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Multilevel Approach To Urban Water Resources Systems Analysis--Application To Medium Size Communities, Planning Groundwater Supply Systems For Urban Growth: Application To West Lafayette, Indiana

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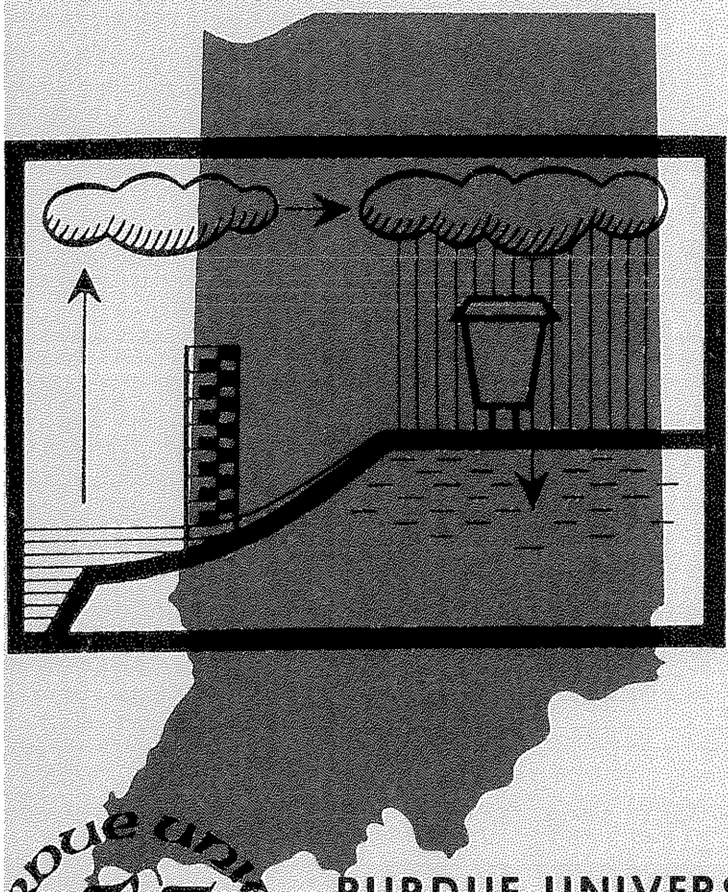

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July 1980

PURDUE UNIVERSITY

WATER RESOURCES RESEARCH CENTER

WEST LAFAYETTE, INDIANA

Water Resources Research Center
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West Lafayette, Indiana 47907

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PREFACE

An urban area may be seen, using the systems approach, as composed of a number of components, where the components themselves depend on the point-of-view of the observer. For example, the urban area may be viewed topologically as being comprised of a set of adjoining watersheds. The same area may be also seen from a political perspective as being divided into overlapping jurisdictions of local, state, and national agencies, planning commissions, and governments. Finally this area may be divided on a water resources basis by function, including water supply, sanitary sewer and storm drainage components.

A multilevel framework has been developed in the previous reports of this project (TR #101,#102) from the topological perspective for storm-drainage planning. The methodology integrates the aspects of water pollution, local flooding and system costs. The algorithm focuses on the optimal allocation of drainage network, storage and treatment facilities among the subbasins (first level) of each watershed (second level) for several growth alternatives of the community. By this means, a unique cost of storm drainage can be associated with each growth scenario. The procedure requires information on projected population growth, water storage and quality limitations in each subbasin and various surface characteristics of each subbasin. The optimization is over the cost of the facilities as well as the cost of local flooding. Constraints to the optimization include the non-analytical relationships introduced by the hydrological simulation model, STORM. The optimization output shows the minimum cost allocation of drainage network capacity and storage and the minimum size of the associated treatment plant. The procedure has been applied to West Lafayette, Indiana, and was shown to converge rapidly to a feasible solution.

The multilevel multiobjective optimization procedure, described in a previous report (TR #121), has been developed to the extent that a water resources manager can interact with a computer to obtain the optimal solution to a complex planning problem expressed in terms of multiple objectives. The manager's role is to provide trade-off information among various objectives at each iteration.

The political perspective can be incorporated into these planning models to the extent that the model parameters and structure depend on geography and function. For example, local regulations may differ regarding the maximum size of runoff retention areas thus altering the value of that parameter for each watershed of a storm drainage model.

The present report involves an extension to the functional perspective in the form of planning procedures for the water supply system.

This is the completion report of Project OWRT-B-083-IND, entitled "Multilevel Approach to Urban Water Resources Systems Analysis - Application to Medium Size Communities."

Previous reports and publications on this project are the following:

PWRRC Tech. Rept. 100, "Urban Growth in Water Resources Planning," by S. A. Dendrou, J. W. Delleur and J. J. Talavage, April 1978, 116 pp.

PWRRC Tech. Rept. 101, "Urban Storm-Drainage System Planning," by S. A. Dendrou, J. J. Talavage and J. W. Delleur, May 1978, 155 pp.

PWRRC Tech. Rept. 121, "Interactive Multiple Objective Optimization," by K. Musselman and J. J. Talavage, February 1979, 196 pp.

"Multilevel Approach to Urban Water Resources Planning," by J. J. Talavage, S. A. Dendrou and J.W. Delleur, Abstract in EOS, Transaction of Amer. Geophysical Union, Vol. 59, No. 12, p. 1071.

"A Tradeoff Cut Approach to Multiple Objective Optimization," by K. Musselman and J. J. Talavage, Journal of ORSA, (to appear).

"Reliability Concepts in Planning Storm-Drainage Systems," by S. A. Dendrou and J. W. Delleur, International Symposium on Risk and Reliability in Water Resources, Univ. of Waterloo, Ontario, June 1978, Vol. 1, pp. 390-410.

"Systematic Planning of Urban Storm Drainage Utilities," by S. A. Dendrou, J. W. Delleur and J. J. Talavage, International Symposium on Urban Storm Water Management, Univ. of Kentucky, Proceedings, July 1978, pp. 229-234.

"Planning Storm-Drainage Systems for Urban Growth," by S. A. Dendrou, J. W. Delleur and J. J. Talavage, Jour. of the Water Resources Planning and Management Division, ASCE, Vol. 104, No. WR1, Nov. 1978, pp. 1-16.

"Optimal Planning for Urban Storm-Drainage Systems," by S. A. Dendrou, J. J. Talavage and J. W. Delleur, Jour. of Water Resources Planning and Management Division, ASCE, Vol. 104, No. WR1, Nov. 1978, pp. 17-33.

"The Design Storm Concept: Is It a Sufficient Criterion to Determine the Reliability of Modern Storm Drainage Systems?," by J. W. Delleur in "The Design Storm Concept," Proceedings of a seminar at Ecole Polytechnique de Montreal, Rept. EP80-R-GREMU 79102, Dec. 1979, pp. 16-24.

"Planning Groundwater Supply Systems for Urban Growth: A Multilevel Approach," by J. W. Delleur, S. A. Dendrou and G. V. Loganathan, IFAC Symposium - Water and Related Land Resources Systems, Case Western Univ., Preprints, Pergamon Press, pp. 239-250, 1980.

"An Interactive Tradeoff Cutting Plane Approach to Continuous and Discrete Multiple Objective Optimization," by K. Musselman, Ph.D. Thesis, Nov. 1978, 150 pp.

"Multilevel Approach to Urban Storm Water Systems Planning," by S. A. Dendrou, Ph.D. Thesis, Dec. 1977, 283 pp.

ABSTRACT

In water supply systems, predicting the spatial disaggregation of the demand is one of the major concerns. In this report the landuse allocation model developed in Tech. Rept. No. 100, has been used for this purpose. The future zonewise development of different landuse activities is converted into an equivalent water demand based on the requirements of the activities projected.

Other important facets are the determination of optimal locations of water wells and of distribution reservoirs along with the optimal flow values and pipe sizes. The annualized cost of the well field, distribution reservoirs, pipes and pumping is minimized. The indivisibility requirement of the number of wells and reservoirs and the continuous variation of flow values require the use of a Mixed Integer Programming (MIP) approach for the optimization. The nonlinear head losses result in nonlinear objective function and constraints. The nonlinearity is circumvented by making use of empirical formulas and well design criteria. The new wells should be located so that the additional drawdowns do not adversely affect the existing system. A two level coordination scheme is used to optimally distribute the facilities and to guarantee a safe exploitation of the aquifer in the future. This task is performed by the program WATSUP, which includes a finite element formulation of the groundwater flow. The methodology is applied to an actual situation in West Lafayette, Indiana.

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CHAPTER 1

INTRODUCTION

1.1 Introduction

Water supply has traditionally figured among the more important factors governing the establishment and growth of human settlements. Today, municipal water supply is the second largest single industry in the United States, surpassed only by the electric power industry.

The urban population in the United States is growing very rapidly. The largest growth occurs in the suburbs and is the result of population growth and of migration attributed to social factors - escape from urban centers of big cities, and is also the result of regional economic and industrial development and other factors. In this context, the urban planning authority may be required to make decisions regarding changes due to urban growth which in turn affect further growth.

It has been a long and well established fact that where there exist economies of scale and expectations of future growth, it is optimal to plan and build ahead of demand (Russel, Arey, Kates, 1970). The scarcity of natural resources warrants the need to optimally plan for their allocation. In the case of water supply, a new dimension was added to the above argument by the enactment of the Safe Drinking Water Act (PL 93-523) by the United States Congress in December 1974, (U.S. Congress). The requirements of PL 93-523 are estimated to apply approximately to 40,000 community water systems. According to Russel (1978), compliance with this act has been estimated to cost for an average family of four, depending on the size of the community, $78.00 per year per household ($/yr/hs) for a 1 million gallons per day (mgd) system, 26 $/yr/hs for a 10 mgd system, and 14 $/yr/hs for a 100 mgd system.

Planning for municipal water supply systems under such circumstances exceeds the narrow approach of developing a master plan for orderly system development in response to the changing needs of the local community. Rather, planning efficiently for future public utilities will have to link explicitly the study of the urban growth process and the demand to the availability and the distribution of the corresponding resource. In the United States many medium size communities depend wholly or in part on groundwater for their water supplies. Analysis of groundwater flow has been recognized as one of the main aspects of regional planning. This report concentrates on safe exploitation of aquifer in view of putting in a new well field.

1.2 Nature of the Problem

From the viewpoint of spatial and temporal variability of demand it is customary to partition urban agglomeration into service zones with equalizing reservoirs and booster pumps, Figure 1.1. Water is stored in equalizing reservoirs to take care of the fluctuations in demand and to furnish water for emergencies such as fire fighting or accidental breakdowns. Booster pumps may be used in order to meet the local pressure drops.

There are two subsets in a water supply system, namely the distribution subset and the supply subset. Two decision problems are thus seen to emerge: (1) at the local level the determination of the pipe distribution network constrained by minimum pressure and discharge criteria; and

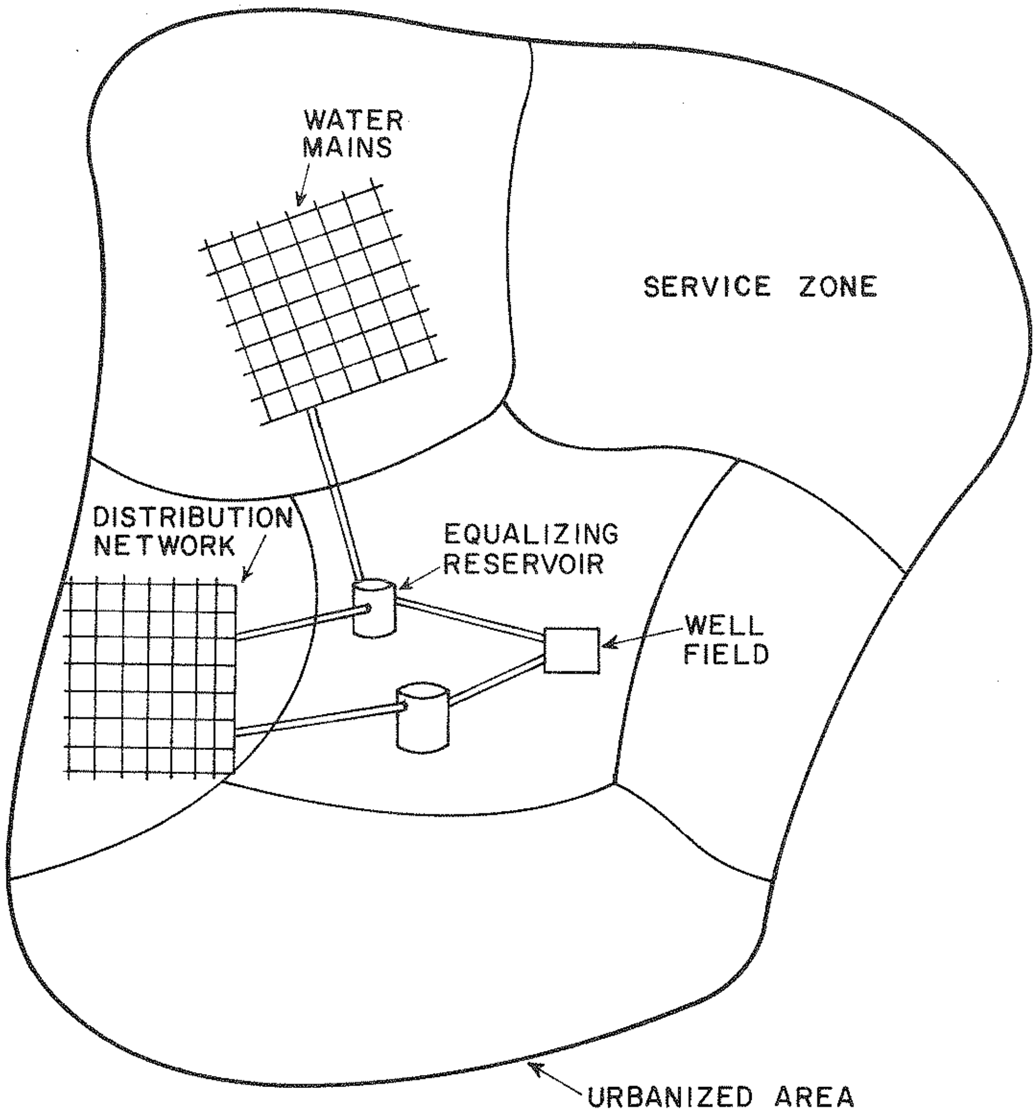


Figure 1.1. Schematic Representation of an Urban Water Supply System

(2) at the larger level the determination of the number, size and location of fresh water sources, capacity and location of equalizing reservoirs and booster pumps. The above facilities are to be provided at the minimal cost. This report addresses the second problem.

Another important point which is part of the second problem considered in this report is the effect that new wells will have on the aquifer. Additional drawdown in piezometric levels due to the new well system should not adversely affect the existing system. These requirements form a series of decision making problems differing in both spatial and temporal levels of integration that need to be coordinated.

1.3 Outline of this Report

(1) Estimation of demand is the first step in water supply systems analysis. It is not sufficient to know the gross demand; the spatial disaggregation of demand is needed also. The Model LANDUSE provides the above information and will be discussed in Chapter 2.

(2) The higher the piezometric level the better is the location for new wells. Numerical methods have proven to be highly effective in the groundwater flow problems which otherwise require stringent assumptions for analytical solution. The Model WATSUP, a finite element based optimization model, uses the finite element method to compute piezometric levels with the input provided by LANDUSE. The Model WATSUP is an interactive program. Based on the piezometric contours the decision maker is required to select a set of feasible subsystem locations and the Model WATSUP processes the set of feasible points to produce the set of optimal locations for the new system of wells.

(3) The theory of the finite element method as applied to groundwater flow problems is presented in Chapter 3.

(4) A Mixed Integer Programming formulation minimizing costs is reported in Chapter 4.

(5) The Model WATSUP uses a two level coordination scheme to check the minimum pressure criterion and to predict the effect of new wells on the existing system of wells. A detailed description of the two level coordination and of the model WATSUP may be found in Chapter 5.

(6) The application of above methodology to West Lafayette, Indiana, is presented in Chapter 6.

(7) Finally, the summary and conclusions of the study are presented in Chapter 7.

CHAPTER 2

LAND USE PLANNING AND WATER DEMAND

2.1 Introduction

In water supply systems planning, it is not sufficient to know the total demand but its spatial disaggregation also must be known. The spatial disaggregation of the demand will enable the decision maker to select probable new locations for the facilities (wells and reservoirs). Hence, there is a need for a methodology by which spatial disaggregation of the demand can be studied. For this purpose a Land Use planning model can be effectively used. Future land use allocations can then be converted to equivalent water demand.

2.2 Land Use Planning and Model LANDUSE

2.2.1 Land Use Planning

Future land use allocations of a geomorphologically and sociologically described region is extremely difficult to forecast because of the subjective decisions involved in such problems. Land Use Planning is a major breakthrough in that respect. It enables the planner to gain:

1. greater objectivity,
2. greater precision, and
3. ability to consider alternatives.

