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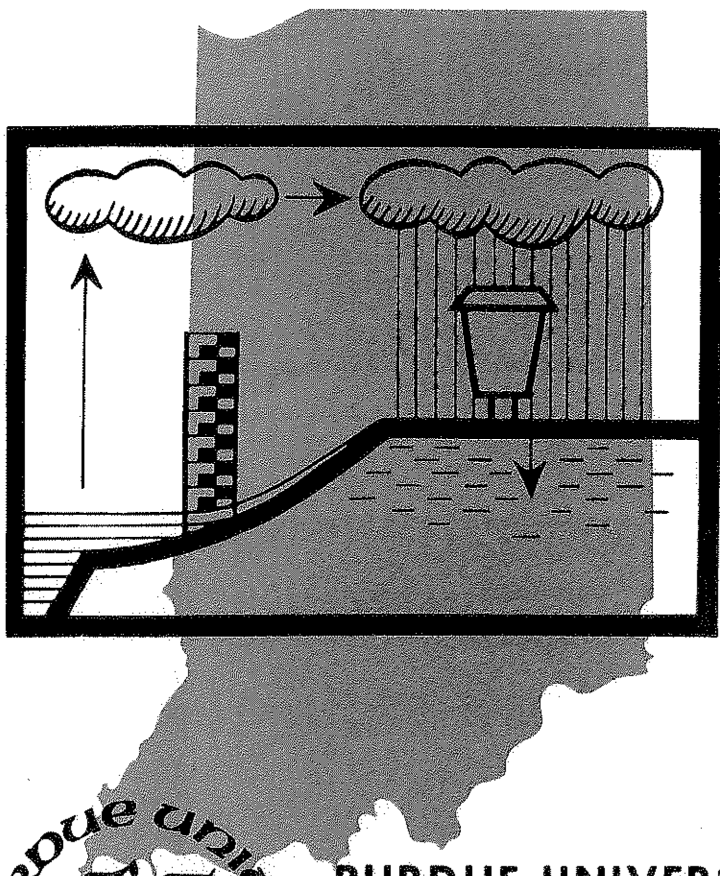
MULTIPLE OBJECTIVE PLANNING OF LAND/WATER INTERFACE IN MEDIUM SIZE CITIES: APPLICATION TO WEST LAFAYETTE, INDIANA



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March 1982

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PREFACE

The work presented in this report is closely related to that performed under project OWRT-B-083-IND entitled "Multilevel Approach to Urban Water Resources System Analysis-Application to Medium Size Cities." The present report is essentially the counterpart of Technical Report No. 131, entitled "Planning Ground Water Supply Systems for Urban Growth: Application to West Lafayette, Indiana, by G.V. Loganathan, J.W. Delleur and J.J. Talavage, which was concerned with the determination of the optimal location of water wells and of distribution reservoirs as well as optimal flow rates and pipe sizes for the water supply of a growing town. This report is concerned with the conflicting goals of land use expansion and the control of the quality of urban runoff effluent. A probability distribution approach is used to obtain the distribution of the overflows and of pollutant concentration levels in the receiving stream. At the planning level this approach is simpler than the simulation approach developed in the previous research project and reported in Technical Report No. 101 entitled "Urban Storm-Drainage Systems Planning" by S.A. Dendrou, J.J. Talavage and J.W. Delleur. The present reports also extends the theory of Multi Criteria Decision Making, and the application to West Lafayette shows that it is a viable tool which deserves further investigation.

TABLE OF CONTENTS

TABLE OF CONTENTS (continued)

LIST OF TABLES

LIST OF FIGURES

LIST OF FIGURES (continued)

Figure | Page

LIST OF SYMBOLS

Symbol	Chapter	Definition
A_j	II	area available in zone j
B_j	II	capital for zone j
C_R	II,III	concentration of pollutant in river
C_e	II,IV	concentration of effluent
C_m	II,IV	mixed concentration
C_o	II,IV	limiting concentration
C_{ij}	II	cost for land use i in zone j
$C_1(a)$	II	cost of treatment
$C_2(b)$	II	cost of storage
D	II	set of zones satisfying min. distance ρ
D_1	II	discrepancy set
M	II	large positive number
X_{ij}	II	number of land use activities of type i in zone j
X_i	II	total number of activities of type i
X_1	II,IV	runoff volume after urbanization
X_2	II,IV	duration of runoff event
X_3	II,IV	intermittent time
Y	II,IV	overflow volume
ZC_{kj}	II	element of zone characteristic vector

LIST OF SYMBOLS (continued)

Symbol	Chapter	Definition
a_i	II	area required for land use type i
d_{ij}^+	II	deviational variable
d_{ij}^-	II	deviational complement
d_{lm}	II	distance between zones l and m
m	II	total number of types of activities
n	II	total number of zones
u	II,IV	urbanization factor
w_j	II	weight of degree of disturbance
δ_{ij}	II	0-1 integer
ϵ	II	risk of pollution
T	III	objective space
U	III	utility function
X	III	feasible region
Y	III	special set $\{y: y \leqq f(x) \text{ for some } x \epsilon X\}$
Z	III	gradient objective function
f	III	vector objective function
g	III	constraint function
h	III	tradeoff constraint
x	III	feasible solution
w_i	III	tradeoff value
y	III	element of the set Y
A	IV	area of watershed
BOD_ℓ	IV	ultimate BOD
C_R	IV	concentration of pollutant in the river

LIST OF SYMBOLS(continued)

Symbol	Chapter	Definition
C_e	IV	effluent concentration
C_m	IV	mixed concentration
C_p	IV	pollution accumulation rate
C_r, C_u	IV	retardance coefficients
D_o	IV	initial oxygen deficit
F(.)	IV	Gaussian hypergeometric function
I	IV	impervious area
K	IV	a parameter in overflow distribution
K_1, K_2	IV	rate constants for Streeter-Phelps equation
L	IV	length of the kinematic plane
L_p	IV	pollutant load
Q	IV	peak flow
Q_R	IV	river flow rate
S	IV	rescaled pollution concentration[C_R/C_o]
S_1	IV	receiving stream concentration (extreme case)
T	IV	random variable = $k_1 Z$
T_b	IV	time base of hydrograph
T_c	IV	critical time in oxygen sag
T_R	IV	time of rise of hydrograph
T_r	IV	duration of runoff before urbanization
T_1	IV	effluent concentration(extreme case)
V_R	V	volume of flow in time T_c

LIST OF SYMBOLS(continued)

Symbol	Chapter	Definition
V_r	IV	volume of runoff before urbanization
W	IV	maximum of S_1, T_1
W_{50}	IV	width of hydrograph at 50% peak flow
W_{75}	IV	width of hydrograph at 75% peak flow
X_1	IV	volume of runoff after urbanization
X_2	IV	duration of runoff after urbanization
X_3	IV	intermittent time
Y	IV	over low volume
Z	IV	ratio of random variables[Y/V_R]
a	IV	treatment rate
a_2	IV	parameter for runoff before urbanization
a_3	IV	parameter for duration before urbanization
b	IV	storage volume
d	IV	average left over pollutant
i_*	IV	steady uniform effective rainfall
m	IV	kinematic wave exponent
n_r, n_u	IV	Manning n values for rural and urban situations respectively
p	IV	parameter in beta distribution for S
q	IV	parameter in beta distribution for S
q_{max}	IV	maximum discharge per unit width
t_c	IV	time of concentration
t_p	IV	time to peak

LIST OF SYMBOLS(continued)

Symbol	Chapter	Definition
t_r	IV	rainfall duration
u	IV	urbanization factor
y	IV	depth of flow
y_{max}	IV	maximum depth
α	IV	parameter in exponential distribution for X_1
α_r, α_u	IV	kinamatic wave parameers for urban and rural conditions respecyively
β	IV	parameter in exponential distribution for X_2
ϵ	IV	very small number
γ	IV	parameter in exponential distribution for X_3
ϕ	IV	conveyance factor
ρ	IV	parameter in gamma distribution for V_R
θ	IV	parameter in gamma distribution for V_R

CHAPTER I

INTRODUCTION

1.1 Introduction: Ever growing urban cities with relatively scarce land and water resources, require proper planning for the enhancement of living conditions for the city dwellers. As the population increases, the demand for land and water increases. The expansion of cities produces changes in the land /water interface. As the result of the need to keep ahead of the impending problems, research on the interaction of land use development and stormwater planning is necessary. This research coordinates many facets of land use planning with the areas of stormwater runoff and wastewater collection.

1.2 Stormwater Modeling: Urban stormwater management is one of the areas of active research in hydrology. It is concerned with the cause and effect relationships of the quantity and quality of stormwater and optimal control alternatives so that the effects can be controlled at a desired level (Medina,1979). Models describing stormwater management are varying in detail depending upon the level of accuracy desired. In general these models fall into three categories, namely:

(1) Design Storm Approach

(2) Simulation Modeling, and

(3) Derived Distribution Approaches

Design Storm Approach: This approach provides a means of estimating rainfall depth or intensity for a specified duration and given frequency which will be used in estimating runoff peaks and volumes. The design storm is obtained from frequency-duration-intensity curves or from other statistical means based on rainfall records. The design storm is usually coupled with the rational formula or a unit hydrograph method to obtain the runoff. This approach neglects the storage carryover effect that may exist in the drainage system by ignoring the time interval between storms. Often an intense short duration storm may be completely contained by the system rather than a closely spaced less intense storm series. In the latter case the system is overtaxed and an overflow occurs.

Simulation Modeling: This is considered to be the refined way of stormwater modeling. This approach simulates the entire physical system,recognizing not only the properties of a storm but also the cumulative effect of close spaced storms. The storage carryover effect is completely depicted and additional information like quality of the effluent may be obtained. These models are expensive, data intensive and require a large core memory in the computer.

Derived Distribution Approaches: These methods are based on the statistical distributions of storm variables. Using the hydrological relationships, distributions are derived for the dependent variables such as runoff and overflow. This approach very much depends on how well the distributions of the original variables can be hypothesized. These methods are intended to approximate the simulation modeling. They may yield closed form solutions and are useful for preliminary planning and design.

1.3 Land Use Planning: Another consideration in stormwater management is the effect of the degree and type of urbanization. Changes in land use patterns affect both quantity and quality of stormwater runoff. On the other hand it has been well realized that, for a comprehensive land use plan the inclusion of variables pertaining to natural resources is necessary. This interaction between water resources and land use planning needs to be modelled to represent truly the response of the urban drainage system. Any land use model must be capable of matching the available characteristics (soil type, slope, etc.) of parcels of vacant land with the demand characteristics (water supply, transportation, etc.) of different land use activities (industries, residential units, etc.).

1.4 Multiple Objective Optimization: It was stated that optimal control alternatives would be necessary to regulate the cause/effect relationships. The limited resources impose an optimal tradeoff between competing needs. In urban water resources, minimizing pollution increases the cost of control alternatives. In land use planning matching the supply and demand also increases the cost. This observation naturally leads to simultaneous consideration of different objectives. These objectives are, in general, incommensurable, thus ruling out the possibility of traditional single objective optimization. In addition the uncertainty inherent in the hydrologic system requires the modeling of land/water interface as a multiobjective optimization problem under uncertainty.

1.5 Previous Work : In this section reference is made to several review papers. As the review papers are self-contained and comprehensive, those topics will not be discussed in detail.

Urban Hydrology: Delleur and Dendrou(1980) provide an excellent review of different techniques involved in modeling the runoff processes in urban areas. Delleur(1981) clearly explains the various effects of urbanization on stormwater runoff.

Derived Distribution Approaches in Stormwater Management: Howard(1976) assumes that the storm volumes and intermittent times between storms are exponentially distributed. This article's central idea is the derivation of analytical expressions for overflows and related variables. This paper does not take into account the duration of runoff events. Di Toro and Small(1979) propose a derived distribution for stormwater overflows. The flows are assumed to be uniform over the duration. Flows, duration and intermittent time are assumed to be gamma distributed. In the formulation several expressions do not have analytical solutions and are numerically evaluated. This often arises in derived distribution modeling of stormwater runoff. Chan and Bras(1979) propose a distribution for overflows based on kinematic routing. This method does not consider carryover storage. This formulation also requires numerical evaluation for end results; however it has the advantage of depicting the time distribution of runoff. Smith(1980) takes into consideration the duration of storms. The storm volumes, duration and intermittent time are assumed to be exponentially distributed. The storage level in the reservoir is also considered as a random variable. The expression for the distribution of storage level rules out a strictly analytical solution. This work takes into consideration many of the criticisms of Howard's(1976) paper. Schwarz and Adams(1981) also assume exponential distributions for storm volumes, duration and intermittent time. This paper provides analytical expressions for spill volumes from two detention storage reservoirs in series.

Land Use Planning: Dendrou et.al(1978) review various land use planning models. The model which is more suitable for water resources applications is the modified version of DYLAM called LANDUSE. The model LANDUSE assumes an implicit preferential ordering in allocating land use types because of its sequential allocation of land use types.

Dendrou(1977) proposes a multilevel approach for urban storm drainage planning. The land use model is used as an input generator for the storm drainage block using a two level coordination scheme. Bammi and Bammi(1979) present a multiobjective formulation for comprehensive land use planning. A composite objective function is generated as the weighted sum of individual objective functions and the problem is solved. Nijkamp and Vos(1977) suggest a variant of concordance analysis to choose among alternative projects which have multiple objectives to be maxmized. A land use planning project is illustrated.

Multiobjective optimization: Hwang and Masud(1979) present a state of the art survey. The various intricacies involved in Multiple Criteria Decision Making(MCDM) are well explained along with a compendium of references. Stadler(1979) presents a review of Vector Maximization Problem(VMP) solving methods. In the area of water resources, Cohon and Marks(1975) is the familiar review paper exposing the MCDM problem. Haimes, Hall and Freedman(1975) indicate the wide range of MCDM problems in water resources with the Surrogate Worth Tradeoff method proposed by Haimes and Hall(1974). Major(1977) presents a few case studies in water resources systems. Cohon(1978) presents different solution procedures for solving MCDM problem and contains a chapter on water resources applications. Keeney and Wood(1977) illustrate an application of Multiattribute Utility Theory in water resources planning. Musselman and Talavage(1980) propose a tradeoff cutting plane algorithm and a stormwater management problem is solved as an illustration of the method.

1.6 Organization of the Thesis: Chapter II contains the problem statement. This Chapter explains the logic in the formulation of the problem. Chapter III presents a new algorithm to solve Multi Criteria Decision Making problems. There are also example problems solved using the new algorithm. Chapter IV contains the analytical treatment of storm drainage planning. Closed form, tractable solutions are obtained. Chapter V illustrates the application of the methodology to West Lafayette. Chapter VI contains the conclusions.

CHAPTER II

PROBLEM STATEMENT

2.1 Introduction: In this chapter the land/water interface problem is described. The land use planning portion of the problem is presented in section 2.2. The water resources part of the problem is explained in section 2.3 and the whole problem is presented in section 2.4.

2.2 Land Use Planning: Land is heterogeneous in nature. Each parcel of land is characterised by physical elements such as soil type and slope. In addition to the physical characteristics , man made changes like transportation facilities, water supply, also affect the value of land. It has been well established that a proper comprehensive land use plan must involve the interrelationship between the environment and the urban development.(Dendrou et al.,1978).

The land use needs are estimated based on population projection for a future time. The land use needs are to be achieved with maximum satisfaction at a minimum cost. The notion of satisfaction is involved because certain land use types have specific need for certain physical and sociological characteristics.(e.g. land use type 'school' may require a low noise environment with good transportation facilities). Hence there is a supply side pertaining to the characteristics of the vacant land and a demand side depicting the characteristics requirement of the different land use types. This leads to 'characteristic matching'. This sort of matching naturally results into a location - allocation problem which requires a 'minimum cost plan'.

2.3 Formulation for Land Use Planning: The land use demand estimates are based on population projections obtained from the standard OBERS projections (combination of Office of Business Economics(OBE) , U.S.Department of Commerce and the Economic Research Service(ERS),U.S.Department of Agriculture). The supply of land units is described by a set of attributes that characterizes the zones approximating the natural areas and neighborhoods. Examples of attributes are physical-topographic characteristics(e.g. soil type, depth to bedrock), and characteristics describing the availability of community services and facilities(e.g. transportation accessibility, availability of water supply and sewer).

On the demand side the loosely coordinated private locational decisions are aggregated into several land use categories, for example, industrial, commercial, housing etc. These activities require different attributes with different levels of importance. Some attributes may be critically needed and some are not(e.g. water supply is critical for housing units). A matching between demand requirement and supply availabilities is possible at the level of attributes if both the supply and the demand are characterized by the same set of attributes. Based on this logic the following formulation is presented:

A_j = total area available for development in zone j

B_j = total capital available for development of zone j

D = set of pairs of zones (l,m), such that the distance between them is less than or equal to ρ, $\{(l,m) \mid d_{lm} \leq \rho \}$.

D_1 = Discrepancy set which contains all land use activities i and zones j which will result in mismatches if the allocations are made. For example land use type 1, industry requires industrial water. But zone 1 does not have industrial water. Hence (1,1) will be a member in D_1. This set is constructed by comparing the available characteristics by zones and the required characteristics of the different land use activities.

Umax = ratio of the abstraction storage and the mean runoff volume corresponding to 100% urbanization.

w_j = weight to indicate the degree of disturbance (e.g. street flooding) in zone j.

M = large positive number

ZC_{kj} = 0 if a particular characteristic or attribute k is absent in zone j (e.g.no water supply)
if the characteristic k is available

X_i = projected number of required land use activities of type i.
(e.g. if 3 schools are needed and i=1=school, then $X_1=3$)

X_{ij} = number of land use activities of type i assigned to zone j.

a_i = area required for land use activity of type i.

C_{ij} = cost of locating land use activity of type i in zone j

d_{lm} = distance between zones l and m

m = total number of different types of activities, e.g. { 2 different types: (1) school (2) industry }

n = total number of zones

u = urbanization factor{fractional runoff volume gain because of increased imperviousness and decreased abstraction storage}

d_{ij}^{+} = deviational variable for characteristic matching between ZC_{kj} and δ_{ij}.
This variable permits activities to be assigned to a zone j where the required characteristic k

need not be present. For example zone 1 may not have industrial water (characteristic 1), $ZC_{11} = 0$. But land use type 1, industry which requires characteristic 1 might still be allocated to zone 1. Consider $\delta_{11} - d_{11}^{+} + d_{11}^{-} = 0$, then $\delta_{11} = 1$ implies $d_{11}^{+} = 1$ which is a mismatch.

d_{ij}^{-} = deviational complement
This variable permits a particular land use type i not to be assigned to a zone j which has the required characteristic k for the land use type i. For example land use type 1 industry may not be assigned to zone 1 which has industrial water available (characteristic 1) Consider $\delta_{11} - d_{11}^{+} + d_{11}^{-} = 1$, then $\delta_{11} = 0$ implies $d_{11}^{-} = 1$.

δ_{ij} = 1 if land use type i is allocated to zone j; 0 otherwise

The characteristic matching portion of the problem may be described as follows. The discrepancy set D_1 is identified based on the pairs (i,j), indicating that allocating i'th type of land use in zone j will result in a mismatch (e.g. locating commercial center(type i), far off from the city (zone j) is a mismatch). Such allocations are to be minimized. These allocations may be weigted with number of such activities allocated or the total number of mismatches (sum of all such d_{ij}^{+}) may be counted. In the present analysis the weighted objective function results in nonconvex objective function and hence is not used. Only d_{ij}^{+} deviations are considered. Because of the convergence criterion the d_{ij}^{+} terms are squared to obtain a strictly convex objective function. Thus the objective function for land use planning may be written in two parts. The first part is the characteristic matching which is written as a minimization of sum of the squares of the deviational variables and a minimization of the cost. The mathematical formulation of the land use problem is as follows.

Min $\Sigma_i \Sigma_j (d_{ij}^{+})^2$ { characteristic matching }

Min $\Sigma_i \Sigma_j C_{ij} X_{ij}$ { cost minimization}

subject to:

$\delta_{ij} \leqq ZC_{kj}$ (critically needed characteristic) (1)

for some values of i,j

$\delta_{ij} - d_{ij}^{+} + d_{ij}^{-} = ZC_{kj}$ (2)

for all $i,j \in D_1$

$d_{ij}^{+} d_{ij}^{-} \leqq 0$ (3)

for all $i,j \in D_1$

$u = [\Sigma_i \Sigma_j a_i w_j X_{ij} / \Sigma_j A_j] U\max$ (4)

for i=1,2,....,m; j=1,2,...,n.

$\Sigma_i a_i X_{ij} \leqq A_j$ (area constraint) j=1,2,.....,n (5)

$\Sigma_i C_{ij} X_{ij} \leqq B_j$ (budget constraint) j = 1,2,.....n (6)

$X_{kl} - M\Sigma\delta_{im} - M\delta_{il} \leqq 0$ (min. distance) for $(l,m) \in D$ (7)

$X_{ij} - M(1 - \delta_{rj}) \leqq 0$ (compatibility) (8)

$\Sigma_j X_{ij} \geqq X_i$ (total number of land use activities)

$(i = 1,2,.....m)$ (9)

$X_{ij} \leqq M\delta_{ij}$ (10)

$X_{ij} \geqq \delta_{ij}$ (11)

$\delta_{ij} \leqq 1$ (12)

$X_{ij}, \delta_{ij} \geqq 0$ and integer

Constraint (1) restricts the land use types to be located in a zone only if the critical characteristic is present. Constraint (2) allows for flexibility in noncritical characteristics. Constraint (3) restricts only one of the deviational variables to be present or both can be absent. Constraint (4) computes the urbanization factor. The urbanization factor u is computed as a fraction of the 100% urbanization factor Umax depending upon the land use allocations. Constraint (5) accounts for the land availability.

Constraint (6) ensures the budget is satisfied.

Constraint (7) says that the land use type k will be located in zone l if and only if δ_{kl} is positive and either at least one of the zones in D has land use type i or zone l itself has land use type i. For example it is preferable to have residential units and a school together. ie. land use types k and i must be close to each other.

Constraint (8) imposes land use type i to be absent if land use type r is present.

Constraint (9) restricts the number of land use activities of each type.

Constraints (11) and (12)impose the conditions

when $\delta_{ij} = 1$, $X_{ij} > 0$

when $\delta_{ij} = 0$, $X_{ij} = 0$

2.4 Urban Runoff Management Planning: Figure 2.1 shows the stormwater runoff process. The runoff volume X_1(in) which occurs over a perod of X_2(hr) is treated at the treatment rate 'a'(in/hr). If the runoff volume is less than the amount that can be treated in X_2 hours there is no need for storage. Otherwise the excess runoff is to be stored so that it can treated at a later time. If a second storm occurs in quick succession X_3 hours after the end of the previous runoff event it is quite possible only part of the storage will be available. If the second runoff event excess volume is greater than the available storage , an overflow Y(in), occurs. Because X_1, X_2 and X_3 are random variables, Y is also a random variable. In the present analysis BOD is considered to be the pollutant. The overflow containing the pollutant of concentration C_e(mg/l) reaches the the receiving body. The receiving body also contains the pollutant with concentration C_R(mg/l). In general, the government regulations require the minimization of mixed pollutant concentration C_m(mg/l) in the stream. This can be interpreted as, the exceedence probability of some threshold concentration must be a minimum.

