to (6) requires an expression for  $V(I, z, u, v)$  and for  $W(I, z, v)$ , each of which corresponds to a new maximization problem over the market 2 decision space, given the market 1 decisions  $z$  and  $I$  as well as the information revealed at the end of the first selling season. Therefore, we next develop  $V(I, z, u, v)$  and  $W(I, z, v)$  by considering that each represents the optimal value function of a distinct sub-problem in which an initial level of inventory is on-hand, leftovers from market 1 may be transferred to market 2, but no additional units may be

procured. The key difference between the two sub-problems is that one includes a deterministic demand function (the sub-problem in which  $V(I, z, u, v)$  denotes the optimal value function) and the other includes an uncertain demand function (the sub-problem in which  $W(I, z, v)$  denotes the optimal value function).

#### 4.0 Computation of $V(I, z, u, v)$ : A Deterministic Sub-Problem With Limited Capacity

If  $\varepsilon < z$  (i.e., if leftovers remain after the first selling season), then no uncertainty surrounds  $\varepsilon$  at the start of the second selling season. In other words, the presence of leftovers after the first selling season implies that the number of sales in market 1 is equivalent to the amount demanded. And, given that the demand for market 1 is known, the firm can deduce the value of  $\varepsilon$ . In particular,  $\varepsilon = \text{market 1 demand} - y(p_1)$ . Equivalently, the firm can determine  $\varepsilon$  from the number of leftovers remaining from the first selling season:  $\varepsilon = \text{market 1 leftovers} - z$ . For the purpose of this section, we assume that the firm observes at the beginning of the second selling season that  $\varepsilon = u$  (i.e., there are  $z - u$  leftovers remaining from market 1).

Since there are  $z - u$  leftovers remaining from market 1, the firm can supplement its initial inventory on-hand (of  $I$  units) for market 2 by transferring  $x \leq z - u$  units from market 1 at a per-unit cost of  $\tau$ . Thus, given that at the beginning of the second selling season the firm sets its selling price at  $p_2$  and observes that  $\eta = v$  (for the foreign exchange rate), it enters market 2 with  $I + x$  units available in stock to satisfy a market demand of  $D(p_2|v) = y(p_2|v) + u = a - bvp_2 + u$  units, where  $u$  is known. Since demand for market 2 is a deterministic function of the selling price, we invert  $D(p_2|v)$ , making  $D_2$  the decision variable and  $p(D_2) = (a + u - D_2)/bv$  the associated (deterministic) demand-dependent price function. Correspondingly,

$$\text{market 2 sales} \equiv S(D_2, x, I) = \min\{D_2, I+x\} \quad (7)$$

The contribution to profit associated with market 2, given that an optimal recourse policy is followed (for this sub-problem, which is defined for  $z > \varepsilon \equiv u$ ) now can be written as:

$$\begin{aligned}
V(I, z, u, v) = & \max_{D_2, x} \{p(D_2)S(D_2, x, I) - \tau x\} \\
& \text{s.t. } 0 \leq x \leq z - u
\end{aligned} \tag{8}$$

In effect, the firm has two sources of stock for market 2. The first source (S1) refers to initial inventory on-hand and the second source (S2) refers to the leftovers remaining from market 1. But, since the capacity of the first source is  $I$  units and the capacity of the second source is  $z - u$  units, the firm can satisfy demand only up to the maximum amount of  $D_2 = I + z - u$  units.

Next, define  $x^*$  and  $D_2^*$  as the values of  $x$  and  $D_2$ , respectively, that satisfy (8), and notice from (7) and (8) that  $D_2^* \leq I$  implies  $x^* = 0$ . Intuitively, this is because the firm should not pay  $\tau$  in order to ship a unit from market 1 to market 2 if there already are enough units on-hand at market 2 to meet all demand. An alternative explanation is that the firm should not plan intentionally for a transferred unit to be a leftover after the second selling season since the per-unit cost of such a leftover is  $\tau$ . Similarly, we conclude that  $D_2^* \leq I + x^*$  because  $p(D_2)$  is decreasing in  $D_2$  and  $S(D_2, x, I)$  is constant in  $D_2$  for  $D_2 \geq I + x$ . In other words, the firm should not plan intentionally for shortages in market 2 since the penalty of such a shortage is forfeited revenue for each of the sales. Further analysis of (8) indicates that the firm's optimal recourse policy for the case when  $z > \varepsilon$  includes four possibilities.

*Theorem 1. Given that at the beginning of the second selling season  $\eta = v$ ,  $\varepsilon = u$ ,  $z - u$  is the number of leftovers remaining after the first selling season, and  $I$  is the initial inventory on-hand, the firm's optimal recourse policy,  $(x^*, p_2^*)$  is:*

$$(x^*, p_2^*) = \begin{cases} (z - u, p(I + z - u)) & \text{if } I < D(\tau) - (z - u) \\ (D(\tau) - I, p(D(\tau))) & \text{if } D(\tau) - (z - u) \leq I \leq D(\tau) \\ (0, p(I)) & \text{if } D(\tau) < I < D(0) \\ (0, p(D(0))) & \text{if } D(0) \leq I \end{cases}$$

where  $D(\kappa) \equiv (a + u - b v \kappa)/2$  and  $p(D) = (a + u - D)/b v$ .

Proof: The marginal revenue associated with selling unit  $D_2$  is:

$$MR(D_2) \equiv \frac{d}{dD_2} [p(D_2)D_2] = \frac{d}{dD_2} \left[ \frac{(a+u-D_2)D_2}{bv} \right] = \frac{a+u-2D_2}{bv}$$

which is a decreasing linear function of  $D_2$ . The marginal cost associated with acquiring unit  $D_2$  depends on the source from which the unit is obtained. If the unit is acquired from S1 (i.e., from the initial inventory on-hand), the marginal cost is 0:  $MC_{S1} = 0$ . If the unit is acquired from S2 (i.e., from the leftovers remaining from market 1), then the marginal cost is  $\tau$ :  $MC_{S2} = \tau$ . Thus, the marginal cost of acquiring unit  $D_2$  is 0 if  $D_2 \leq I$  and  $\tau$  if  $I < D_2 \leq I + z - u$  (recall that  $I + z - u$  is the maximum number of units that the firm can sell).