Land Use Planning can be divided into two Phases. Phase I includes: (1) a decision regarding the methods to be used, (2) inventories and forecasts of population and employment, and (3) inventories of vacant and renewable land and of existing and substandard land uses. Phase II includes:

(1) estimation of future land requirements based on:

  a. location requirements (fixed requirements)

  b. activity requirements (serving requirements)

(2) allocation of land use.

2.2.2 Description of Model LANDUSE

The land use demand estimates are based on population projections obtained from the standard "OBERS" projections. (Combination of "Office of Business Economics" (OBE), U.S. Department of Commerce and the "Economic Research Service" (ERS), U.S. Department of Agriculture.)

The model LANDUSE transforms aggregated land use demand estimates of a morphologically and socioeconomically described region into actual allocations at the end of the planning horizon, Figure 2.1. These land allocations are performed by simulating the matching procedure between the supply of available land and the demand. The supply of land units is described by a set of attributes that characterizes the elements of a rectangular grid approximating the natural areas and neighborhoods. Examples of attributes are physical-topographic characteristics (soil type, slope, depth to bedrock), and characteristics describing the availability of community services and facilities (e.g., transportation accessibility, availability of water supply and sewer).

On the demand side, the loosely coordinated private locational decisions are aggregated in several land use categories, for example, industrial, commercial, housing, recreational, etc. It is assumed that similar activities require locations with similar attributes. A matching

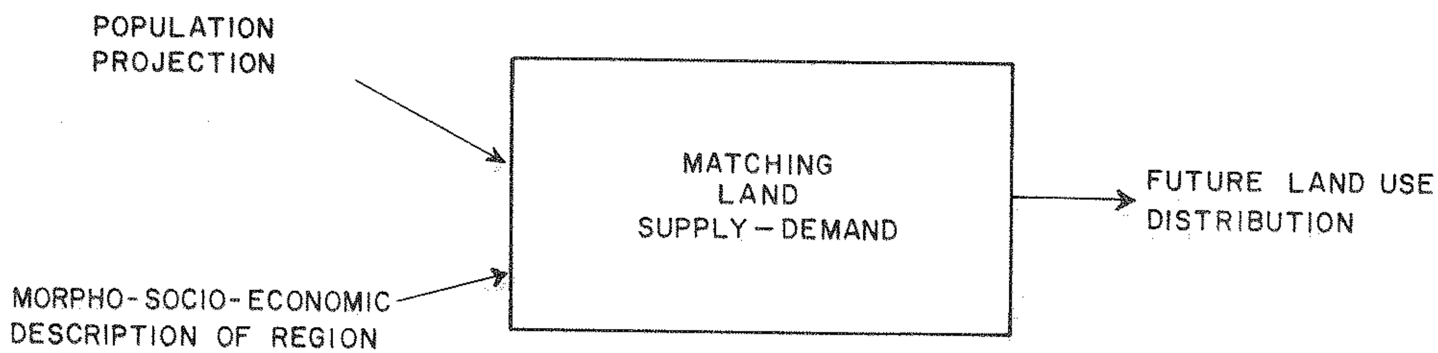


Figure 2.1 Functional Representation of Model LANDUSE

between demand requirement and supply availabilities is possible at the level of attributes if both supply and demand are characterized by the same set of attributes.

The matching mechanism operates as follows: The planning horizon is divided into a number of periods during which the demand is specified in terms of land increments per land use per period. All land use types are sequentially addressed and the available lots are allocated to the land uses that best meet the desired characteristics, until as much of the demand is satisfied as possible. This matching mechanism is assumed to simulate a free (without any controls) real estate market. Controls justifiably qualified as "public policies" can also be introduced in the model by exogenously imposing modifications in the attributes of portions of the area under urbanization, at specified times. The primary purpose of selecting the public policy option is to isolate all independent actions that are likely to influence the urban development, such as zoning, and to compare their effect on the growth without such influences. Dendrou, Delleur and Talavage have given detailed descriptions of the LANDUSE model (1978a,b) in a previous report of this research project.

2.3 Estimation of Water Demand

The water demand is predicted as an equivalent of the land use allocations made by the model LANDUSE. The following is a list of different land use types and the number of people served by each land use (Greater Lafayette Area Transportation and Development Study, 1975).

1. Commercial - all trade activities like wholesale, food, motels, retail and restaurants are classified as the land uses numbered 50 to 59 by the Bureau of Public Roads (BPR).

2. Light industry - food, furniture, textile, printing, paper, clothing--BPR #21, 22, 23, 25, 26, 27, 35, 39--2 acres serving 1,000 people.

3. Heavy industry - rubber, chemicals, plastics, lumber, stone, clay, primary metals, resource production--BPR #8, 24, 28, 29, 31, 32, 33, 34--10 acres for 1,500 people.

4. Parks and Recreation - all entertainment but not eating or drinking, includes cultural and amusement--BPR #70's--15 acres for 1,500 people.

5. Primary school - BPR #68--10 acres for 4,000 people.

6. Government and institutions - all transportation, institutions, universities, hospitals, services, professional--BPR #60's and 40's--4 acres for 1,000 people.

7. Single family residential - BPR #11--15 acres for 250 people.

8. Apartments - 15 acres for 750 people.

The per capita water consumption is taken as (Steel and McGhee (1979), Seidel (1978)):

	gpcd
residential	75
commercial and industrial	60
public (institutions, streets, parks, etc.)	20

Knowing the number of people engaged in one activity and the per capita consumption, water demand per activity can be estimated. In this way the spatial variation of land use activities can be converted into a spatial distribution of water demand.

The above procedure estimates the quantity of water demand. Activities like fire fighting not only require a minimum discharge, but also a minimum pressure. This can be satisfied either by larger booster pumps or by storing the supplied water at an elevation so that enough gravity head can be obtained.

CHAPTER 3

FINITE ELEMENT ANALYSIS OF GROUNDWATER FLOW

3.1 Introduction

Once the disaggregation of the demand is done as discussed in the previous chapter, the next step is the evaluation of alternative locations of wells and distribution reservoirs so as to optimally satisfy the demand. This optimal allocation problem requires the prior knowledge of the groundwater piezometric levels as a decision instrument. This chapter summarizes the theory of finite elements to evaluate the piezometric levels of a groundwater system. The optimization procedure is developed in Chapter 4. The various solution methods available for groundwater flow problems fall into either one of the following categories:

a. Analytical methods

b. Simulation methods

Various analytical solutions are available for the general groundwater flow problem based on different assumptions. Gambolati (1976) has reviewed most of the available analytical methods.

When a problem becomes too complicated for analytical treatment, either a physical model is built or numerical methods may be used. Herbert (1968) describes the use of resistance network analogues. Prickett (1976) gives a very good account of various physical and numerical models.

Among numerical methods the finite difference and finite element methods are widely used. Remson et.al. (1971) explain in great detail both methods as applied to subsurface hydrology. Taylor and Luthin (1969), Marious Todsen (1971), Verma and Brutsaert (1971), and Lakshminarayana and Rajagopalan (1977) have used the finite difference method with different configurations of the problem. Desai (1975) has reviewed various articles in these two methods as applied to flow in porous media.

Finite element method has been used by Neuman and Witherspoon (1971) to solve the problem of unsteady flow to water table wells. This paper is an extensive study on this topic. France et.al. (1971) have used isoparametric elements to solve the problem. Cheung and Skjolingstad (1974) have used three dimensional elements. Marino (1976) has applied finite element method for aquifer dynamic responses.

3.2 Preliminaries

The finite element method is a numerical procedure for solving differential equations. A region where the distribution of a state variable is to be determined is subdivided into subdomains or finite elements. A function is chosen to define uniquely the state variable within each element in terms of the values of the function and/or its derivatives at some specific points called nodes.

The function within each element depends upon the coordinates of the nodes forming the element. The relations of dependence are known as the shape functions of the element.

Considering any element e in the region R the function h within the element is represented in terms of nodal values as

$$\{h\} = [N] \{h^e\} \qquad (3.1)$$

where $\{h\}$ is the unknown function and $\{h^e\}$ represents a column matrix of the nodal values of the function, and $[N]$ is a square matrix representing element shape functions in terms of nodal coordinates.

The function defining the state variable within each element must satisfy convergence criteria. The element shape functions have to be chosen such that at element interfaces the values of h and of its derivatives of one order less than that occurring in the nodal equations are both continuous. The element shape functions must also be such that, with a suitable choice of $\{h^e\}$, they represent constant values of the state variable or of its derivatives as the element size shrinks to zero.

3.3 Linear Triangular Element:

The flow region R is divided into triangular elements. The hydraulic head h within each element e can be expressed as a function of the coordinates

$$h = A + Bx + Dy \tag{3.2}$$

where, A,B, and D are constants for each element. A, B, and D are evaluated depending on the nodal coordinates i, j, k, and on the values of h evaluated at the nodes. The function h can be expressed as, Figure 3.1,

$$h = [N_i \; N_j \; N_k] \begin{Bmatrix} h_i \\ h_j \\ h_k \end{Bmatrix} \tag{3.3}$$

where: $N_i = (a_i + b_i x + d_i y)/2\Delta$ (3.3a)

$$N_j = (a_j + b_j x + d_j y)/2\Delta$$

$$N_k = (a_k + b_k x + d_k y)/2\Delta$$

$$a_i = x_j y_k - x_k y_j \tag{3.3b}$$

$$b_i = y_j - y_k$$

$$d_i = x_k - x_j$$

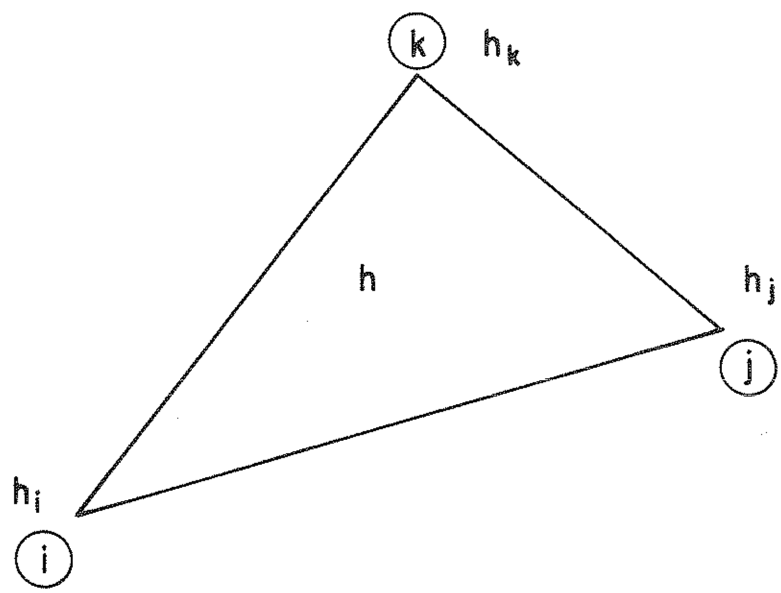


Figure 3.1. Linear Triangular Element 'e'

$a_j = x_k y_i - x_i y_k$

$b_j = y_k - y_i$

$d_j = x_i - x_k$

$a_k = x_i y_j - x_j y_i$

$b_k = y_i - y_j$

$d_k = x_j - x_i$

h_i, h_j, h_k = heads at nodes i, j, and k respectively

x_i = x-coordinate of node i

y_i = y-coordinate of node i

Δ = area of element e

The area of element e may be written as

$$\Delta = 1/2 \begin{vmatrix} 1 & x_i & y_i \\ 1 & x_j & y_j \\ 1 & x_k & y_k \end{vmatrix}$$

Nodes i, j, and k are arranged in counter clockwise order around the element.

3.4 Variational Formulation

The calculus of variations is that branch of mathematics which treats the selection of an unknown function appearing in the integrand of an integral such that the value of the integral is made either a maxinum or a minimum. Problems in subsurface hydrology do not naturally lead to variational formulations. The procedure used in such problems is to find a functional that yields the governing differential equation as its Euler-Largrange equation and to operate with the functional itself to determine the solution (The solution thus minimizes the functional as well as satisfies the governing differential equation and the boundary conditions).

In this report only a special form of the general groundwater flow equation, namely the confined steady groundwater flow equation, will be studied. The equation is,

$$k_{xx} \frac{\partial^2 h}{\partial x^2} + k_{yy} \frac{\partial^2 h}{\partial y^2} + Q = 0 \qquad (3.4)$$

where: k_{xx}, k_{yy} = coefficients of permeability

h = piezometric head measured from the bottom of the aquifer

Q = recharge (positive) or pumping (negative)

The boundary conditions are

$$h = h_o \qquad (3.5)$$

where the head is prescribed on the boundary and

$$k_{xx} \frac{\partial h}{\partial x} \ell_x + k_{yy} \frac{\partial h}{\partial y} \ell_y + q_o = 0 \qquad (3.6)$$

where ℓ_x and ℓ_y are direction cosines in the x and y directions and the flow q_o is specified on the boundary C.

For the above differential equation together with the boundary conditions, the variational functional is:

$$\chi = \iint_R 1/2 \left[k_{xx} \left(\frac{\partial h}{\partial x}\right)^2 + k_{yy} \left(\frac{\partial h}{\partial y}\right)^2 - 2Qh\right] dR + \int_C q_o \, h \, ds \qquad (3.7)$$

where R is the region of interest and C its contour.

The basic idea of the finite element method is to discretize the region of interest into finite elements and to analyze each element. The problem solution, χ, is obtained by assembling the solutions of $\chi^{(e)}$ of all the individual elements. To enable such a procedure the integral χ is separated into its element components. Thus,

$$\chi = \sum_{e=1}^{N} \chi^{(e)} \tag{3.8}$$

where:

$$\chi^{(e)} = \iint_{R^{(e)}} 1/2 \left[k_{xx} \left(\frac{\partial h}{\partial x}\right)^2 + k_{yy} \left(\frac{\partial h}{\partial y}\right)^2 - 2Qh\right] dR + \int_{C^{(e)}} q_o \, h \, ds$$

and N = total number of elements in the region R.

For $\chi^{(e)}$ to be an extremum

$$\frac{\partial \chi^{(e)}}{\partial \{h^{(e)}\}} = \begin{Bmatrix} \dfrac{\partial \chi^{(e)}}{\partial h_i} \\ \dfrac{\partial \chi^{(e)}}{\partial h_j} \\ \dfrac{\partial \chi^{(e)}}{\partial h_k} \end{Bmatrix} = 0 \tag{3.9}$$

The term $\frac{\partial \chi^{(e)}}{\partial h_i}$ is expanded as follows:

$$\frac{\partial \chi^{(e)}}{\partial h_i} = \iint_{R^{(e)}} \left[k_{xx} \frac{\partial h}{\partial x} \frac{\partial}{\partial h_i} \frac{\partial h}{\partial x} + k_{yy} \frac{\partial h}{\partial y} \frac{\partial}{\partial h_i} \frac{\partial h}{\partial y} - Q \frac{\partial}{\partial h_i} (h)\right] dx\, dy$$

$$+ \int_{C^{(e)}} q_o \frac{\partial}{\partial h_i} (h) \, ds \tag{3.10}$$

Substituting $h = [N_i \; N_j \; N_k] \begin{Bmatrix} h_i \\ h_j \\ h_k \end{Bmatrix}$

in the previous equation

$$\frac{\partial \chi^{(e)}}{\partial h_i} = \iint_{R^{(e)}} \left\{ k_{xx} \left[\frac{\partial N_i}{\partial x} \; \frac{\partial N_j}{\partial x} \; \frac{\partial N_k}{\partial x} \right] \frac{\partial N_i}{\partial x} \begin{Bmatrix} h_i \\ h_j \\ h_k \end{Bmatrix} + k_{yy} \left[\frac{\partial N_i}{\partial y} \; \frac{\partial N_j}{\partial y} \; \frac{\partial N_k}{\partial y} \right] \frac{\partial N_i}{\partial y} \begin{Bmatrix} h_i \\ h_j \\ h_k \end{Bmatrix} \right.$$

$$\left. - Q N_i \right\} dx\, dy + \int_{C^{(e)}} q_o N_i \, ds \qquad (3.11)$$

Making use of equation (3.3a)

$$\frac{\partial \chi^{(e)}}{\partial h_i} = \iint_{R^{(e)}} \frac{k_{xx}}{4\Delta^2} \left[b_i \quad b_j \quad b_k \right] b_i \begin{Bmatrix} h_i \\ h_j \\ h_k \end{Bmatrix} dx\, dy$$

$$+ \iint_{R^{(e)}} \frac{k_{yy}}{4\Delta^2} \left[d_i \quad d_j \quad d_k \right] d_i \begin{Bmatrix} h_i \\ h_j \\ h_k \end{Bmatrix} dx\, dy$$

$$- \iint_{R^{(e)}} Q N_i \, dx\, dy + \int_{C^{(e)}} q_o N_i \, ds \qquad (3.12)$$

The determination of $\frac{\partial \chi^{(e)}}{\partial \{h^e\}}$ involves the integration of the shape functions N over the element. The solution procedure can be greatly simplified by using local coordinate systems: one for each element with the center of coordinates within the element.

The evaluation of the first two terms in the r.h.s. of equation (3.12) is straight forward and will be performed at a later stage. It remains to find evaluation procedures for the third and fourth terms of equation (3.12).