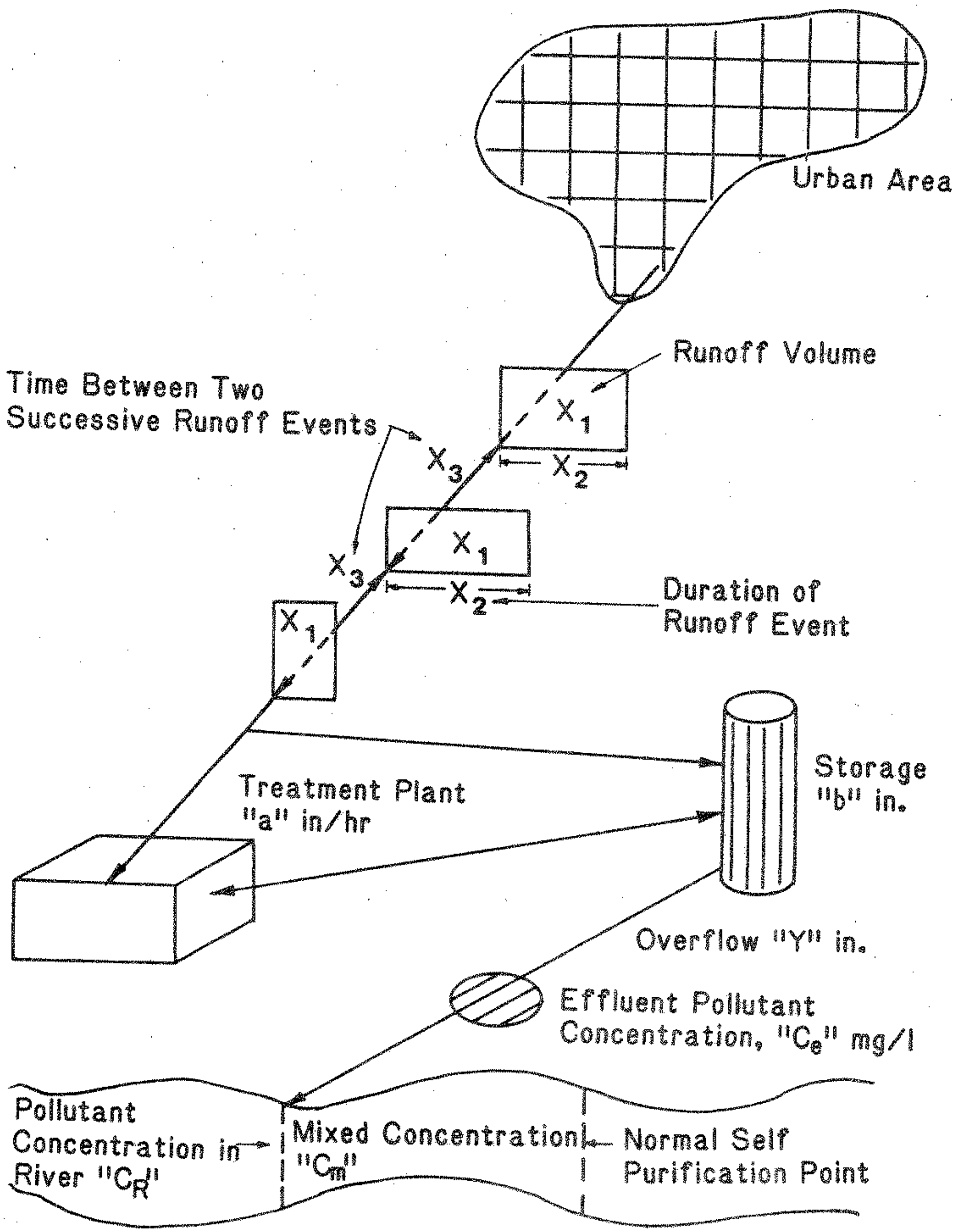

Figure 2.1 Schematic Representation of Urban Stormwater Runoff Process.

Minimizing the pollutant concentration requires larger storage and increased treatment capacity, which in turn results in higher cost. The hydrologic portion of the problem may be stated as,

min $C_1(a) + C_2(b)$ (drainage cost minimization)

min ϵ (risk minimization)

subject to: $P(C_m \geqq C_o) \leqq \epsilon$

where:

$C_1(a)$ = cost of treatment process (dollars)

$C_2(b)$ = cost of storage (dollars)

C_m = mixed concentration (mg/l)

C_o = threshold concentration (mg/l)

ϵ = very small probability of exceedence (e.g. 0.01)

<u>2.5 Land/Water Problem:</u> The whole problem may be stated as follows.

min $\Sigma_i \Sigma_j (d_{ij}^+)^2$ (characteristic matching)

min $\Sigma_i \Sigma_j C_{ij} X_{ij}$ (land development cost minimization)

min $C_1(a) + C_2(b)$ (drainage cost minimization)

min ϵ (risk minimization)

subject to:

(1) land use constraints (section 2.2)

(2) probability constraint (section 2.3)

Because the interest lies only on the total cost, the land use cost and drainage cost can be combined into one. This is a three objective optimization problem. In Chapter III a methodology will be developed to solve a general multiobjective problem. Chapter IV contains the theory underlying the probability constraint development.

CHAPTER III

MULTIOBJECTIVE OPTIMIZATION: AN INTERACTIVE CUTTING PLANE ALGORITHM

3.1 Introduction: An algorithm to solve a general Multi Criteria Decision Making (MCDM)problem is presented in this chapter. Various definitions and approaches of solving MCDM problems are presented in section 3.2. The nonconvexity of the objective space under concave mapping from the feasible region is explained in section 3.3. A few useful results for the set $Y = \{y| \ y \leqq f(x) \text{ for some } x \in X\}$ and its relations with the objective space are established in section 3.3. The actual algorithm is presented in section 3.4. Section 3.5 contains the proofs for the efficiency of the generated points and for the convergence of the algorithm.

3.2 General Multiple Criteria Decision Making Problem:

Multiple Criteria Decision Making(MCDM), Vector Maximization Problem(VMP), Multiobjective Optimization Problem(MOOP) all mean the same problem. The Vector Maximum Problem may be stated as (vector maximization is denoted by, V - Max)

(VMP): $V - \text{Max } f(x) = [f_1(x), f_2(x), \ldots\ldots, f_p(x)]$

subject to: $g_i(x) \leqq 0$, $i = 1,2,\ldots\ldots,m$

where: x is an n dimensional vector of decision variables

$f_i(x)$, $i = 1,2,\ldots\ldots,p$ are the p objective or criteria functions

$g_i(x)$, $i = 1,2,\ldots\ldots,m$ are the m constraint functions

The objective functions are noncommensurable. If the objective functions are commensurable the problem reduces to a scalar maximization problem. The following definitions will be used for the development of the algorithm.

1) A Feasible Solution is a vector x satisfying the constraints $g_i(x) \leqq 0$, $i = 1,2,\ldots\ldots,m$.

2) Feasible Region denoted by X, is the set of all feasible solutions. i.e. $X=\{x|\ g_i(x) \leqq 0,\ i = 1,2,\ldots\ldots,m\}$.

3) Objective Space denoted by T, is the set of points mapped by the vector objective function f, from the feasible region, i.e. $T = \{t|\ f(x)=t,\ x\in X\}$

4) An Efficient Solution(Nondominated Solution, Pareto Optimal Solution) denoted by x^0 is a feasible solution such that there does not exist another feasible solution x, which can improve at least one objective function without hurting

at least one other objective function. i.e. if $f_i(x) > f_i(x^0)$, then there exists at least one other j such that $f_j(x^0) > f_j(x)$

5) A Best Compromise Solution is an efficient solution that is best with respect to the Decision Maker's(DM) preference structure

6) A Superior Solution denoted by x^s is a feasible solution which maximizes all the objectives simultaniosly, i.e. $f(x^s) \geqq f(x)$, for all $x \in X$

7) A Utility Function(Preference Function) denoted by U, is a scalar valued function which assigns higher values for more preferable points (actions) within the set over which U is defined.

Note: x^* is the best compromise solution if and only if $U(x^*) \geqq U(x)$, for all $x \in X$

Methods which solve MCDM problems fall into three categories:

1) Methods which do not use any knowledge of DM's preferences. (E.g. methods which generate all the efficient solutions)

2) Methods which use completely prespecified preferences of the DM. (E.g. (a) Goal Programming (b) Multiattribute utility theory)

3) Methods which use progressively revealed preferences of the DM. (E.g. Interactive methods)

Why efficient solution?: It is assumed that the DM prefers higher $f(x)$ values for every $x \in X$. If $f(x^1) > f(x^2)$ then the DM will choose x^1 over x^2. This assumption implies that the solution of the VMP is among the efficient points and all the dominated solutions can be deleted from consideration.

An evaluation:

1) Methods which do not use any knowledge of DM's preferences: In these methods all the efficient points are generated. Usually the VMP is formulated as a parametric scalar maximization problem. In general these problems generate an infinite number of efficient points. This implies that analyzing all the efficient points is a tedious process. An obvious inference will be to couple the efficient point generation scheme with a preferential structure so that only a subset of the efficient set needs to be considered for the best compromise solution.

2) Methods which use completely prespecified preferences of the DM: In these methods the DM is clearly able to state his preferences. Because the preferences are known explicitly, it is possible to reduce the VMP to a single scalar maximum problem. The amount of subjectivity involved in this approach needs careful attention.

3) Interactive methods: These methods involve a progressive dialogue between the DM and the analyst during problem solving. The interactive methods involve the following steps:

a) generation of a feasible point (efficient point is preferable)

b) adjusting for compatibility with DM's preference choice.

c) flipping back and forth between steps (a) and (b) until the DM is satisfied.

By incorporating efficient point generation scheme as step (a) the cognitive burden on the DM can be reduced because each iteration leads to a 'nothing is lost' situation because of the efficiency of the generated point.

<u>A few observations</u>

a) Even though MCDM problems involve solution of vector maximum problems, these problems can be solved as single scalar maximization problems (methods in category(2)) or a series of scalar maximization problems(methods in categories(1) and(3)).

b) The efficiençy of the MCDM solution procedure also depends on the type and amount of information required from the DM.

c) Many times MCDM problems involve solving a series of nonlinear programming (NLP) problems. Naturally the efficiency of MCDM solution procedure depends on the efficiency of the NLP code used.

In NLP problems the gradient based methods have better convergence properties than the direct search techniques. It also turns out that, though the utility function is only implicitly known to the DM, the information regarding the gradient of the utility function U, with respect to its components symbolically denoted by ∇U, can be easily provided by the DM. There are a number of interactive methods which take advantage of this observation. In the following section an algorithm is presented which eliminates an unwanted portion of the feasible region, along with the generation of an efficient point at each iteration with the use of gradient information

Notation: $f(x^k)$ will be denoted as f^k; the components will be denoted by f_i^k.

3.3 Preliminary Analysis: Consider the (VMP),

(P1): $V - \text{Max } f(x) = [f_1(x),\ f_2(x), \ldots .. f_p(x)]$

subject to: $x \in X$

In general a utility function $U(f_1, f_2, \ldots .., f_p)$ is defined on the objective space T (range under the mapping f from the domain X) for the scalarization of VMP.

i.e. (P2): Max $U[f_1, f_2, \ldots ..., f_p]$

subject to: $x \in X$

Solution to (P2) is the best compromise solution.

Assumptions:

1) X is a convex, compact set.

2) $f_i(x)$ are differentiable and concave on X.

3) U is strictly increasing and differentiable.

It turns out that the range space T need not be a convex set when f is concave.

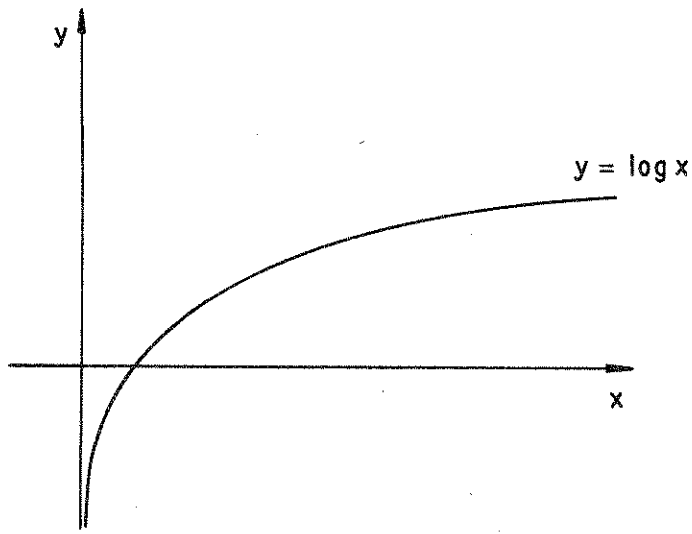

Figure 3.1 Nonconvex Objective Space

<u>Example(Moeseke,1965):</u> Let $X = \{x|\ x > 0\}$, convex set

$f_1(x) = x$, concave

$f_2(x) = \ell og\ x$, concave

The objective space T, is the $\ell og\ x$ graph which is not a convex set, Figure 3.1.

In general it is desired that the function U be concave, but it is ill defined on T if T is not a convex set. One way to overcome this difficulty is to create a new set which is

(1) convex (Theorem 3.1)

(2) the optimal solutions of the new set and of the range space must be the same (Theorem 3.2)

A special set Y is presented which has the above two properties. Let,

$$Y = \{y|\ y \leqq f(x),\ \text{for some } x \in X\}$$

<u>Note:</u> T is a subset of Y. $T \subset Y$

The following Theorem and the proof can be found in (Moeseke,1965).

<u>Theorem 3.1(Moeseke,1965):</u> If X is a convex set and f is concave on X, then Y is a convex set.

<u>Proof:</u> Let $y^1, y^2 \in Y$

To show: $\lambda y^1 + (1 - \lambda)y^2 \in Y$; $0 \leqq \lambda \leqq 1$

$y^1 \in Y$ implies that there exists $x^1 \in X$ such that $y^1 \leqq f(x^1)$

$y^2 \in Y$ implies that there exists $x^2 \in X$ such that $y^2 \leqq f(x^2)$

Because X is a convex set, $x \in X$ where $x = \lambda x^1 + (1 - \lambda)x^2$

f concave on X implies,

$f(x) \geqq \lambda f(x^1) + (1 - \lambda)f(x^2) \geqq \lambda y^1 + (1 - \lambda)y^2 = y$,

implies $y \in Y$. Hence Y is convex. Q.E.D.

Following Sadagopan(1980) we will prove the following Theorem for the general case.

Theorem 3.2: Let (P3): Max $U[f(x)]$

subject to: $f(x) \in T$

and let (P4): Max $U[y]$

subject to: $y \in Y$

The optimal solution set T^* of (P3) and optimal solution set Y^* of (P4) are equal.

Proof:

Case1: To show $T^* \subset Y^*$

Let $t^* \in T^*$ which implies $t^* \in Y$ because $T \subset Y$

claim: $t^* \in Y^*$

assume $t^* \notin Y^*$ which implies that there exists

$y^* \in Y^*$ such that $U(y^*) > U(t^*)$

By construction of Y there exists $f(x^*) \in T$

such that $f(x^*) \geqq y^*$ which implies

$U[f(x^*)] \geqq U(y^*) > U(t^*)$

contradicting the fact $t^* \in T^*$

Hence $t^* \in Y^*$ implies $T^* \subset Y^*$.

Case 2: To show $Y^* \subset T^*$. let $y^* \in Y^*$.

Since $Y \supset T$, if we show y^* is in T,

that will imply $y^* \in T^*$.

claim: $y^* \in T$

By construction of Y for $y^* \in Y^*$ there exists,

$f(x^*) \in T$ such that $f(x^*) \geqq y^*$

If $f(x^*) > y^*$ then $U[f(x^*)] > U(y^*)$

Also $f(x^*) \in T \subset Y$ contradicting $y^* \in Y^*$.

Hence $f(x^*) = y^*$ implying $y^* \in T$ which implies $y^* \in T^*$

Hence $Y^* \subset T^*$. Q.E.D.

In the following section an algorithm which makes use of a tradeoff cut proposed by Musselman and Talavage(1981) is presented. The algorithm progressively eliminates portions of the feasible region where the maximum of the objective can not lie(lemma 3.1). It is shown that the algorithm generates only efficient points(Theorem 3.3) and eventually converges to the best compromise solution in a finite number of steps(Theorem 3.4) or in the limiting sense(Theorem 3.5).

3.4 Algorithm:

The objective function is

(P2) Max $U[f]$

subject to: $x \in X$

where: U = utility function

f = vector objective function

X = feasible region

Assumptions:

1) X is a convex, compact set in $\mathbb{R}^n$
2) U is not known explicitly
3) U is concave on the convex set Y
4) U is strictly increasing in its components
5) U and f are differentiable

6) Each f_i is concave on the convex set X and at least one f_i is strictly concave.

<u>Tradeoff cut generation:</u>

<u>Lemma 3.1(Musselman and Talavage,1980):</u> Let U(y) be concave on the convex set Y.

If $U(y) > U(y^0)$ then $\nabla U(y^0).(y - y^0) > 0$

Lemma 3.1 implies that by concentrating on the half space,

$$\Sigma \; (\partial U/\partial f_i) \; (f_i - f_i^k) \geqq 0 \qquad (3.1)$$

the improved values of U can be found. Since $(\partial U/\partial f_i)$ is unknown, inequality (3.1) in the present form is not useful. However, by dividing (3.1) with a positive number $(\partial U/\partial f_l)$ which is also unknown, the following inequality is obtained. (Trade off cut):

$$\Sigma \; w_i^o \; (f_i - f_i^k) \geqq 0 \qquad (3.2)$$

where: $w_i \;\; = (\partial U/\partial f_i)/(\partial U/\partial f_l)$

$\qquad\qquad\qquad$ = tradeoff value

w_i is the number of units the DM is willing to forego in the objective f_l for a unit gain in objective f_i to keep up the same amount of utility. It is hoped that the DM can easily provide this information from his experience. $w_i > 0$ because U is strictly increasing. It is also possible that the inequality (3.2) can be defined on the feasible region X by expressing f in terms of x.

<u>Central idea:</u> Inequality (3.2) called tradeoff cut when transformed on the feasible region indicates in which direction one should move to increase the utility, Figures 3.2 and 3.3.

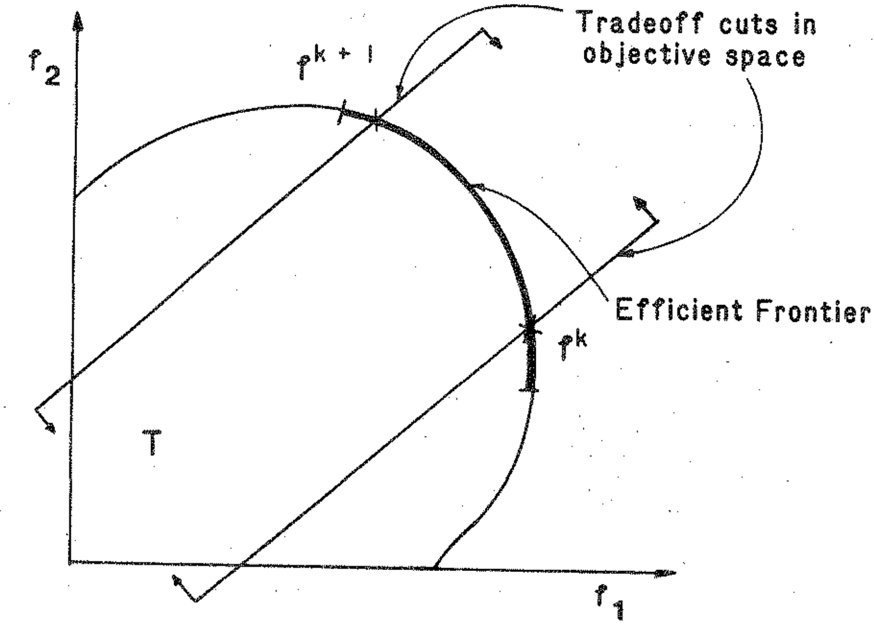

Figure 3.2 Tradeoff cut on objective space

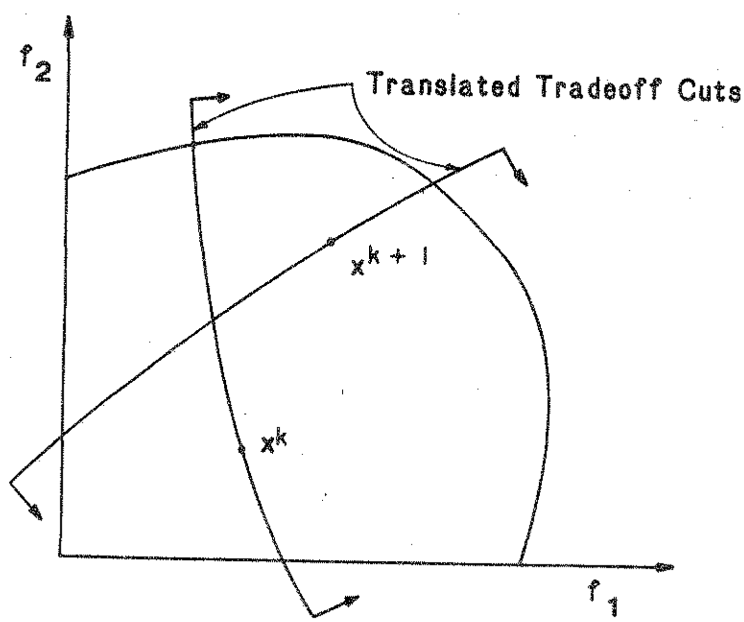

Figure 3.3 Tradeoff cut over the Feasible Region

More explicitly it removes a portion of the feasible region where the maximum can not lie. One interpretation of the tradeoff cut can be to move in the tradeoff direction as far as possible for maximum improvement. Another interpretation can be because U is increasing in f, increase f as much as possible. Using w_i as the tradeoff between objective 1 and objective i, U can be expressed in terms of f_1 alone. Thus to maximize U, we need to maximize f_1 according to the local tradeoffs.

The Algorithm:

Step 0: Ask the DM for tradeoff values at x^k. Establish a tradeoff cut at x^k.

Step 1: Solve the following problem

(P5) Max $Z^k = \Sigma_{i=1}^{p} w_i^k (f_i - f_i^k)$

subject to: $x \in X$

$$h_j(x) = \Sigma_{i=1}^{p} w_i^j (f_i - f_i^j) \geqq 0,$$

for $j = 1,\ldots,k$

Feasible region $X^k = X \cap \{x | h_j(x) \geqq 0\}$,

for $j = 1,\ldots,k$

Let x^{k+1} be the solution for (P5).

Step 2: If at iteration k $Z^k = 0$, then x^k is the best compromise solution. Otherwise $x^k = x^{k+1}$. Go to Step 0.

3.5 Theory of the Algorithm:

The following theorems prove the efficiency of the generated points and the stopping rule for the algorithm.

Theorem 3.3: Optimal solution of (P5) x^{k+1} is an efficient solution for (P1)

Proof: Assume x^{k+1} is not efficient for (P1). Then there exists $x^0 \in X$ such that

$$f_i(x^0) \geqq f_i(x^{k+1}) \text{ for all } i \neq j$$

and $\quad f_j(x^0) > f_j(x^{k+1})$ for atleast one j $\qquad$ (3.3)

Because x^{k+1} is feasible in (P5), x^{k+1} satisfies

$\Sigma_{i=1}^{p}\ w_i^j\ (f_i(x^0) - f_i(x^j)) \geqq 0,\ j = 1,\ldots,k$

From (3.3) it follows that

$\Sigma_{i=1}^{p}\ w_i^j\ (f_i(x^0) - f_i(x^j)) \geqq 0,\ j = 1,\ldots,k$ Hence $x^0 \in X^k$.

Also by (3.3)

$\Sigma_{i=1}^{p}\ w_i^k\ (f_i(x^0) - f_i(x^k)) > \Sigma_{i=1}^{p}\ w_i^k\ (f_i(x^{k+1}) - f_i(x^k))$

Contradicting x^{k+1} is optimal in (P5)

Hence x^{k+1} is efficient in (P1). Q.E.D.

Lemma 3.2: Let U be concvave on the convex set Y. Given $\nabla U(y^0).(y - y^0) \leqq 0$ for all $y \in Y$ for some $y^0 \in Y$, then y^0 is optimal for U in Y.