Now, consider Figure 1:

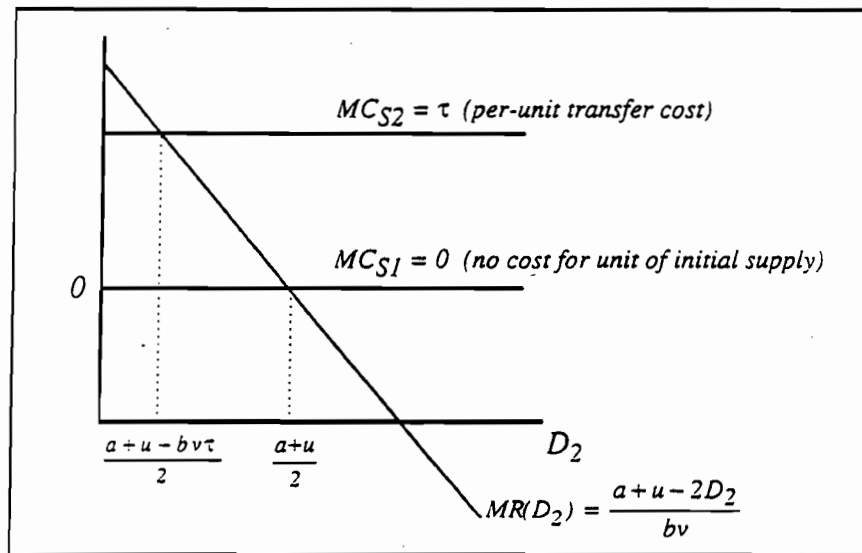


Figure 1. Marginal Revenue vs. Marginal Cost

In general, it behooves the firm to increase  $D_2$  as long as the marginal revenue associated with selling an additional unit is no less than the marginal cost of acquiring the unit. According to Figure 1, then, it is optimal for the firm to continue acquiring additional units from S1 to satisfy demand as long as  $D_2 \leq (a+u)/2$  (because then the marginal cost of acquiring an additional unit from S1 is less than or equal to the marginal revenue associated with selling the unit). Similarly,



it is optimal for the firm to continue acquiring additional units from S2 to satisfy demand as long as  $D_2 \leq (a+u-bv\tau)/2$  (but, only if the firm first exhausts the total supply of S1).

Therefore, given that  $I$  is the capacity of S1, there are three scenarios. If  $I \geq (a+u)/2$ , then  $D_2^* = (a+u)/2$  and all units needed to satisfy  $D_2^*$  are acquired from S1. If  $(a+u-bv\tau)/2 < I < (a+u)/2$ , then  $D_2^* = I$  and all units needed to satisfy  $D_2^*$  again are acquired from S1. (In this case, the firm exhausts S1's capacity, but it does not turn to S2 for additional supply because the marginal cost of acquiring an additional unit from S2 exceeds the marginal revenue associated with selling the unit). However, if  $I \leq (a+u-bv\tau)/2$ , then  $D_2^* = \min\{(a+u-bv\tau)/2, I+(z-u)\}$ , where the first  $I$  units needed to satisfy  $D_2^*$  are acquired from S1 and the remainder ( $x^*$ ) are acquired from S2. (In this case, the firm wants to acquire a total of  $(a+u-bv\tau)/2 - I$  units from S2, but it is restricted by S2's limited supply of  $z - u$  units).  $\square$

We now can characterize  $V(I, z, u, v)$ , which represents the optimal value of the firm's recourse policy given that  $\varepsilon < z$ . From (8), (7), and Theorem 1:

$$V(I, z, u, v) = \begin{cases} [(a+2u-I-z)(I+z-u)]/bv - \tau(z-u) & \text{if } I < D(\tau) - (z-u) \\ [(a-bv\tau+u)^2]/4bv + \tau I & \text{if } D(\tau) - (z-u) \leq I \leq D(\tau) \\ [(a+u-I)I]/bv & \text{if } D(\tau) < I < D(0) \\ [(a+u)^2]/4bv & \text{if } D(0) \leq I \end{cases} \quad (9)$$

We conclude this section with the following lemma in order to characterize the behavior of  $V(I, z, u, v)$  as a function of  $I$  and as a function of  $z$ . Given (9), the proof is a straightforward derivation. Consequently, it is omitted.

*Lemma 2. Given  $u$ ,  $v$ , and  $z$ ,  $V(I, z, u, v)$  is a continuous, non-decreasing, concave function of  $I$ , and it is differentiable everywhere. The same is true for  $V(I, z, u, v)$  as a function of  $z$  for a given  $u$ ,  $v$ , and  $I$ .*

### 5.0 Computation of $W(I, z, v)$ : An Uncertainty Sub-Problem With Fixed Capacity

If  $\varepsilon \geq z$  (i.e., if no leftovers remain after the first selling season), then uncertainty continues to surround the value of  $\varepsilon$  at the start of the second selling season, but the firm uses the information obtained from market 1 to update its characterization of  $\varepsilon$  at that time. In particular, the firm replaces  $A$ , the initial lower bound for the range of possible values of  $\varepsilon$ , with  $z$ ; and revises the probability density function assigned to represent the likelihood that  $\varepsilon = u$  from  $f(u)$  to  $g_z(u) \equiv f(u)/[1-F(z)]$  (see Lemma 3 of Petruzzi, 1995). Consequently, the updated cumulative distribution function is  $G_z(\bullet) \equiv \int_z^\bullet g_z(u) du = [F(\bullet)-F(z)]/[1-F(z)]$ . However, the corresponding hazard rate,  $h_g(\bullet)$ , does not change:

$$h_g(\bullet) = \frac{g_z(\bullet)}{1-G_z(\bullet)} = \frac{f(\bullet)/[1-F(z)]}{[1-F(\bullet)]/[1-F(z)]} = \frac{f(\bullet)}{1-F(\bullet)} = h(\bullet)$$