3.5 Local Coordinate System

Area Coordinates: The surface integral (3rd term on the r.h.s. of equation (3.12) requires the introduction of area coordinate systems. Consider the triangle i, j, k shown in Figure 3.2 and 3.3.

$$\text{area } Pij = 1/2 \begin{vmatrix} 1 & x & y \\ 1 & x_i & y_i \\ 1 & x_j & y_j \end{vmatrix}$$

$$= \frac{(x_i\, y_j - y_i\, x_j) + x\,(y_i - y_j) + y\,(x_j - x_i)}{2} \tag{3.13}$$

$$\text{define } L_k = \frac{\text{area of triangle } Pij}{\text{area of triangle } kij} \tag{3.14}$$

$$= \frac{\Delta k}{\Delta}$$

Substituting (3.13) in (3.14) and making use of the definitions 3.3a and 3.3b

$$L_k = N_k$$

Similarly defining

$$L_j = \frac{\Delta j}{\Delta}$$

it is seen that

$$L_j = N_j$$

Likewise, defining

$$L_i = \frac{\Delta i}{\Delta}$$

then

$$L_i = N_i$$

and it follows that

$$L_i + L_j + L_k = \frac{\Delta i + \Delta j + \Delta k}{\Delta} = \frac{\Delta}{\Delta} = 1 \tag{3.15}$$

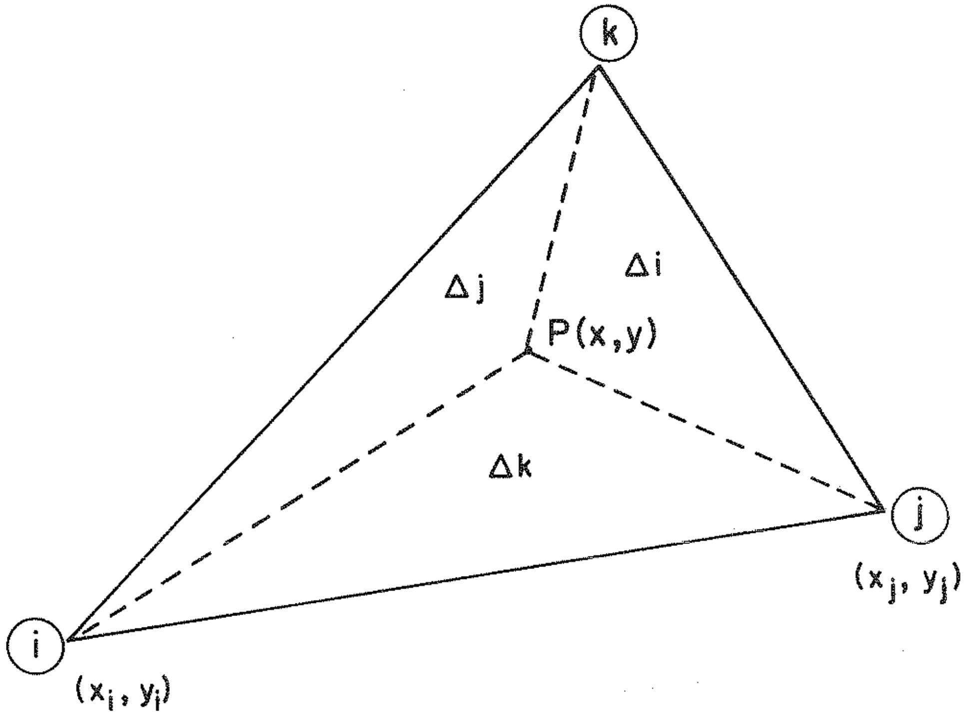
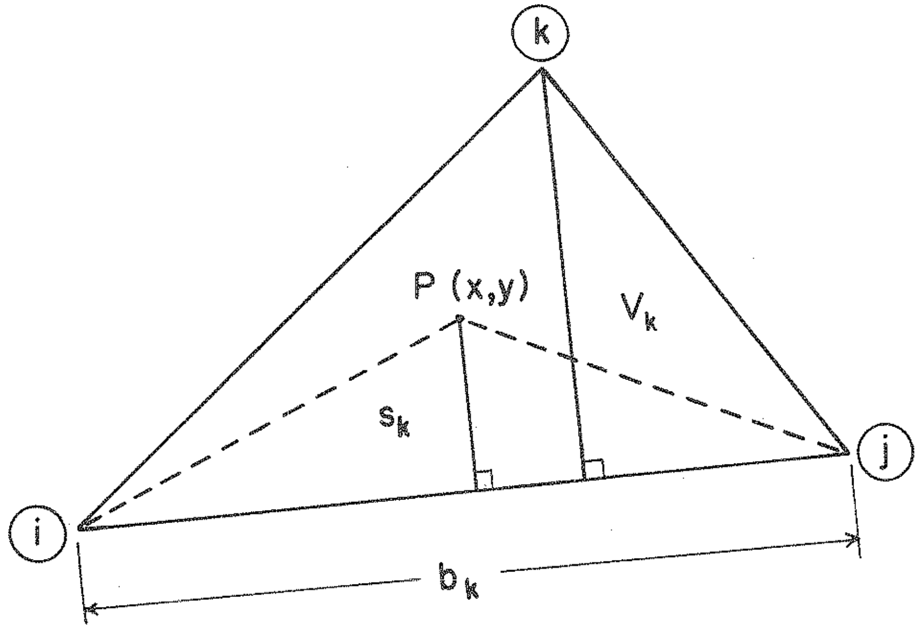


Figure 3.2. Area Coordinate System



$$L_k = \frac{\text{area } Pij}{\text{area } kij} = \frac{\frac{1}{2} s_k b_k}{\frac{1}{2} V_k b_k} = \frac{s_k}{V_k}$$

Figure 3.3. Area Coordinates as the Ratios of Vertex Heights

Also the coordinates of any point P (x,y) inside the triangle ijk in terms of local coordinates L_i, L_j, L_k can be described as

$$x = L_i x_i + L_j x_j + L_k x_k$$
$$y = L_i y_i + L_j y_j + L_k y_k \qquad (3.16)$$

It is to be noticed the L_i is unity at node i where L_j and L_k are zeros. Similarly at node j, L_j is unity and L_k and L_i are zeros; at node K, L_k is unity and L_i and L_j are zeros. Considering the Figure 3.3

$$L_k = \frac{1/2\ s_k\ b_k}{1/2\ V_k\ b_k} = \frac{s_k}{V_k} \qquad (3.17)$$

Similarly

$$L_j = \frac{s_j}{V_j}$$

and

$$L_i = \frac{s_i}{V_i}$$

where:

s_i, s_j, s_k = perpendicular heights from bases opposite to the nodes i, j, k to point P respectively,

V_i, V_j, V_k = perpendicular heights from nodes i, j, k to their opposite bases respectively and

b_i, b_j, b_k = length of bases opposite to nodes i, j, and k respectively.

This transformation of coordinates simplifies the evaluation of the integral of the second term of the r.h.s. of equation (3.12) which is of the form

$$\iint_{R^{(e)}} N_i \, dx\, dy = \iint_{R^{(e)}} L_i \, dx\, dy \qquad (3.18)$$

This can be evaluated with the aid of relationships derived above.

Consider the general form (Eisenburg and Malvern, 1973),

$$\iint_{R^{(e)}} L_i^a \, L_j^b \, L_k^c \, dR \tag{3.19}$$

where dR is shown in Figure 3.4. From the figure geometry

$$dR = \frac{ds_i}{\sin \alpha_j} \, ds_k = \frac{V_i \, dL_i \, V_k \, dL_k}{\sin \alpha_j} \tag{3.20}$$

$$= 2 \, \Delta \, dL_i \, dL_k \tag{3.21}$$

and the general form becomes

$$\iint_{R^{(e)}} L_i^a \, L_j^b \, L_k^c \, dR = 2\Delta \int_0^1 \left[\int_0^{1-L_i} L_i^a \, L_k^c \, (1-L_i-L_k)^b \, dL_k \right] dL_i \tag{3.22}$$

Letting $u = \frac{L_k}{1-L_i}$, $du = \frac{1}{1-L_i} \, dL_k$

The previous integral becomes

$$2\Delta \int_0^1 \int_0^{1-L_i} L_i^a \, L_k^c \, (1-L_i-L_k)^b \, dL_k \, dL_i$$

$$= 2\Delta \int_0^1 L_i^a \, (1-L_i)^{c+b+1} \, dL_i \int_0^1 u^c \, (1-u)^b \, du \tag{3.23}$$

where each of the integrals on the right hand side is in the form of the beta function

$$B(\alpha,\beta) = \int_0^1 t^{\alpha-1} \, (1-t)^{\beta-1} \, dt$$

$$= \frac{\Gamma(\alpha) \, \Gamma(\beta)}{\Gamma(\alpha+\beta)}$$

where $\Gamma(\alpha)$ denotes the gamma function, and for α an integer $\Gamma(\alpha) = (\alpha-1)!$

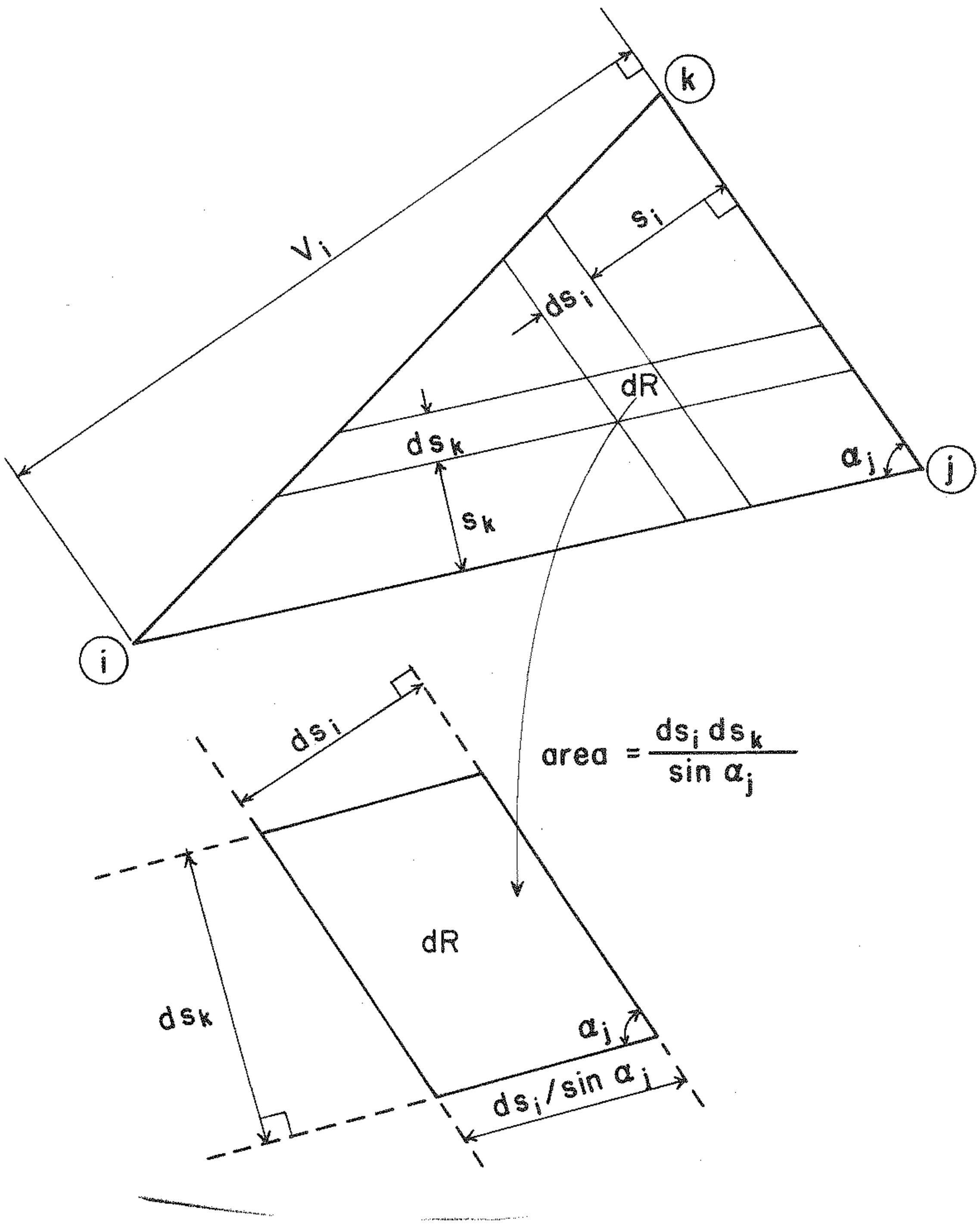


Figure 3.4. Domain of Integration

This gives the following result

$$2\Delta \iint_{R^{(e)}} L_i^{\,a} L_j^{\,b} L_k^{\,c}\, dR$$

$$= \frac{\Gamma(a+1)\ \Gamma(b+c+2)\ \Gamma(c+1)\ \Gamma(b+1)}{\Gamma(a+b+c+3)\ \Gamma(b+c+2)}\ 2\Delta$$

$$= \frac{a!\ b!\ c!}{(a+b+c+2)\ !}\ 2\Delta \qquad (3.24)$$

3.6 Local Coordinate System for a Line Element

The line integral (4th term on the r.h.s. of equation (3.12)) requires the introduction of line coordinate systems. From Figure 3.5 for the line element define

$$L_i = \frac{\ell_i}{\ell}$$
$$L_j = \frac{\ell_j}{\ell} \qquad (3.25)$$

where $\ell_i = x_j - x$ and $\ell_j = x - x_i$ (3.25a)

$$L_i + L_j = 1$$

$$L_i\ x_i + L_j x_j = x$$

At node i, L_i is unity and L_j is zero. Likewise at node j, L_j is unity and L_i is zero.

For a line element defining

$$h = \alpha_1 + \alpha_2\ x \qquad (3.26)$$

$$h = \left[\frac{x_j - x}{\ell}\right] h_i + \left[\frac{x - x_i}{\ell}\right] h_j \qquad (3.27)$$

$$h = [N_i\ \ N_j]\begin{Bmatrix} h_i \\ h_j \end{Bmatrix} \qquad (3.28)$$

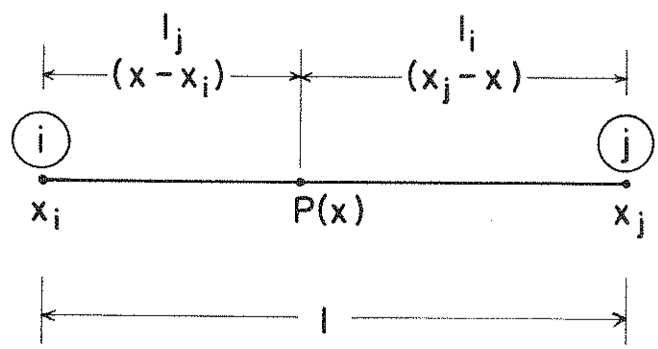


Figure 3.5. Line Element with Local Coordinates

where $N_i = \frac{x_j - x}{\ell} = L_i$

$$N_j = \frac{x - x_i}{\ell} = L_j$$

Note that the N_i's for the line elements are different from those used previously for the area elements.