Proof: If y^0 is not optimal, then there exists $y^1 \in Y$ such that $U(y^1) > U(y^0)$. By lemma 3.1 $\nabla U(y^0).(y - y^0) > 0$ which is a contradiction. Hence y^0 is optimal in Y. Q.E.D.

Theorem 3.4: If $\Sigma_{i=1}^{p}\ w_i^k\ (f_i(x) - f_i(x^k)) \leqq 0$, for all $x \in X^k$ then x^k is the best compromise solution.

Proof:

claim: x^k is optimal in X^k. Assume x^k is not optimal in X^k. Then there exists x^1 such that

$\Sigma_{i=1}^{p}\ w_i^k\ (f_i(x^1) - f_i(x^k)) > 0$ which contradicts the fact $\Sigma_{i=1}^{p}\ w_i^k\ (f_i(x) - f_i(x^k)) \leqq 0$ for all $x \in X^k$. Hence x^k is optimal in X^k.

Let $Y^k = \{y|\ y \leqq f(x)$ for some $x \in X^k\}$. Then Y^k is a convex set by Theorem 3.1. Fix $y^k = f(x^k)$

Also $\Sigma_{i=1}^{p}\ w_i^k\ (f_i(x) - f_i(x^k)) \leqq 0$ for all $x \in X^k$ which implies for all $y \in Y^k$

$\Sigma_{i=1}^{p}\ w_i^k\ (y_i - y_i{}^k) \leqq \Sigma_{i=1}^{p}\ w_i^k\ (f_i(x) - f_i(x^k)) \leqq 0$

By lemma 3.2 y^k is optimal in Y^k. By Theorem 3.2

$U[f(x^k)] \geqq U[f(x)]$ for all $x \in X^k$. (3.4)

By construction of X^k, $\{X - X^k\}$ contains only inferior solutions. Hence,

$U[f(x^k)] \geqq U[f(x)]$ and x^k is the best compromise solution.

Q.E.D.

Theorem 3.5: The sequence $\{x_n\}$ of optimal solutions of subproblems of (P5) has a subsequence $\{x_{\eta_k}\}$ converging to the best compromise solution.

Proof: By construction of subproblems we have,

$X^1 \supset X^2 \supset \ldots\ldots X^k \supset X^{k+1} \ldots\ldots$

Each X^k is nonempty. Because each X^k contains all its boundary points by way of its construction, each X^k is a closed set. Now X is compact. There exists a subseqence $\{x_{\eta_k}\}$ of $\{x_n\}$ such that,

$$\lim_{k\to\infty} \{x_{\eta_k}\} = x^* \in X$$

$n_k \geqq k$ implies, x_{η_k}, $x_{\eta_{k+1}}, \ldots\ldots$ all lie in X^k.

As X^k is closed and x^* is a limit point of X^k, $x^* \in X^k$.

Because k was fixed but arbitrary in the above argument,

$$x^* \in X^k, \text{ for all } k \geqq 1$$

Therefore $x^* \in \cap X^k$ of all $k \geqq 1$

By construction of X^k , $X - X^\infty$

where: $$X^\infty = \bigcap_{k\geq 1} X^k$$

contains only inferior solutions. Since atleast one f_i is strictly concave and each w_i is positive, Z is strictly concave.

<u>Claim:</u> x^* is the best compromise solution.

Proof by contradiction. Assume x^∞ is the best compromise solution. By construction of X^ks, $x^\infty \in X^\infty$. Since $U(f(x^\infty)) \geqq U(f(x))$ for all $x \in X^\infty$ it implies that $Z(f(x^\infty)) = 0$. But x^* also belongs to X^∞ which implies that

$$\Sigma_{i=1}^{p} w_i^\infty (f_i(x^*) - f_i(x^\infty)) = 0 \qquad (3.5)$$

if $\Sigma_{i=1}^{p} w_i^\infty (f_i(x^*) - f(x^\infty)) > 0$,then it will violate that x^∞ is the best compromise solution. From (3.5) it is seen that

$$\Sigma_{i=1}^{p} w_i^\infty f_i(x^*) = \Sigma_{i=1}^{p} w_i^\infty f_i(x^\infty)$$

which contradicts the fact that Z is strictly concave. Hence x^* must be the best compromise solution.

<u>Q.E.D.</u>

3.6 Advantages of the Algorithm:

(1) Only efficient points are generated.

(2) For a utility function linear in the objectives(objectives may be nonlinear in x) only one iteration is required because the local tradeoffs are also global.

(3) No line search is required. This implies the DM's response is needed only once for each iteration; otherwise

the DM must state his preference during the line search.

(4) Use of gradient direction and regional elimination by tradeoff cuts might lead to quick convergence.

<u>Example 3.1:</u>

(P6): Max $f = -(x_1 - 8)^2 - (x_2 - 2)^2$

subject to:

$0.1\ x_1^2 - x_2 \leqq 0$

$x_2 + 0.33\ x_1 - 4.5 \leqq 0$

$x_1, x_2 \geqq 0$

Optimal solution to (P6) $x^* = (5.258147, 2.764811)$

<u>Solution:</u>

starting point, $x^1 = (0,0)$

Gradient at $x^1 = (16,4)$

(P7): Max $Z^1 = 16x_1 + 4x_2$

subject to:

$0.1x_1^2 - x_2 \leqq 0$

$x_2 + 0.33x_1 - 4.5 \leqq 0$

$16x_1 + 4x_2 \geqq 0$

Optimal solution, $x^* = (5.258147, 2.764811)$

Gradient at $x^2 = (5.483706, -1.529622)$

(P8): Max $z^2 = 5.483706x_1 - 1.529622x_2$

subject to:

$0.1x_1^2 - x_2 \leqq 0$

$x_2 + 0.33x_1 - 4.5 \leqq 0$

$16x_1 + 4x_2 \geqq 0$

$$5.483706x_1 - 1.529622x_2 - 24.605017 \geqq 0$$

Optimal solution = (5.258147,2.764811)

Objective function value $Z^2 = 0$.

Extension to Mixed Integer MCDM:

As an extension to mixed integer MCDM the problem (P5) can be solved as a nonlinear mixed integer program. Branch and bound procedure can be used for the purpose(Gupta,1980). The example 3.1 is solved as an integer program for illustrating the idea.

Example 3.2:

(P9): Max $f = -(x_1 - 8)^2 - (x_2 - 2)^2$

subject to:

$0.1\ x_1^2 - x_2 \leqq 0$

$x_2 + 0.33\ x_1 - 4.5 \leqq 0$

x_1, $x_2 \geqq 0$, and x_1 is integer

Optimal solution $x^* = (5,2.5)$

Solution:

Starting point, $x^1 = (0,0)$

Gradient at $x^1 = (16,4)$

The resulting problem is same as (P7) in example 3.1

Solving (P7) as a mixed integer program the following optimal solution is obtained.

Optimal solution $x^2 = (5,2.85)$

Gradient at $x^2 = (6.0,-1.7)$

(P10): max $Z^2 = 6x_1 - 1.7x_2$

subject to:

$6x_1 - 1.7x_2 - 25.155 \geqq 0$

$0.1\ x_1^2 - x_2 \leqq 0\ x_2 + 0.33\ x_1 - 4.5 \leqq 0$

$x_1, x_2 \geqq 0$, and x_1 is integer

Optimal solution $x^2 = (5,2.5)$ same as the optimal solution to(P9)

The example problem 3.1 is also solved for pure integer case.

Initial point, $x^1 = (0,0)$

All integer optimal solution to (P6), $x^2 = (4,3)$

Gradient at $x^2 = (8,-2)$

(P11): Max $Z^2 = 8x^1 - 2x_2$

subject to:

$0.1\ x_1^2 - x_2 \leqq 0\ x_2 + 0.33\ x_1 - 4.5 \leqq 0$

$x_1, x_2 \geqq 0$, x_1 and x_2 are integers

Optimal solution = (4,2) same as the pure integer solution to (P6).

CHAPTER IV

URBAN STORMWATER MANAGEMENT: A DERIVED DISTRIBUTION APPROACH

4.1 Introduction: In this chapter a derived distribution approach is presented. The stormwater runoff process along with the impact on the quality of the downstream receiving body is analyzed. Tractable closed form solutions are obtained. An attempt has been made to model future urbanization effects on stormwater management. In general the current data on runoff volume, duration of events, etc. need to go through a transformation to account for future urbanization activities in a developing urban environment.

4.2 Modeling Urbanization Effects: The effects of urbanization on stormwater management have been well studied. The increased urbanization has the following effects on the hydrology of the study area (Delleur and Dendrou ,1980; Delleur,1981).

1) decrease in infiltration storage

   (increase in imperviousness)

2) decrease in depression storage

3) increase in volume of runoff

4) increase in peak flow

5) decrease in time to peak

Volume Increase: The above effects indicate that the volume of runoff after urbanization X_1 can be expressed as,

$$X_1 = (1+u)V_r \tag{4.1}$$

where: u = urbanization factor
{fractional volume gain because of increased imperviousness and decreased depression storage}

V_r = volume of runoff before urbanization (in)

Reduction in Time to Peak: Urbanization also provides for rapid drainage, which results in the reduction of time to peak. Two methods are suggested to model this phenomenon:

(1) Kinematic flow routing

(2) 10 minute unit hydrograph

Kinematic Flow Routing: The urban watershed is viewed as a single uniform plane catchment with flow length L. Following Eagleson(1970) two main cases can be considered.

case 1: $t_c < t_r < \infty$

where: t_c = time of concentration

t_r = rainfall duration

Let i_* = steady uniform effective rainfall

then maximum depth, $y_{max} = i_* t_c$

maximum discharge per unit width, $q_{max} = i_* L$

Therefore for constant net effective rainfall i_*,

$$q_{max} = i_* L = \alpha(i_* t_c)^m \tag{4.2}$$

$$t_c = [Li_*^{(1-m)}/\alpha]^{1/m} \tag{4.3}$$

where: α and m are the kinematic wave parameters (for Manning formula $m=5/3$)

It should be noted that, in this case t_c can be treated as time to peak

Assuming the time base of the hydrograph to be given by

$$T_b = \text{const} \times t_c$$

(E.g. As in USBR(1973) $T_b = 2.67T_p$, T_p = time to peak)

it can be obtained from (4.3) that,

$$T_b{}^r/T_b{}^u = t_c{}^r/t_c{}^u$$

$$=(\alpha_u/\alpha_r)^{1/m} = (n_r/n_u)^{1/m}$$

where: $T_b{}^r, T_b{}^u$ = hydrograph time bases under rural and urban conditions, respectively

n_r = Manning n under rural conditions

n_u = Manning n after urbanization

Typical values:

n_r = 0.06 (natural channels)

n_u = 0.013 (smooth asphalt)

$T_b{}^r = (0.06/0.013)^{1/m}\ T_b{}^u$

$T_b{}^r = 2.5\ T_b{}^u$

Using Izzard's retardance coefficients, C in place of Manning n (Linsley, Kohler and Paulhus,1975),

C_r = 0.06 (blue grass turf)

C_u = 0.007 (smooth asphalt)

it follows that $T_b{}^r = (0.06/0.007)^{1/m}\ T_b{}^u$

and $T_b{}^r = 3.63\ T_b{}^u$

<u>case 2:</u> $t_r < t_c$

Using the expression for time to peak given by Eagleson(1970),

Time to peak, $t_p = t_r + (1/m)(t_c{}^* - t_r)$ (4.4)

$$t_c{}^* = L[1/(\alpha\ y_{\ell r}{}^{(m-1)})]$$

where: $y_{\ell r} = i_* \ t_r$ (4.5)

Equation(4.4) can be written as,

$$t_p = [(m-1)t_r + t_c{}^*]/m \qquad (4.6)$$

Dividing both the numerator and the denominator of (4.6) by t_r and using the relation (4.5) it is obtained that,

$$t_p = [(m - 1) + \{L/(\alpha(i_* \ t_r)^{m-1} \ t_r)\} \ ]/(m/t_r)$$

The second term in the numerator can be further simplified by multiplying and dividing by i_* and $t_c{}^m$

$$t_p = [(m - 1) + \{L \ i_*/(\alpha(i_* \ t_c)^m)\} (t_c/t_r)^m]/(m/t_r) \qquad (4.7)$$

But it is known from (4.2) that $i_*L = \alpha(i_*t_c)^m$. Hence equation (4.7) simplifies to,

$$t_p = [(m - 1) + (t_c/t_r)^m]/(m/t_r) \qquad (4.8)$$

From equation (4.8) it is obtained that,

$$t_p{}^r/t_p{}^u = [(m - 1) + (t_c{}^r/t_r)^m] /[(m - 1) + (t_c{}^u/t_r)^m] \qquad (4.9)$$

Because (1) $t_c > t_r$ then $t_c/t_r > 1$.

And (2) $(t_c{}^r/t_c{}^u)^m = (\alpha_u/\alpha_r) = (n_r/n_u)$

is about 4.61.

Under conditions (1) and (2) the contribution of $(m - 1)$ $=0.667$, in (4.9) is not significant and hence can be dropped. Thus (4.9) simplifies to

$$(t_p^{u}/t_p^{r}) \simeq (t_c^{u}/t_c^{r})^m \qquad (4.10)$$

Hence it is obtained that,

$T_b^{r} = 4.61\ T_b^{u}$

<u>10 min.unit hydrograph:</u> Following Delleur and Dendrou(1980), the 10 min. unit hydrograph parameters shown in Figure 4.1 are estimated by means of the following equations:

$$T_R = 3.1L^{0.23}S^{-0.25}I^{-.18}\phi^{1.57} \qquad (4.11)$$

$$Q = 31.62(10^3)A^{0.96}T_R^{-0.95} \qquad (4.12)$$

$$T_B = 125.89(10^3)AQ^{-0.95} \qquad (4.13)$$

$$W_{50} = 16.22(10^3)A^{0.93}Q^{-0.92} \qquad (4.14)$$

$$W_{75} = 3.27(10^3)A^{0.79}Q^{-.78} \qquad (4.15)$$

where: L = the total distance(ft.) along the main channel from the point being considered to the u/s boundary

S = the main channel slope

I = impervious area

ϕ = conveyance factor given by $\phi = \phi_1 + \phi_2$

A = area of watershed

T_R = time of rise of unit hydrograph

Q = peak flow

T_B = time base of hydrograph

W_{50} = width of hydrograph at 50% of Q

W_{75} = width of hydrograph at 75% of Q

ϕ_1 = 0.6 for extensive channel improvement

= 0.8 for some channel improvement

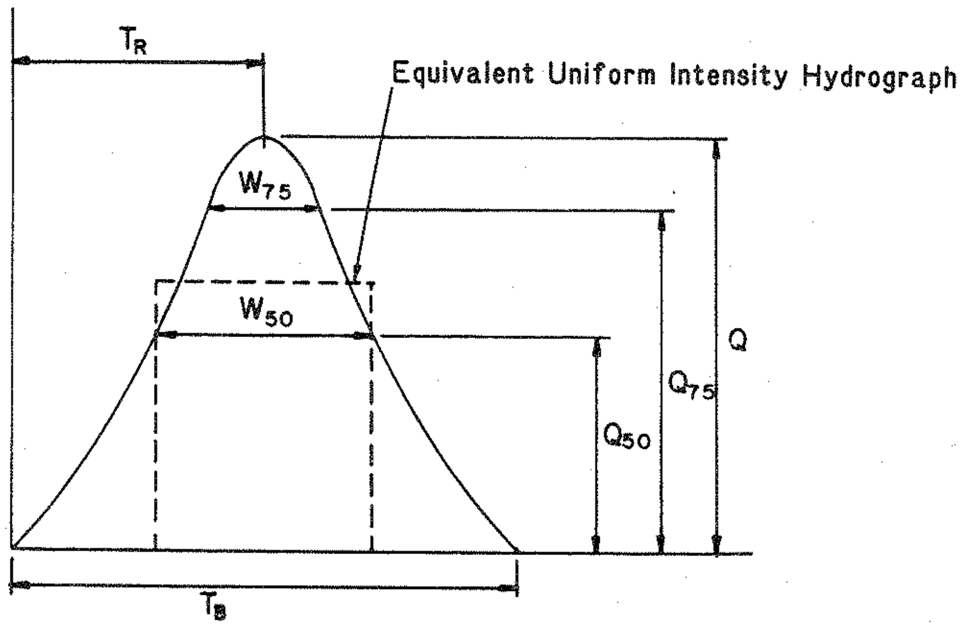

Figure 4.1 Ten Minute Unit Hydrograph

= 1.0 for natural conditions

ϕ_2 = 0.0 for no channel vegetation

= 0.1 for light channel vegetation

= 0.2 for moderate channel vegetation

= 0.3 for heavy channel vegetation

The regression equations (4.11) through (4.15) were obtained for the following range of the parameters:

$$0.0128 < A < 15 \text{ mi}^2$$

$$555 < L < 35600 \text{ ft}$$

$$0.006 < S < 0.0193 \text{ ft/ft}$$

$$2 < I < 100\%$$

$$0.6 < \phi < 1.28$$

For the equivalent uniform intensity rectangular hydrograph(Morris and Wiggert,1972) the time base of the hydrograph can be taken as W_{50}. Thus equation (4.14) can be rewritten as

$$W_{50} = T_B = 16.22(10^3)A^{0.93}Q^{-0.92}.$$

Substituting for Q from equation (4.12) and within (4.10) substituting for T_R from equation (4.11) the expression for the time base of the hydrograph becomes

$$T_B = I^{-0.177192}\phi^{1.545508}$$

Using I_r and ϕ_r as the imperviousness and conveyance parameters and I_u and ϕ_u as the values of these parameters after urbanization , the ratio of the eqivalent uniform intensity rectangular hydrographs in rural and urban conditions is given by,

$$T_b^r/T_b^u = (I_r/I_u)^{b1}(\phi_r/\phi_u)^{b2}$$

where: $b1 = -0.177192$

$b2 = 1.545508$

Let $I_r = 10\%$

$I_u = 80\%$

$\phi_{1r} = 1.0$ for natural conditions

$\phi_{2r} = 0.3$ for heavy vegetation

$\phi_r = \phi_{1r} + \phi_{2r} = 1.3$

$\phi_u = 0.6$

$$T_b^r/T_b^u = (10/80)^{b1}(1.3/0.6)^{b2} \quad (4.16)$$

$$T_b^r = 4.775\ T_b^u$$

The time base of hydrograph after urbanization is seen to be a constant fraction of the runoff event before urbanization

$$T_b^r = c_1 T_b^u$$

where: c_1 = constant

4.3 Stormwater Runoff Modeling(after urbanization):

Assumptions:

1) In the present analysis the eqivalent uniform intensity hydrographs(Morris and Wiggert,1972) will be used. Even though a triangular approximation would be more appropriate, it does not render a closed form tractable solution in the proposed analysis.

2) The random variables, volume of runoff before urbanization V_r, duration of runoff event before urbanization T_r, and time between successive runoff events X_3 are statistically independent (Di Toro and Small,1979; Padmanabhan and Delleur,1978) and exponentially distributed

(Howard,1976; Smith, 1980).

3) The previous storm completely fills the storage.

The following notation is used:

X_1 = volume of runoff after urbanization (L^3)

X_2 = duration of runoff event (T)

X_3 = time between successive runoff events (T)

Y = overflow volume (L^3)

a = treatment rate (L^3/T)

b = storage volume (L^3)

The stormwater runoff process is shown in schematically Figure 4.2. The overflow Y can be expressed as,

$$Y = X_1 - aX_2 - \min(aX_3,\ b) \quad \text{(for } X_1 - aX_2 > \min(aX_3,\ b)\text{)} \qquad (4.17)$$

$$= 0 \quad \text{elsewhere}$$

Using relationship (4.17) it is possible to compute the probability that the overflow volume exceeds some threshold volume.ie. $P(Y > y) < \alpha^1$

4.4 Quality Modeling:

Effluent Pollutant: The overflow can be related to the pollutant loading, by multiplying by the average concentration (Mueller and Anderson,1979).

$$\text{Total Pollutant Load, } L_p = C_e Y \qquad (4.18)$$

where: C_e = average concentration of the pollutant

Using relationship (4.18) it is possible to compute the pollutant exceedence probability.ie. $P(L_p > \ell) < \alpha^2$

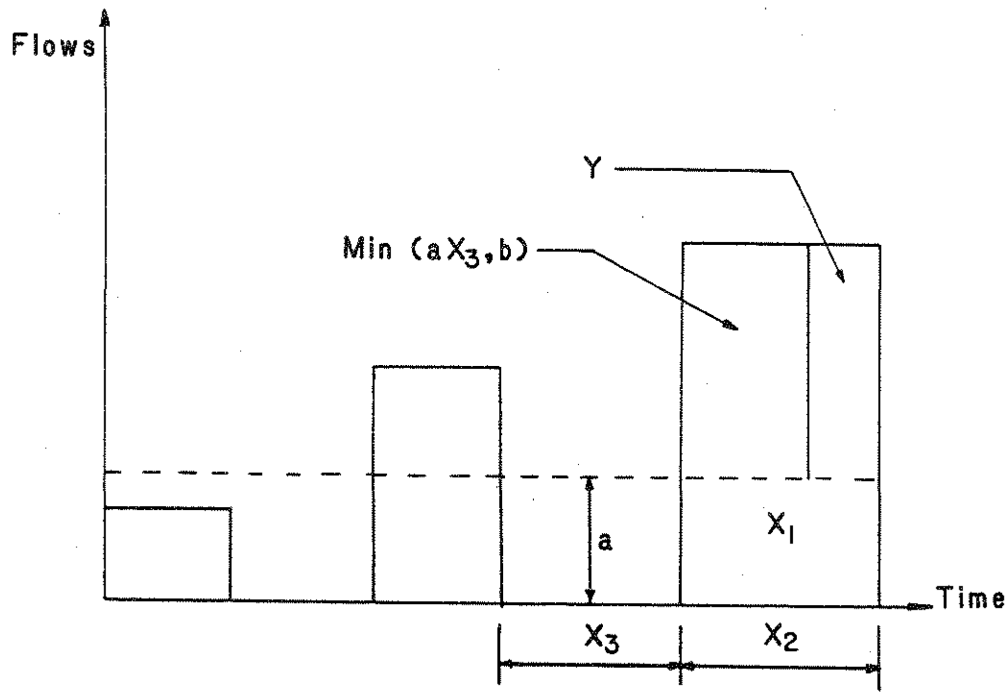

Figure 4.2 Definition Sketch for Overflow Volume.

<u>Receiving Body Pollutant:</u> Thus far the analysis is fairly straight forward. Now there is a difficulty in finding the correct distribution for the pollutant in the receiving water body . In appreciation of the central limit theorem the Normal density function is used in the literature. In this regard the remarks of Beckers et al.(1972) are pertinent. A correct distribution for a water quality parameter should have zero value for values of the parameter less than zero or greater than the specified maximum value. The Gaussian distribution does not fit these requirements. Becker et al. suggest the Rayleigh distribution. However the Rayleigh distribution also does not accurately portray the actual physical properties.

The hydrologic variables are mostly described by exponential families. In general the overflow pollutant load can be described in terms of the hydrologic variables and pollutant concentration. The resulting distribution is quite likely to have an exponential related structure. Appealing to the small particle statistics, the natural downstream pollution process may be described by a Lognormal distribution. A truncated lognormal distribution may be used for a finite range. However, convolution of lognormal with exponential related families is cumbersome. Most importantly a tractable closed form solution may not be possible for the convolution. To overcome some of these difficulties the following procedure is proposed. In general some governing body specifies the standards for the pollutant concentration in the receiving end. Assuming every user obeys the rule, the pollutant concentration varies between zero and some maximum value. Because of the absence of a widely accepted probability density function for the pollution variables, a natural recourse is to fit an empirical distribution to the available data. This approach can be fairly generalized if some distribution can approximate just about any empirical distribution. The Beta distribution is one such distribution because of its various shapes and finite range. In the present analyis the beta distribution is used for the receiving stream quality modeling.