Since no leftovers remain from market 1, the firm cannot supplement its initial inventory on-hand (of  $I$  units) for market 2 by transferring units from market 1, which implies that  $x^* = 0$ . Thus, given that at the beginning of the second selling season the firm sets its selling price at  $p_2$ , and observes that  $\eta = v$ , it enters market 2 with  $I$  units available in stock to satisfy a market demand of  $D(p_2|v) = y(p_2|v) + \varepsilon = a - bvp_2 + \varepsilon$  units, where  $\varepsilon$  is unknown. Correspondingly,

$$\text{expected market 2 sales} = E[S(p_2, I, z, v)] = \int_z^{I-y(p_2|v)} D(p_2|v) g_z(u) du + \int_{I-y(p_2|v)}^B I g_z(u) du$$

Alternatively, we can define  $E_z[D(p_2|v)]$ ,  $\Lambda_z(I-y(p_2|v))$ , and  $\Theta_z(I-y(p_2|v))$  as the expected demand, expected leftovers, and expected shortages, respectively, associated with market 2 (for this sub-problem, which is defined for  $\varepsilon \geq z$ ); and simplify the expression for  $E[S(p_2, I, z, v)]$ :

$$E[S(p_2, I, z, v)] = E_z[D(p_2|v)] - \Theta_z(I-y(p_2|v)) \quad (10)$$

or

$$E[S(p_2, I, z, v)] = I - \Lambda_z(I-y(p_2|v)) \quad (11)$$

where

$$E_z[D(p_2|v)] \equiv \int_z^B (y(p_2|v) + \varepsilon) g_z(u) du \quad (12)$$

$$\Lambda_z(I - y(p_2|v)) \equiv \int_z^{I - y(p_2|v)} [I - (y(p_2|v) + \varepsilon)] g_z(u) du \quad (13)$$

and

$$\Theta_z(I - y(p_2|v)) \equiv \int_{I - y(p_2|v)}^B [(y(p_2|v) + \varepsilon) - I] g_z(u) du \quad (14)$$

The contribution to profit associated with market 2 given that an optimal recourse policy is followed for this sub-problem now can be written as:

$$W(I, z, v) = \max_{p_2} Q(p_2, I, z, v) \quad (15)$$

where

$$Q(p_2, I, z, v) = p_2 E[S(p_2, I, z, v)] \quad (16)$$

Since  $I$  is determined prior to the first selling season, the stocking quantity for market 2 is fixed much like the number of seats on an airplane or the number of rooms in a hotel is fixed at the time that the pricing decision is made. Consequently, in this case, the only way that the firm can establish a desired safety stock for market 2 is by adjusting the expected demand (via its selling price), which again is analogous to the airline and hotel examples. In effect, the firm's management situation at the beginning of the second selling season (given that demand uncertainty still exists) resembles a yield management problem. Accordingly, our analysis of this case reveals that the optimal decision rule for setting the market 2 selling price parallels the familiar newsboy fractile rule that provides the optimal policy for establishing a safety stock in the inverse situation in which the expected demand for a firm's product is fixed, but the stocking quantity is a decision variable.

*Theorem 2. Given that at the beginning of the second selling season  $\eta = v$ ,  $\varepsilon \geq z$ , there are no leftovers remaining after the first selling season, and  $I$  is the initial inventory on-hand, the firm's optimal recourse policy is  $(x^*, p_2^*) = (0, p(I, z, v))$ , where  $p(I, z, v)$  satisfies:*

$$G_z(I - a + bvp(I, z, v)) = \frac{E[S(p(I, z, v), I, z, v)]/bv}{p(I, z, v)}$$

Proof: Consider the first and second partial derivatives of  $Q(p_2, I, z, v)$  taken with respect to  $p_2$ .

From (16), (11), (13), and the definition of  $y(p_2|v)$ :

$$\frac{\partial Q(p_2, I, z, v)}{\partial p_2} = E[S(p_2, I, z, v)] - bvp_2 G_z(I - a + bvp_2)$$

and

$$\frac{\partial^2 Q(p_2, I, z, v)}{\partial p_2^2} = -2bv G_z(I - a + bvp_2) - (bv)^2 p_2 g_z(I - a + bvp_2) < 0$$

Thus,  $Q(p_2, I, z, v)$  is concave in  $p_2$ , which implies that  $p_2^*$  is determined implicitly for given values of  $I$ ,  $z$ , and  $v$  as the unique value of  $p_2$  that satisfies  $\partial Q(p_2, I, z, v)/\partial p_2 = 0$ . That is, the market 2 optimal selling price for this case is  $p_2^* \equiv p(I, z, v)$ , where:

$$G_z(I - a + bvp(I, z, v)) = \frac{E[S(p(I, z, v), I, z, v)]/bv}{p(I, z, v)} \quad \square$$

We offer the following “newsboy-type” interpretation of  $p(I, z, v)$ . Since  $E_z[D(p_2|v)] = a - bvp_2 + \int_z^B u g_z(u) du$ , an increase of  $1/bv$  in  $p_2$  corresponds to a unit decrease in expected demand. Thus, consider  $1/bv$  as the base unit for incrementing  $p_2$  and interpret a decision to increase  $p_2$  by a single increment instead as a decision to decrease expected demand by one unit. Suppose, then, that the firm chooses to decrease expected demand by one unit and the one less demand results in a leftover. This means that the firm loses a sale (since, if the firm does not decrease demand by one unit, a unit of the firm's product exists in stock to satisfy it). Consequently, the firm loses  $p_2$ , the per-unit revenue associated with that lost sale. However,

since the decrease in expected demand of one unit corresponds to an increase in price of  $1/bv$ , the firm generates an extra revenue of  $1/bv$  for each sale that it does make, which offsets the lost revenue of  $p_2$ . Thus, the net per-unit expected cost associated with the risk of having a unit decrease in expected demand result in a unit leftover is  $(p_2 - E[S(p_2, I, z, v)]/bv)$ . We designate this as the per-unit overage cost.