In order to evaluate the last integral of equation (3.12), consider the general expression

$$\int_0^{\ell} L_i^{\,p} \, L_j^{\,q} \, d\ell$$

Substituting equations (3.25) and (3.25a) in the general expression gives

$$\int_0^{\ell} L_i^{\,p} \, L_j^{\,q} \, d\ell = \int_{x_i}^{x_j} \left(\frac{x_j - x}{x_j - x_i}\right)^p \left(\frac{x - x_i}{x_j - x_i}\right)^q dx \tag{3.29}$$

$$= \frac{1}{(x_j - x_i)^{p+q}} \int_{x_i}^{x_j} (x_j - x)^p \, (x - x_i)^q \, dx$$

Integrating by parts

$$\int_0^{\ell} L_i^{\,p} \, L_j^{\,q} \, d\ell = \frac{1}{(x_j - x_i)^{p+q}} \, (x_j - x)^p \, \frac{(x - x_i)^{q+1}}{q+1} \Bigg|_{x_i}^{x_j}$$

$$+ \int_{x_i}^{x_j} p \, (x_j - x)^{p-1} \, \frac{(x - x_i)^{q+1}}{q+1} \, dx \tag{3.30}$$

Integrating by parts again and again P times

$$= \frac{1}{(x_j-x_i)^{p+q}} \frac{p(p-1)\ldots(p-\overline{p-1})}{(q+1)(q+2)\ldots(q+p)} \int_{x_i}^{x_j} (x-x_i)^{q+p}\, dx$$

$$= \frac{1}{(x_j-x_i)^{p+q}} \frac{p(p-1)\ldots(p-\overline{p-1})}{(q+1)(q+2)\ldots(q+p)} \left. \frac{(x-x_i)^{q+p+1}}{(q+p+1)} \right|_{x_i}^{x_j} \tag{3.31}$$

$$= \frac{(x_j-x_i)^{p+q+1}}{(x_j-x_i)^{p+q}} \frac{1.2.\ldots.q.p(p-1).\ldots.1}{1.2.\ldots.q(q+1).\ldots.(q+p+1)} \tag{3.32}$$

$$= \ell \frac{p!\, q!}{(p+q+1)!} \tag{3.33}$$

3.7 Evaluation of the Integrals of Equation (3.12)

Considering the first integral on the r.h.s. of equation (3.12)

$$\iint_{R^{(e)}} \frac{k_{xx}}{4\Delta^2} [b_i \; b_j \; b_k] \, b_i \begin{Bmatrix} h_i \\ h_j \\ h_k \end{Bmatrix} dx\, dy$$

$$= \frac{k_{xx}}{4\Delta^2} [b_i \; b_j \; b_k] \, b_i \begin{Bmatrix} h_i \\ h_j \\ h_k \end{Bmatrix} \iint_R dx\, dy$$

using $\iint_R dx\, dy = \Delta$, the r.h.s. of the previous equation becomes

$$\frac{k_{xx}}{4\Delta} [b_i \; b_j \; b_k] \, b_i \begin{Bmatrix} h_i \\ h_j \\ h_k \end{Bmatrix} \tag{3.34}$$

The surface integral is evaluated next

$$\iint_{R^{(e)}} Q\, N_i \, dx\, dy = \iint_R Q\, L_i \, dR$$

Substituting equation (3.24) with a = 1, b = 0, and c = 0, the previous integral becomes, $\frac{Q}{6}\,2\Delta = \frac{Q}{3}\,\Delta$ (3.35)

The line integral

$$\int_{C^{(e)}} q_o\, N_i\, ds = \frac{\ell}{2}\, q_o \tag{3.36}$$

Now the above derivations can be used to evaluate $\partial\chi^{(e)}/\partial h_i$ as

$$\frac{\partial\chi^{(e)}}{\partial h_i} = \frac{k_{xx}}{4\Delta}\,[b_i \;\; b_j \;\; b_k]\; b_i \begin{Bmatrix} h_i \\ h_j \\ h_k \end{Bmatrix} + \frac{k_{yy}}{4\Delta}\,[d_i \;\; d_j \;\; d_k]\; d_i \begin{Bmatrix} h_i \\ h_j \\ h_k \end{Bmatrix}$$

$$- \frac{Q\Delta}{3} + \frac{q_o \ell_{ij}}{2} \tag{3.37}$$

where $\ell_{ij} = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2}$. The above equation assumes that the seepage q_o occurs between nodes i and j. The general form (3.12) becomes

$$\frac{\partial\chi^{(e)}}{\partial\{h^e\}} = \left[\frac{k_{xx}}{4\Delta} \begin{bmatrix} b_i b_i & b_i b_j & b_i b_k \\ b_j b_i & b_j b_j & b_j b_k \\ b_k b_i & b_k b_j & b_k b_k \end{bmatrix} + \frac{k_{yy}}{4\Delta} \begin{bmatrix} d_i d_i & d_i d_j & d_i d_k \\ d_j d_i & d_j d_j & d_j d_k \\ d_k d_i & d_k d_j & d_k d_k \end{bmatrix} \right] \begin{Bmatrix} h_i \\ h_j \\ h_k \end{Bmatrix}$$

$$- \frac{Q\Delta}{3} \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} + \frac{q_o}{2} \left\{ \begin{matrix} \ell_{ij} \begin{Bmatrix} 1 \\ 1 \\ 0 \end{Bmatrix} \\ \ell_{jk} \begin{Bmatrix} 0 \\ 1 \\ 1 \end{Bmatrix} \\ \ell_{ki} \begin{Bmatrix} 1 \\ 0 \\ 1 \end{Bmatrix} \end{matrix} \right. \tag{3.38}$$

The last term on the r.h.s. of the previous equation takes one of the indicated three forms depending upon the side through which q_o occurs. In a simplified form

$$\frac{\partial \chi^{(e)}}{\partial \{h^e\}} = [k^e] \{h^e\} + \{f^e\} = 0 \qquad (3.39)$$

Now for the solution of the whole region R the element matrices should be assembled. For *N* number of elements the general system of equations can be written as

$$[K] \{U\} = \{F\} \qquad (3.40)$$

where

$$[K] = \sum_{e=1}^{N} [k^e]$$

$$\{F\} = - \sum_{e=1}^{N} \{f^e\}$$

Equation (3.40) is solved for the vector {U} which is the global head distribution.

3.8 Summary

This chapter summarizes the FEM in groundwater hydrology. Starting with the basic groundwater flow equation (3.4) and the boundary conditions (3.5) and (3.6), the variational formulation (3.7) is obtained. The variational formulation, upon minimization with respect to the state variable (piezometric head), yields the set of simultaneous equations (3.40) which are solved for the unknown head values. As mentioned earlier the decision maker uses this information in selecting a set of feasible well locations for the optimization of the water supply system. The decision maker may choose a new well field with higher piezometric levels. The optimization procedure processes the set of feasible subsystem locations to pick the combination of wells and reservoirs together with flows and pipe sizes resulting in minimum cost. In chapter 4 a mixed integer programming formulation is developed for this purpose.

CHAPTER 4

MIXED INTEGER PROGRAMMING FORMULATION

4.1 Introduction

Systems engineering as defined by Hall and Dracup (1970) is "The art and science of selecting from a large number of feasible alternatives, involving substantial engineering content, that particular set of actions which will accomplish the overall objectives of the decision makers, within the constraints of law, morality, economics, resources, political and social pressures, and laws governing the physical life and other natural sciences." In chapter 1 it was stated that the decision maker had to select an optimal set of solutions from a set of feasible choices available to him. The decision maker is required to make his decisions under various types of constraints. Mathematical Programming is an effective way to solve such problems. The indivisibility requirement of some decision variables (wells, reservoirs, etc.) and the continuous variation of other decision variables (flow values) require the use of a Mixed Integer Programming (MIP) approach. The nonlinearity of the objective function and of the constraints, due to nonlinear head losses, together with the requirement that some variables need to be integers make the problem highly complex. The nonlinearity of the objective function and of the constraints has been circumvented by means of empirical relationships (i.e. Manning's formula), for head losses and design criteria for pipe sizes. As mentioned in chapter 1, the water supply system is made up of several components. In this chapter each of these components is explicitly taken into consideration in the formulation of the minimum cost system.

4.2 Mixed Integer Programming Formulation

The three main components of an urban water supply system are:

(i) the sources,
(ii) the storage reservoirs, and
(iii) the service zones.

Associated with each of the above components are fixed costs and operating costs. In the present formulation, the treatment cost is added to the pumping cost since all the pumped water must be treated. It is decided to minimize the annualized costs.

The investment costs can be annualized by an equal-payment-series capital-recovery factor defined as,

$$R = P * (1+r)^n * \left[\frac{r}{(1+r)^n - 1}\right] \tag{4.1}$$

where: R = a single payment, in a series of n equal payments, made at the end of each annual interest period.

P = a present principal sum

r = the nominal annual interest rate

n = the number of annual interest periods

The operating costs are summed up over the planning horizon and brought down to a present principal sum which is expressed as

$$P = \frac{c}{(1+f)} + \frac{c(1+r)}{(1+f)^2} + \dots\dots + \frac{c(1+r)^{n-1}}{(1+f)^n} \tag{4.2}$$

where: c = operating cost in current year.

f = inflation rate

If $$r = f, \text{ then } P = \frac{nc}{(1+r)} \qquad (4.3)$$

The annualized cost of the well field, the reservoir field, pumping and pipe cost from the wells to the reservoirs and from these to demand points is to be minimized, as shown in eqn. (4.4) which makes use of the following notation:

CIP_k = cost of pipe of size index k per unit length (\$/L) (k=1,2,3...).

CP_{ij} = cost of pipe between well field i and reservoir field j per unit length, (\$/L).

CIR_n = cost of reservoir with size index n, (\$). (n = 1,2,3,...)

CIW_i = cost of having one well at site i, (\$)

H_{ij} = head between the well at site i and reservoir j, (L) (obtained from Piezometric contours of GRNDFLO ground-water subroutine of WATSUP and topographic maps)

QDZ_p = demand at zone p, (L^3 in a day)

QW^iR_j = flow rate from well field i to reservoir field j, (L^3/T)

QR^jZ_p = flow quantity from reservoir field j to demand zone p, (L^3)

R_{jn} = total number of reservoirs at site j with size index n

$R^jZ_pS_k$ = pipe of size index k carrying flow from reservoirs at site j to demand zone p. (0-1 variable, 1: exists, 0: does not exist)

W_i = total number of wells at site i

W_{ij} = number of wells which pump from well field i to reservoir field j

Y = yield of aquifer, (L^3/T)

c = unit pumping and treatment cost, (\$/HP)

d^* = pipe diameter producing minimum head loss in (L), (See derivation in the following section).

k^* = head loss per unit length, (L/L)

ℓ_{ij} = length of pipe from well field at i to reservoir field j, (L, known quantity).

ℓ_{jp} = length of pipe from reservoir field at j to demand zone p, (L, known quantity).

m^* = a constant which determines the optimal pipe size from reservoir to demand zone. (See derivation in the following section).

t = time period, (T, 1 yr for annualized costs)

δt_i = duration of pumping at well field i, (T)

α_n = capacity of reservoir with size index n, (L^3)

β_k = diameter of pipe with index k (commercially available discrete pipe sizes, known quantity, L)

γ = specific weight of water in (F/L^3)

η = efficiency of the pump. (constant)

Minimize $$\sum_i CIW_i \cdot W_i + \sum_n \sum_j CIR_n \cdot R_{jn} + \sum_i \sum_j c \frac{\gamma QW^iR_j}{\eta 550} (H_{ij} + k^* \ell_{ij})\, t$$

$$+ \sum_i \sum_j CP_{ij} \cdot \ell_{ij} \cdot W_{ij} + \sum_j \sum_p \sum_k CIP_k \cdot R^jZ_pS_k \cdot \ell_{jp} \qquad (4.4)$$

Subject to:

$$QW^iR_j \leq Y \cdot W_{ij} \qquad \forall\ i,j \qquad (4.5a)$$

$$\sum_j W_{ij} = W_i \qquad \forall\ i \qquad (4.5b)$$

$$\sum_i QW^iR_j \cdot \delta t_i = \sum_p QR^jZ_p \qquad \forall\ j \qquad (4.6)$$

$$\sum_j QR^jz_p \geq QDZ_p \qquad \forall\ p \qquad (4.7)$$

$$\sum_n \alpha_n \cdot R_{jn} - \sum_i QW^iR_j \cdot \delta t_i \geq 0 \qquad \forall\ j \qquad (4.8)$$

$$\sum_k \beta_k \cdot R^jZ_pS_k - d^* \geq 0 \qquad \forall\ p,j \qquad (4.9)$$

$$\sum_k R^jZ_pS_k \leq 1 \qquad \forall\ j,p \qquad (4.10)$$

Equation (4.9) can also be written as:

$$\sum_k (\beta_k)^{8/3} \cdot R^j Z_p S_k - m^* Q \geq 0$$

The meaning of the constraints is given below:

Eq. (4.5) The rate of pumping should not exceed the aquifer yield.

Eq. (4.6) The quantity pumped to the reservoir must be equal to the amount released from the reservoir over one day.

Eq. (4.7) Quantity released from the reservoirs must be greater than or equal to the demand in each zone.

Eq. (4.8) Total capacity of reservoirs must be greater than or equal to the quantity pumped to the reservoirs.

Eq. (4.9) Only available discrete pipe sizes are to be chosen equal to or greater than theoretical pipe size.

Eq. (4.10) Only one pipe size is permitted.

4.2.1 Derivation of d^*

Constraint (4.9) requires a relationship between available discrete pipe diameters and the theoretical pipe size for minimum head loss. The Darcy - Weisbach equation for head loss in pipe flow is given by:

$$h_f = \frac{f\,\ell}{d}\,\frac{V^2}{2g} \qquad (4.11)$$

where:

h_f = head loss
f = friction factor
ℓ = length of the pipe
d = diameter of the pipe
V = flow velocity
g = acceleration due to gravity

For gravity flow from reservoir to demand points, under atmospheric pressure at both the ends, the velocity head is

$$\frac{V^2}{2g} = H - h_f \qquad (4.12)$$

where H is the head differential between the water surface in the reservoir and the demand point. Thus equation (4.11) can be written as

$$h_f = \frac{f\,\ell}{d}\,(H-h_f) \tag{4.13}$$

The head loss on the right hand side may be replaced by an empirical formula such as the Hazen-Williams or the Manning formula. The latter is chosen in this case, and the numerical constants are for English units (ft, lb, sec).

Manning's formula is given as

$$V = \frac{1.486}{N}\;R^{2/3}\;S_f^{\;1/2} \tag{4.14}$$

where:

N = Manning's roughness coefficient
R = Hydraulic radius
S_f = Slope of the energy grade line

substituting $S_f = \frac{h_f}{\ell}$ in equation (4.14) we obtain,

$$h_f = \frac{N^2V^2}{(1.486)^2}\left(\frac{4}{d}\right)^{\frac{4}{3}}\;\ell \tag{4.14a}$$

Replacing V in terms of Q in (4.14a) and substituting the resulting expression for h_f, on the r.h.s. of equation (4.13) give

$$\left\{h_f = \frac{f\,\ell}{d}\;H - \frac{f\,\ell}{d}\quad\frac{4^2\;N^2\;(4)^{4/3}}{\pi^2\;(1.486)^2}\quad\frac{Q^2\ell}{d^{16/3}}\right\} \tag{4.15}$$

where Q = flow rate

For given Q, H, and ℓ requiring h_f to be minimum

$$\frac{dh_f}{dd} = 0 \tag{4.16}$$

Substituting equation (4.15) in equation (4.16) and calling optimal d as d^* give

$$(d^*)^{8/3} = (1.887)^{8/3} N \left(\frac{\ell}{H}\right)^{1/2} Q \qquad (4.17)$$

Defining

$$(d^*)^{8/3} = m^* Q \qquad (4.18)$$

then

$$m^* = (1.887)^{8/3} N \left(\frac{\ell}{H}\right)^{1/2} \qquad (4.19)$$

4.2.2 Determination of k^*

In the objective function, the pumping cost requires the evaluation of the head loss per unit length of pipe between the pump and the reservoir. This diameter is generally fixed by the design criteria. According to Walton (1970), "The diameter of the production well casing should be two nominal sizes larger than the bowl size of the pump to prevent the pump shaft from binding, to reduce head losses and to allow measurement of water levels in the well". Trade manuals such as the reference book published by Johnson Division of the Universal Oil Products (1972) give the nominal size of the pump bowls based on the well design criteria. Hence the diameter of the pipe is fixed by the design of the well. Once the diameter is chosen the maximum head loss occurs when the discharge is maximum for a specificed length. Therefore, substituting $Q_{max} = Y$ gives,

$$h_{f_{max}} = k^* \ell \qquad (4.20)$$

$$k^* = (16.fY^2) / (2g\pi^2 d^5) \qquad (4.21)$$

4.3 Summary

In this chapter a mixed integer programming formulation is developed for the optimization of well and reservoir locations and for the flow values and the pipe sizes which are the components of an urban water supply system. As indicated in Section 1.2, the local distribution network is not part of this optimization. The Darcy-Weisbach equation for head losses which is non-linear, results in a non-linear objective function and non-linear constraints. The use of Manning's formula results in equation (4.15) for the head loss. Minimization of equation (4.15) yields the least head loss diameter d^* given by equation (4.18). For known values of the pipe length and diameter and head, equation (4.18) is a function of Q and is linear. The well design criteria result in equation (4.20) which is a function of length of the pipe alone. Equation (4.20) is used to compute head losses for the known length of pipes. The two equations (4.18) and (4.20) transform the original complex optimization problem into regular MIP problem which can be solved by existing algorithms.

CHAPTER 5
TWO LEVEL COORDINATION SCHEME AND DESCRIPTION OF MODEL WATSUP

5.1 Introduction

In Chapter 1 the importance of studying the effect that the proposed new well system will have on the aquifer was emphasized. It was noted that the excessive drawdown of the piezometric contours due to the new system of wells should not adversely affect the existing well system. In this chapter a predictor - corrector two level coordination scheme is proposed to study additional drawdown due to the new well system and to satisfy local pressure requirements. This chapter also includes a description of the model WATSUP. This model is developed in the framework of the two level coordination scheme, based on the finite element analysis of groundwater flow discussed in Chapter 3 and on the mixed integer programming formulation described in Chapter 4 for the optimization of pipe sizes, wells and reservoir locations,and flow values.

5.2 Two Level Coordination Scheme

There are different pressures to be maintained at different points of a distribution system. The pressure requirements are based on the following needs:

(1) to supply water to high rise buildings

(2) to have sufficient pressure for direct fire hydrant service

(3) to support domestic needs i.e. automatic sprinkler service, etc.

(4) to supply a marginal pressure to take care of sudden drafts and offset losses due to clogging, etc.