4.5 Derivation of the Distribution Functions:

The following notation is used:

X_1 = volume of runoff event after urbanization (in)

X_2 = duration of runoff event after urbanization (hr)

X_3 = time between successive runoff events (hr)

a = treatment rate (in/hr)

b = storage volume (in)

V_r = volume of runoff event before urbanization (in)

T_r = duration of runoff event before urbanization (hr)

n_r, n_u = Manning n for rural and urban conditions

C_o = limiting concentration of the pollutant (mg/l)

C_e = concentration of effluent pollutant (mg/l)

C_R = concentration of receiving stream pollutant (mg/l)

V_R = volume of receiving stream flow over critical duration (in)

S = rescaled pollutant concentration, $[C_R/C_o]$

T_c = critical duration (critical time in oxygen sag) (hr)

Y = overflow volume (in)

u = urbanization factor from (4.1)

The variables V_r, T_r, X_3 are assumed to be exponentially distributed with parameters a_2, a_3 and γ, respectively. Thus,

$$P(V_r \leqq v_r) = 1 - \exp(-a_2 v_r) \quad (4.19)$$

$$P(T_r \leqq t_r) = 1 - \exp(-a_3 t_r) \quad (4.20)$$

$$P(X_3 \leqq x_3) = 1 - \exp(-\gamma x_3) \quad (4.21)$$

Using the relationship, $X_1 = (1+u) V_r$, and (4.19) the distribution function for the volume of runoff after urbanization is expressed as

$$F(x_1) = P(X_1 \leqq x_1) = P((1 + u)V_r \leqq x_1) = 1 - \exp(-\alpha x_1)$$

where: $\alpha = a_2/(1 + u)$

By differentiating $F(x_1)$ with respect to x_1 the density function for volume of runoff after urbanization is obtained as

$$f(x_1) = \alpha \exp(-\alpha x_1), \; x_1 > 0$$
$$= 0 \; , \; x_1 \leqq 0 \quad (4.22)$$

Similarly using the relationship, $X_2 = c\, T_r$

where: $c = (n_r/n_u)^{1/m}$

and (4.20) the density function for the duration of runoff events after urbanization is obtained as

$$f(x_2) = \beta \exp(-\beta x_2), \quad x_2 \geqq 0$$
$$= 0 \text{ , elsewhere} \qquad (4.23)$$

where: $\beta = a_3/c$

Using the density functions for X_1, X_2 and X_3 it is possible to derive the distribution function for Y.

Let U be the excess of the volume of runoff after urbanization over the volume that can be treated during the runoff event, namely

$$U = X_1 - aX_2 \qquad (4.24)$$

and let V be the available storage for later treatment. V is the least of the volume treated between consecutive runoff events and the whole storage depending upon the intermittent time. V is expressed as,

$$V = \min (aX_3, b) \qquad (4.25)$$

The overflow volume can then be expressed as,

$$Y = U - V \text{ , for } U > V$$
$$= 0 \text{ , for } U \leqq V \qquad (4.26)$$

In order to find the distribution of the overflow volumes Y, the distribution functions of U and V will be computed first.

<u>Distribution of V:</u>

Since V is the least of $\{aX_3 , b\}$ the exceedence probability can be written as

$P(V > v) = P(aX_3 > v)\ P(b > v)$

Since X_3 is assumed to be exponentially distributed with parameter γ , it is obtained that

$$P(V > v) = \exp(-\gamma v/a)\ 1 \ ,\text{if } v < b$$
$$= 0 \text{ if } v \geqq b \qquad (4.27)$$

The distribution of V is expressed as

$$P(V \leqq v) = 1 - \exp(-\gamma v/a),\ \text{for } v < b$$
$$= 1\ ,\ \text{otherwise} \qquad (4.28)$$

Note that the distribution of V has a point mass at b. The density function for V is written as

$$f(v) = (\gamma/a)\ \exp(-\gamma v/a)\ ,\ \text{for } v < b$$
$$= \exp(-\gamma b/a)\ ,\ \text{for } v = b \qquad (4.29)$$

Since only positive overflows are of concern and V is nonnegative ,with regard to the exceedence probability of U, only a positive threshold value, $u > 0$, needs be considered.

<u>Exceedence probability for U:</u> $P(U > u)$,$u > 0$

From the definition of U,(4.24) the exceedence probability of U is expressed as

$$P(U > u) = P(X_1 - aX_2 > u)$$
$$= P(X_1 > u + aX_2)$$
$$= \int P(X_1 > u + ax_2)\ dF(x_2) \qquad (4.30)$$

With the probability distribution of X_1, (4.22) and the density of X_2, (4.23) the expression (4.30) becomes

$$P(U > u) = {}_0\int^{\infty} \exp(-\alpha(u + ax_2))\ \beta\ \exp(-\beta x_2)\ dx_2$$
$$= \{\ \beta/(\beta + \alpha a)\}\ \exp(-\alpha u)\ ,\ \text{for } u > 0 \qquad (4.31)$$

<u>Distribution of overflow, Y:</u>

Since U and V are independent due to the independence assumption on X_1, X_2 and X_3, the distribution of $Y = U - V$ for $U > V$ can be expressed as

$P(0 < Y \leqq y) = P(0 < U-V \leqq y) = P(V < U \leqq V+y)$

$P(0 < Y \leqq y) = {}_0\int^b f(v) \ {}_v\int^{v+y} f(u)\ du\ dv$

(for $y > 0$) , i.e. $U > V$

With the use of the exceedence probability of U, (4.31) and the density of V, (4.29) the following expression is obtained:

$$P(0 < Y \geqq y) = {}_0\int^{b-} (\gamma/a)\exp(-\gamma v/a)$$
$${}_v\int^{v+y} \alpha\beta/(\alpha a + \beta)\exp(-\alpha u)\ du\ dv$$
$$+ \exp(-\gamma b/a)\{\beta/(\beta + \alpha a)\}$$
$$[1- \exp(-\alpha\ (b + y)) -1 + \exp(-\alpha b)] \qquad (4.32)$$

The last term in (4.32) results from the point mass at b. The term $\exp(-\gamma b/a)$ is the jump at b and the other product term is the value of the integral with respect to u at b. Thus performing the integration in(4.32) and rearranging the expressions (4.33) and (4.34) are obtained.

$$P(0 < Y \leqq y) = K(1 - \exp(-\alpha y)) \qquad (4.33)$$

where: $K = \{[\beta\gamma/((\gamma + \alpha a)(\beta + \alpha a))]$

$$[1 - \exp(-b(\alpha + \gamma/a))]\}$$
$$+ \beta/(\beta + \alpha a)\ [\exp(-b(\alpha + \gamma/a))] \qquad (4.34)$$

The probability density of the overflows is expressed as

$$f(y) = K\ \alpha\ \exp(-\alpha y)\ ,\ \text{for } y > 0$$
$$= 1 - K\ ,\ \text{for } y = 0 \qquad (4.35)$$

This density is physically meaningful. Consider the case b = 0 and 'a' is very,very small. Since storage b is zero and very little treatment is available, all the runoff will overflow. That is the probability of overflows is the same as probability of runoff. By substituting b = 0 and a very,very small value for 'a' in (4.34) it is obtained that $K = 1$. When $K = 1$ expression (4.35) reduces to the density function for the runoff volumes given by (4.22).

Water Quality Distribution:

In Figure 4.3 the total effluent pollutant load, L_p discharged into the receiving stream is given by

$L_p = C_e Y$, where C_e is the effluent concentration.

In BOD analysis Streeter - Phelps equation provides a critical distance x_c at which the minimum dissolved oxygen occurs. The flow travel time to point x_c is called(Figure 4.4) the critical time. The critical time period is given by (Metcalf and Eddy,1972),

$$T_c = 1/(K_2 - K_1)\{ \ln(K_2/K_1)\}$$
$$[1 -\{ D_o(K_2 - K_1)/(K_1 BOD_\ell)\}] \qquad (4.36)$$

where:

BOD_ℓ = ultimate BOD (mg/l)

K_1 = BOD rate constant (per day)

K_2 = reaeration constant (per day)

D_o = initial oxygen deficit (mg/l)

Let V_R be the volume of flow in the stream during the critical period.

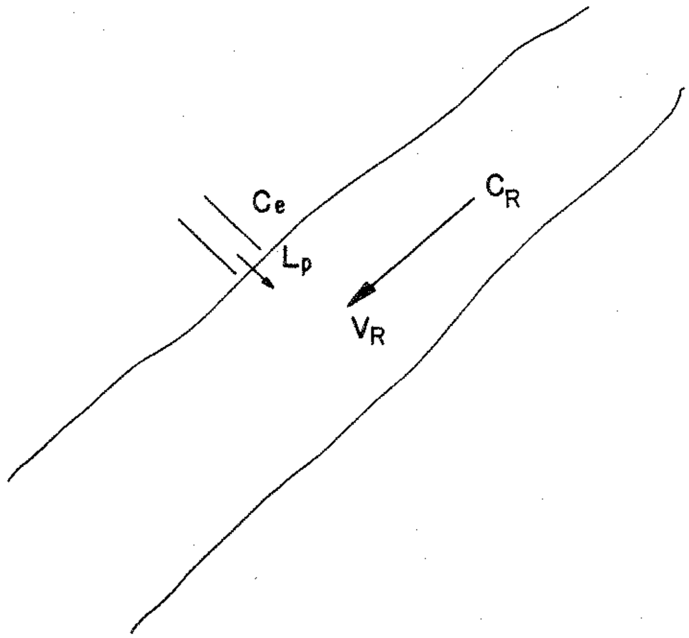

Figure 4.3 Definition Sketch for Mixing

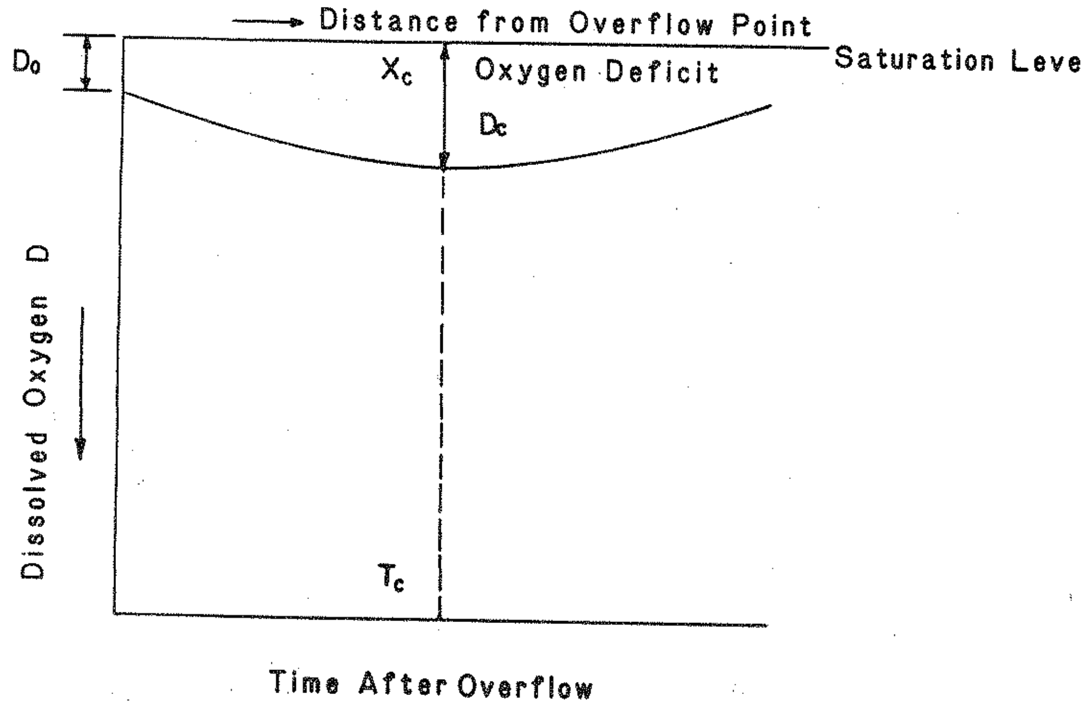

Figure 4.4 Oxygen Sag Curve

<u>Assumption:</u> V_R is Gamma distributed with scaling parameter ρ and shape parameter θ.

$$f(v_R) = (\rho^{\theta}/\Gamma(\theta))v_R^{\theta-1}\exp(-\rho v_R) \quad (\rho > 0,\ \theta > 0 \text{ and } v_R > 0)$$
$$= 0,\ \text{for } v_R \leqq 0 \qquad (4.37)$$

Letting C_R be the pollutant concentration in the receiving stream of the overflow point, the concentration of pollutant after mixing may be written as

$$C_m = (C_eY + C_RV_R)/(Y + V_R) \qquad (4.38)$$

<u>Assumption:</u> In general, the receiving flow volume is much larger than the overflow volume during the critical time period T_c, i.e. $V_R >> Y$. Hence we may approximate $Y + V_R$ by V_R.

Therefore

$$C_m = C_e(Y/V_R) + C_R \qquad (4.39)$$

It is desired that the probability that C_m exceeds a limiting threshold, C_o, be very small, say not larger than ϵ, i.e. $P(C_m \geqq C_o) \leqq \epsilon$ which, with the use of equation (4.39), may be rewritten as

$$P[(C_e/C_o)(Y/V_R) + C_R/C_o \geqq 1] \leqq \epsilon \qquad (4.40)$$

<u>Assumption:</u> There is very little information available on the probability distribution of C_R but it is expected that information will gradually become available as the EPA/USGS Nationwide Urban Runoff Program (NURP) nears completion. A flexible probability distribution is therefore chosen for C_R so that it can reasonably be expected to fit field observations. For this reason the distribution of C_R/C_o is

assumed to be Beta distributed. There are two reasons for rescaling C_R with respect to C_o. First it enables the range to be (0,1) so that the Beta distribution can be fitted. The second reason is the rescaling coincidently simplifies the procedure in finding the exceedence probability as shown in (4.40). To obtain the exceedence probability in (4.40), the distributions of

$$Z = (Y/V_R),\ k_1 Z = (C_e/C_o)(Y/V_R)\ ,\ S = C_R/C_o \tag{4.41}$$

are computed successively.

<u>Distribution of, $Z = (Y/V_R)$:</u>

Since the overflow, Y is positive, and V_R is positive, z is positive.

$$P(Z \leqq z) = P(Y/V_R \leqq z) = P(\ Y \leqq zV_R)\ ,\ z > 0$$

$$= \int P(Y \leqq zv_R)\ f(v_R)\ dv_R$$

Using (4.33) and (4.35) it is written as,

$$P(Z \leqq z) = {}_0\int^{\infty}[(1 - K) + K(1 - \exp(-\alpha z v_R))]\ f(v_R)\ dv_R$$

Which by introduction of the density for V_R (4.37) yields,

$$P(Z \leqq z) = (1/\Gamma(\theta))\ {}_0\int^{\infty}[1-K \exp(-\alpha z v_R)]\rho^{\theta}\ v_R{}^{\theta-1}\exp(-\rho v_R)\ dv_R$$

The integration yields

$$P(Z \leqq z) = 1 - K\rho^{\theta}/(\rho + \alpha z)^{\theta}\ ,\ \text{for } z > 0 \tag{4.42}$$

<u>Distribution of $(C_e/C_o)Z$:</u>

$$\text{Let } k_1 = C_e/C_o\ , k_1 > 0 \tag{4.43}$$

$$\text{Let } T = k_1 Z \tag{4.44}$$

$$P(T \leqq t) = P(Z \leqq t/k_1)\ ,\text{for } k_1 > 0\ ;\ t > 0$$

Substituting (t/k_1) for z in (4.42) it is obtained that

$$P(T \leqq t) = 1 - K(\rho k_1)^{\theta}/(\rho k_1 + \alpha t)^{\theta}\ ,\ \text{for } k_1 > 0\ ;\ t > 0$$

(4.45)

Distribution of Mixed Concentration:

Let $S = C_R/C_o$ (4.46)

As S is assumed to be beta distributed with parameters p and q, $p > 0$, $q > 0$

$$f(s) = K_p\, s^{p-1}(1-s)^{q-1}, \text{ for } 0 < s < 1$$
$$= 0, \text{ elsewhere} \quad (4.47)$$

where:

$$K_p = \Gamma(p + q)/(\Gamma(p)\Gamma(q)) \quad (4.48)$$

From the expression (4.40) the critical exceedence probability is stated as

$$P(T + S \geqq 1) = 1 - P(T + S \leqq 1)$$
$$= 1 - P(T \leqq 1-s) \text{ , } \underline{\text{Note:}}\ (1 - s) > 0$$

Using the expression (4.45) it is obtained that

$$P(T + S \geqq 1) = 1 - {}_o\int^1 [1 - \{K(\rho k_1)^\theta/(\rho k_1 + \alpha(1-s))^\theta\}\, f(s)\, ds$$
$$= 1 - 1 + K(\rho k_1)^\theta\ {}_o\int^1 [1/(\rho k_1 + \alpha - \alpha s)^\theta]\, f(s)\, ds$$

let $k_2 = \rho k_1 + \alpha$ and $k_3 = \alpha/k_2$

Hence

$$P(T + S \geqq 1) = K(\rho k_1)^\theta\ {}_o\int^1 [1/(k_2 - \alpha s)^\theta]\, f(s)\, ds$$

Using k_3 it is obtained

$$P(T + S \geqq 1) = K(\rho k_1/k_2)^\theta\ {}_o\int^1 [1/(1 - k_3 s)\theta]\, f(s)\, ds$$
$$= K(\rho k_1/k_2)^\theta\, K_p\ {}_o\int^1 s^{p-1}(1-s)^{q-1}(1-sk_3)^{-\theta}\, ds$$
$$= K(\rho k_1)^\theta\, F(\theta,p;r;k_3)/(k_2)^\theta \quad (4.49)$$

where:

$k_1 = C_e/C_o$

$k_2 = \rho k_1 + \alpha$

$k_3 = \alpha/k_2$, Note: $0 < k_3 < 1$

$r = p + q$

$$F(\theta,p;r;k_3) = K \int_0^1 s^{p-1}(1-s)^{r-p-1}(1-sk_3)^{-\theta}\, ds \qquad (4.50)$$

$F(\theta,p;r;k_3)$:(Abramowitz and Stegun,1972;Johnson and Kotz,1969)

$F(\theta,p;r;k_3)$ in (4.50) can be expressed in terms of an infinite series.

$$F(\theta,p;r;k_3) = \Sigma\theta^{[j]}\, p^{[j]}\, k_3{}^{j}/(r^{[j]}\, j!) \qquad (4.51)$$

where: $x^{[j]} = x(x + 1)(x + 2)......(x + j- 1)$

This series converges for $|k_3| < 1$

These results are summarised in the following theorems.

Theorem 3.1: X_1,X_2 and X_3 are exponentially distributed with α,β and γ as parameters.

Let $Y = X_1 - aX_2 - \min(aX_3,b)$ for $Y > 0$

$=0$, otherwise

Then Y has the following distribution

$$f(y) = K\,\alpha \exp(-\alpha y)\ , \text{ for } y > 0$$
$$= 1 - K\ , \text{ for } y = 0 \qquad (4.35)$$

where: $K = \{[\beta\gamma/((\gamma + \alpha a)(\beta + \alpha a))]$
$[1 - \exp(-b(\alpha + \gamma/a))]\}$
$+ \beta/(\beta + \alpha a)\ [\exp(-b(\alpha + \gamma/a))]$ (4.34)

Theorem 3.2: Let Y be distributed as in Theorm 3.1. Also let V_R be gamma distributed with parameters θ and ρ and let k_1 be some constant. Then $T = k_1(Y/V_R)$ has the following cumulative probability distribution function.

$$P(T \leqq t) = 1 - K(\rho k_1)^{\theta}/(\rho k_1 + \alpha t)^{\theta}\ , \text{ for } k_1 > 0\ ;\ t > 0$$

<u>Theorem 3.3:</u> Let T have a CPDF as in Theorem 3.2. Also let S be beta distributed with parameters p,q. Then the following result holds.

$$P(T + S \geqq 1) = K(\rho k_1)^{\theta} F(\theta,p;r;k_3)/(k_2)^{\theta} \qquad (4.49)$$

where:

$k_1 \quad = C_e/C_o$

$k_2 \quad = \rho k_1 + \alpha$

$k_3 \quad = \alpha/k_2$, <u>Note:</u> $0 < k_3 < 1$

$r \quad = p + q$

$F(\theta,p;r;k_3) \quad = K \int_0^1 s^{p-1}(1-s)^{r-p-1}(1-sk_3)^{-\theta}\, ds$

<u>Summary of Results:</u>

With reference to Figure 2.1, the results may be summarized as follows:

X_1 = volume of runoff after urbanization; $E(X_1) = 1/\alpha$

X_2 = duration of runoff event after urbanization;

$$E(X_2) = 1/\beta$$

X_3 = time between successive runoff events, $E(X_3) = 1/\gamma$

Y = overflow volume after urbanization

$= X_1 - aX_2 - \min(aX_3, b)$

where: a = treatment rate

b = storage volume

then $f(y) = K\alpha \exp(-\alpha y)$ for $y > 0$

$= 1 - K$ for $y = 0$ (probability of no overflow)

where: $K = f(\alpha,\beta,\gamma,a,b)$ (4.34)

V_R = volume of flow in stream during critical period.

$\sim \text{Gamma}(\rho,\theta)$

C_e = concentration of pollutant in overflow event

C_o = maximum acceptable concentration of pollutant

C_R = concentration of pollutant in the receiving stream before mixing with overflow; $C_o/C_R \sim Beta(p.q)$

C_m = concentration of mixed overflow and receiving water

$P[(C_m/C_o) \geqq 1] \leqq \epsilon$ = probability of violation

$P[(C_m/C_o) \geqq 1] = K[C_e/(C_c+(\alpha C_o)/\rho)]^{\theta}F(.)$

$F(.) = f(\theta,\rho,p,q,\alpha,C_e,C_o)$ (4.51)

Example of An Extreme Case: Consider the situation wherein both C_e and C_R are very close to C_o. There is no or little dilution possible. In such circumstances the controlling criterion can be none of the pollutant concentrations should violate the threshold value C_o. Since there is very little control one can exercise on the pollutant concentration C_R, the only possible alternative is to keep the effluent concentration C_e to a minimum.