Now suppose that the firm chooses to increase expected demand by one unit and the one extra demand results in a shortage. Since this is equivalent to the firm decreasing its per-unit selling price by  $1/bv$ , the firm sacrifices revenue in the amount of  $E[S(p_2, I, z, v)]/bv$ . But, since the extra unit of demand ends up exceeding its capacity, the firm does not make an additional sale and consequently, it receives no additional revenue. Thus, the net per-unit expected cost associated with the risk of having a unit increase in demand result in a shortage is  $E[S(p_2, I, z, v)]/bv$ . We designate this as the per-unit underage cost.

Therefore, the decision rule given in Theorem 2 parallels the familiar newsboy fractile rule: it equates the probability of a leftover with the ratio of the firm's per-unit underage cost and the sum of the per-unit underage and overage costs in order to determine the optimal safety stock (i.e.,  $p_2^*$  is determined from  $Pr\{\text{leftover in market 2}\} = \text{underage}/[\text{underage} + \text{overage}]$ ). Note, however, that  $p(I, z, v)$  is more difficult to compute than the solution to a typical newsboy problem because the fractile (i.e., the RHS of the decision rule given in Theorem 2) is not a constant; it varies with the decision variable.

We now can characterize  $W(I, z, v)$ , which represents the optimal value of the firm's recourse policy given that  $\epsilon \geq z$ . From (16), (15), and Theorem 2:

$$W(I, z, v) = p(I, z, v)[I - \Lambda_z(I - a + bvp(I, z, v))] \quad (17)$$

We conclude this section with the following two lemmas. They are useful for characterizing the behavior of  $W(I, z, v)$  as a function of  $I$  and as a function of  $z$ .

*Lemma 3. Given  $v$  and  $z$ ,  $p(I, z, v)$  is in general a unimodal function of  $I$ , first increasing and then decreasing. However, if the condition  $(a+z)h(z) \geq 1$  is satisfied, then  $p(I, z, v)$  is non-increasing for all  $I$ .*

Proof: For the purpose of this proof, define  $p_I = p(I, z, v)$  and  $w_I = I - a + bvp_I$ . Given these definitions, cross multiply the expression in Theorem 2, apply (11), and rearrange terms in order to express  $p_I$  explicitly as a function of  $w_I$ :

$$p_I = \frac{I}{bv} \left[ \frac{I - \Lambda_z(w_I)}{G_z(w_I)} \right] = \frac{I}{bv} \left[ \frac{w_I + a - bvp_I - \Lambda_z(w_I)}{G_z(w_I)} \right]$$

or

$$p_I = \frac{I}{bv} \left[ \frac{a + w_I - \Lambda_z(w_I)}{I + G_z(w_I)} \right] \equiv N(w_I) > 0$$

Thus,

$$\begin{aligned} \frac{\partial p_I}{\partial I} &= \frac{\partial N(w_I)}{\partial w_I} \frac{\partial w_I}{\partial I} = \frac{\partial N(w_I)}{\partial w_I} \left[ I + bv \frac{\partial p_I}{\partial I} \right] \\ \Rightarrow \frac{\partial p_I}{\partial I} &= \frac{\partial N(w_I)/\partial w_I}{1 - bv(\partial N(w_I)/\partial w_I)} \end{aligned} \quad (18)$$

But,

$$\frac{\partial N(w_I)}{\partial w_I} = \frac{I}{bv} \left[ \frac{1 - G_z(w_I)}{I + G_z(w_I)} \right] - \frac{g_z(w_I)}{I + G_z(w_I)} N(w_I) = \frac{I - G_z(w_I)}{bv(I + G_z(w_I))} [1 - bvN(w_I)h(w_I)] \leq \frac{I}{bv}$$

Since  $\partial N(w_I)/\partial w_I \leq I/bv$ , the denominator in the expression for  $\partial p_I/\partial I$  is non-negative; therefore, the sign of  $\partial p_I/\partial I$  depends only on the sign of the numerator,  $\partial N(w_I)/\partial w_I$ . However, notice that, except possibly at the boundary point,  $w_I = B$ , the sign of  $\partial N(w_I)/\partial w_I$  is determined solely by the sign of  $\tilde{N}(w_I) \equiv 1 - bvN(w_I)h(w_I)$ .

We verify next that  $\tilde{N}(w_I)$  has at most one sign change, from positive to negative. This means that in general,  $\partial N(w_I)/\partial w_I$  is first positive and then negative; consequently, the same is true for  $\partial p_I/\partial I$ . That is, given that  $\tilde{N}(w_I)$  has at most one sign change, from positive to negative,  $p_I$  is unimodal in  $I$ , first increasing and then decreasing.

$$\frac{\partial \tilde{N}(w_I)}{\partial w_I} = -bvN(w_I)\frac{\partial h(w_I)}{\partial w_I} - bvh(w_I)\frac{\partial N(w_I)}{\partial w_I} \leq -\frac{g_z(w_I)}{1+G_z(w_I)}\tilde{N}(w_I)$$

where the inequality follows because  $\partial h(w_I)/\partial w_I \geq 0$  (by the non-decreasing hazard rate assumption) and  $\partial N(w_I)/\partial w_I = [(1-G_z(w_I))/(bv(1+G_z(w_I)))]\tilde{N}(w_I)$ . Consequently,

$$\tilde{N}(w_I) \geq 0 \quad \Rightarrow \quad \frac{\partial \tilde{N}(w_I)}{\partial w_I} \leq 0$$

In other words, once (or if)  $\tilde{N}(w_I)$  becomes negative, it cannot become positive again.

Therefore,  $\tilde{N}(w_I)$  changes sign at most one time -- from positive to negative -- which establishes the claim that  $p_I$  is unimodal in  $I$ .