Having chosen a well field and a distribution reservoir system, there are two preferred ways to increase the pressure:

(i) to provide booster pumps

(ii) to change the elevation and/or locations of various reservoirs

Obviously these two solutions incur extra expenditures. Hence, whenever a new system of wells and reservoirs is proposed a pressure check is to be made for adequacy. The alterations of elevations and locations of wells and reservoirs, naturally lead to an iterative cost comparison scheme. This is called, the first level coordination or local coordination, shown in Figure 5.1.

The decision maker proposes several trial locations for the wells and reservoirs based on the existing piezometric contour pattern and the demand points. Among these preferred locations, the set with the least cost arrangement is chosen with the aid of an optimization scheme. This particular set of wells and reservoir locations is checked for pressure. If the required pressure limits are satisfied the local coordination is complete. Otherwise the decisionmaker is required to change the trial locations until the pressure check is satisfied.

As mentioned earlier in regional planning one has to study the aquifer response for the proposed new system of wells. The additional drawdown and lowering of piezometric contours due to the system of new wells, may affect the existing well system. The new well system should be located in such a way that the existing system of well fields is not adversely affected.

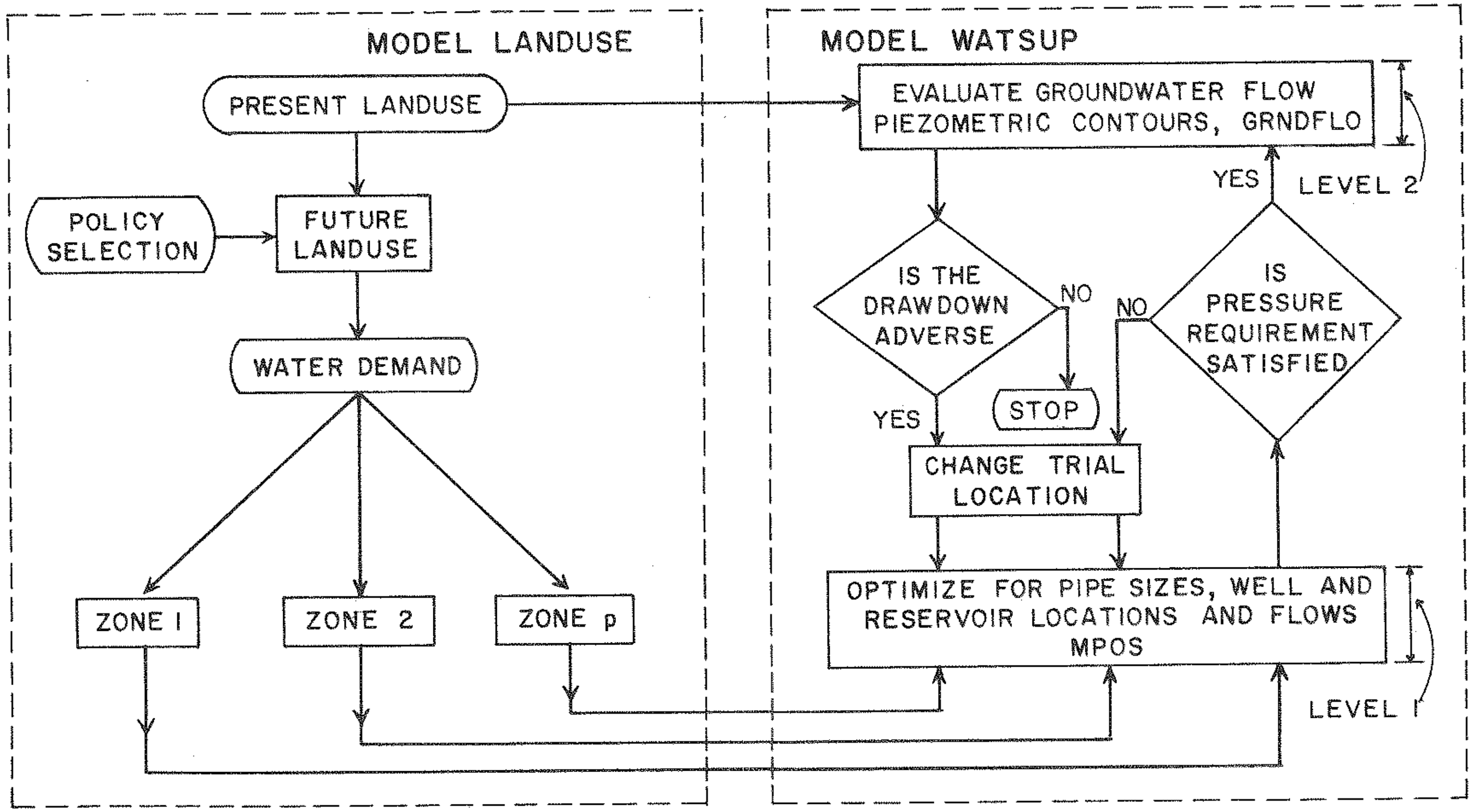


Figure 5.1. Schematic Representation of Two Level Coordination Scheme

This effect can be studied by imposing the new system of wells along with the existing wells in the groundwater piezometric contour evaluation. A comparison of the new piezometric contours with the old one (corresponding to the existing well system alone) will show the influence of the new well system. This is the second level coordination or global coordination, Figure 5.1. Once the local coordination is passed, the proposed new locations for the wells are incorporated in the groundwater piezometric contour evaluation. The resulting piezometric contours are checked for the excessive drawdown. If the effect of lowering piezometric contours is adverse, the trial fields are relocated to start afresh from the local coordination level, Figure 5.2.

5.3 Description of Model WATSUP

The model WATSUP includes a groundwater model subroutine GRNDFLO and an optimization system routine MPOS. The flow chart of model WATSUP and its interaction with the model LANDUSE is shown in Figure 5.3.

5.3.1 Groundwater Model GRNDFLO

The model GRNDFLO uses linear triangular elements for the finite element solution. The region of interest is divided into coarse quadrilateral subregions. An automatic mesh generation scheme subdivides these subregions into finer triangular elements. This information is used to fix the nodes of the various source (recharge) and sink (wells) points. The coordinates of various nodes, the hydraulic conductivity data and recharge-pumpage data are input to GRNDFLO.

The model GRNDFLO functions on the theory discussed in Chapter 3. It yields the piezometric head contours. The matrix [K] of equation (3.40) has non-zero entries lying in a band and the entries outside the

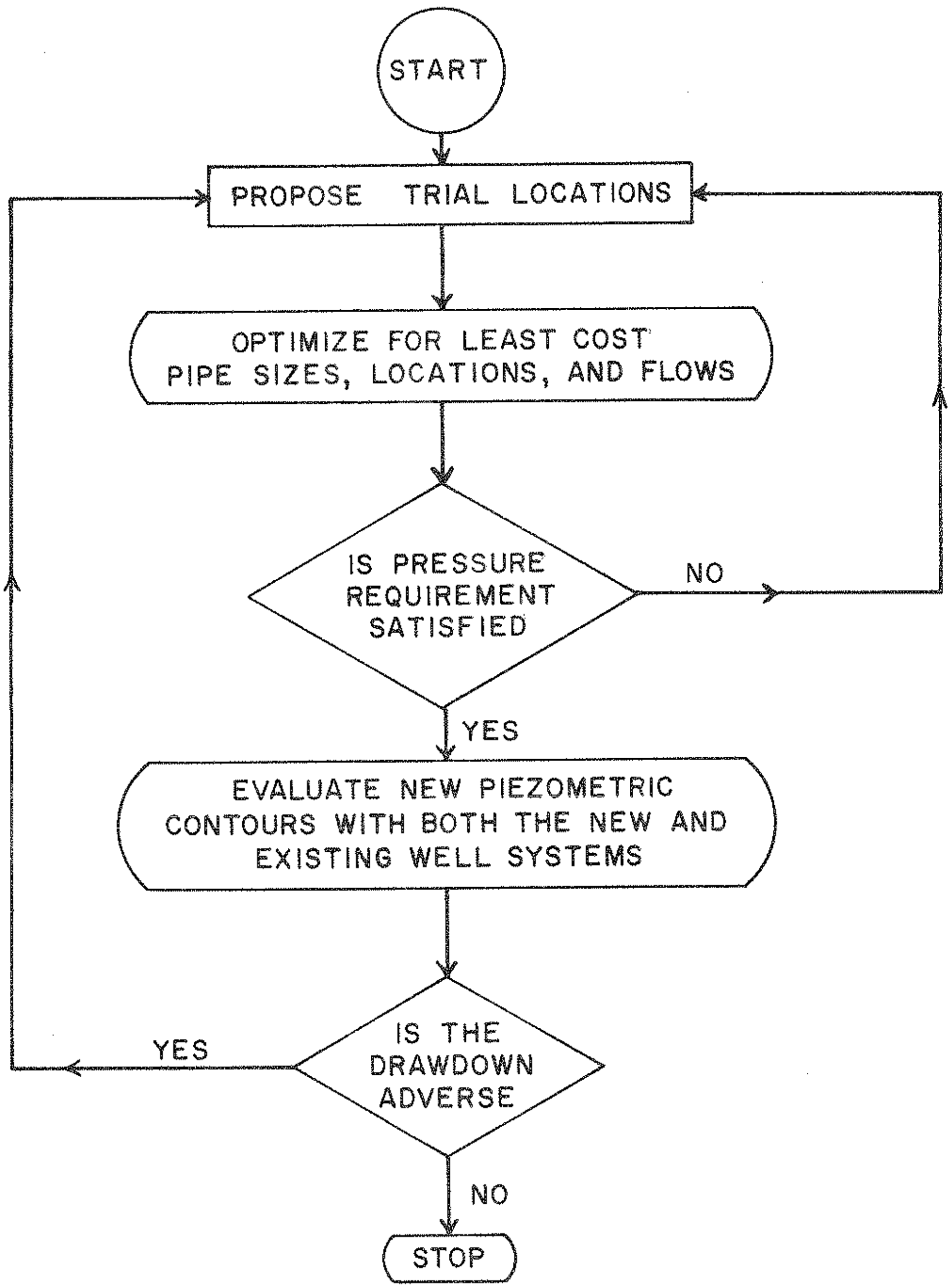


Figure 5.2. Flow Chart of Two Level Coordination Scheme

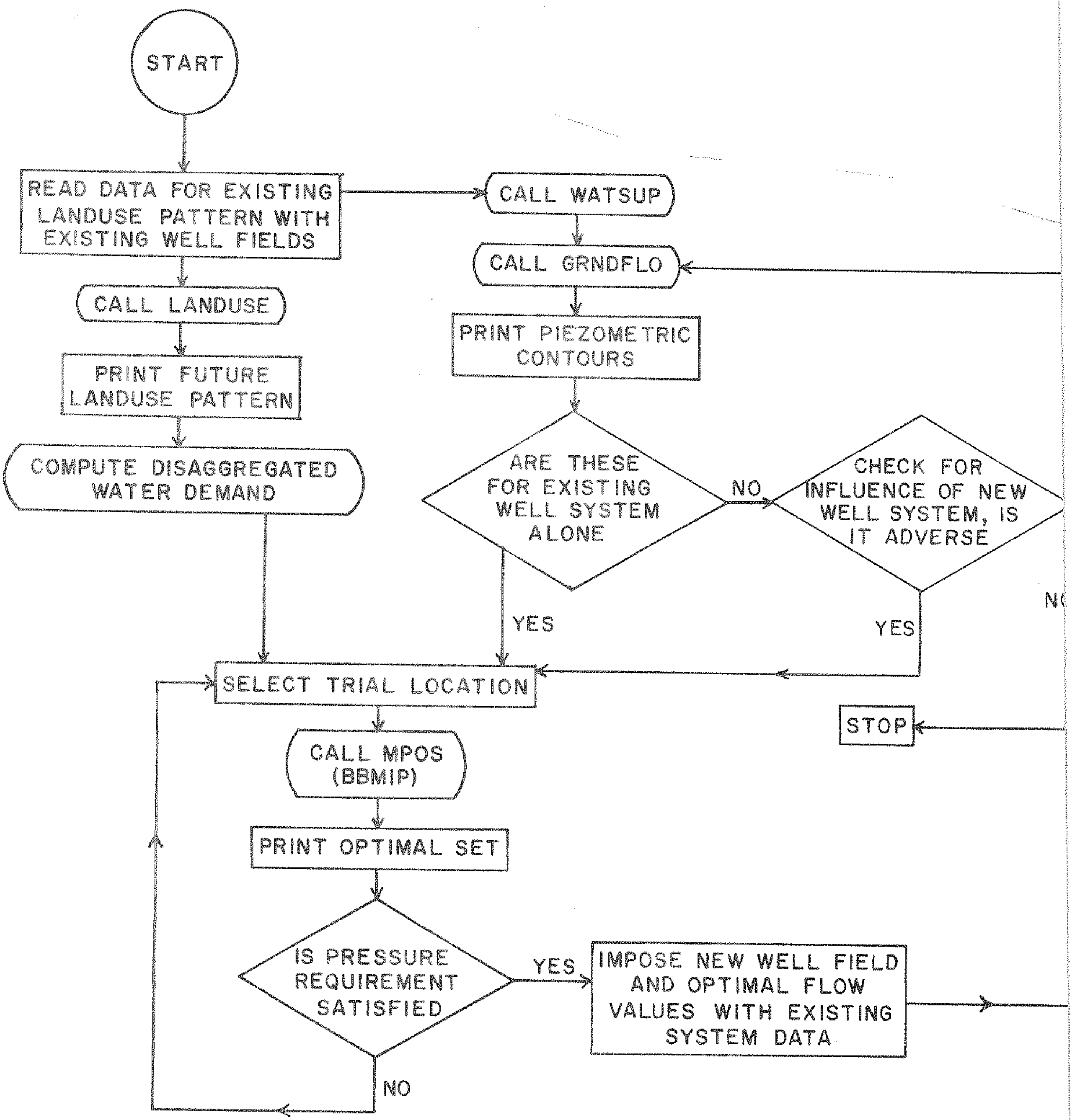


Figure 5.3. Flow Chart of WATSUP with LANDUSE

band are zeros. The model GRNDFLO takes advantage of this configuration and stores the entries of matrix [K] in a rectangular array, even though [K] is a square matrix, resulting in less memory storage. Portions of GRNDFLO are based on program written by Segerlind (Chapter 18, 1976).

5.3.2 Integration of GRNDFLO with LANDUSE

The model LANDUSE processes the existing landuse pattern as the input and predicts the future landuse activities. Water demand is computed as an equivalent of projected landuse activities and is disaggregated into demand zones based on future landuse pattern.

The model GRNDFLO is run with the input of the existing system of wells to obtain existing piezometric head contours. The predicted water demand (from future landuse pattern of the model LANDUSE) and the generated piezometric contours for the existing system of wells (from GRNDFLO) enable the decision maker to judiciously select locations for the new wells and for equalizing reservoirs. The decision maker may select the higher piezometric levels for the well field locations and both well and reservoir locations can be chosen as close to demand zones as the topography of the study region permits.

5.3.3 Multipurpose Optimization System (MPOS) and Mixed Integer Programing (MIP) Code

The model WATSUP calls the MPOS routine to obtain the optimal locations for the wells and reservoirs, sizes of the pipes, and flow values. The MPOS, MIP code uses Branch and Bound algorithm (BBMIP) for the optimization. Since the method examines the various branches of the solution tree, a substantial memory and computer time may be required. Once the trial locations are selected, the MPOS routine is used to select the optimal locations and optimal flow values with optimal pipe sizes. The optimal well and reservoir locations are checked for pressure needs, the first level coordination. If the pressure needs are satisfied, the second level coordination is sought. Otherwise new trial locations are selected and the optimization scheme is run again. This process is repeated until the pressure criterion is satisfied.

For the second level coordination, the new system of wells are identified with the nodes of the groundwater model GRNDFLO. New pumpage data is also fed in, in accordance with the optimal flow values of the new wells. The new piezometric contours are obtained from GRNDFLO. The discrepancy between the new one and the old one (Corresponding to the existing system alone) are checked with the permissible limits. If the effect is excessive, the decision maker is required to select new trial locations. The whole process is repeated until the effect of the new well system on the existing well system is within allowable limits.

5.4 Outline of the Methodology

In Chapter 5 a method has been devised to coordinate the different aspects of the solution procedure. Chapter 2 describes a method for estimation and disaggregation of the water demand. Chapter 3 describes the procedure to analyze the drawdowns due to pumping of wells. Chapter 4 presents the optimization procedure for a water supply system. Chapter 5 coordinates these different solution aspects of the water supply system to obtain the global optimal solution.