Let C_p = pollutant accumulation rate per unit of time

The total accumulated pollutant load between runoff events can be expressed as,

$$\text{Total load}, L_p = C_p X_3 + d \qquad (4.52)$$

where: d = average leftover pollutant load

Since X_3 is exponentially distributed with parameter γ, the density function for L_p can be expressed as,

$$f(\ell) = (1/C_p)\gamma \exp(-\gamma((\ell - d)/C_p)) \text{ , for } \ell \geqq d$$
$$= 0 \text{ , elsewhere} \qquad (4.53)$$

the concentration is expressed as

$$T_1 = \text{load/volume} = L_p/X_1 \qquad (4.54)$$

Distribution of T_1:

Using (4.54)

$$P(T_1 \leqq t) = P(L_p \leqq tX_1)$$

$$= P(X_3 \leqq (tX_1 - d)/C_p)$$

Since X_3 is exponentially distributed with parameter γ

$$P(T_1 \leqq t) = {}_{(d/t)}\int^{\infty}[1 - \exp(-\gamma((tx_1 - d)/C_p))]\ f(x_1)\ dx_1 \qquad (4.55)$$

By (4.22) X_1 is exponentially distributed with parameter α it is obtained that

$$P(T_1 \leqq t) = \exp(-\alpha d/t)$$

$$- {}_{(d/t)}\int^{\infty} \alpha \exp[-x_1(\alpha+(\gamma t\ /C_p))+(\gamma d/C_p)]\ dx_1$$

$$= \exp(-\alpha d/t) - (\alpha C_p/(\alpha C_p+\gamma t))\ \exp(-\alpha d/t)$$

Hence it is obtained that

$$P(T_1 \leqq t) = [\gamma t/(\alpha C_p + \gamma t)]\ \exp(-\alpha d/t) \qquad (4.56)$$

Concentration in Receiving Water, S_1:

From small particle statistical theory lognormal distribution can also be assumed as an alternative to the beta distribution for the pollutant concentration in the receiving stream. The preliminary results from NURP also indicate that the receiving stream pollutant may be lognormally distributed. Hence in this example case it is assumed that S_1 is lognormally distributed.

$$f(s) = K_s \exp(-1/2((\ell og\ s - \mu_{\ell ogs})/\sigma_{\ell ogs})^2)$$

$$(\text{for } s \leqq 0)$$

$$= 0\ ,\ \text{elsewhere} \qquad (4.57)$$

where: $K_s = 1/(s\ \sigma_{\ell ogs}\ (2\pi)^{1/2})$ (4.58)

Mixed concentration, $C_m \leqq \max(T_1, S_1)$

Let $W = \max(T_1, S_1)$

It is desired that $P(W \geqq C_o)$ to be small.

$$P(W \geqq C_o) = 1 - P(T_1 \leqq C_o)\ P(S_1 \leqq C_o)$$

$$= 1 - \{[\gamma C_o/(\alpha C_p + \gamma C_o)]\exp(-\alpha d/C_o)\}\ \Phi(\ell og\ C_o) \quad (4.59)$$

where: $\Phi(\ell og\ C_o)$ = corresponding area under standard Normal curve

<u>Digression:</u> The special circumstance considered above also indicates that it is always better to control the source than increasing the capacity of the treatment plant or of the storage facility. In the above analysis the only controllable parameter is the pollution accumulation rate, C_p. By implementing better source control practices it is possible to minimize the parameter C_p. There is also another case when the receiving body of water is a lake with no or little outflow. In that case it is the accumulation of the pollution load that governs, possibly with some decay.

<u>4.6 Advantages of the Derived Distribution:</u> The newly developed probability distributions provide a simple but powerful methodology in solving urban stormwater problems. These probability functions can be applied to stormwater problems in estimating the size of the detention basin, and treatment plant for a given reliability level. Also the degree of control needed to contain the pollution source can

be found. Closed form, tractable analytical solutions are simple to use and may be viewed as a better table top technique in solving urban water management problems.

CHAPTER V

APPLICATION TO WEST LAFAYETTE

The methodology developed in the previous chapters is used for West Lafayette. Section 5.1 contains the development of hydrologic constraints. Section 5.2 contains the whole formulation and results.

5.1 Hydrologic Constraints: There are two sets of data used for the study. The rainfall data for the study are taken at the Purdue Agronomy Farm, approximately 5 miles northwest of the West Lafayette city hall. The rainfall data for the periods 1953 to 1974 and 1977 to 1979 are analysed. The rainfall values are converted into runoff values using a runoff of coefficient 0.21 and a maximum depression storage 0.18 inch (Sautier and Delleur,1978). The runoff data for the period 1953 to 1974 are used only to check the performance of the analytical model against the simulation model STORM. The runoff data for 1977 to 1979 are used for the whole analysis since this would reflect a more realistic picture of current West Lafayette situation. Tables 5.1 and 5.2 contain the information concerning the runoff data. The river water quality data are obtained from the Indiana State

Board of Health yearly publications. BOD concentration is taken as the criterion for the analysis. Table 5.3 contains the needed water quality information. The average BOD concentration of combined sewer effluent is computed from the yearly data 1977 to 1979 supplied by the city engineer's office. This information is shown in Table 5.3. The daily river flows for the years 1977 to 1979 are obtained from the USGS Water Resources Data yearly publications. For consistency in units, the flow values are converted to basin inches per hour from cfs. The mean and variance of the flow values are listed in Table 5.3.

Table 5.1. Runoff Data(1953-1974)

location	West Lafayette
period of record	1953 to 1974
total number of events[1]	2538
area[2]	3052 acres
mean runoff volume, $E(X_1)$	0.06 in
mean duration, $E(X_2)$	2.1 hr
mean intermittent time, $E(X_3)$	70 hr

Table 5.2. Runoff Data(1977-1979)

location	West Lafayette
period of record	1977 to 1979
total number of events[1]	366
area[2]	3052 acres
mean runoff volume, $E(V_r)$	0.06 in
mean duration, $E(T_r)$	1.73 hr
mean intermittent time, $E(X_3)$	58.61 hr

1. Runoff events from rainfall using runoff coefficient .21 and depression storage .18 inch.
2. Total sewered area of West Lafayette(December 1981)

Table 5.3 Quality and River Flow Data

Mean BOD concentration in river, $E(C_R)$	4.1 mg/ℓ
EPA limiting BOD concentration, C_o	35 mg/ℓ
Rescaled pollutant concentration, S	C_R/C_o
Mean rescaled concentration, E(S)	0.117
Var(S)	0.004
Mean storm runoff concentration, C_e	170mg/ℓ
Mean hourly hourly flow[1]	1.91 in/hr
Variance of hourly flow[1]	5.42 $(in/hr)^2$

1. converted to inches over basin per hour.

Verification of Assumptions:

The assumptions on the statistical distributiuons of the runoff data are verified. The exponential distributions for runoff volume, duration, and intermittent time are checked by plotting the log cumulative exceedence probability and the corresponding threshold values. It must plot as a straight line. Figures 5.1 to 5.3 prove the hypotheses of exponential distribution for runoff volume, duration and intermittent time. The independence assumption is tested using the definition of statistical independence,

ie. $P(X_1 \leqq x_1, X_2 \leqq x_2, X_3 \leqq x_3) = P(X_1 \leqq x_1)\ P(X_2 \leqq x_2)\ P(X_3 \leqq x_3)$

Table 5.4. Statistical Independence

$(x_1.x_2,x_3)$	$F(x_1,x_2,x_3)$	$F(x_1)F(x_2)F(x_3)$
(.04,2,20)	.41925	.33922
(.06,3,40)	.54968	.49945
(.08,4,60)	.63354	.60898
(.1,5,80)	.73602	.72874
(.12,6,100)	.80434	.79799
(.14,7,120)	.84782	.84281

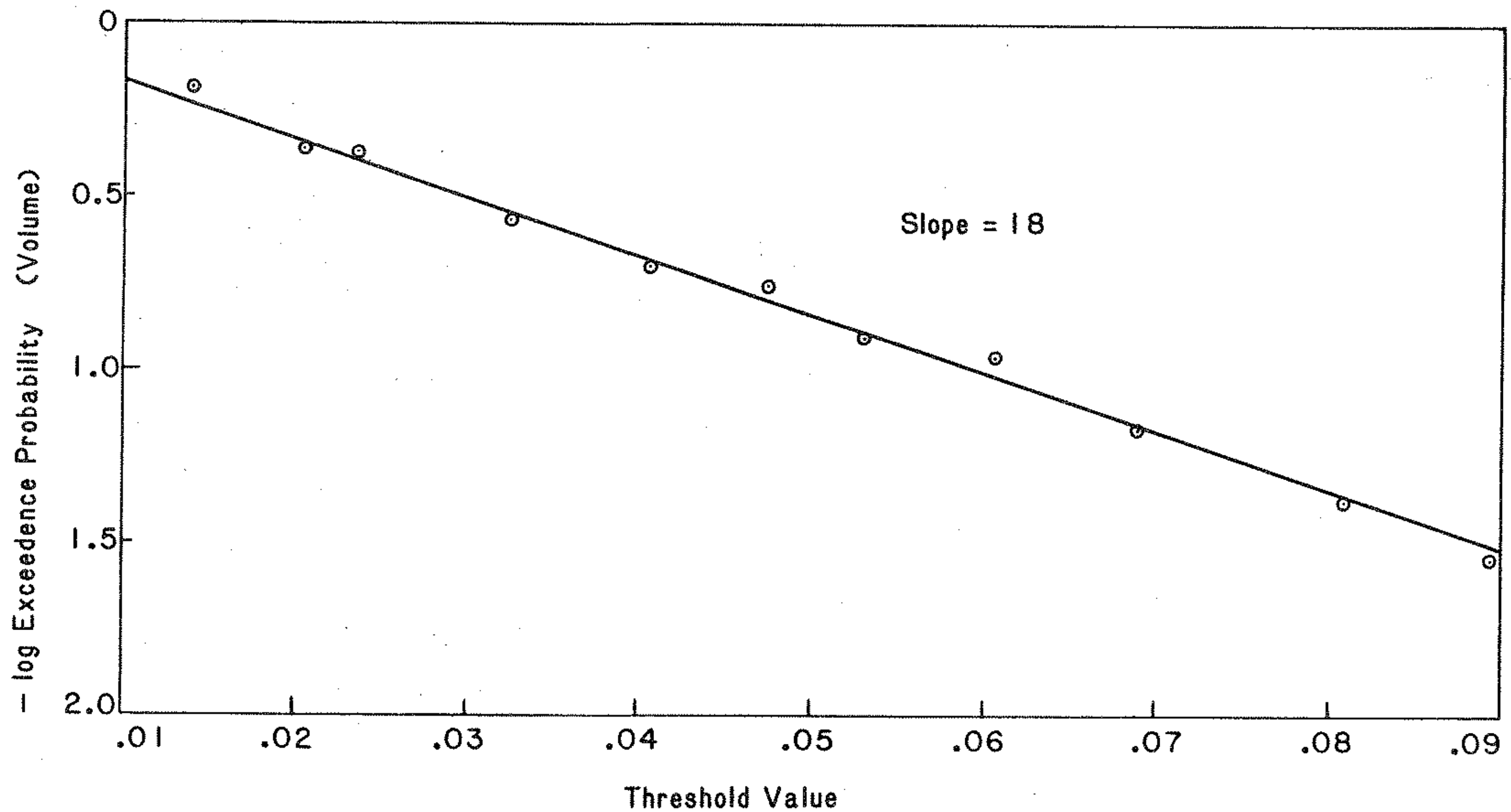

Figure 5.1 Plot of log Exceedence Probability

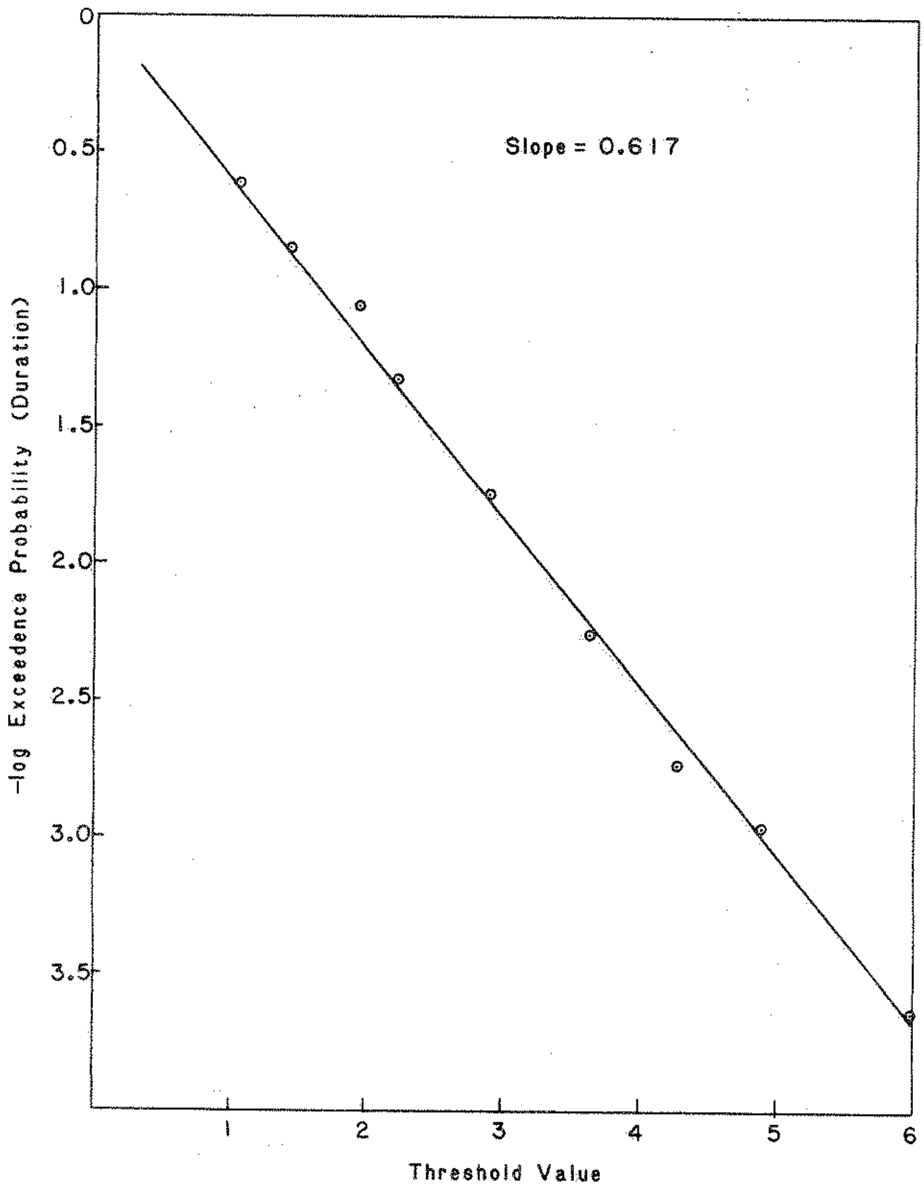

Figure 5.2 Plot of log Exceedence Probability

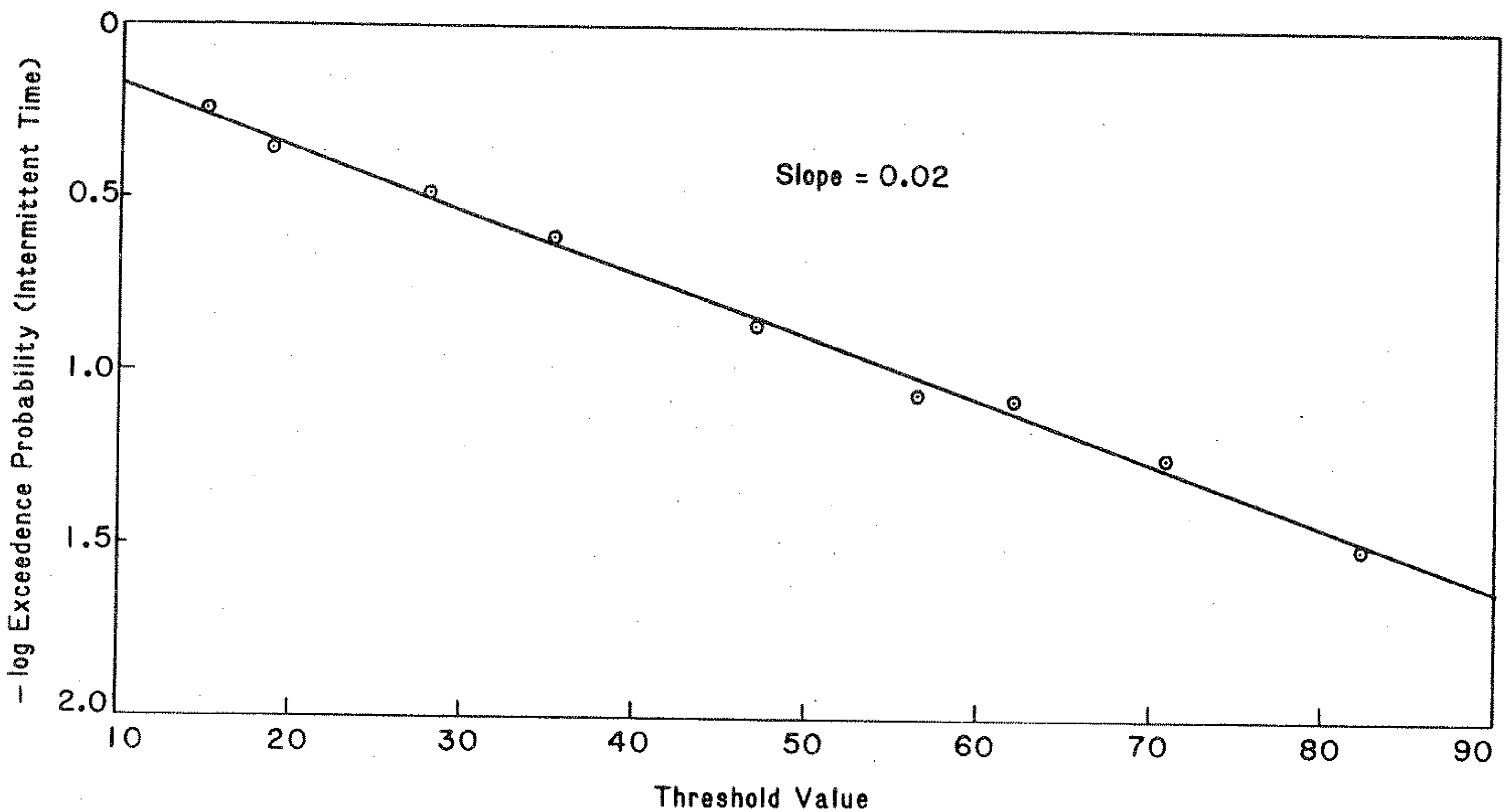

Figure 5.3 Plot of log Exceedence Probability

Because, as shown in Table 5.4, the joint probabilities and product of the marginals are close it is concluded that the variables are independent. As a further analysis of independence the following theorem is used.

Theorem 5.1(Kumar Jogdeo,1968):

Let $F(x_1,x_2,x_3) \geqq F(x_1)F(x_2)F(x_3)$. Then $E(X_iX_j)=EX_iEX_j$, $i \neq j$, and $E(X_1X_2X_3)= EX_1EX_2EX_3$; implies that X_1,X_2,X_3 are independent.

In the present analysis

$E(X_1X_2X_3)=9.149$ and $EX_1EX_2EX_3 =8.82$

$E(X_1X_2)=.174$ and $EX_1EX_2=.126$

$E(X_1X_3)=3.405$ and $EX_1EX_3=4.2$

$E(X_2X_3)=92.52$ and $EX_2EX_3=147.0$

From these values and Theorem 5.1. it is concluded that the the random variables, runoff volume, duration, and intermittent time are statistically independent.

Comparison of the analytical and simulation model STORM: In the follwing section the analytical model developed in Chapter III is compared with the simulation model STORM. The overflow probability is given by the expression,

$$P(Y = 0) = 1 - K$$

$$P(0 < Y \leqq y) = K(1 - \exp(-\alpha y)) \qquad (4.33)$$

where: $K = \{[\beta\gamma/((\gamma + \alpha a)(\beta + \alpha a))]$

$(1 - \exp(-b(\alpha + \gamma/a))\}$

$+ \beta/(\beta + \alpha a) [\exp(-b(\alpha + \gamma/a))] \qquad (4.34)$

Also from Table 5.1, for 1953 to 1974 data it is obtained,

$\alpha = 1/E(X_1) = 1/0.06 = 16.7$

$\beta = 1/E(X_2) = 1/2.1 = 0.4761$

$\gamma = 1/E(X_3) = 1/70 = 0.014$

Using the equation (4.34) K is computed as

$$K(a,b) = 4[0.4761 \times 0.014\ [1 - \exp\{(-b(16.7+0.014/a)\}]$$
$$/(0.014 + 16.7a)(0.4761 + 16.7a)]$$
$$+[0.4761 \exp\{-b(16.7+0.014/a\}/(0.4761 + 16.7a)]$$

For existing conditions in West Lafayette

$a = 0.006$ and $b = 0$; $K(.006,0) = 0.8261$

and $P(0 < Y \leqq y) = 0.8261[1 - \exp(-16.7y)]$

For varying treatment rates and storage capacities the analytical model results are plotted in Figures 5.4 to 5.7 along with the results of model STORM. The parameters used in the simulation analysis are shown in Table 5.5. These parameters are taken from Padmanabhan and Delleur(1978) and Sautier and Delleur(1978). The model STORM is used only in quantity analysis mode.