Given the shape of  $\tilde{N}(w_I)$ , a sufficient condition for  $p_I$  to be monotone non-increasing for all  $I$ , is for  $\tilde{N}(w_I) \leq 0$  for all  $w_I$ . And since  $\tilde{N}(w_I)$  cannot change sign from negative to positive, if  $\tilde{N}(w_I)$  is less than or equal to zero when evaluated at  $w_I = z$ , then it is non-positive for all  $w_I$  of interest (because  $z$  is  $w$ 's lower bound). That is,  $p_I$  is non-increasing in  $I$  if  $\tilde{N}(z) \leq 0$ , which is guaranteed if  $I \leq bvN(z)h(z)$ . But,  $bvN(z)h(z) = (a+z)h(z)$ . Therefore,

$$(a+z)h(z) \geq I \quad \Rightarrow \quad bvN(z)h(z) \geq I \quad \Rightarrow \quad \frac{\partial p_I}{\partial I} \leq 0 \quad \square$$

*Corollary.*  $\partial p(I, z, v)/\partial I \geq -1/bv$

Proof: From (18):

$$1 + bv\frac{\partial p_I}{\partial I} = \frac{\partial p_I/\partial I}{\partial N(w_I)/\partial w_I}$$

where  $p_I \equiv p(I, z, v)$ . But, recall from the proof of Lemma 3 that  $\partial p_I/\partial I \geq 0 \Leftrightarrow \partial N(w_I)/\partial w_I \geq 0$  (i.e.,  $\partial p_I/\partial I$  and  $\partial N(w_I)/\partial w_I$  share the same sign for all  $I$ ). Consequently,  $1 + bv(\partial p_I/\partial I) \geq 0$ .  $\square$

We find the first part of Lemma 3 somewhat curious. Intuitively, we expect  $p(I, z, v)$  and  $I$  to be correlated negatively. This is because  $I$  represents the supply of the firm's product and

$p(I, z, v)$  represents the firm's control over demand for its product. Thus, it seems reasonable to believe that the firm should respond to a decrease in  $I$  (i.e., a decrease in supply) by correspondingly triggering a decrease in demand, which is accomplished by a price increase. Given this reasoning, the second part of Lemma 3, which establishes a condition ensuring that  $p(I, z, v)$  is non-increasing in  $I$ , seems more natural. We interpret the sufficient condition (i.e.,  $(a+z)h(z) \geq I$ ) as a requirement on the minimum size of the firm's market; and we argue by example in the last section of this paper that this requirement is not restrictive. Therefore, we posit that only under rare circumstances, characterized by cases in which the average size of the firm's market is small relative to the variation in size due to uncertainty, does the complication arise in which  $p(I, z, v)$  and  $I$  are correlated positively for some range of  $I$ .

*Lemma 4. The following two relationships are true:*

$$(a) \quad \frac{\partial W(I, z, v)}{\partial I} = p(I, z, v) [1 - G_z(I - a + bvp(I, z, v))] \geq 0$$

$$(b) \quad \frac{\partial W(I, z, v)}{\partial z} = h(z) \{W(I, z, v) - p(I, z, v)[a - bvp(I, z, v) + z]\} \geq 0$$

*Further, if for a given  $z$ ,  $(a+z)h(z) \geq I$ , then the following also is true:*

$$(c) \quad \frac{\partial^2 W(I, z, v)}{\partial I^2} \leq 0$$

*Proof.* To establish part (a) of the lemma, apply (17) and Theorem 2:

$$\begin{aligned} \frac{\partial W(I, z, v)}{\partial I} &= \frac{\partial p(I, z, v)}{\partial I} [I - \Lambda_z(I - a + bvp(I, z, v))] + p(I, z, v) \left[ 1 - G_z(I - a + bvp(I, z, v)) \left( 1 + bv \frac{\partial p(I, z, v)}{\partial I} \right) \right] \\ &= p(I, z, v) \left\{ 1 - G_z(I - a + bvp(I, z, v)) + bv \left[ \frac{E[S(p(I, z, v), I, z, v)]/bv}{p(I, z, v)} - G_z(I - a + bvp(I, z, v)) \right] \frac{\partial p(I, z, v)}{\partial I} \right\} \\ &= p(I, z, v) [1 - G_z(I - a + bvp(I, z, v))] \geq 0 \end{aligned}$$



To establish part (b), first notice from the definitions of  $g_z(u)$  and  $\Lambda_z(I-y(p_2|v))$  that the following identities hold:

$$\frac{\partial g_z(u)}{\partial z} = \frac{f(u)\dot{f}(z)}{[1-F(z)]^2} = h(z)g_z(u)$$

and

$$\begin{aligned} \frac{\partial \Lambda_z(I-y(p_2|v))}{\partial z} &= \int_z^{I-y(p_2|v)} [I-(y(p_2|v)+u)] \frac{\partial g_z(u)}{\partial u} du - (I-y(p_2|v)-z)g_z(z) \\ &= h(z)[\Lambda_z(I-y(p_2|v)) - (I-y(p_2|v)-z)] \end{aligned}$$

Then, given these identities, apply (17) and Theorem 2:

$$\begin{aligned} \frac{\partial W(I, z, v)}{\partial z} &= \frac{\partial p(I, zv)}{\partial z} [I - \Lambda_z(I-a+bvp(I, zv))] \\ &+ p(I, zv) \left\{ -h(z)[\Lambda_z(I-y(p_2|v)) - (I-y(p_2|v)-z)] - G_z(I-a+bvp(I, zv))bv \frac{\partial p(I, zv)}{\partial z} \right\} \\ &= h(z) \{ W(I, z, v) - p(I, zv)[a - bvp(I, zv) + z] \} \\ &= bvp(I, zv) \left[ \frac{E[S(p(I, zv), I, z, v)]/bv}{p(I, zv)} - G_z(I-a+bvp(I, zv)) \right] \frac{\partial p(I, zv)}{\partial z} \\ &= h(z) \{ W(I, z, v) - p(I, zv)[a - bvp(I, zv) + z] \} \geq 0 \end{aligned}$$

where the inequality follows because from (17) and (13).