CHAPTER 6

APPLICATION

6.1 Introduction

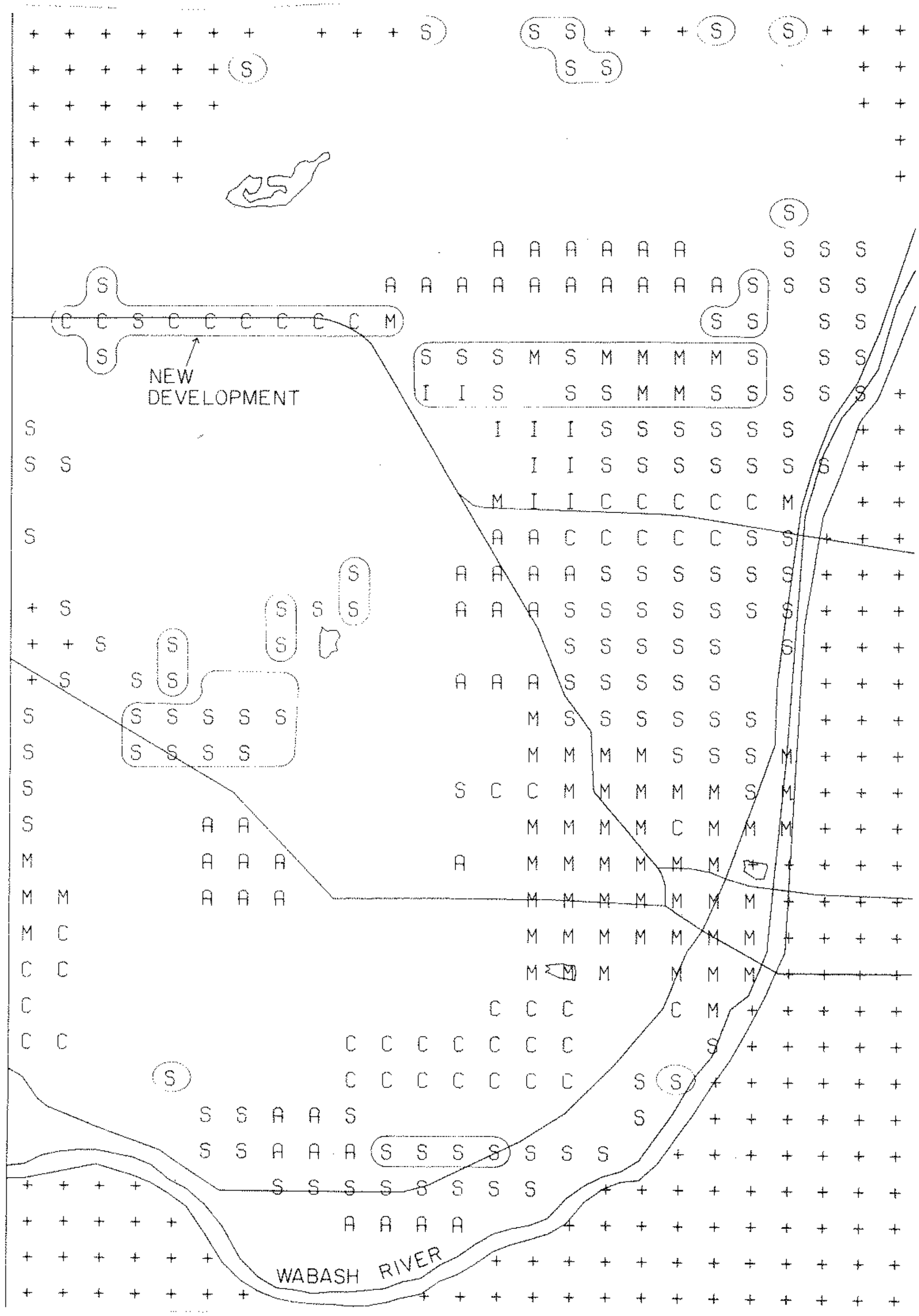
The methodology developed in the previous chapters is used for West Lafayette, Indiana. West Lafayette is situated 60 miles northwest of Indianapolis, the state capital, and 126 miles southeast of Chicago. West Lafayette is approximately at the center of Tippecanoe County, located in west-central Indiana. West Lafayette's water supply is from its groundwater. Regarding subsurface geology, Tippecanoe county had three major episodes of Pleistocene continental glaciation. The three major drift sheets are Pre-Illinoian, Illinoian and Wisconsinan age. Unconsolidated glacial deposits of the county range in thickness from 0 to 450 feet with average about 200 feet. The glacial drift contains significant aquifers. The Pre-Illinoian, Illinoian, and Wisconsinan outwash aquifers are the major sources of groundwater in the county. The Pre-Illinoian aquifer is extensive and is covered by Illinoian till which acts as a confining bed. A short description of the geology of Tippecanoe County can be found in Delleur et al. (1976). For the present analysis the wells are assumed to be located in confined aquifer.

There are three major pumping centers in the study area. These are: (1) Lafayette Water Works, (2) The West Lafayette Water Company, and (3) Purdue University. The first two meet the city requirements and Purdue University accounts for the campus requirements. Purdue University is the major employer in West Lafayette. The growth of West Lafayette primarily depends on Purdue University. Purdue Research Foundation (PRF) plans the growth of the univeristy which in turn affects the city's growth. The above account is a bird's-eye view of the situation in West Lafayette.

6.2 Land Use Projection and Water Demand

PRF owns the major share of land in and around West Lafayette. PRF proposes a scheme of areas reserved for light industrial development, commercial development, as well as residential areas and open space. Based on the population estimate of 25,000 and the PRF policy, it is estimated that by A.D. 2000 West Lafayette will require 76 hectars of multi-family residential areas, 544 hectars of single family dwelling, 30 commercial acres, and 2 industrial acres. The model LANDUSE is run for the above mentioned future demand, which is in accordance with PRF policy. The present land use activities and future land use allocations are shown in Figure 6.1. Detailed accounts of modeling LANDUSE with different policies can be found in a previous report by Dendrou, Delleur, and Talavage (1978a).

The future land use allocations are divided into 3 zones as shown in Figure 6.2. In dividing these zones a few allocations belonging to the southern and northern extremities of the city have been omitted because of the presumption that they can be incorporated in the existing system design for the water supply needs. The water demand is computed zonewise, as an equivalent of land use allocations in zones 1, 2, and 3. The zonewise water demand is shown in Table 6.1.



LEGEND

= non-occupied lot
+ = dummy lot
I = industrial development (1)
O = projected development
C = commercial development (2)
M = multiple family dwelling (3)
S = single family dwelling (4)
A = agricultural area (5)

Figure 6.1 Existing and Projected Land Use Patterns

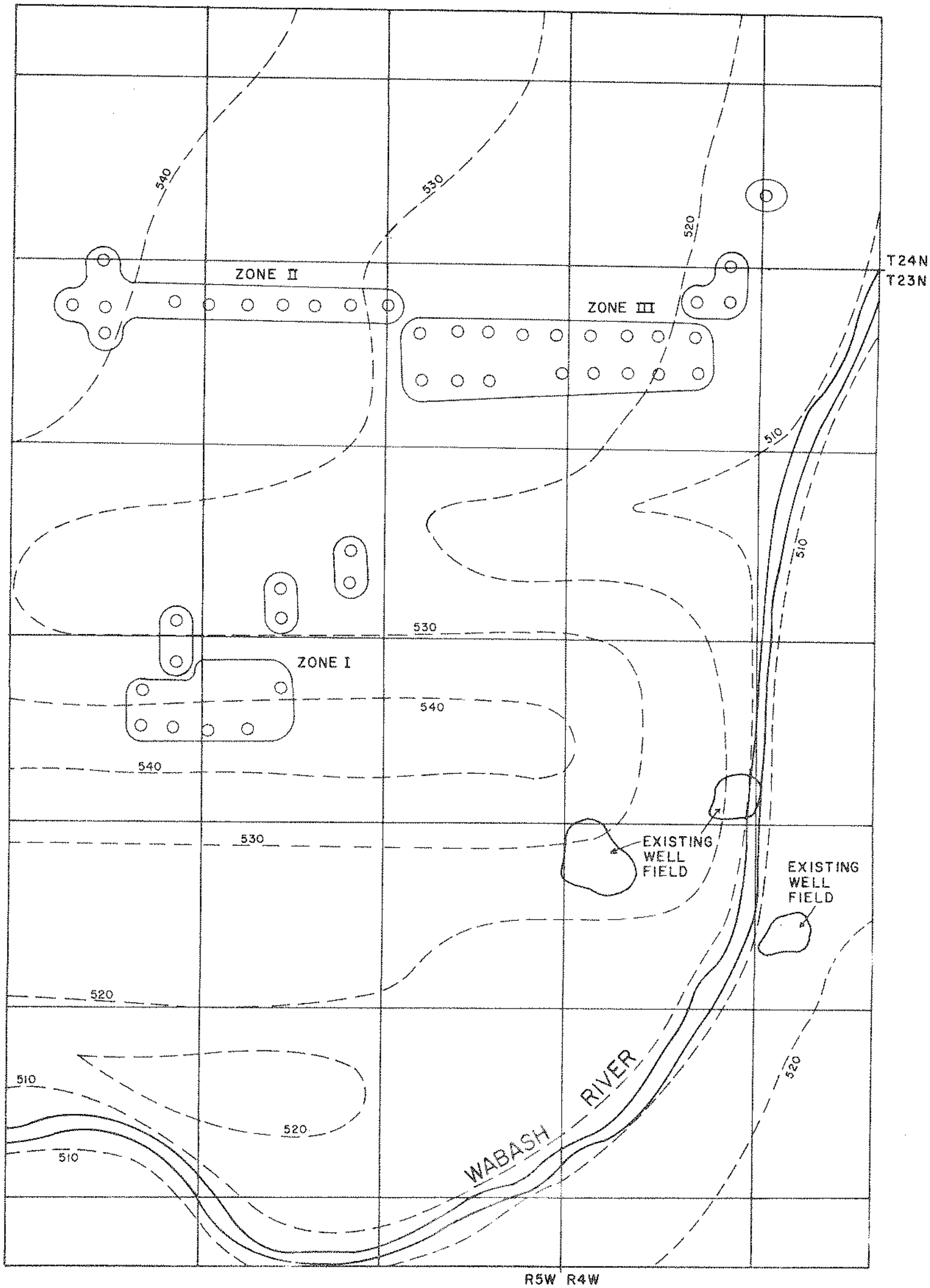


Figure 6.2 Piezometric Contours for the Existing Well System

Table 6.1 Zonewise Water Demand

Zone	Water Demand ft^3/day
1	44,290
2	19,730
3	122,566

6.3 Groundwater Flow Modeling

The groundwater subroutine GRNDFLO of the model WATSUP requires three types of data:

(i) geological data

(ii) hydrological data

(iii) pumping data.

The geological data provide information regarding the types of aquifers involved, and location of wells by township, range and section numbers. Most of the geological data are obtained from Marrouf and Melhorn (1975). In regard to hydrological data, in addition to rainfall there are several recharge areas in the study area. The Purdue gravel pit, for example, has considerable recharge to the groundwater basin. The Wabash River base-flow and flow across the boundaries of the study region constitute an important part in the hydrologic data. The study region has an average hydraulic conductivity of 230 ft/day. The yield of the confined aquifer of the region is 2 ft^3/sec. The pumpage information and the above mentioned hydrological information are obtained from Bathala, Spooner and Rao (1976), and from private communications with the water companies and Purdue University physical plant. The data are listed in Table 6.2. The model GRNDFLO was run with the above data. The resulting piezometric contours are shown in Figure 6.2.

Table 6.2 Data for Groundwater Model

S. No.	Item	Recharge [mgd]	Discharge [mgd]
1	Rainfall	17.4	
2	Purdue Gravel Pit	15.9	
3	Other ponds	2.2	
4	Wabash River (baseflow)		54.2
5	Flow across boundaries	31.5	0.1
6	Pumpage		15.6

6.4 Optimization Scheme

Based on the demand locations, Zones 1, 2 and 3, and on the existing piezometric contours of Figure 6.2, the trial well and reservoir fields are located in each zone. The necessary information is shown in Tables 6.3-6.8 with the calculations. A pumping duration of 15 hours, an interest rate of 13%, and a planning horizon of 20 years are used in the computations. A commercially available Multi-Purpose Optimization System (MPOS) has been used for the MIP code making use of the Branch and Bound algorithm. The optimal solution for the problem is given below:

optimal value = $222,626.29

Zone 1: - Total # of wells at site 1, $W_1 = 1$

- # of wells which pump from well field at site 1 to reservoirs at site 1, $W_{11} = 1$
- $W_{12} = W_{13} = 0$
- # reservoirs at site 1 of size 3, $R_{13} = 1$
- $R_{11} = R_{12} = 0$

- Pipe of size index 1* carrying flow from reservoirs at site 1 to demand Zone 1, $R^1Z_1S_1 = 1$
- $R^1Z_2S_1 = 1$
- Rate of pumping from wells at site 1 to reservoirs at site 1, $QW^1R_1 = 1.186$ ft^3/sec
- $QW^1R_2 = QW^1R_3 = 0$
- Quantity of water released from reservoirs at site 1 to demand zone 1, $QR^1Z_1 = 44{,}290$ ft^3/day
- $QR^1Z_2 = 19{,}730$ ft^3/day
- $QR^1Z_3 = 0$

*size index 1 means 12" pipe, size index 2 means 16" pipe.

Zone 2: - $QR^1Z_2 = 19{,}730$ ft^3/day, $R^1Z_2S_1 = 1$

Zone 3: - $W_3 = 2$, $W_{33} = 3$, $R_{33} = 2$, $R^3Z_3S_1 = 1$, $QW^3R_3 = 2.27$ ft^3/sec

- $QR^3Z_3 = 122{,}566$ ft^3/day.

6.5 Two Level Coordination

The optimal solution is checked for pressure criterion and is found to be satisfactory. In the second level it is decided to check for fire fighting water demand too. The rate of flow for fire fighting as specified by the American Insurance Association (1969) is:

$$Q = 1020 \sqrt{P}\,(1 - 0.1 \sqrt{P})$$

in which Q is the rate of flow in gpm and P is the population in thousands. The fire demand for each zone are given in Table 6.9.

The demand can be met in three ways:

(i) Provide new wells and reservoirs for the flow requirement and provide booster pumps for pressure requirement.

(ii) Provide new wells and reservoirs of required elevation to meet not only the flow requirements but also the pressure needs.

(iii) Provide new wells in each zone and pump directly into the water mains during the period of fire in each zone.

In the case of West Lafayette the high costs of reservoirs lead to the rejection of choices (i) and (ii). With a yield of 2 ft^3/sec or 900 gpm 3 wells are required to meet the 2100 gpm fire flow requirement for Zone 1. Similarly 2 and 4 wells are required for Zones 2 and 3, respectively. These are added to the number of water supply wells obtained by the MIP solution. This is the worst possible situation. The model GRNDFLO is run with the new system of water supply and fire fighting wells. The new piezometric contours are shown in Figure 6.3. It is found that the existing system of wells will not be adversely affected by the new wells.

Table 6.3 Annualized Costs

$$R = P * (1+r)^n \left[\frac{r}{(1+r)^n - 1}\right] \qquad R = P * (1.13)^{20} * \left\{\frac{0.13}{(1.13)^{20} - 1}\right\}$$

$$= 0.14P$$

Description	R Annualized Cost [$]
Pump and well at zone 1	8,050
Pump and well at zone 2	8,120
Pump and well at zone 3	8,680
26,600 ft^3 reservoir	25,200
39,900 ft^3 reservoir	32,900
66,500 ft^3 reservoir	44,800
12" diameter pipe/unit length	2.51
16" diameter pipe/unit length	3.49

Table 6.4 Length of Pipes (ft) Between Reservoirs (R) and Demand Zones (Z) and Between Well Fields (W) and Reservoir Fields (R)

		(Z_1)	(Z_2)	(Z_3)
		R_1	R_2	R_3
(R_1)	W_1	1,000	10,000	12,000
(R_2)	W_2	10,000	1,000	8,000
(R_3)	W_3	10,000	6,000	1,000

Table 6.5 Annualized Cost of Pipes ($)

Total Cost = cost/unit length * Total length

= $2.51 × 1000 = $2510

		(Z_1)	(Z_2)	(Z_3)
		R_1	R_2	R_3
(R_1)	W_1	2,510 (3,490)*	25,100 (34,900)	30,120 (41,880)
(R_2)	W_2	25,100 (34,900)	2,510 (3,490)	20,080 (27,920)
(R_3)	W_3	25,100 (34,900)	15,060 (20,940)	2,510 (3,490)

* Quantities in parentheses indicate costs for 16" pipe. Otherwise 12" pipe.