Table 5.5. Parameters for STORM

computation of runoff by coefficient method
Runoff coefficient(pervious) = 0.08
Runoff coefficient(impervious) =0.34
Maximum depression storage = .18 inch
Percentage imperviousness = 50%
Street sweeping efficiency =0.7
Number of land uses = 1
Washoff decay coefficient = 2.0
Also information in Table 5.1

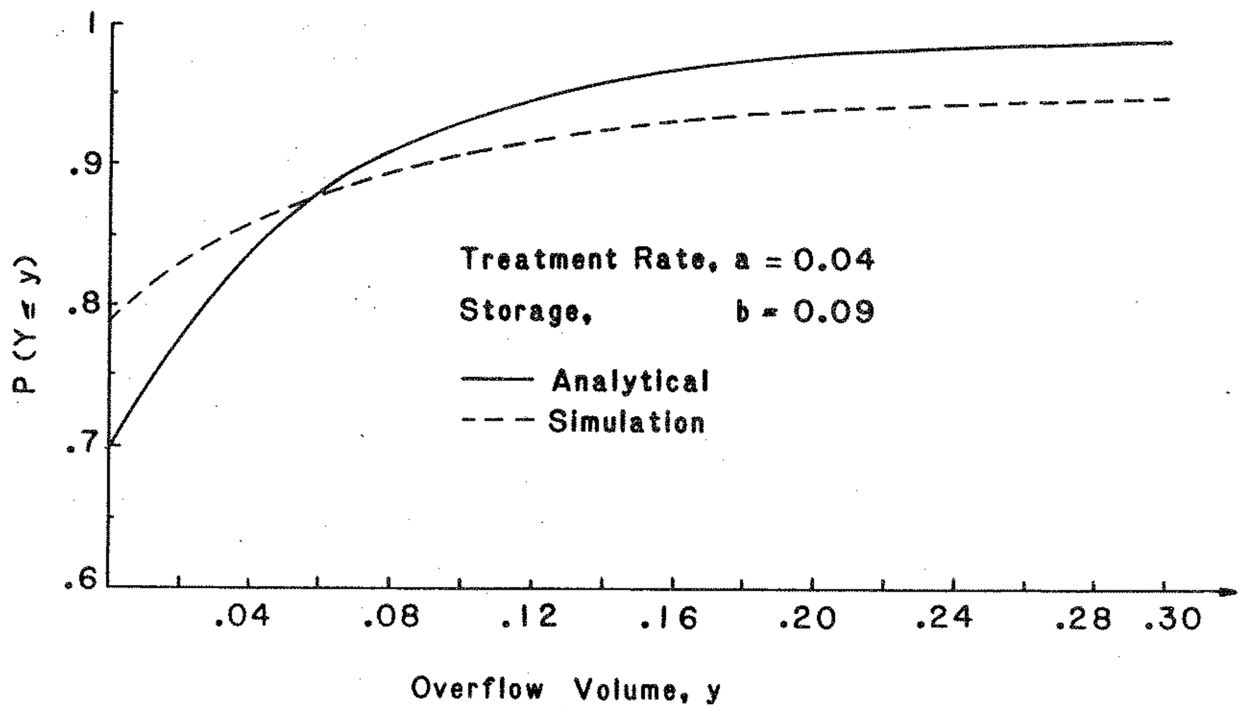

Figure 5.4 Comparison of Analytical and Simulation Results

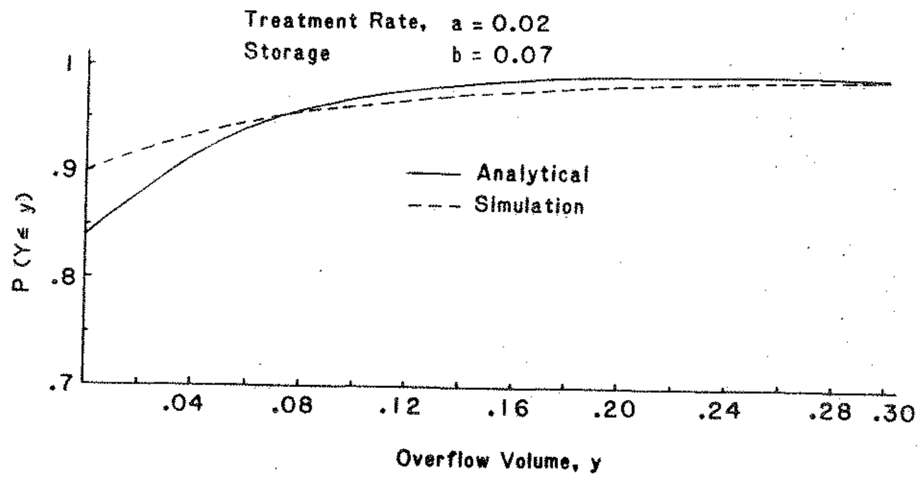

Figure 5.5 Comparison of Analytical and Simulation Results

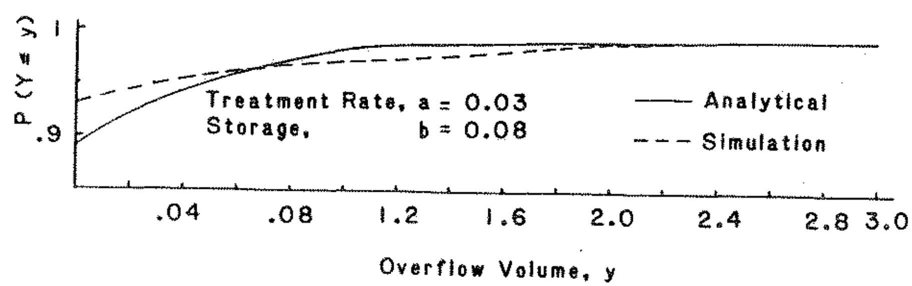

Figure 5.6 Comparison of Analytical and Simulation Results

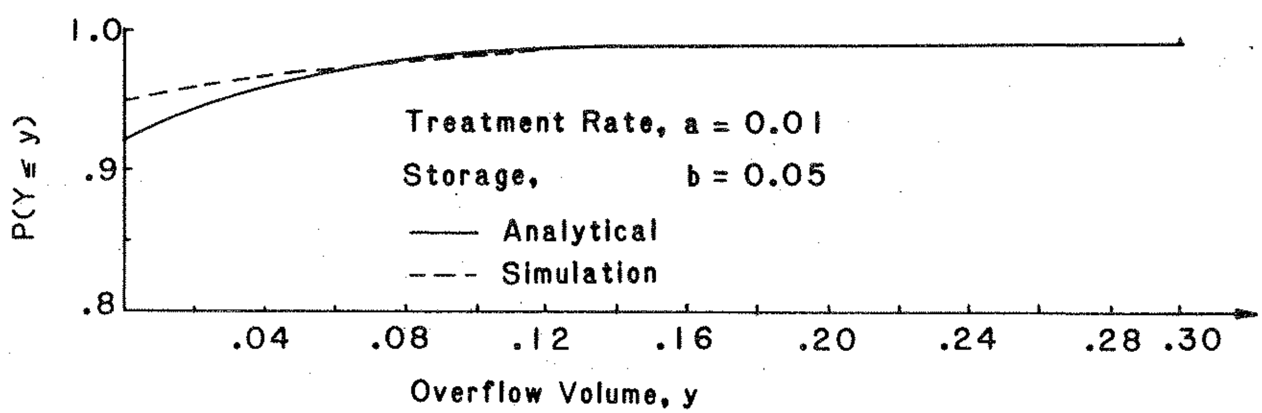

Figure 5.7 Comparison of Analytical and Simulation Results.

<u>General overflow probability computation:</u>

For the general analysis the 1977 to 1979 data are used. The parameters are evaluated as shown.

$\alpha = a_2/(1+u)$ with $a_2 = 1/E(V_R)$, thus from Table 5.2, $\alpha = (1/.06)\times(1/(1+u))$

$\alpha = 16.7/(1+u)$

$\beta = a_3/$(coefft. of time base reduction)

In the case of West Lafayette the total vacant land available for growth from Table 5.9 is 6755 acres. Thus the current imperviousness ratio for the urbanized area of 3052 acres can be computed. It is obtained that $I_r = 0.31$. Similarly from Tables 5.7 and 5.8 assuming that the demand is exactly satisfied, it can be obtained that $I_u = 0.39$. The conveyance factors are computed as the weighted average of urbanized and nonurbanized fractions. For the existing condition of 0.31 urbanized and 0.69 nonurbanized the compound conveyance factor, for the values given in section 4.2 may be estimated as

$\phi_r = 0.31\times0.6+0.69\times1.3 = 1.083$

For the conditions after urbanization the compound conveyance factor is computed as,

$\phi_u = 0.39\times0.6+0.61\times1.3 = 1.027$

Using the equation 4.16 the time base reduction factor can be computed.

$$T_b{}^r/T_b{}^u=(.31/.39)^{-0.177192}\ (1.083/1.027)^{1.545508}$$

$$= 1.131$$

Hence β can be computed as follows.

From Table 5.2, $a_3 = 1/E(T_R) = 1/1.73 = 0.578$,

thus $\beta = .578/1.131 = 0.511$

$\gamma = 0.0171$

Using the above parameters the expression for K becomes,

$$K = [0.0087(1+u)^2\{1-\exp[-b\{(16.7/(1+u))+ (.0171/a)\}]\}$$
$$/\{(0.0171(1+u)+16.7a)(0.511(1+u)+16.7a)\}]$$
$$+[.511/(.511+16.7a/(1+u))]$$
$$\exp\{-b[\{16.7/(1+u)\}+ \{.0171/a\}]\}$$

Beta Distribution for S: The rescaled river BOD concentration value is fitted with the beta distribution. The parameters are computed from the mean and variance of S shown in Table 5.3.

$$f(s) = K_p\, s^{p-1}(1-s)^{q-1}\ , \text{ for } 0 < s < 1$$
$$= 0\ , \text{ elsewhere} \qquad (4.47)$$

where:

$$K_p = \Gamma(p + q)/(\Gamma(p)\Gamma(q)) \qquad (4.48)$$

Using the data from Table 5.3 it is obtained, $E(S) = 0.117 = p/(p + q)$

$Var(S) = 0.004 = pq/[(p + q)^2 (p + q + 1)]$

From the above relations it is obtained,

$p = 2.956$, $q = 22.058$

Computation of critical time for reoxygenation: The Streeter-Phelps equation is used for the recovery time of self purification of the river.

$$T_c = 1/(K_2 - K_1)\{\ \ln(K_2/K_1)\}$$

$$[\,1 - \{\, D_o(K_2 - K_1)/(K_1 BOD_\ell)\}] \qquad (4.36)$$

The data for West Lafayette situation are used. The data are obtained from Prof. Bell in Environmental Engineering at Purdue. The data are $K_1 = 0.19$, $K_2 = 1.99$, $D_o = 0.88$(mg/l), $BOD_\ell = 9.34$(mg/l) Using the above data in equation (4.36) it is obtained,

$$T_c = [1/(1.99-0.19)]\ \ell n[1.99/0.19]$$
$$\times[1 - 0.88(1.99 - 0.19)/(0.19 \times 9.34)]$$
$$= 0.14 \text{ days} = 3.4 \text{ hrs.}$$

Gamma distribution for river flow volume, V_R: The volume of water passing through the critical time period T_c is V_R. V_R is assumed to be gamma distributed.

$$f(v_R) = (\rho^\theta/\Gamma(\theta))v_R{}^{\theta-1}\exp(-\rho v_R)$$
$$(\ \rho > 0\ ,\ \theta > 0 \text{ and } v_R > 0)$$
$$= 0\ ,\ \text{for } v_R \leqq 0 \qquad (4.37)$$

ρ and θ can be estimated from the mean and variance of the flow values in Table 5.3. Let the river flow rate be denoted by Q_R.

Then, $V_R = Q_R T_c$, implies $E(V_R) = T_c E(Q_R)$ and $Var(V_R) = T_c{}^2\ Var(Q_R)$. From these relations the parameters ρ and θ are obtained as follows.

$$\rho = E(V_R)/Var(V_R) = [1/T_c][E(Q_R)/Var(Q_R)]$$
$$(1/3.4)(1.91/5.42) = 0.104$$
$$\theta = \rho \times E(V_R) = 0.672$$

Computation of critical exceedence probability: The critical exceedence probability is computed using the parameters evaluated before.

$$P(T+S\geqq 1) = K(\rho k_1)^{\theta}\ F(\theta,p;r;k_3)/(k_2)^{\theta} \tag{4.49}$$

$k_1 = C_e/C_o = 170/35 = 4.8$

$k_2 = \rho k_1 + \alpha = 0.104 \times 4.8 + \{16.7/(1+u)\}$

$= [0.5(1+u)+16.7]/(1+u)$

$k_3 = \alpha/k_2 = 16.7/[16.7+0.5(1+u)]$

The function F(.) can be approximated as follows,(Abromowitz and Stegun, p556 and p272,1972)

$F(\theta,p;r;k_3) = \Gamma(r)\Gamma(r-p-\theta)/[\Gamma(r-\theta)\Gamma(r-p)]$

$=1.131$

$P(T+S \geqq 1) = 1.131K[0.5(1+u)/\{0.5(1+u)+16.7)\}]^{.672}$

5.2 Formulation and Results: West Lafayette is divided into four zones, as shown in Figure 5.8. Dendrou et al.(1978) , and Dana Hall et al.(1975) have analyzed the future growth of West Lafayette. Based on that information four types of land use activities are chosen. These activities are shown in Table 5.6. Based on the population projection of 25,000 by A.D.2000 the required minimum number of land use activities are determined (Dana Hall et al.,1975). These are shown in Table 5.7. The area requirements of the different land use activities are shown in table 5.8. Table 5.9 shows the vacant land availability in each zone. The cost information in 1975 dollars for different land use types (Dana Hall et al.,1975) are shown in Table 5.10.

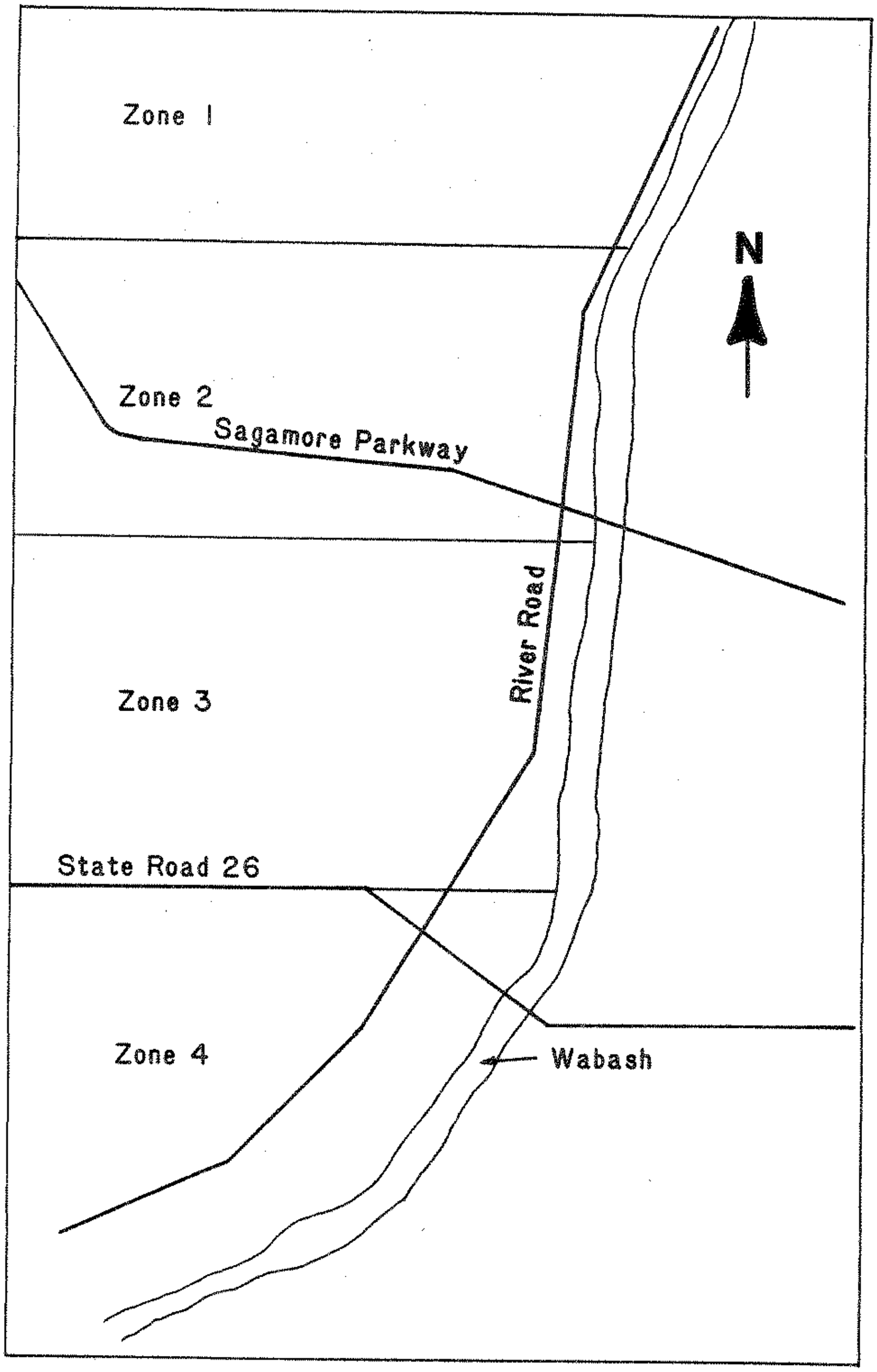

Figure 5.8 West Lafayette Zonal Discussion

Table 5.6. Land Use Types

Land use i	Description
1	Commercial centers
2	Light industries
3	Institutions and service
4	Residential units

Table 5.7. Number of Activities required by A.D.2000

Land use i	Number required, X_i
1	3
2	19
3	12
4	55

Table 5.8. Area required by Land Use Type

Land use i	Area required per activity, a_i (acres)
1	10
2	2
3	10
4	10

Table 5.9. Area available by Zones

Zone j	Area available, A_j (acres)
1	2800
2	255
3	200
4	3500

Table 5.10. Cost by Land Use Type and Zone

Zone	Land use	Cost(dollars)
1	1	1187500
1	2	285000
1	3	71250
1	4	1425000
2	1	1187000
2	2	284874
2	3	71000
2	4	1423000
3	1	1250000
3	2	300000
3	3	75000
3	4	1500000
4	1	1187500
4	2	285000
4	3	71250
4	4	1425000

Drainage Cost(Heany et al.,EPA-/600-2-77-064,1977):

1 in of storage = 9500000 dollars

i in/hr of treatment = 34500000 dollars

(260280 dollars for 15mgd capacity)

A critically needed characteristic for apartment units is good water supply and sewer facility. In West Lafayette zone 1 does not meet these requirements and hence zone 1 will not be considered for housing units. The characteristics of the different zones are shown in table 5.11. The characteristics requirement of different land use activities are shown in table 5.12.

Designation of characteristics:

1)Close to downtown

2)Heavy duty power available

3)Closeness to highway

4)Indutrial water available

Table 5.11 . Available Characteristics by Zones ZC_{kj}*

Zone	Characteristics, k			
j	1	2	3	4
1	0	1	1	0
2	1	0	1	0
3	1	1	1	0
4	0	1	0	1

* 1 means characteristic is present

0 means characteristic is absent

Table 5.12. Required Characteristics by Land Use

Land use	Characteristics k*			
i	1	2	3	4
1	1	0	1	0
2	0	1	1	1
3	1	0	0	0
4	1	0	0	0

* used in constructing the set D_1

<u>Formulation:</u> These data are used in the formulation of the whole problem. In the case of West Lafayette budgetary limitations, and compatibility are not imposed because of the restriction on number of land use activities and widespread boundaries of the zones respectively. Also because the distance between zones is small, no minimum distance requirement is imposed. All other constraints from Chapter 2 are included. The complete problem is thus formulated as

Min $f_1 = \epsilon$ (risk)

Min $f_2 = \Sigma_i \Sigma_j\ C_{ij}\ X_{ij} + C_1(a) + C_2(b)$

(cost)

Min $f_3 = \Sigma_i \Sigma_j\ (d_{ij}^+)^2$

subject to:

Area constraints(constraint 5, Section 2.3):

$10X_{11}+2X_{21}+10X_{31}+10X_{41} \leqq 2800$

$10X_{12}+2X_{22}+10X_{32}+10X_{42} \leqq 255$

$10X_{13}+2X_{23}+10X_{33}+10X_{43} \leqq 200$

$10X_{14}+2X_{24}+10X_{34}+10X_{44} \leqq 3500$

Land use requirement(constraint 9, Section 2.3):

$X_{11}+X_{21}+X_{31}+X_{41} \geqq 3$

$X_{12}+X_{22}+X_{32}+X_{42} \geqq 19$

$X_{13}+X_{23}+X_{33}+X_{43} \geqq 12$

$X_{14}+X_{24}+X_{34}+X_{44} \geqq 55$

Critically needed characteristic(constraint 1, Section 2.3):

$\delta_{41} \leqq 0$. Zone 1 does not have good sewer facility. Hence δ_{41} will be zero and no land use type 4 (residential units) will be allocated to zone 4.

Characteristic matching(constraint 2, Section 2.3):

$\delta_{11}-d_{11}^{+}+d_{11}^{-} = 0$

Zone 1 is away from down town. But land use type 1 (commercial center) needs to be close to down town. Hence if $\delta_{11} = 1$, then there is a mismatch and d_{11}^{+} equals 1. $d_{11}^{+}d_{11}^{-} \leqq 0$ implies d_{11}^{-} will be made zero. Similar restrictions for other zones are {(1,1), (2,1), (3,1); (2,2); (2,3) }.

Urbanization factor(constraint 4, Section 2.3):

Accoriding to the definition given in Chapter IV the urbanization factor can be written as,

$u = [\Sigma_i \Sigma_j w_j a_i X_{ij} / \Sigma_j A_j]$ Umax

where: Umax = mean depression storage/mean runoff volume

By definition,

Vol. of runoff after urbanization

=Vol. of runoff before urbanization + depression storage as a fraction of original volume.

For an average depression storage .09 and mean runoff .06, Umax equals 1.5. Because of street flooding and related effects(Haimes et al.,1980) w_j for an urban area may be given a higher value compared to a nonurbanized area. Some zones may not be within the watershed boundary, in such cases w_js may be made zero because no contribution to runoff is added from the depression storage.

For the case of West Lafayette, the weights for zones 2 and 3 the weights are $w_2 = w_3 = 1.2$ and for zones 1 and 4 the weights are $w_1 = w_4 = 1.0$.

Bounds(constraint 10,11 and 12; Section 2.3):

$X_{ij} - 9999\delta_{ij} \leqq 0$,for all i and j

$X_{ij} \geqq \delta_{ij}$,for all i and j

$\delta_{ij} \leqq 1$

Probability constraint:

$1.31K[0.5(1+u)/\{0.5(1+u)+16.7)\}]^{.672} \leqq \epsilon$

Results and Discussion: The solution methodology is the same as in example 3.2 of Chapter 3. The problem is solved using Branch and Bound Nonlinear Mixed Integer Program code (BBNLMIP, Gupta,1980). The code is essentially the superposition of Branch and Bound procedure on the Generalized Reduced Gradient Method. The starting point for the analysis is (.90,88.4,3) where the components are reliability,total cost and mismatch value respectively. The reliability is defined as the probability of being less than or equal to the safe limit, the cost is measured in million dollars and mismatch value is the total number of mismatches. Mr. Chris Burke, graduate student in Hydrology acted as the Decision Maker. He gave the tradeoff vector as (0.1,0.025,0.02), where the components indicate that the DM is willing to give up .025 in reliability provided 100000 dollars can be gained. Similarly the DM is willing to forego .02 units of reliability for .1 unit of mismatch at the current point. Here reliability is considered to be the base objective and the second component is measured in reliability per dollars, and the third component is measured in reliability per mismatches. This tradeoff vector yields, (0.95,88.82,3) as the final solution. The rest of the results are tabulated in Table 5.13. Table 5.14 contains the cost of the system. Table 5.15 presents the treatment rates and storage values for different reliability levels. Finally Table 5.16 presents the various land use patterns.

Table 5.13. Summary of Results

I[1]	Ini.Point[2]	Tradeoff[3]	Endpoint
1	(.90,88.4,3)	(.1,.025,.02)	(.95,88.82,3)
2	(.95,88.82,3)	(.1,.065,.03)	(.96,88.88,2)
3	(.96,88.88,2)	(.1,.07,.035)	(.96.88.88,2)

1. Iteration number
2. (reliability,cost,mismatches)
3. gradient vector with units reliability per reliability, reliability per .1 million dollars and reliability per .1 mismatch.

Table 5.14. Cost Analysis

I[1]	Drainage Cost (dollars)	Land Use Cost (dollars)
1	.21(10^6)	88.2(10^6)
2	.66(10^6)	88.16(10^6)
3	.73(10^6)	88.15(10^6)

1. Iteration number

Table 5.15. Treatment Rate and Storage

I[1]	Treatment Rate, a (in/hr)	Storage, b (in)
1	.006	0
2	.006	.047
3	.006	.055

1. Iteration number

Table 5.16. Land Use Pattern

I[1]	Zone j	Land Use			
1	1	0	0	12	0
	2	3	19	0	8
	3	0	0	0	0
	4	0	0	0	47
2	1	0	0	12	0
	2	3	19	0	18
	3	0	0	0	0
	4	0	0	0	37
3	1	0	11	2	0
	2	0	0	0	25
	3	0	0	0	0
	4	3	8	10	30

1. Iteration number

In the case of West Lafayette the polluted water goes to the Wabash river . The self purification capacity of Wabash is very high which is also indicated by the small value for critical time in oxygen sag curve. The statistical parameters are influenced by these facts and hence a higher reliability level is acceptable. Also a point of interest is the quick convergence. The reason is attributed to the gradient vector. The fact gradient gives the maximum rate of increase may be the reason. Also the DM is seemed to be biased towards 96% reliability level because the tradeoffs are not widely different. The characteristic deviational variables for the end point 1 are given as,

for δ_{31}, $d_{11}^{+}=1$; for δ_{22}, $d_{22}^{+}=1$

implying institution and service are not close to down town and absence of heavy power respectively. Also for δ_{22}, $d_{42}^{+} = 1$, implying industrial water not available in zone 2. Similarly for endpoint 2 (same as endpoint 3)

for δ_{21}, $d_{41}^{+} = 1$; for δ_{31}, $d_{11}^{+} = 1$

implying absence of industrial water and institute and service are located at a far off place respectively. The urbanization factors are .17,.174,.175 respectively for the three iterations. Even though the treatment rate remains the same the storage increases with the urbanization factor. The expression for K also indicates for increased urbanization, more treatment and/or storage would be necessary for a fixed reliability level. Hence it can be seen that the simultaneous treatment of urban growth and storm drainage planning provides a feed back loop, which results in changing land use patterns and storm drainage capacity. It is also seen that the land use cost stays fairly even while the drainage cost varies with the reliability level. This is due to the stringent, requirement constraints on land use activities.