To establish part (c), notice from part (a) that:

$$\begin{aligned} \frac{\partial^2 W(I, z, v)}{\partial I^2} &= \frac{\partial p(I, zv)}{\partial I} [I - G_z(I-a+bvp(I, zv))] - p(I, zv)g_z(I-a+bvp(I, zv)) \left( 1 + bv \frac{\partial p(I, zv)}{\partial I} \right) \\ &\leq \frac{\partial p(I, zv)}{\partial I} [I - G_z(I-a+bvp(I, zv))] \end{aligned}$$

since, from the corollary to Lemma 3,  $\partial p(I, zv)/\partial I \geq -1/bv$ . But, from Lemma 3,  $\partial p(I, zv)/\partial I \leq 0$  if  $(a+z)h(z) \geq 1$  for a given  $z$ . Therefore, in such a case,  $\partial^2 W(I, z, v)/\partial I^2 \leq 0$ .  $\square$

## 6.0 Analysis of the Firm's Objective Function

Given the characterizations of  $V(I, z, u, v)$  and  $W(I, z, v)$  developed in the previous two sections, we now return to the firm's objective function. Recall from (16) that the firm desires to maximize jointly over  $z$  and  $I$ , the function  $\Pi(z, p(z), I)$ , where  $p(z)$  is the value of  $p_I$  that maximizes  $\Pi(z, p_I, I)$  for given values of  $z$  and  $I$ . We proceed by taking the first partial derivatives of  $\Pi(z, p(z), I)$  with respect to  $I$  and with respect to  $z$ . From (1)-(4):

$$\begin{aligned} \frac{\partial \Pi(z, p(z), I)}{\partial I} &= \frac{\partial \pi(z, I)}{\partial I} \\ &= -c + \int_{\alpha}^{\beta} \left[ \int_A^z \frac{\partial V(I, z, u, v)}{\partial I} f(u) du + [1 - F(z)] \frac{\partial W(I, z, v)}{\partial I} \right] \phi(v) dv \end{aligned} \quad (19)$$

and

$$\begin{aligned} \frac{\partial \Pi(z, p(z), I)}{\partial z} &= - \frac{\partial L(z, p_I)}{\partial z} \Big|_{p_I = p(z)} + \left( \frac{\partial [\Psi(p_I) - L(z, p_I)]}{\partial p_I} \Big|_{p_I = p(z)} \frac{\partial p(z)}{\partial z} \right) + \frac{\partial \pi(z, I)}{\partial z} \\ &= - \frac{\partial L(z, p_I)}{\partial z} \Big|_{p_I = p(z)} + [(a + bc + E[\varepsilon] - \cancel{\Theta(z)} - 2bp(z))] \frac{\partial p(z)}{\partial z} \\ &\quad + \int_{\alpha}^{\beta} \left[ \int_A^z \frac{\partial V(I, z, u, v)}{\partial z} f(u) du + V(I, z, z, v) f(z) + [1 - F(z)] \frac{\partial W(I, z, v)}{\partial z} - W(I, z, v) f(z) \right] \phi(v) dv \end{aligned}$$

Or, applying Lemma 4:

$$\frac{\partial \Pi(z, p(z), I)}{\partial z} = - \frac{\partial L(z, p_I)}{\partial z} \Big|_{p_I = p(z)} + \int_{\alpha}^{\beta} K(I, z, v) \phi(v) dv \quad (20)$$

where,

$$K(I, z, v) \equiv \int_A^z \frac{\partial V(I, z, u, v)}{\partial z} f(u) du + [V(I, z, z, v) - p(I, z, v)[a - bvp(I, z, v) + z]] f(z) \quad (21)$$

We point out that we do not take the second partials because we are unable to establish any property, such as joint concavity, that guarantees the existence of an efficient method for

identifying  $I^*$  and  $z^*$ . Thus, in the worst case, an exhaustive search over both decision variables,  $z$  and  $I$ , is required to ensure the optimality of a particular policy. However, we note that it is possible to determine  $I^*$  analytically as a function of  $z$ , thereby reducing significantly the complexity of the search procedure, if we impose more restrictive conditions on the model parameters. We offer two such possibilities as examples, the first of which we mention only briefly, but the second of which we pursue thoroughly because it is central to the purpose of this paper.

As the first example, suppose that foreign exchange risk did not exist; that is, assume that the value of  $v$  is static, and it is known at the beginning of the first selling season (for example, because the firm locks into an exchange rate at the beginning of the first selling season or because both markets are in the same country). In this case, the integral over  $\phi(v)$  in (19) reduces to its integrand, evaluated at the single value of  $v$  that represents the deterministic foreign exchange rate. As a result, using Lemmas 3 and 4, we could verify that in general, given  $z$ ,  $\partial \Pi(z, p(z), I) / \partial I = 0$  is satisfied for at most two values of  $I$ , the larger of which corresponds to  $I^*$ . Thus, in the absence of foreign exchange risk, the firm's objective function reduces to a maximization problem over the single variable  $z$ .

Unfortunately, we are unable to verify a similar result in general (i.e., without further assumptions) when uncertainty accompanies the foreign exchange rate, regardless of the form of  $\phi(v)$ . Therefore, we proceed instead as follows: first we establish a lower bound for  $z^*$  (which applies even if  $v$  is fixed), and then we use that lower bound to provide a sufficient condition that, if satisfied, ensures concavity of  $\Pi(z, p(z), I)$  in  $I$  for a given  $z$  in the region of interest. Thus, if the sufficient condition is satisfied -- and we argue in the next section that the condition is a mild one -- then the problem of jointly maximizing  $\Pi(z, p(z), I)$  over two variables ( $z$  and  $I$ ), when foreign exchange risk exists, requires only a search over a truncated region of one of the variables ( $z$ ).