Table 6.6 Total Head Distribution

$H_T = H + k^* \ell$

$$k^* = \frac{16\ f\ Y^2}{2g\ \pi^2\ d^2}$$

Well design requires d to be 12".

$$k^* = \frac{16 \times 0.03 \times 2^2}{2 \times 32.2 \times 3.14^2 \times 1^2} = 0.002$$

	R_1	R_2	R_3
W_1	212	230	264
W_2	235	217	261
W_3	245	237	257

Table 6.7 Annualized Operating Costs ($)

Present Unit Power Cost = 2.46 cents*

$$\text{Present Annual Power Cost (\$)} = \frac{2.46}{100} \times \frac{\gamma Q H_T}{737 \times .9} \times 365 \times \delta t$$

For Duration of Pumping δt = 15 hours

Present Annual Power Cost ($) = 12.17 QH_T

$$\text{Total Power Cost (\$) over the Planning Horizon} = \frac{nc}{1 + r} = \frac{20 \times 12.17\ QH_T}{1.13} = 215.4\ \text{Q}$$

Annualized Power Cost = 215.4 QH_T × 0.14 = 30.15 QH_T

	R_1	R_2	R_3
W_1	6392	6935	7959
W_2	7085	6543	7869
W_3	7387	7146	7749

*EPA-600/2-79-147a,b Managing Small Water Systems: A Cost Study - Vols. I&II

Table 6.8 Values of m*

$$m^* = (1.887)^{8/3} N\left(\frac{\ell}{H}\right)^{\frac{1}{2}} \quad \text{for } N = 0.012$$

	Z_1	Z_2	Z_3
R_1	0.20	0.65	0.63
R_2	0.65	0.20	0.51
R_3	0.78	0.61	0.20

Table 6.9 Fire Fighting Water Demand

Zone	Fire Flow Durations [hours]	Rate [gpm]	Total Demand	Head [ft]
1	6	2,100	756,000	115
2	4	1,400	336,000	175
3	10	3,250	1,950,000	175

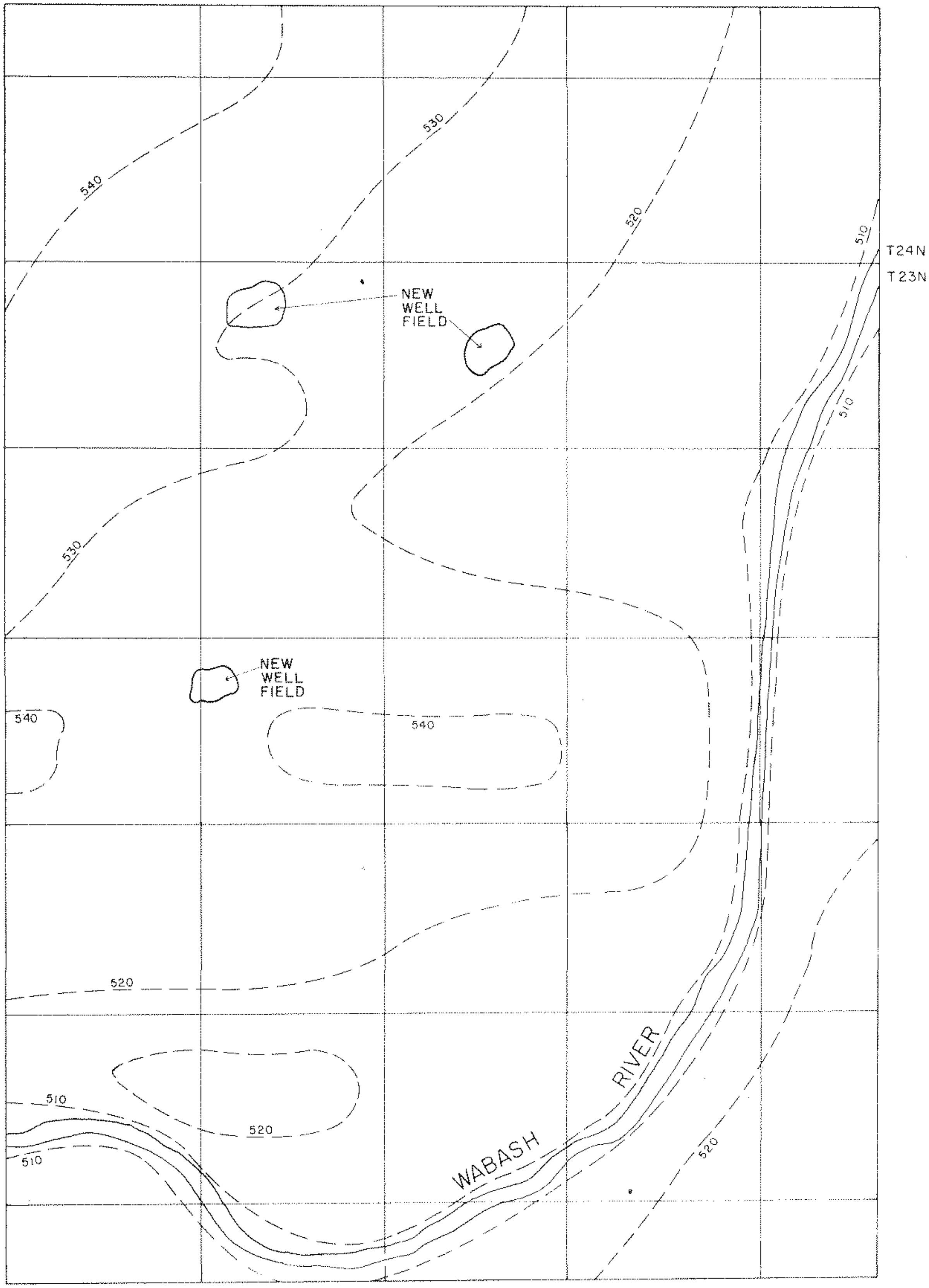


Figure 6.3 Piezometric Contours for the Proposed Well System

CHAPTER 7

SUMMARY AND CONCLUSIONS

7.1 Summary of the Study

In the present work, a landuse allocation model has been successfully used for the prediction of spatial disaggregation of water demand. Theoretical foundations have been developed for solving the original nonlinear optimization problem with integer requirements on several variables into an equivalent integer linear programming problem. Use of Manning's formula in the head loss relationship naturally leads to optimal theoretical pipe diameters. This report has also emphasized the need to plan for future urban water supply systems and for a safe long term exploitation of the aquifer by taking into consideration explicitly the various patterns of urban growth. This emphasis resulted in the recognition of a multilevel coordination scheme, satisfying the pressure requirements at the local level; and guaranteeing safe exploitation of the aquifer at the global level.

7.2 Conclusions

(1) The model LANDUSE makes it possible to test and to compare alternate growth scenarios and their corresponding patterns of landuse.

(2) The multilevel coordination scheme provides a unified approach in linking the various facets of urban water resources.

(3) The multilevel coordination scheme provides comprehensive growth patterns of the land/water interface in urban areas.

(4) Finally from a technical stand point, multilevel coordination is a successful way to obtain a tractable solution to network problems which are inherently complex.

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APPENDIX A - OPTIMIZATION PROGRAM

Optimization Program: MPOS (Multi Purpose Optimization System) is an integrated system of computer programs to solve optimization problems on CDC 6000/CYBER computers.

The copyright of the program is vested with

Vogelback Computing Center
Northwestern University
Evanston, Illinois 60201, USA

The system permits the user to state the mathematical programming problem in English and algebraic notation. It has the following algorithms for solving Linear Programming, Integer Programming and Quadratic Programming problems.

LP algorithms:	REGULAR	2-phase simplex
	REVISED	revised simplex
	PREVISED	packed revised simplex
	DUAL	dual simplex
	MINIT	primal-dual algorithm
	GENERAL	generalized upper bounds
IP algorithms:	BBMIP	branch and bound mixed integer
	DSZLIP	direct search 0-1 integer programming
	GOMORY	Gomory's cutting plane
QP algorithms:	WOLFE	Wolfe's quadratic simplex
	BEALE	Beale's algorithm
	LEMKE	Lemke's complementary pivot algorithm
	SYMQUAD	Van de Panne and Whinston's symmetric algorithm

The BBMIP program was used in this research.