Difficulties Encountered: The initial point indicated as iteration 1 is not arbitrarily chosen. Since the land use portion of the problem predominantly contains linear constraints it is solved using a linear programming code MPOS. Based on this optimal solution the initial point is chosen. The method uses Generalized Reduced Gradient method to solve the intermediate continuous problems. The method is unable to handle large size problems. The present formulation has 68 variables and 74 constraints without including the bounds. The author dropped many deviational and either or variables and reduced the problem size to 40 variables and 61 constraints. That also did not help. The OPTLIB manual admits that it is possible to have singular matrix in nonbasic variables and the suggestion is to change the order of the constraints. That is also of no help. There is a need for an efficient nonlinear programming code.

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions: The main contributions in this research are

(a) Development of new probability distribution functions for overflows and receiving body pollutant concentration levels.

(b) Development of the algorithm to solve a general MCDM problem.

These points are briefly elaborated in the following.

(a) Development of the distribution function: The newly developed probability distributions provide a simple but powerful methodology in solving urban stormwater problems. The very close matching of results of the analytical model with the simulation model STORM provides the justification. These distribution functions can be applied to stormwater problems in estimating the size of the required detention storage, in estimating the reliability of the treatment system and sizing the treatment plant for a given reliability level. Also the need to control the pollution source by better abatement practices is brought out in the example of the extreme case. Closed form, tractable analytical solutions are provided in this analysis. The distribution function may be viewed as a better table top technique in solving the urban water management problem. The verification of the statistical independence of the hydrologic variables provides the product of the marginals as the joint density. Otherwise the problem is very complicated. the verification of the exponential distribution provides a strong evidence that the hydrologic variables runoff volume,duration, and intermittent time are indeed exponentially distributed.

Development of the algorithm for general MCDM problem: By their very nature interactive algorithms are iterative. In such a case the most favored aspect of the algorithm is to generate efficient points at each iteration. The new cutting plane algorithm developed has this desired property. Unlike other gradient based methods no line search is necessary. This also implies that there is only one interaction with the DM for each iteration; otherwise the DM's response is needed to pick the favoured point from the line search. Since tradeoff cuts eliminate part of the region in each iteration better rate of convergence is anticipated. For certain structured problems like linear utility function only one iteration is needed to solve the problem. The application of the method to West Lafayette proves that for real life problems Multi Criteria Decision Making is a viable tool. It also needs to be mentioned that for large scale problems an efficient nonlinear programming code is highly necessary.

6.2 Recommendations:

(1) The probability distribution for the receiving stream pollutant concentration is not well established in the literature. Research in that direction is already in progress(Athayde,1981) and is necessary.

(2) The possibility of developing joint distributions for different hydrologic variables may be studied.

(3) The possibility of incorporating tradeoff intervals instead of point estimates may be explored.