*Theorem 3.  $z^* \geq z_s$ , where  $z_s$  denotes the optimal value of  $z$  if  $z$  were chosen myopically at the beginning of the first selling season, without regard to its effect on the profit associated with market 2. In other words,  $z_s = \operatorname{argmax}\{\Psi(p(z)) - L(z, p(z))\}$ , which is equivalent to the solution of a corresponding single period problem.*

Proof: The cornerstone of the proof is that  $K(I, z, v) \geq 0$ ; consequently, we begin by establishing that inequality. Notice from (21) that, given Lemma 2:

$$K(I, z, v) \geq [V(I, z, z, v) - p(I, z, v)[a - bvp_2 + z]]f(z)$$

Now consider the meaning of  $V(I, z, z, v)$ : it represents the optimal value of the firm's recourse policy for the degenerate case in which the demand for the second selling season is deterministic as a function of the selling price ( $D = a + bvp_2 + z$ ) and an initial stock of inventory is on-hand ( $I$ ), but no leftovers remain from market 1. In this case, since no units exist to transfer from market 1 to market 2 (i.e., since  $x^* = 0$ ),  $V(I, z, z, v)$  can be expressed in reduced form, regardless of the value of  $I$ . In particular, from (7), (8), and the earlier conclusion that  $D_2^* \leq I + x^*$ :  $V(I, z, z, v) = p(D_2^*)D_2^* = \max_{D_2}\{p(D_2)D_2\}$ , where  $p(D_2) = (a + z - D_2)/bv$ . Alternatively, we can invert  $p(D_2)$  and write:  $V(I, z, z, v) = \max_{p_2}\{p_2(a - bvp_2 + z)\}$ . In other words, for a given value of  $I$ ,  $V(I, z, z, v)$  maximizes the function  $p_2[a - bvp_2 + z]$  and therefore,  $V(I, z, z, v) \geq p_2[a - bvp_2 + z]$  for all values of  $p_2$ , including  $p_2 = p(I, z, v)$ . This implies that  $K(I, z, v) \geq 0$ .

Given that  $K(I, z, v) \geq 0$ , from (20):

$$\frac{\partial \Pi(z, p(z), I)}{\partial z} \geq - \left. \frac{\partial L(z, p_1)}{\partial z} \right|_{p_1 = p(z)} = \frac{d[\Psi(p(z)) - L(z, p(z))]}{dz}$$

This implies that  $z^*$ , the value of  $z$  that maximizes  $\Pi(z, p(z), z)$  is no less than  $z_s$ , the value of  $z$  that maximizes  $\Psi(p(z)) - L(z, p(z))$  (which follows because the slope of  $\Pi(z, p(z), z)$  is greater than or equal to the slope of  $\Psi(p(z)) - L(z, p(z))$  at every value of  $z$ ; see Topkis (1978)).  $\square$

We attribute the result of Theorem 3 to the strategic role that inventory plays in market  $I$ . By increasing its safety stock in the first selling season above the level it would set if future considerations were not included (where  $z$  serves as a substitute for safety stock), the firm increases the likelihood that it will observe the value of  $\varepsilon$  and consequently, operate in the second selling season with perfect information regarding the demand function. Thus, there is an economic incentive in terms of increased future profit to choose a higher safety stock in the first selling season. And, although there is a cost associated with increasing  $z$  because the increase means higher expected leftovers in market  $I$ , the cost is mitigated because the leftovers provide a value in terms of greater flexibility since the firm has the recourse option of transferring some or all of the leftovers to the second market for sale there.

Given Theorem 3 together with Lemmas 2 and 4, we next establish a condition that ensures that  $\Pi(z, p(z), z)$  is concave in  $I$  over the region of  $z$  for which we are interested. Interpretations of the condition are discussed in the next section.

*Theorem 4. If  $(a + z_s)h(z_s) \geq 1$ , then  $\Pi(z, p(z), z)$  is concave in  $I$  for any value of  $z \geq z_s$ .*

*Proof:* First assume that  $(a + z_s)h(z_s) \geq 1$  and notice that the function  $(a + z)h(z)$  is non-decreasing in  $z$  since  $h(\cdot)$ , the hazard rate associated with  $\varepsilon$ , is non-decreasing. This implies that  $(a + z)h(z) \geq 1$  for all  $z \geq z_s$ .

Next, consider the second partial derivative of  $\Pi(z, p(z), z)$  taken with respect  $I$ . From (19):

$$\frac{\partial^2 \Pi(z, p(z), I)}{\partial I^2} = \int_{\alpha}^{\beta} \left[ \int_A^z \frac{\partial^2 V(I, z, u, v)}{\partial I^2} f(u) du + [1 - F(z)] \frac{\partial^2 W(I, z, v)}{\partial I^2} \right] \phi(v) dv$$

But,  $\partial^2 V(I, z, z, v)/\partial I^2 \leq 0$  for all  $z$  (by Lemma 2) and  $\partial^2 W(I, z, v)/\partial I^2 \leq 0$  for  $z \geq z_s$  (by Lemma 4 since  $(a + z)h(z) \geq 1$  for  $z \geq z_s$ ). Therefore,  $\partial^2 \Pi(z, p(z), I)/\partial I^2 \leq 0$  for  $z \geq z_s$ , which implies that  $\Pi(z, p(z), I)$  is concave in  $I$  over that region of  $z$ .  $\square$

## 7.0 Discussion

This paper focuses on a specific management situation in which a monopolist having only a single opportunity to procure operates in two distinct markets that are separated by international borders and have non-overlapping selling seasons. At the beginning of the first selling season, the firm makes three decisions: the quantity to procure for market 1, the quantity to procure for market 2, and the selling price to set for market 1. Then, at the beginning of the second selling season, the firm implements two recourse decisions: the number of leftovers remaining from market 1 to transfer to market 2 and the selling price to set for market 2.

A key result of the analysis is that this problem can be reduced to one involving only two principal decision variables:  $I$ , the procurement quantity for market 2; and  $z$ , a surrogate representation of the safety stock for market 1. Given  $I$  and  $z$ , the firm's optimal course of action then can be established. In particular, the optimal selling price and procurement quantity for market 1 are determined myopically as a function of  $z$ . Similarly, the firm's optimal recourse policy for market 2 is characterized completely in terms of  $I$  and  $z$  (the characterization, however, depends on two observations made at the end of the first selling season: market 1 sales and the value of the foreign exchange rate). The problem reduces further, to one that requires a maximization over only the single variable,  $z$ , if the condition  $(a + z_s)h(z_s) \geq I$  is satisfied, where  $z_s$  represents the solution to a corresponding single-period problem. We now provide evidence as justification for the earlier claims that this sufficiency condition is rather mild.