APPENDIX B - FINITE ELEMENT PROGRAM

```
      PROGRAM GRNDFLO (INPUT,OUTPUT,TAPE50,TAPE51,TAPE60=INPUT,TAPE61=OU
     1PUT)
C
C     ******************************************************
C
C                                         VERSION  DNGF778
C
C     PROGRAM  GRNDFLO   USES  A FINITE  ELEMENT  APPROACH
C     TO SOLVE  A  TWO  DIMENSIONAL GROUND   WATER  FLOW PROBLEM
C
C     ------------------------------------------------------
C
C      INPUT  PHYSICAL  PARAMETERS
C
C               HCX        HYDR. COND. IN X  DIRECTION
C               HCY        HYDR. COND. IN Y  DIRECTION
C
C      INPUT  MESH  CHARACTERISTICS
C
C               NE         NUMBER  OF  FINITE  ELEMENTS
C               NBN         NUMBER  OF NODES
C               NBDW        BANDWIDTH
C               NVL        NUMBER  OF  LOAD  VECTORS
C
C      ------------------------------------------------------
C
      DIMENSION NC(3), CDM(3,3), D1(3), D2(3), PSI(3), ST(13100)
      COMMON /TLE/ TITLE(20)
C
C
      IN=60
      IO=61
      NVL=1
      IDV=0
      IEL=50
C
C
      NDIM=3
      NLIMIT=13100
C
C      READ  INPUT  CHARACTERISTICS
C
C     ------------------------------------------------------
C
      CALL INPCH (IN,NBN,NE,NBDW,HCX,HCY,IRH,ISLH,ILIM,NVL,ST,NLIMIT,IO)
C
C     ------------------------------------------------------
C
C      EVALUATION  OF  STIFFNESS  MATRIX
C
C     ------------------------------------------------------
C
      CALL STIFF (CDM,D1,D2,NC,NDIM,ST,NLIMIT,X1,Y1,X2,Y2,X3,Y3,NBN,HCX,
     1HCY,ISLH,IO,IEL,NE)
C
C     ------------------------------------------------------
C
C
      REWIND IEL
C
C      EVALUATION  OF  RIGHT   HAND SIDE  (LOAD VECTORS)
C
C     ------------------------------------------------------
C
      CALL RGHSIDE (ST(IRH+1),NBN,NVL)
C
C     ------------------------------------------------------
C     INSERT   BOUNDARY  CONDITIONS
C
C     ------------------------------------------------------
C
      CALL BDCON (ST(ISLH+1),ST(IRH+1),NBN,NBDW,NVL)
C
C     ------------------------------------------------------
C
C
C      RESOLUTION  OF  THE  LINEAR  SYSTEM  OF  EQUATIONS
C
C     ------------------------------------------------------
C
```

```
      CALL TRIGM (ST(ISLH+1),NBN,NBDW)
      CALL SOLVE (ST(ISLH+1),ST(IRH+1),ST(1),NBN,NBDW,NVL,IDV)
C
C     ------------------------------------------------------------
C
C
C      EVALUATION   OF  VELOCITIES AT  EACH  ELEMENT
C
C
C     ------------------------------------------------------------
C
      CALL VELOC (D1,D2,NDIM,ST,NLIMT,NE,TITLE,NC,PSI,HCX,HCY,IEL,IO)
C
C     ------------------------------------------------------------
C
      STOP
C
      END
      SUBROUTINE INPCH (IN,NBN,NE,NBDW,HCX,HCY,IRH,ISLH,ILIM,NVL,ST,NLIM
     1IT,IO)
C
C     THIS  SUBROUTINE  READS  THE  INPUT  CHARACTERISTICS  AND
C     DEFINES  INITIAL  AND LIMIT  PARAMETERS
C
C       *************************************************************
C
      DIMENSION ST(NLIMIT)
C
C
      DIMENSION TITLE(20)
C
      READ (IN,102) TITLE
      READ (IN,103) NBN,NE,NBDW,HCX,HCY
C
C     INITIALIZATION  STAGE
C
      IRH=NBN*NVL
      ISLH=IRH*2
      ILIM=ISLH+NBN*NBDW
      DO 101 I=1,ILIM
  101 ST(I)=0.0
C
C
      WRITE (IO,106)
      WRITE (IO,105) TITLE
      WRITE (IO,104) HCX,HCY
C
C     WRITE(IO,5)
C
C     5    FORMAT (                                              1X,78HNE
C     1  NODE NUMBER        X(1)       Y(1)       X(2)       Y(2)       X(3)
C     2      Y(3)    )
C
      RETURN
C
  102 FORMAT (20A4)
  103 FORMAT (3I3,1X,2F10.5)
  104 FORMAT (/1X,5HHCX   ,F9.1/1X,5HHCY   ,F9.1//)
  105 FORMAT (/1X,20A4/)
  106 FORMAT (1H1)
C
      END
      SUBROUTINE STIFF (CDM,D1,D2,NC,NDIM,ST,NLIMIT,X1,Y1,X2,Y2,X3,Y3,NB
     1N,HCX,HCY,ISLH,IO,IEL,NE)
C
C      *************************************************************
C
C      THIS  SUBROUTINE  EVALUATES  THE  STIFFNESS   MATRIX
C      AND  STORES  IT  IN  VECTOR  ST(
C
      DIMENSION CDM(NDIM,NDIM), D1(NDIM), D2(NDIM), NC(NDIM), ST(NLIMIT)
C
      DO 104 KK=1,NE
         READ (60,105) NEL,NC,X1,Y1,X2,Y2,X3,Y3
         WRITE (IO,106) NEL,NC,X1,Y1,X2,Y2,X3,Y3
         D1(1)=Y2-Y3
         D1(2)=Y3-Y1
         D1(3)=Y1-Y2
         D2(1)=X3-X2
         D2(2)=X1-X3
```

```
          D2(3)=X2-X1
          AREA=(X2*Y3+X3*Y1+X1*Y2-X2*Y1-X3*Y2-X1*Y3)*2.
          DO 101 I=1,3
          DO 101 J=1,3
  101     CDM(I,J)=(HCX*D1(I)*D1(J)+HCY*D2(I)*D2(J))/AREA
C
C
      DO 104 I=1,3
          II=NC(I)
          DO 103 J=1,3
              JJ=NC(J)
              JJ=JJ-II+1
              IF (JJ) 103,103,102
  102         K1=ISLH+(JJ-1)*NBN+II
              ST(K1)=ST(K1)+CDM(I,J)
  103     CONTINUE
  104 CONTINUE
C
      RETURN
C
  105 FORMAT (4I3,6F10.4)
  106 FORMAT (1X,I3,2X,3I4,1X,6(2X,F8.1))
C
      END
      SUBROUTINE RGHSIDE (RV,NBN,NUL)
C
C     ***********************************************************
C
C     THIS  SUBROUTINE  EVALUATES  THE  RIGHT  HAND  SIDE
C     OF  THE  LINEAR   SYSTEM  OF  EQUATIONS
C
      DIMENSION RV(NBN,NUL), IDD(6), BV(6)
      COMMON /TLE/ TITLE(20)
      DATA IN/60/,IO/61/,INFL/51/
      WRITE (IO,108) TITLE
C
      WRITE (IO,109)
      DO 107 JM=1,NVL
          ID1=0
          INK=0
          II2=(NBN/6)+1
  101     READ (IN,110) IDD,BV
          DO 102 L=1,6
  102     BV(L)=BV(L)*1440.*0.133
          ID=0
          DO 103 L=1,6
              IF (IDD(L).LE.0) GO TO 104
              ID=ID+1
              I=IDD(L)
  103     RV(I,JM)=BV(L)+RV(I,JM)
          GO TO 105
  104     INK=1
          IF (ID.EQ.0) GO TO 107
  105     IF (ID1.EQ.1) GO TO 106
          WRITE (IO,111) JM
  106     WRITE (IO,112) (IDD(L),BV(L),L=1,ID)
          IF (INK.EQ.1) GO TO 107
          ID1=1
          GO TO 101
  107 CONTINUE
C
      RETURN
C
  108 FORMAT (1H1,///////,1X,20A4)
  109 FORMAT (/1X,15HBOUNDARY VALUES//1X,12HNODAL  LOADS)
  110 FORMAT (6I3,2X,6F10.5)
  111 FORMAT (1X,12HLOADING CASE,I2)
  112 FORMAT (1X,6(I3,E14.5,2X))
C
      END
      SUBROUTINE BDCON (SLM,RV,NBN,NBDW,NUL)
C
C     *************************************************************
C
C     THIS  SUBROUTINE  INTRODUCE  THE  BOUNDARY  CONDITIONS
C     AND  REDUCES   ADEQUATELY  THE  LINEAR  SYSTEM  OF  EQUAT.
C
      DIMENSION SLM(NBN,NBDW), RV(NBN,NUL), IB(6), BV(6)
      DATA IN/60/,IO/61/,INFL/51/
C
```

```
      WRITE (IO,112)
      INK=0
  101 READ (IN,110) IB,BV
      ID=0
      DO 107 L=1,6
         IF (IB(L).LE.0) GO TO 108
         ID=ID+1
         I=IB(L)
         BC=BV(L)
C
C       REDUCTION  STAGE
C
         K=I-1
         DO 105 J=2,NBDW
            M=I+J-1
            IF (M.GT.NBN) GO TO 103
            DO 102 JM=1,NUL
  102       RV(M,JM)=RV(M,JM)-SLM(I,J)*BC
            SLM(I,J)=0.0
  103       IF (K.LE.0) GO TO 105
            DO 104 JM=1,NUL
  104       RV(K,JM)=RV(K,JM)-SLM(K,J)*BC
            SLM(K,J)=0.0
            K=K-1
  105    CONTINUE
         IF (SLM(I,1).LT.0.05) SLM(I,1)=500000.
         DO 106 JM=1,NUL
  106    RV(I,JM)=SLM(I,1)*BC
  107 CONTINUE
      GO TO 109
C
C
  108 INK=1
      IF (ID.EQ.0) RETURN
  109 WRITE (IO,111) (IB(L),BV(L),L=1,ID)
      IF (INK.EQ.1) RETURN
      GO TO 101
C
  110 FORMAT (6I3,2X,6F10.5)
  111 FORMAT (1X,6(I3,E14.5,2X))
  112 FORMAT (////,1X,24HPRESCRIBED NODAL  VALUES)
C
      END
      SUBROUTINE TRIGM (SLM,NBN,NBDW)
C
C     ***********************************************************
C
      DIMENSION SLM(NBN,NBDW)
      IO=61
      NBN1=NBN-1
      DO 102 I=1,NBN1
         MJ=I+NBDW-1
         IF (MJ.GT.NBN) MJ=NBN
         NJ=I+1
         MK=NBDW
         IF ((NBN-I+1).LT.NBDW) MK=NBN-I+1
         ND=0
         DO 101 J=NJ,MJ
            MK=MK-1
            ND=ND+1
            NL=ND+1
         DO 101 K=1,MK
            NK=ND+K
  101    SLM(J,K)=SLM(J,K)-SLM(I,NL)*SLM(I,NK)/SLM(I,1)
  102 CONTINUE
      RETURN
C
      END
      SUBROUTINE SOLVE (SLM,RV,X,NBN,NBDW,NUL,ID)
C
C     ***********************************************************
C
      DIMENSION SLM(NBN,NBDW), RV(NBN,NUL), X(NBN,NUL)
      COMMON /TLE/ TITLE(20)
      IO=61
      NBN1=NBN-1
      DO 104 KK=1,NUL
         JM=KK
C
C     DECOMPOSITION OF THE COLUMN VECTOR RV( )
```

```
C
      DO 101 I=1,NBN1
         MJ=I+NBDW-1
         IF (MJ.GT.NBN) MJ=NBN
         NJ=I+1
         L=1
      DO 101 J=NJ,MJ
         L=L+1
  101 RV(J,KK)=RV(J,KK)-SLM(I,L)*RV(I,KK)/SLM(I,1)
C
C     BACKWARD  SUBSTITUTION
C
      X(NBN,KK)=RV(NBN,KK)/SLM(NBN,1)
      DO 103 K=1,NBN1
         I=NBN-K
         MJ=NBDW
         IF ((I+NBDW-1).GT.NBN) MJ=NBN-I+1
         SUM=0.0
         DO 102 J=2,MJ
            N=I+J-1
  102    SUM=SUM+SLM(I,J)*X(N,KK)
  103 X(I,KK)=(RV(I,KK)-SUM)/SLM(I,1)
C
C     CALCULATED NODAL VALUES
C
      IF (ID.EQ.1) GO TO 104
      WRITE (IO,105) TITLE,KK
      WRITE (IO,106) (I,X(I,KK),I=1,NBN)
  104 CONTINUE
      RETURN
C
  105 FORMAT (1H1////1X,20A4//1X,26HNODAL VALUES, LOADING CASE,I2)
  106 FORMAT (1X,I3,E14.5,3X,I3,E14.5,3X,I3,E14.5,3X,I3,E14.5,3X,I3,E14.
     15)
C
      END
      SUBROUTINE VELOC (D1,D2,NDIM,ST,NLIMIT,NE,TITLE,NC,PSI,HCX,HCY,IEL
     1,IO)
C
C     ***********************************************************
C
C     THIS  SUBROUTINE  EVALUATES  THE  VELOCITIES  AT  THE
C     CENTROID  OF  EACH  ELEMENT
C
      DIMENSION D1(NDIM), D2(NDIM), ST(NLIMIT), NC(NDIM), PSI(NDIM)
C
      IN=60
      DO 104 IJ=1,NE
         READ (IN,105) NEL,NC,X1,Y1,X2,Y2,X3,Y3
C
         IF (NEL.LT.0) STOP
         IF (IJ.GT.1) GO TO 101
         WRITE (IO,106) TITLE
         WRITE (IO,107)
C
  101    DO 102 I=1,3
            II=NC(I)
  102    PSI(I)=ST(II)
C
C     CALCULATION OF THE VELOCITY COMPONENTS
C
         D1(1)=Y2-Y3
         D1(2)=Y3-Y1
         D1(3)=Y1-Y2
         D2(1)=X3-X2
         D2(2)=X1-X3
         D2(3)=X2-X1
         AR2=(X2*Y3+X3*Y1+X1*Y2-X2*Y1-X3*Y2-X1*Y3)
         GRADX=0.0
         GRADY=0.0
         DO 103 I=1,3
            GRADX=GRADX+D1(I)*PSI(I)/AR2
  103    GRADY=GRADY+D2(I)*PSI(I)/AR2
         VELX=-HCX*GRADX
         VELY=-HCY*GRADY
  104 WRITE (IO,108) NEL,VELX,VELY
C
C
      RETURN
C
```

```
  105 FORMAT (4I3,6F10.4)
  106 FORMAT (1H1////1X,20A4//1X,27HELEMENT VELOCITY   VECTORS //)
  107 FORMAT (5X,37HELEMENT         VEL(X)            VEL(Y))
  108 FORMAT (7X,I3,5X,E12.5,5X,E12.5)
C
      END
```

```
C**********************************************************************
C     THIS SUBROUTINE GENERATES A FINITE ELEMENT MESH FOR A GIVEN REGION
C     ------------------------------------------------------------------
C     COARSELY DIVIDED QUADRILATERAL SUB-REGIONS ARE FINELY DIVIDED INTO
C     TRIANGULAR ELEMENTS
C     ------------------------------------------------------------------
C
C     INPUT PARAMETERS FOR COARSE SUB-REGIONS
C*******************************************
C
C     INLC=NUMBER OF SUB-REGIONS
C     INBN=TOTAL NUMBER OF BOUNDARY NODES FOR THE WHOLE REGION
C      (8-NODES FOR EACH SUB-REGION)
C     ICOD=OUTPUT OPTION
C     XBN(I)=X-COORDINATE OF BOUNDARY NODE
C     YBN(I)=Y-COORDINATE OF BOUNDARY NODE
C     NLC=SUB-REGION NUMBER
C     KCM(I)=CONNECTIVITY DATA(SUB-REGIONS SURROUNDING SUB-REGION-I-)
C      (SURROUNDING SUB-REGIONS ARE NUMBERED COUNTER-CLOCKWISE)
C      (4-SUB-REGIONS CORRESPONDING TO THE 4-SIDES OF THE QUADRILATERAL)
C     NHR=NUMBER OF HORIZONTAL PARTITIONS INTENDED
C     NVER=NUMBER OF VERTICAL PARTITIONS INTENDED
C     NAB=BOUNDARY NODE NUMBERS CONSTITUTING THE SUB-REGION(8 NODES)
C      (START NUMBERING FROM LOWER LEFT-HAND CORNER AND PROCEED COUNTER
C      CLOCKWISE)
C
C     ------------------------------------------------------------------
C
      PROGRAM MAIN (INPUT,TAPE60=INPUT,OUTPUT,TAPE61=OUTPUT,TAPE62)
      DIMENSION TITLE(10), XBN(100), YBN(100), XRG(9), YRG(9), N(8), NAB
     1(8)
      DIMENSION NN(21,21), YC(21,21), XC(21,21), NSAV(20,4,21), KCM(20,4
     1)
      DIMENSION LB(3), NE(400), XE(400), YE(400), NR(4), ICOMP(4,4)
      REAL N
      DATA ICOMP/-1,1,1,-1,1,-1,-1,1,1,-1,-1,1,-1,1,1,-1/
      DATA IN/60/,IO/61/,IP/62/,NBDW/0/,NB/0/,NEL/0/
      READ (IN,109) TITLE
      READ (IN,110) INLC,INBN,ICOD
      READ (IN,111) (XBN(I),I=1,INBN)
      READ (IN,111) (YBN(I),I=1,INBN)
      DO 101 I=1,INLC
  101 READ (IN,112) NLC,(KCM(NLC,J),J=1,4)
      WRITE (IO,113) TITLE
      WRITE (IO,114) (I,XBN(I),YBN(I),I=1,INBN)
      WRITE (IO,115)
      WRITE (IO,116)
      DO 102 I=1,INLC
  102 WRITE (IO,117) I,(KCM(I,J),J=1,4)
      DO 108 KK=1,INLC
         READ (IN,118) NLC,NHOR,NVER,NAB
         WRITE (IO,119) NLC,NHOR,NVER,(NAB(I),I=1,8)
C
C     ------------------------------------------------------------------
C     THIS SUBROUTINE
C*******************
C     GENERATES GLOBAL COORDINATES
C**********************************************************************
C
         CALL GNODC (NHOR,TR,NVER,NAB,XRG,XBN,YRG,YBN,N,XC,YC)
C
C     ------------------------------------------------------------------
C     THIS SUBROUTINE
C*******************
C     GENERATES GLOAL NODE NUMBERING
C**********************************************************************
C
         CALL GRNNB (NLC,NHOR,NVER,KCM,NN,NSAV,ICOMP,KN1,KN2,KS1,KS2)
C
C     ------------------------------------------------------------------
C
         IF (KN1.GT.KN2) GO TO 107
         IF (KS1.GT.KS2) GO TO 107
         DO 103 I=KN1,KN2
         DO 103 J=KS1,KS2
            NB=NB+1
  103    NN(I,J)=NB
         DO 104 I=1,NVER
            NSAV(NLC,1,I)=NN(NHOR,I)
  104    NSAV(NLC,3,I)=NN(1,I)
```

```
      DO 105 I=1,NHOR
         NSAV(NLC,2,I)=NN(I,NVER)
  105    NSAV(NLC,4,I)=NN(I,1)
      WRITE (IO,120)
      DO 106 I=1,NHOR
  106 WRITE (IO,121) (NN(I,J),J=1,NVER)
  107 WRITE (IO,122)
C
C     -----------------------------------------------------------------
C     THIS SUBROUTINE
C*******************
C     CALCULATES BAND-WIDTH
C*********************************************************************
C
      CALL GTRELM (NHOR,NVER,NEL,NBDW,NELBW,XE,XC,YE,YC,NE,NN,NR,LB,I
     1COD,IO,IP)
C
C     -----------------------------------------------------------------
C
  108 CONTINUE
      WRITE (IO,123) NBDW,NELBW
      STOP
C
  109 FORMAT (10A8)
  110 FORMAT (3I3)
  111 FORMAT (8F10.4)
  112 FORMAT (5I3)
  113 FORMAT (1H1////1X,10A8//1X, 18HGLOBAL COORDINATES,//1X, 27HNUMBER
     1       XCORD     YCORD)
  114 FORMAT (2X,I3,7X,F8.2,5X,F8.2)
  115 FORMAT (//1X,17HCONNECTIVITY DATA/1X,41HREGION      SIDE     1
     12      3       4 )
  116 FORMAT (1X, 41H*****************************************,/)
  117 FORMAT (2X,I3,14X,4(I2,5X))
  118 FORMAT (11I3)
  119 FORMAT (1H1///1X,12H***  REGION ,I2,6H  ****//10X,I2,5H ROWS,10X,I
     12,7HCOLUMNS//10X,21HBOUNDARY NODE NUMBERS,10X,8I5)
  120 FORMAT (//1X,19HREGION NODE NUMBERS/)
  121 FORMAT (1X,20I5)
  122 FORMAT (//3X,17HNEL  NODE NUMBERS,9X,4HX(1),8X,4HY(1),8X,4HX(2),8X
     1,4HY(2),8X,4HX(3),8X,4HY(3))
  123 FORMAT (///1X,21HBANDWIDTH QUANTITY IS,I4, 31H   CALCULATED IN
     1    ELEMENT,I4)
C
      END
      SUBROUTINE GNODC (NHOR,TR,NVER,NAB,XRG,XBN,YRG,YBN,N,XC,YC)
      DIMENSION NAB(9), XRG(9), YRG(9), XBN(100), YBN(100), N(8)
      DIMENSION XC(21,21), YC(21,21)
      REAL N
      DO 101 I=1,8
         II=NAB(I)
         XRG(I)=XBN(II)
  101 YRG(I)=YBN(II)
      XRG(9)=XRG(1)
      YRG(9)=YRG(1)
      TR=NHOR-1
      DETA=2./TR
      TR=NVER-1
      DSI=2./TR
      DO 102 I=1,NHOR
         TR=I-1
         ETA=1.-TR*DETA
      DO 102 J=1,NVER
         TR=J-1
         SI=-1.+TR*DSI
         N(1)=-0.25*(1.-SI)*(1.-ETA)*(SI+ETA+1.)
         N(2)=0.5*(1.-SI**2)*(1.-ETA)
         N(3)=0.25*(1.+SI)*(1.-ETA)*(SI-ETA-1.)
         N(4)=0.50*(1.+SI)*(1.-ETA**2)
         N(5)=0.25*(1.+SI)*(1.+ETA)*(SI+ETA-1.)
         N(6)=0.5*(1.-SI**2)*(1.+ETA)
         N(7)=0.25*(1.-SI)*(1.+ETA)*(ETA-SI-1.)
         N(8)=0.5*(1.-SI)*(1.-ETA**2)
         XC(I,J)=0.
         YC(I,J)=0.
      DO 102 K=1,8
         XC(I,J)=XC(I,J)+XRG(K)*N(K)
  102 YC(I,J)=YC(I,J)+YRG(K)*N(K)
      RETURN
C
```

```
      END
      SUBROUTINE GRNNB (NLC,NHOR,NVER,KCM,NN,NSAV,ICOMP,KN1,KN2,KS1,KS2)
      DIMENSION KCM(20,4), NN(21,21), NSAV(20,4,21), ICOMP(4,4)
      KN1=1
      KS1=1
      KN2=NHOR
      KS2=NVER
      DO 107 I=1,4
         NRT=KCM(NLC,I)
         IF (NRT.EQ.0.OR.NRT.GT.NLC) GO TO 107
         DO 101 J=1,4
  101    IF (KCM(NRT,J).EQ.NLC) NRTS=J
         K=NVER
         IF (I.EQ.2.OR.I.EQ.4) K=NHOR
         JL=1
         JK=ICOMP(I,NRTS)
         IF (JK.EQ.-1) JL=K
         DO 106 J=1,K
            GO TO (102,103,104,105), I
  102       NN(NHOR,J)=NSAV(NRT,NRTS,JL)
            KN2=NHOR-1
            GO TO 106
  103       NN(J,NVER)=NSAV(NRT,NRTS,JL)
            KS2=NVER-1
            GO TO 106
  104       NN(1,J)=NSAV(NRT,NRTS,JL)
            KN1=2
            GO TO 106
  105       NN(J,1)=NSAV(NRT,NRTS,JL)
            KS1=2
  106    JL=JL+JK
  107 CONTINUE
      RETURN
C
      END
      SUBROUTINE GTRELM (NHOR,NVER,NEL,NBDW,NELBW,XE,XC,YE,YC,NE,NN,NR,L
     1B,ICOD,IO,IP)
      DIMENSION XE(400), XC(21,21), YE(400), YC(21,21), NE(400)
      DIMENSION NN(21,21), NR(4), LB(3)
      K=1
      DO 101 I=1,NHOR
      DO 101 J=1,NVER
         XE(K)=XC(I,J)
         YE(K)=YC(I,J)
         NE(K)=NN(I,J)
  101 K=K+1
      L=NHOR-1
      DO 105 I=1,L
      DO 105 J=2,NVER
         DIAG1=SQRT((XC(I,J)-XC(I+1,J-1))**2+(YC(I,J)-YC(I+1,J-1))**2)
         DIAG2=SQRT((XC(I+1,J)-XC(I,J-1))**2+(YC(I+1,J)-YC(I,J-1))**2)
         NR(1)=NVER*I+J-1
         NR(2)=NVER*I+J
         NR(3)=NVER*(I-1)+J
         NR(4)=NVER*(I-1)+J-1
      DO 105 IJ=1,2
         NEL=NEL+1
         IF ((DIAG1/DIAG2).GT.1.02) GO TO 102
         J1=NR(1)
         J2=NR(IJ+1)
         J3=NR(IJ+2)
         GO TO 103
  102    J1=NR(IJ)
         J2=NR(IJ+1)
         J3=NR(4)
  103    LB(1)=IABS(NE(J1)-NE(J2))+1
         LB(2)=IABS(NE(J2)-NE(J3))+1
         LB(3)=IABS(NE(J1)-NE(J3))+1
         DO 104 IK=1,3
            IF (LB(IK).LE.NBDW) GO TO 104
            NBDW=LB(IK)
            NELBW=NEL
  104    CONTINUE
         WRITE (IO,106) NEL,NE(J1),NE(J2),NE(J3),XE(J1),YE(J1),XE(J2),YE
     1   (J2),XE(J3),YE(J3)
         IF (ICOD.EQ.0) GO TO 105
         WRITE (IP,107) NEL,NE(J1),NE(J2),NE(J3),XE(J1),YE(J1),XE(J2),YE
     1   (J2),XE(J3),YE(J3)
  105 CONTINUE
      RETURN
```

```
C
  106 FORMAT (1X,4I5,3X,6F12.4)
  107 FORMAT (4I3,6F10.4)
C
      END
```

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