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APPENDIX

```
*       READ ORIGINAL NWRC TAPE
*       AND CONVERT TO FORTRAN READABLE FORM
*
C       FILES,PFKFX,C=S0.
C       ATTACH,INFILE,PFKFX,C=S0.
C       IF(S0=0)GOTO(AOK)
C       FILES,PFKFX,T=X.
C       ATTACH,INFILE,PFKFX,T=X.
C       REQUEST(TAPE,3254,C=800)
C       COPYCF,TAPE,INFILE,,R,BF=10.
C       RETURN,TAPE.
C       SAVE,INFILE.PFKFX.
C       ATTACH,INFILE,PFKFX.
C       -AOK.
C       MNF,F.
C       USE(L=200)
C       COPYSBF,OUTFILE,,,RIB.
C       -EOR
C
      PROGRAM RAIN (INFILE,OUTFILE,OUTPUT,TAPE1=INFILE,TAPE2=OUTFILE)
     1,OUTFILE=2
      PARAMETERINFILE=1,OUTFILE
      LOGICAL ERR
      DIMENSION DIGITS(12), VALUES(24)
      NIN=0
      NREJ=0
      NOUT=0
*
*     MAIN LOOP - READ FIRST CARD
C
      NIN=NIN+1
      STATE1=STATE
      STATN1=STATION
      DATE1=DATE
*
*     MUST START WITH CARD NUMBER 1: OTHERWISE, IGNORE
C
      IF (CARD.NE.1) GO TO 102
*
*     CONVERT DIGITS TO VALUES FOR 1ST 12 HOURS OF THE DAY
C
      CALL CONVERT (DIGITS,VALUES(1),ERR)
      IF (ERR) GO TO 102
*
*     READ THE SECOND CARD
C
*
*     MAKE SURE THIS DATA GOES WITH THE FIRST CARD
C
      BACKSPACE INFILE
      GO TO 102
C
  101 FORMAT (I2,I4,I6,I1,12A3,T79,I2)
C
      END
      NIN=NIN+1
*
*     CONVERT DIGITS TO VALUES FOR 2ND 12 HOURS OF THE DAY
C
      CALL CONVERT (DIGITS,VALUES(13),ERR)
      IF (ERR) GO TO 101
*
*     WRITE THE RESULTS
C
      WRITE (OUTFILE,102) STATE,STATION,DATE,VALUES,NEXT
      NOUT=NOUT+1
      GO TO 104
*
*     HERE FOR REJECTS DUE TO CARDS MISSING OR OUT OF SEQUENCE
C
  101 PRINT 103, NIN,STATE1,STATN1,DATE1,CARD
      NREJ=NREJ+1
      GO TO 104
*
*     END OF FILE ENCOUNTERED
C
C
  102 FORMAT (I2.2,I4.4,I6.6,3X,24I4,I6.2)
  103 FORMAT ( 22H REJECTED CARD NUMBER ,I4,  2H: ,I2.2,I4.4,I6.6,I1)
C
```

```
      END
      SUBROUTINE CONVERT (DIGITS,VALUES,ERR)
      LOGICAL ERR
*
*     SUBROUTINE CONVERTS DIGITS INTO VALUES
*
*     EACH INPUT VALUE CONSISTS OF 3 DIGITS, NOMINALLY REPRESENTING
*     THE AMOUNT OF RAINFALL WITH THE IMPLIED DECIMAL POINT AFTER
*     THE FIRST DIGIT.  THE FIRST DIGIT MAY BE ~OVERPUNCHED~ ON THE
*     INPUT DATA; AN 11 OVERPUNCH MEANS ~ADD 10~ AND A 12 OVERPUNCH
*     MEANS ~ADD 20~.  OVERPUNCH COMBINATIONS PRODUCE THE FOLLOWING
*     CHARACTERS WHEN READ:
*
*              DIGIT    11-OVER    12-OVER
*              -----    -------    -------
*                0         ~          <
*                1         J          A
*                2         K          B
*                3         L          C
*                4         M          D
*                5         N          E
*                6         O          F
*                7         P          G
*                8         Q          H
*                9         R          I
*
*     THE THIRD DIGIT IS LEFT BLANK FOR DATA FROM THE
*        ~FISCHER PORTER~ PRECIPITATION GUAGES.
*
*     THE FOLLOWING SPECIAL COMBINATION PUNCHES ARE USED:
*
*        BLANK/BLANK/BLANK => ZERO
*        MINUS/BLANK/BLANK => TRACE AMOUNT (CONVERTED TO ZERO)
*        ZERO /MINUS/BLANK => ACCUMULATED DATA FOLLOWS LATER
*                               (CONVERTED TO ZERO)
*     ----------------------------------------------------------------
*
      DIMENSION DIGITS(12), VALUES(12), LEGAL(30)
      DATA LEGAL/1R0,1R1,1R2,1R3,1R4,1R5,1R6,1R7,1R8,1R9,1R~,1RJ,1RK,1RL
     1,1RM,1RN,1RO,1RP,1RQ,1RR,1R<,1RA,1RB,1RC,1RD,1RE,1RF,1RG,1RH,1RI/
*
      ERR=.F.
*
*     LOOP THROUGH 12 VALUES
C
      DO 102 IV=1,12
         FIELD=DIGITS(IV)
         VALUES(IV)=0
*
         IF (FIELD.EQ.≠≠.OR.FIELD.EQ.≠-≠.OR.FIELD.EQ.≠0-≠) GO TO 102
*
*        CHECK FOR LEGAL DIGITS AND CONVERT TO PROPER VALUE
C
         DO 101 I=1,3
            INV=SHIFT(FIELD,I*6).AND.77B
*
*           ALLOW LAST DIGIT TO BE BLANK
C
            J=1
            GO TO 101
C
      END
      DO 101 J=1,30
         IF (INV.EQ.LEGAL(J)) GO TO 102
  101 CONTINUE
      ERR=.T.
      PRINT 104, FIELD
      GO TO 103
  102 VALUES(IV)=VALUES(IV)*10+J-1
*
  103 CONTINUE
      RETURN
C
  104 FORMAT ( 16H INVALID FIELD: ,A3)
C
      END
```

```
C      PFILES(GET,OPTLIB,ID=BNV)
C      RFL(60000)
C      MNF(T,N,L=0)
C      LOADX(LGO,OPTLIB,RUNLIB2)
C      -EOR
C
       PROGRAM MAIN (INPUT,OUTPUT,TAPE5=INPUT,TAPE6,TAPE7=OUTPUT)
       DIMENSION XO(50), XMAXO(50), XMINO(50)
C
       COMMON D(20000)
       COMMON /IP/ X(100,50),XMIN(100,50),INODE(100),XMAX(100,50),ZVALUE(
      1100),XOPT(50),Y(50),NORDER(100),PCU(50),PCL(50),KPCL(50),KPCU(50),
      2XSTAR(50),V(50),E(100),SAVXO(50),LOC(200)
       COMMON /PARI/ CRIT,EPS,IPR,MAXM,IDATA,NE,NI,LBD,NCON,EPSLS,EPSBD
       COMMON /IP1/ KK,LIFO,M,NNN
       COMMON /OPT003/ ISING,IFEAS
       COMMON /S/ INTSOLN,NHRST,XLIM,INDEX,LBL,NINT,NSTORE
       COMMON /H2/ T1,NNOPT,NFN,NCN
       COMMON /1/ NF,NC
C
C
C***   SPECIFY PARAMETERS FOR OPT
C
C
       N=44
       NE=0
       NI=45
       IPR=1
       MAXM=50
       EPS=1.E-4
       CRIT=1.E-4
       IDATA=1
       LBD=0
       EPSLS=1.E-4
       EPSBD=1.E-4
C
C ***** M= THE NUMBER OF INTEGER VARIABLES
C
       M=40
C
C ***** VARIABLE DESCRIPTIONS ARE *****
C
C
       XO(1)=3.
       XO(2)=11.
       XO(3)=2.
       XO(4)=0.
       XO(5)=0.
       XO(6)=0.
       XO(7)=0.
       XO(8)=25.
       XO(9)=0.
       XO(10)=0.
       XO(11)=3.
       XO(12)=0.
       XO(13)=0.
       XO(14)=8.
       XO(15)=10.
       XO(16)=30.
       XO(17)=1
       XO(18)=1
       XO(19)=1
       XO(20)=0
       XO(21)=0
       XO(22)=0
       XO(23)=0
       XO(24)=1
       XO(25)=0
       XO(26)=0
       XO(27)=0
       XO(28)=0
       XO(29)=0
       XO(30)=1
       XO(31)=1
       XO(32)=1
       XO(33)=1
       XO(34)=0
       XO(35)=0
       XO(36)=0
       XO(37)=1
```

```
      XO(38)=0
      XO(39)=0
      XO(40)=0
      XO(41)=.006
      XO(42)=.055
      XO(43)=.18
      XO(44)=.03
      DO 101 I=1,44
         XMINO(I)=0.
         XMAXO(I)=1.
         IF (I.LE.16) XMAXO(I)=55.
  101 CONTINUE
C
C ***** BRANCH AND BOUND PARAMETERS ARE SET HERE *****
C
      NNN=1
      KK=2
      LIFO=2
      NHRST=1
C
C     ****  BRANCH AND BOUND CODE IS CALLED HERE ****
C
      CALL IP (XO,XMAXO,XMINO,N)
      STOP
C
      END
      FUNCTION F(X)
      DIMENSION X(50)
      COMMON /1/ NF,NC
      NF=NF+1
C
C     F=296875.*X(1)+71250.*X(2)+17812.*X(3)+356250.*X(4)+
C     1296750.*X(5)+71218*X(6)+17750.*X(7)+355750.*X(8)+
C     2312500.*X(9)+75000.*X(10)+18750.*X(11)+375000.*X(12)+
C     3236873.*X(13)+71250.*X(14)+17812.*X(15)+356250.*X(16)+
C     41000000.*X(44)+8625000.*X(41)+2375000.*X(42)
C
      F=771875.*X(1)+185250.*X(2)+46312.*X(3)+926250.*X(4)+771550.*X(5)+
     1185167.*X(6)+46150.*X(7)+924950.*X(8)+812500.*X(9)+195000.*X(10)+4
     28750.*X(11)+975000.*X(12)+771875.*X(13)+185250.*X(14)+46312.*X(15)
     3+926250.*X(16)+1000000.*X(44)+24150000.*X(41)+6650000.*X(42)+30000
     40.*X(33)+300000.*X(35)+300000.*X(37)+300000.*X(39)
      RETURN
C
      END
      SUBROUTINE CONST (X,CON)
      DIMENSION X(50), CON(50)
      COMMON /1/ NF,NC
      NC=NC+1
      CON(1)=X(1)+X(5)+X(9)+X(13)-3.
      CON(2)=X(2)+X(6)+X(10)+X(14)-19.
      CON(3)=X(3)+X(7)+X(11)+X(15)-12.
      CON(4)=X(4)+X(8)+X(12)+X(16)-55.
      CON(5)=(2800.-10.*X(1)-2.*X(2)-10.*X(3)-10.*X(4))/100.
      CON(6)=(255.-10.*X(5)-2.*X(6)-10.*X(7)-10.*X(8))/100.
      CON(7)=(200.-10.*X(9)-2.*X(10)-10.*X(11)-10.*X(12))/100.
      CON(8)=(3500.-10.*X(13)-2.*X(14)-10.*X(15)-10.*X(16))/100.
      CON(9)=X(44)*(0.0171*(1.+X(43))+16.7*X(41))*(0.511*(1.+X(43))+16.7
     1*X(41))/(1.131*(0.5*(1.+X(43))/(0.5*(1.+X(43))+16.7))**0.672)-(.00
     287*(1.+X(43))*(1.+X(43))*(X(42)*(16.7/(1.+X(43))+.0171/X(41)))+.51
     31*(.0171*(1.+X(43))+16.7*X(41))*(1.-X(42)*(16.7/(1.+X(43))+.0171/X
     4(41))))
C
C     CON(9)=560.*X(44)*X(41)*X(41)*X(41)+4.6*X(44)*X(41)*X(41)+
C    10.004*X(41)*X(44)-0.01*X(42)*X(41)-0.0012*X(43)*X(42)*X(41)
C    2-0.00001*X(42)-0.7348*X(41)*X(41)+6.18*X(42)*X(41)*X(41)
C    3+0.0067*X(42)*X(41)-0.7348*X(41)*X(41)*X(43)
C    4+6.18*X(42)*X(41)*X(41)*X(43)+0.0067*X(42)*X(41)*X(43)
C    5+0.001-0.000748*X(41)+0.0063*X(41)*X(42)+0.00001*X(42)
C
      CON(10)=X(43)-.000222*(10.*X(1)+2.*X(2)+10.*X(3)+10.*X(4)+12.*X(5)
     1+2.4*X(6)+12.*X(7)+12.*X(8)+12.*X(9)+2.4*X(10)+12.*X(11)+12.*X(12)
     2+10.*X(13)+2.*X(14)+10.*X(15)+10.*X(16))+.01
      CON(11)=(22915886-296875.*X(1)-71250.*X(2)-17812.*X(3)-356250.*X(4
     1)-296750.*X(5)-71218*X(6)-17750.*X(7)-355750.*X(8)-312500.*X(9)-75
     2000.*X(10)-18750.*X(11)-375000.*X(12)-296875.*X(13)-71250.*X(14)-1
     37812.*X(15)-356250.*X(16)-1000000.*X(44)-8625000.*X(41)-2375000.*X
     4(42)-300000.*(X(33)+X(35)+X(37)+X(39)))/1000.
      CON(12)=90.*X(17)-X(1)
      CON(13)=90.*X(18)-X(2)
```

```
      CON(14)=90.*X(19)-X(3)
      CON(15)=90.*X(20)-X(4)
      CON(16)=90.*X(21)-X(5)
      CON(17)=90.*X(22)-X(6)
      CON(18)=90.*X(23)-X(7)
      CON(19)=90.*X(24)-X(8)
      CON(20)=90.*X(25)-X(9)
      CON(21)=90.*X(26)-X(10)
      CON(22)=90.*X(27)-X(11)
      CON(23)=90.*X(28)-X(12)
      CON(24)=90.*X(29)-X(13)
      CON(25)=90.*X(30)-X(14)
      CON(26)=90.*X(31)-X(15)
      CON(27)=90.*X(32)-X(16)
      CON(28)=X(17)-X(33)+X(34)
      CON(29)=X(19)-X(33)+X(34)
      CON(30)=X(22)-X(35)+X(35)
      CON(31)=X(29)-X(39)+X(40)
      CON(32)=X(18)-X(37)+X(38)
      CON(33)=(62950650-771875.*X(1)-185250.*X(2)-46312.*X(3)-926250.*X(
     14)-771550.*X(5)-185167.*X(6)-46150.*X(7)-924950.*X(8)-812500.*X(9)
     2-195000.*X(10)-48750.*X(11)-975000.*X(12)-771875.*X(13)-185250.*X(
     314)-46312.*X(15)-926250.*X(16)-1000000.*X(44)-24150000.*X(41)-6650
     4000.*X(42)-300000.*(X(33)+X(35)+X(37)+X(39)))/100000.
      CON(34)=X(1)-X(17)
      CON(35)=X(2)-X(18)
      CON(36)=X(3)-X(19)
      CON(37)=X(4)-X(20)
      CON(38)=X(5)-X(21)
      CON(39)=X(6)-X(22)
      CON(40)=X(7)-X(23)
      CON(41)=X(8)-X(24)
      CON(42)=X(13)-X(29)
      CON(43)=X(14)-X(30)
      CON(44)=X(15)-X(31)
      CON(45)=X(16)-X(32)
      RETURN
C
      END
      SUBROUTINE IP (XO,XMAXO,XMINO,N)
      DIMENSION XO(N), XMAXO(N), XMINO(N)
      COMMON /IP/ X(100,50),XMIN(100,50),INODE(100),XMAX(100,50),ZVALUE(
     1100),XOPT(50),Y(50),NORDER(100),PCU(50),PCL(50),KPCL(50),KPCU(50),
     2XSTAR(50),U(50),E(100),SAUXO(50),LOC(200)
      REAL Z,ZUP,ZLOW,ZOPT
      INTEGER ID,KCUNT,INTSOLN,INODE,NNODE,NCONT
      COMMON D(20000)
      COMMON /S/ INTSOLN,NHRST,XLIM,INDEX,LBL,NINT,NSTORE
      COMMON /H2/ T1,NNOPT,NFN,NCN
      COMMON /IP1/ KK,LIFO,M,NNN
      COMMON /OPT003/ ISING,IFEAS
      COMMON /1/ NF,NC
      WRITE (7,190) KK,LIFO,NHRST
      PCT=0
      ZUP=1000000.00
C
C
      NNOPT=0
      NCONT=0
      KOUNT=0
      NNODE=0
      INTSOLN=0
      NSTORE=0
      NFN=0
      NCN=0
C
      DO 101 I=1,M
         PCL(I)=0
         PCU(I)=0
         KPCU(I)=0
         KPCL(I)=0
  101 CONTINUE
C
C       SOLVE BY -OPT-
C
      CALL SECOND (T1)
C
C
      CALL OPT (XO,XMAXO.XMINO,N)
      NNOPT=NNOPT+1
```

```
      NFN=NFN+1
      NCN=NCN+1
C
C      CHECK FOR  NLP FEASIBILITY
C
      IF (IFEAS.EQ.0) GO TO 102
      WRITE (7,170)
      GO TO 169
C
C     THE ORIGINAL NLP IS FEASIBLE
C
  102 Z=F(XO)
      NCONT=NCONT+1
      WRITE (7,171) XO
      WRITE (7,172) Z
C
C     CHECK FOR INTEGER FEASIBILITY
C
      CALL INTFEAS (XO,ID,M)
      IF (ID.NE.0) GO TO 104
C
C     OPTIMAL INTEGER SOLUTION FOUND
C
      DO 103 I=1,M
  103 XOPT(I)=XO(I)
      ZOPT=Z
      WRITE (7,173) XOPT
      WRITE (7,172) ZOPT
      INTSOLN=INTSOLN+1
      GO TO 168
C
C*******************************************
C
  104 IF (NHRST.NE.2) GO TO 106
      LBL=0
      MSAVE=M
  105 CALL HRSTC2 (NE,NI,M,N,XO,XMINO,XMAXO,Y,ZUP,INTSOLN,LBL,XOPT,INODE
     1,X,XMAX,XMIN,ZVALUE,Z,KOUNT,NNODE,NSTORE)
C
C*******************************************
C********************************************
C
  106 IF (NHRST.NE.1) GO TO 108
C
C**** FIX ALL INTEGER VALUES FOR CORRESPONDING INTEGER VARIABLES
C
      NINT=0
      DO 107 J=1,M
         X1=XO(J)-AINT(XO(J))
         X2=1.0-X1
         X3=AMIN1(X1,X2)
         IF (X3.GT..00001) GO TO 107
         IF (NINT.EQ.0) CALL STORE (KOUNT,INODE,X,XO,XMAX,XMIN,XMINO,XMA
     1   XO,Z,ZVALUE,N,NNODE)
         XMINO(J)=XO(J)
         XMAXO(J)=XO(J)
         NINT=2
  107 CONTINUE
      IF (NINT.EQ.0) NINT=1
C
C******************************************
C
  108 IF (KK.EQ.1) GO TO 109
C
C*****************************************
C
C     THE  NLP SOLUTION IS NOT INTEGER
C
C     SELECT A NON INTEGER VARIABLE TO BRANCH
C     <INDEX> IS THE INDEX OF THE VARIABLE DECIDED TO BE BRANCHED
C
      CALL SELECT (XO,M,INDEX,KK,PCL,PCU,XSTAR,V)
      IF (INDEX.EQ.0) GO TO 167
      GO TO 110
  109 INDEX=ID
  110 WRITE (7,174) INDEX
      XLOW=INT(XO(INDEX))
      HIGH=XMAXO(INDEX)
      XMAXO(INDEX)=XLOW
      DO 111 I=1,M
```

```
  111 SAUXO(I)=XO(I)
      IF (KK.NE.3.AND.LIFO.NE.2) GO TO 112
      OLDX=XO(INDEX)
      OLDZ=F(XO)
C
C***********************
C
  112 IF (NNN.EQ.1) XO(INDEX)=XMAXO(INDEX)
      IF (XMAXO(INDEX).LT.XMINO(INDEX)) GO TO 127
      IF ((XMAXO(INDEX)-XMINO(INDEX)).LT..00001) XO(INDEX)=(XMAXO(INDEX)
     1+XMINO(INDEX))/2.0
C
C******************************
C
      CALL OPT (XO,XMAXO.XMINO,N)
      NNOPT=NNOPT+1
      NFN=NFN+1
      NCN=NCN+1
      IF (IFEAS.EQ.0) GO TO 113
      GO TO 127
  113 Z=F(XO)
      IF (KK.NE.3.AND.LIFO.NE.2) GO TO 114
      IF (KPCL(INDEX).EQ.1) GO TO 114
      CALL PSEUDO (M,KK,INDEX,PCL,PCU,KPCL,KPCU,Z,OLDZ,XO,OLDX)
  114 NCONT=NCONT+1
      WRITE (7,175) NCONT.XO
      WRITE (7,172) Z
      CALL INTFEAS (XO,ID,M)
      IF (ID.EQ.0) GO TO 117
C
C       INTEGER SOLUTION
C
      IF (INTSOLN.GE.1) GO TO 116
      XLIM=XLOW
  115 CALL STORE (KOUNT,INODE,X,XO,XMAX,XMIN,XMINO,XMAXO,Z,ZVALUE,N,NNOD
     1E)
      GO TO 127
  116 IF (Z.GT.ZUP) GO TO 127
      GO TO 115
  117 WRITE (7,176) XO
      WRITE (7,172) Z
      IF (INTSOLN.GE.1) GO TO 125
C
C****************************************
C
      CALL SECOND (FIRST)
      TIM=FIRST-T1
      WRITE (7,178) TIM
      WRITE (7,185) NNODE
      INTSOLN=1
      WRITE (7,186) NNOPT,INTSOLN
      INTSOLN=0
      WRITE (7,188) NFN,NCN
      WRITE (7,189) NSTORE
      WRITE (7,177)
C
C****************************************
C
      IF (NHRST.EQ.1) GO TO 119
      IF (NHRST.EQ.2.AND.LBL.NE.100) GO TO 118
      GO TO 120
C
  118 LBL=100
      M=MSAVE
  119 ISKIP=1
C
C***      FIRST INTEGER SOLUTION FOUND
C
  120 IF (NNODE.EQ.0) GO TO 122
C
C
      DO 121 I=1,NNODE
         IF (INODE(I).NE.2) GO TO 121
         INODE(I)=0
         KOUNT=KOUNT-1
  121 CONTINUE
C
  122 ZUP=Z
  123 INTSOLN=INTSOLN+1
      DO 124 I=1,N
```

```
  124 XOPT(I)=XO(I)
      ZOPT=Z
      CALL REMOVE (KOUNT,ZUP,INODE,ZVALUE,NNODE)
      WRITE (7,181) KOUNT
      GO TO 127
  125 ZUP=AMIN1(Z,ZUP)
      IF (Z.GT.ZUP) GO TO 126
      GO TO 123
  126 INTSOLN=INTSOLN+1
C
C       SOLVE SECOND BRANCH PROBLEM
C
  127 IF (KSKIP.EQ.1) GO TO 146
      XMINO(INDEX)=XLOW+1.0
      XMAXO(INDEX)=HIGH
      DO 128 I=1,N
  128 XO(I)=SAUXO(I)
C
C****************
C
      IF (NNN.EQ.1) XO(INDEX)=XMINO(INDEX)
      IF (XMAXO(INDEX).LT.XMINO(INDEX)) GO TO 129
      IF ((XMAXO(INDEX)-XMINO(INDEX)).LT..00001) XO(INDEX)=(XMAXO(INDEX)
     1+XMINO(INDEX))/2.0
C
C******************
C
      CALL OPT (XO,XMAXO,XMINO,N)
      NNOPT=NNOPT+1
      NFN=NFN+NF
      NCN=NCN+NC
      IF (IFEAS.EQ.0) GO TO 133
C
  129 KKOUNT=0
      IF (NHRST.EQ.2.AND.LBL.NE.100) KKOUNT=1
      IF (KOUNT.EQ.KKOUNT) GO TO 130
      GO TO 146
  130 IF (INTSOLN.GE.1) GO TO 132
      IF (NHRST.EQ.2.AND.LBL.NE.100) GO TO 131
      WRITE (7,179)
      GO TO 168
C
C****************
C
  131 LBL=1
      GO TO 105
C
C****************
C
  132 WRITE (7,173) XOPT
      WRITE (7,172) ZOPT
      GO TO 168
  133 Z=F(XO)
      IF (KK.NE.3.AND.LIFO.NE.2) GO TO 134
      IF (KPCU(INDEX).EQ.1) GO TO 134
      KKSAVE=KK
      KK=40
      CALL PSEUDO (M,KK,INDEX,PCL,PCU,KPCL,KPCU,Z,OLDZ,XO,OLDX)
      KK=KKSAVE
  134 NCONT=NCONT+1
      WRITE (7,175) NCONT,XO
      WRITE (7,172) Z
      CALL INTFEAS (XO,ID,M)
      IF (ID.EQ.0) GO TO 137
      IF (INTSOLN.GE.1) GO TO 136
      XLIM=XLOW+1.0
  135 CALL STORE (KOUNT,INODE,X,XO,XMAX,XMIN,XMINO,XMAXO,Z,ZVALUE,N,NNOD
     1E)
      GO TO 146
  136 IF (Z.LE.ZUP) GO TO 135
      IF (KOUNT.GT.0) GO TO 146
      GO TO 144
  137 WRITE (7,176) XO
      WRITE (7,172) Z
      IF (INTSOLN.GE.1) GO TO 145
C
C******************************
C
      CALL SECOND (FIRST)
      TIM=FIRST-T1
```

```
      WRITE (7,178) TIM
      WRITE (7,185) NNODE
      INTSOLN=1
      WRITE (7,186) NNOPT,INTSOLN
      INTSOLN=0
      WRITE (7,188) NFN,NCN
      WRITE (7,189) NSTORE
      WRITE (7,177)
C
C********************************
C
      IF (NHRST.EQ.2.AND.LBL.NE.100) GO TO 138
      GO TO 139
C
  138 LBL=100
      M=MSAVE
C
C*****  FIRST INTEGER SOLUTION FOUND
C
  139 IF (NNODE.EQ.0) GO TO 141
C
C
      DO 140 I=1,NNODE
         IF (INODE(I).NE.2) GO TO 140
         INODE(I)=0
         KOUNT=KOUNT-1
  140 CONTINUE
  141 ZUP=Z
      INTSOLN=INTSOLN+1
  142 DO 143 I=1,M
  143 XOPT(I)=XO(I)
      ZOPT=Z
      CALL REMOVE (KOUNT,ZUP,INODE,ZVALUE,NNODE)
      WRITE (7,181) KOUNT
      IF (KOUNT.EQ.0) GO TO 144
      GO TO 146
  144 WRITE (7,173) XOPT
      WRITE (7,172) ZOPT
      GO TO 168
  145 INTSOLN=INTSOLN+1
      ZUP=AMIN1(Z,ZUP)
      IF (Z.LE.ZUP) GO TO 142
      IF (KOUNT.EQ.0) GO TO 144
C
C******************************
C
  146 KSKIP=0
      IF (NHRST.EQ.1.AND.NINT.EQ.1) NINT=2
      JJ=1
      IF (NHRST.EQ.2.AND.LBL.NE.100) JJ=2
      IF (INTSOLN.GE.1) GO TO 147
      IF (NHRST.EQ.1) GO TO 163
C
C******************************
C
  147 IF (LIFO.NE.1) GO TO 150
      NTEMP=NSTORE
  148 IF (INODE(LOC(NTEMP)).NE.0) GO TO 149
      NTEMP=NTEMP-1
      GO TO 148
  149 LL=LOC(NTEMP)
      GO TO 159
C
C******************************
C
  150 IF (LIFO.EQ.0) GO TO 156
      DO 153 I=JJ,NNODE
         IF (INODE(I).EQ.0) GO TO 153
         DO 151 J=1,M
            XSTAR(J)=X(I,J)-AINT(X(I,J))
            U(J)=AMIN1(PCL(J)*XSTAR(J),PCU(J)*(1.0-XSTAR(J)))
  151    CONTINUE
         E(I)=ZVALUE(I)
         DO 152 J=1,M
            E(I)=E(I)+U(J)
  152    CONTINUE
  153 CONTINUE
C
C
      DO 155 J=JJ,NNODE
```

```
      IF (INODE(J).EQ.0) GO TO 155
      LL=J
      DO 154 I=LL,NNODE
         IF (INODE(I).EQ.0) GO TO 154
         IF (E(I).GE.E(LL)) GO TO 154
         LL=I
  154 CONTINUE
      GO TO 159
  155 CONTINUE
C
C******************************
C
  156 DO 158 J=JJ,NNODE
      IF (INODE(J).EQ.0) GO TO 158
      ZLOW=ZVALUE(J)
      LL=J
      DO 157 I=LL,NNODE
         IF (INODE(I).EQ.0) GO TO 157
         IF (ZLOW.LE.ZVALUE(I)) GO TO 157
         ZLOW=ZVALUE(I)
         LL=I
  157 CONTINUE
      GO TO 159
  158 CONTINUE
C
C****************
C
  159 IF (PCT.LE..0001) GO TO 164
      IF (INTSOLN.EQ.0) GO TO 164
      DO 161 J=1,NNODE
         IF (INODE(J).EQ.0) GO TO 161
         XZLOW=ZVALUE(J)
         KFIRST=J
         DO 160 I=KFIRST,NNODE
            IF (INODE(I).EQ.0) GO TO 160
            IF (XZLOW.LT.ZVALUE(I)) GO TO 160
            XZLOW=ZVALUE(I)
  160    CONTINUE
         GO TO 162
  161 CONTINUE
  162 IF (ABS(XZLOW).LT..0001) GO TO 164
      XYZ=(ZUP-XZLOW)*100.0/ABS(XZLOW)
      IF (XYZ.LE.PCT) GO TO 166
      WRITE (7,180) XYZ
      GO TO 164
C
C**********************************************
C
  163 CALL HRSTC1 (X,NNODE,INODE,LL,NORDER,M,N)
      IF (LL.NE.0) GO TO 164
      NHRST=0
      JJ=1
      GO TO 147
C
C*****************************
C
  164 DO 165 I=1,N
         XO(I)=X(LL,I)
         XMAXO(I)=XMAX(LL,I)
         XMINO(I)=XMIN(LL,I)
  165 CONTINUE
      CALL INTFEAS (XO,ID,M)
      INODE(LL)=0
      KOUNT=KOUNT-1
      WRITE (7,181) KOUNT
      GO TO 108
  166 WRITE (7,182) XOPT
      WRITE (7,183) ZOPT,XYZ
      GO TO 168
  167 WRITE (7,184)
  168 WRITE (7,185) NNODE
      WRITE (7,186) NNOPT,INTSOLN
C
      CALL SECOND (T2)
      TIME=T2-T1
      WRITE (7,187) TIME
      WRITE (7,188) NFN,NCN
      WRITE (7,189) NSTORE
  169 CONTINUE
      RETURN
```

```
C
  170 FORMAT (1X, 31H THE ORIGINAL NLP IS INFEASIBLE)
  171 FORMAT (  1H-, 28HTHE ORIGINAL NLP IS FEASIBLE.,/,1X, 14HTHE X VECT
     1OR= ,10F10.3)
  172 FORMAT (1X, 36HTHE VALUE OF THE OBJECTIVE FUNCTION=,F10.3,/)
  173 FORMAT (1X, 22HOPTIMAL SOLUTION FOUND,/,1X, 13HTHE X VECTOR=,10F10
     1.3)
  174 FORMAT (5X,  8H BRANCH ,I3, 14H TH   VARIABLE,/)
  175 FORMAT (1X,I5, 28HTH CONTINUOUS SOLUTION FOUND,/,1X, 13HTHE X VECT
     1OR=,10F10.3)
  176 FORMAT (5X, 25HAN INTEGER SOLUTION FOUND,/,1X, 13HTHE X VECTOR=,10
     1F10.3)
  177 FORMAT (  1H1)
  178 FORMAT (  1H0, 33HTIME FOR FIRST INTEGER SOLUTION= ,F10.3)
  179 FORMAT (1X, 33HTHE PROBLEM IS INTEGER INFEASIBLE)
  180 FORMAT (  1H0, 48HTHE BEST INTEGER SOLUTION FOUND SO FAR IS WITHIN
     1,F8.2, 33H  PERCENTAGE OF THE OPTIMAL VALUE)
  181 FORMAT (5X, 33HTHE TOTAL NUMBER OF ACTIVE NODES=,I5,/)
  182 FORMAT (  1H0, 37HAN APPROXIMATE OPTIMAL SOLUTION FOUND,/,  9HX VE
     1CTOR=,10F10.3)
  183 FORMAT (1X, 36HTHE VALUE OF THE OBJECTIVE FUNCTION=,F10.3,5X, 12HI
     1T IS WITHIN,F8.2, 33H  PERCENTAGE OF THE OPTIMAL VALUE)
  184 FORMAT (10X, 39HNO BRANCHING VARIABLE FOUND -SOME ERROR)
  185 FORMAT (  1H-, 26HTHIS PROBLEM USED AT  MOST,I5,  6H NODES)
  186 FORMAT (  1H0, 45HTOTAL NUMBER OF CONTINUOUS NONLINEAR PROBLEMS,
     18H SOLVED=,I3,//,1X, 34HTOTAL NUMBER OF DISCRETE SOLUTIONS, 10H AC
     2HIEVED=,I3)
  187 FORMAT (  1H0, 15HEXECUTION TIME=,F12.3)
  188 FORMAT (  1H0, 30HTOTAL FUNCTIONAL EVALUATIONS =,2X,I5,/,1X, 30HTO
     1TAL CONSTRAINTS EVALUATIONS=,2X,I5)
  189 FORMAT (  1H0, 33HTOTAL NUMBER OF SOLUTIONS STORED=,I5)
  190 FORMAT (  1H1, 42H THIS RUN IS FOR THE STRATEGY WITH OPTIONS,/,2X,
     1  4HKK= ,I2, 10H    LIFO= ,I2, 11H    NHRST= ,I2)
C
      END
      SUBROUTINE INTFEAS (XO,ID,M)
      DIMENSION XO(M)
      DO 101 J=1,M
         X1=XO(J)-AINT(XO(J))
         X2=1.0-X1
         X3=AMIN1(X1,X2)
         IF (X3.GT..0001) GO TO 102
  101 CONTINUE
C
C     INTEGER SOLUTION FOUND
C
      ID=0
      RETURN
  102 ID=J
      RETURN
C
      END
      SUBROUTINE SELECT (XO,M,INDEX,KK,PCL,PCU,XSTAR,V)
      DIMENSION XO(M), PCL(M), PCU(M), XSTAR(M), V(M)
      IF (KK.EQ.3) GO TO 103
  101 XXMIN=0.0
      INDEX=0
      DO 102 J=1,M
         X1=XO(J)-AINT(XO(J))
         X2=1.0-X1
         X3=AMIN1(X1,X2)
         IF (X3.LE..0001) GO TO 102
         IF (X3.LE.XXMIN) GO TO 102
         INDEX=J
         XXMIN=X3
  102 CONTINUE
      RETURN
C
C
  103 DO 104 J=1,M
         V(J)=0
         XSTAR(J)=XO(J)-AINT(XO(J))
         IF (XSTAR(J).LE..0001) GO TO 104
         IF ((1.0-XSTAR(J)).LE..0001) GO TO 104
         V(J)=AMIN1(PCL(J)*XSTAR(J),PCU(J)*(1.0-XSTAR(J)))
  104 CONTINUE
      VA=0.00001
      INDEX=0
      DO 105 J=1,M
         IF (V(J).LE.VA) GO TO 105
```

```
         VA=V(J)
         INDEX=J
  105 CONTINUE
      IF (INDEX.EQ.0) GO TO 101
      RETURN
C
      END
      SUBROUTINE PSEUDO (M,KK,INDEX,PCL,PCU,KPCL,KPCU,Z,OLDZ,XO,OLDX)
      DIMENSION PCL(M), XO(M), PCU(M), KPCL(M), KPCU(M)
      IF (KK.EQ.40) GO TO 101
      PCL(INDEX)=(Z-OLDZ)/(OLDX-XO(INDEX))
      KPCL(INDEX)=1
      RETURN
  101 PCU(INDEX)=(Z-OLDZ)/(XO(INDEX)-OLDX)
      KPCU(INDEX)=1
      RETURN
C
      END
      SUBROUTINE STORE (KOUNT,INODE,X,XO,XMAX,XMIN,XMINO,XMAXO,Z,ZVALUE,
     1N,NNODE)
      DIMENSION INODE(100), X(100,N), XO(N), XMAX(100,N), XMIN(100,N), X
     1MINO(N), XMAXO(N), ZVALUE(100), LOC(200)
C
      COMMON /S/ INTSOLN,NHRST,XLIM,INDEX,LBL,NINT,NSTORE
      LOCAL=0
C
  101 IF (KOUNT.EQ.100) GO TO 108
      DO 102 I=1,100
         IF (INODE(I).EQ.0) GO TO 103
  102 CONTINUE
C
C     -I- TH NODE INACTIVE
C
  103 NODE=I
      KOUNT=KOUNT+1
      INODE(NODE)=1
      NSTORE=NSTORE+1
      LOC(NSTORE)=NODE
C
C
      IF (NHRST.EQ.2.AND.LBL.NE.100.AND.NODE.NE.1) INODE(NODE)=2
C
C
C     STORE THE INFORMATION
C
      DO 104 J=1,N
         X(NODE,J)=XO(J)
         XMAX(NODE,J)=XMAXO(J)
         XMIN(NODE,J)=XMINO(J)
  104 CONTINUE
      ZVALUE(NODE)=Z
C
C********************************
C
      IF (LOCAL.EQ.1) GO TO 105
      IF (NHRST.NE.1) GO TO 106
      IF (NINT.EQ.0) GO TO 106
      IF (INTSOLN.GE.1) GO TO 106
      IF (ABS(XO(INDEX)-XLIM).GT..0001) GO TO 106
      IF (NINT.GE.2) GO TO 105
      LOCAL=1
      GO TO 101
  105 XMAX(NODE,INDEX)=XLIM
      XMIN(NODE,INDEX)=XLIM
      INODE(NODE)=2
C
C********************************
C
  106 IF (NNODE.GE.KOUNT) GO TO 107
      NNODE=KOUNT
  107 WRITE (7,110) NNODE
      WRITE (7,109) KOUNT
      RETURN
  108 WRITE (7,111)
      STOP
C
  109 FORMAT (5X, 33HTHE TOTAL NUMBER OF ACTIVE NODES=,I5,/)
  110 FORMAT (5X, 40HTHE MAXIMUM NUMBER OF NODES USED SO FAR=,I5)
  111 FORMAT (1X, 46HALL THE NODES USED UP, SUPPLY ADDITIONAL NODES)
C
```

```
      END
      SUBROUTINE REMOVE (KOUNT,ZUP,INODE,ZVALUE,NNODE)
      DIMENSION ZVALUE(100), INODE(100)
      IF (KOUNT.EQ.0) RETURN
      NN=0
      DO 102 I=1,NNODE
         IF (INODE(I).EQ.0) GO TO 102
         IF (ZVALUE(I).GT.ZUP) GO TO 101
         GO TO 102
  101    INODE(I)=0
         NN=NN+1
  102 CONTINUE
      KOUNT=KOUNT-NN
      RETURN
C
      END
      SUBROUTINE HPSTC2 (NE,NI,M,N,XO,XMINO,XMAXO,Y,ZUP,INTSOLN,LBL,XOPT
     1,INODE,X,XMAX,XMIN,ZVALUE,Z,KOUNT,NNODE,NSTORE)
      DIMENSION XOPT(N), Y(N), XO(N), XMINO(N), XMAXO(N), CON(1)
      DIMENSION INODE(100), X(100,N), XMAX(100,N), XMIN(100,N), ZVALUE(1
     100)
      COMMON /H2/ T1,NNOPT,NFN,NCN
C
C
      IF (LBL.EQ.1) GO TO 109
      KOUNTER=0
      DO 101 J=1,M
         Y(J)=AINT(XO(J))
         IF ((XO(J)-Y(J)).GE.0.5) Y(J)=Y(J)+1.0
C
         IF (Y(J).LT.XMINO(J)) Y(J)=Y(J)+1
         IF (Y(J).GT.XMAXO(J)) Y(J)=Y(J)-1
C
  101 CONTINUE
      IF (M.EQ.N) GO TO 103
      DO 102 J=M+1,N
         Y(J)=XO(J)
  102 CONTINUE
C
C
  103 CALL CONST (Y,CON)
      IF (NE.EQ.0) GO TO 105
      DO 104 J=1,NE
         IF (ABS(CON(J)).GT..0001) GO TO 109
  104 CONTINUE
  105 IF (NI.EQ.0) GO TO 107
      DO 106 J=NE+1,NE+NI
         IF (CON(J).LT.-.0001) GO TO 109
  106 CONTINUE
C
C
  107 CALL INTFEAS (Y,ID,M)
      IF (ID.NE.0) GO TO 110
      Z=F(Y)
      ZUP=Z
      INTSOLN=INTSOLN+1
      WRITE (7,115) Y
      WRITE (7,116) Z
      CALL SECOND (FIRST)
      TIM=FIRST-T1
      WRITE (7,118) TIM
      WRITE (7,122) NNODE
      WRITE (7,121) NNOPT,INTSOLN
      WRITE (7,119) NFN,NCN
      WRITE (7,120) NSTORE
      WRITE (7,117)
C
C
      DO 108 I=1,N
         XOPT(I)=Y(I)
  108 CONTINUE
      LBL=100
C
C
      RETURN
C
  109 KOUNTER=KOUNTER+1
      Y(KOUNTER)=XO(KOUNTER)
C
C
```

```
      IF (KOUNTER.EQ.M) LBL=100
C
C
      GO TO 103
C
C
  110 IF (LBL.NE.0) GO TO 111
      CALL STORE (KOUNT,INODE,X,XO,XMAX,XMIN,XMINO,XMAXO,Z,ZVALUE,N,NNOD
     1E)
  111 IF (KOUNTER.EQ.M) GO TO 113
      DO 112 J=KOUNTER+1,M
         XMINO(J)=Y(J)
         XMAXO(J)=Y(J)
  112 CONTINUE
  113 DO 114 J=1,N
         XO(J)=Y(J)
  114 CONTINUE
      M=KOUNTER
      RETURN
C
  115 FORMAT (  1H0, 50HUSING HEURISTIC 2 THE FIRST INTEGER SOLUTION FOU
     1NΠ,/,1X,  9HX VECTOR=,10F10.3)
  116 FORMAT (1X, 36HTHE VALUE OF THE OBJECTIVE FUNCTION=,F10.3)
  117 FORMAT (  1H1)
  118 FORMAT (  1H0, 33HTIME FOR FIRST INTEGER SOLUTION= ,F10.3)
  119 FORMAT (  1H0, 30HTOTAL FUNCTIONAL EVALUATIONS =,2X,I5,/,1X, 30HTO
     1TAL CONSTRAINTS EVALUATIONS=,2X,I5)
  120 FORMAT (  1H0, 33HTOTAL NUMBER OF SOLUTIONS STORED=,I5)
  121 FORMAT (  1H0, 45HTOTAL NUMBER OF CONTINUOUS NONLINEAR PROBLEMS,
     118H SOLVED=,I3,//,1X, 34HTOTAL NUMBER OF DISCRETE SOLUTIONS, 10H AC
     2HIEVED=,I3)
  122 FORMAT (  1H-, 26HTHIS PROBLEM USED AT  MOST,I5,  6H NODES)
C
      END
      SUBROUTINE HRSTC1 (X,NNODE,INODE,LL,NORDER,M,N)
      DIMENSION X(100,N), NORDER(1), INODE(1)
      MIN=1000
      LL=0
      DO 101 I=1,NNODE
         IF (INODE(I).EQ.0) GO TO 101
         IF (INODE(I).EQ.1) GO TO 101
         CALL ORDER (X,NORDER,I,M,N)
         IF (NORDER(I).GE.MIN) GO TO 101
         MIN=NORDER(I)
         LL=I
  101 CONTINUE
      RETURN
C
      END
      SUBROUTINE ORDER (X,NORDER,I,M,N)
      DIMENSION X(100,N), NORDER(1)
      NORDER(I)=0
      DO 101 K=1,M
         X1=X(I,K)-AINT(X(I,K))
         X2=1.0-X1
         X3=AMIN1(X1,X2)
         IF (X3.GT..00001) NORDER(I)=NORDER(I)+1
  101 CONTINUE
      RETURN
C
      END
```