Given  $F(\bullet)$ , the subjective probability distribution originally assigned to characterize the uncertainty term,  $\epsilon$ , define  $z_0$  such that  $F(z_0) = 1 - F(z_0) = 1/2$  (i.e.,  $z_0$  is the median). Notice that although it is possible for  $z_s < z_0$ , we argue that this is an unlikely event because such an inequality would mean that in a single period problem, it is optimal for the probability of a stockout to exceed 50%. Consequently, we assume that  $z_s \geq z_0$ , which implies that  $(a + z_s)h(z_s) \geq (a + z_0)h(z_0)$  (recall that  $(a + z)h(z)$  is a non-decreasing function of  $z$ ). This means that if  $(a + z_0)h(z_0) \geq I$ , then  $(a + z_s)h(z_s) \geq I$ . In other words, as long as  $a \geq I/h(z_0) - z_0$ , we can be

confident that the sufficiency condition is satisfied. Basically, we choose  $z_0$  as a conservative estimate of  $z_s$ ; however, we do so only for illustrative purposes since, without specifying particular values for  $a$ ,  $b$ , and  $c$  (in which case, we could compute an actual value for  $z_s$ ) it is more convenient to work with  $z_0$ .

In order to interpret further what it means for  $a \geq 1/h(z_0) - z_0$ , we consider three specifications of  $F(\bullet)$ : the uniform distribution, the exponential distribution, and the normal distribution (technically, the normal distribution is not a valid alternative because it allows for  $\epsilon < -a$ , which is not meaningful; but as long as the probability that  $\epsilon < -a$  approaches zero, the normal distribution can serve as an excellent approximation). Table 1 provides the necessary data for each of these three cases (in the table, we let  $\sigma$  denote the standard deviation).

Table 1. Computation of  $1/h(z_0) - z_0$  for Common Forms of  $F(\bullet)$

<i>Measure</i>	<i>Uniform</i>	<i>Exponential</i>	<i>Normal</i>
$f(z)$	$1/(B-A)$	$\lambda e^{-\lambda z}$	$(1/(2\pi\sigma^2)^{1/2})e^{-(x-\mu)^2/(2\sigma^2)}$
$E[\epsilon]$	$(B+A)/2$	$1/\lambda$	$\mu$
$\sigma$	$(B-A)/(2 \cdot 3^{1/2})$	$1/\lambda$	$\sigma$
$z_0$	$E[\epsilon]$	$(\ln 2)E[\epsilon]$	$E[\epsilon]$
$h(z_0)$	$(1/3^{1/2})/\sigma$	$1/\sigma$	$(2/\pi)^{1/2}/\sigma$
$1/h(z_0) - z_0$	$(3^{1/2})\sigma - E[\epsilon]$	$(2 - \ln 2)\sigma - E[\epsilon]$	$(\pi/2)^{1/2}\sigma - E[\epsilon]$

For the three distribution examples provided, Table 1 indicates that as long as  $a + E[\epsilon] \geq k\sigma$ , where  $k$  is roughly equal to 1.5 ( $k = 3^{1/2}$  for the uniform distribution, etc.), then the sufficiency condition is satisfied (assuming that  $z_s \geq z_0$ ). Expressed differently, the sufficiency test for each of the three examples requires that the coefficient of variation of the size of the market ( $\sigma/(a+E[\epsilon])$ ) be not greater than roughly 2/3 ( $1/k$ ). And since it seems unlikely in any practical situation that the initial uncertainty associated with the firm's market, as measured by  $\sigma$ , would be more than half or two-thirds the expected size of the market, we conclude that the

sufficiency condition identified to reduce the two market problem to a maximization over a single decision variable is rather unrestrictive.

Finally, we point out that the results of this paper are robust with respect to the characterization of the foreign exchange risk -- no assumptions are required on the specific form of  $\Phi(\bullet)$ . We attribute this phenomenon to the manner in which foreign exchange is incorporated into the model: by introducing the exchange rate random variable as a multiplicative operator on the elasticity parameter of the firm's price-dependent demand function, we effectively absorb the monetary uncertainty into demand uncertainty.

## 8.0 Implications to Yield Management

Although the intent of this paper is to model explicitly a two-market/two-selling-season management situation, consider the following single-period extension. A firm operating in a single market has only one chance to procure and, because of a long procurement lead-time, does so based on some preliminary characterization ( $F(\bullet)$ ) of the uncertainty surrounding the scale parameter of its price-dependent demand function (assume, as before, that  $D(p) = a - bp + \varepsilon$ ). However, the firm is able to revise by some method its characterization of  $\varepsilon$  just prior to the start of the actual selling season, at which time it sets its selling price (for example, suppose the firm establishes a better forecast due to updated economic indicators, or due to interest stimulated by preliminary advertising). Consequently, when the firm sets its selling price, it does so given a pre-specified capacity level -- the procurement amount -- and given an updated characterization ( $G(\bullet)$ ) of the uncertainty surrounding the scale parameter of the demand function.

This scenario represents a simplified version of the model developed in this paper. In particular,  $z$  and  $p_I$  are not decision variables since a first selling season does not exist (in this context, by first selling season, we mean a selling season that occurs before the firm revises its characterization of the uncertainty included in the demand function). Correspondingly,  $\Psi(p_I) - L(z, p_I) = 0$ . Consequently, the expected profit function is given by  $\pi(I)$ , where  $\pi(I)$  does not



include the integral over  $\phi(v)$ . In addition, the firm's optimal recourse path is represented completely by the solution to the sub-problem for which the function  $W(\cdot)$  depicts the optimal solution value. In other words, the possibility that the firm sets its selling price in a deterministic environment does not exist (in which case the function  $V(\cdot)$  also would apply).

In summary, the results of this paper ensure that the problem described here -- which might apply to certain yield management problems in which the firm chooses a stocking quantity (i.e., capacity level) at some point in time prior to establishing a demand level (i.e., selling price) -- can be reduced to a maximization of a single variable function. The solution to the resulting maximization problem then requires either an exhaustive search (in the worst case) or the application of an analytical formula (depending on the model parameters). One example of a firm for which this management situation might apply is a grower of seasonal or holiday plants and flowers since, due to the plant-development-time, the grower must decide the quantity of plants to produce well in advance of establishing the selling price.

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