Analysis of four-bar planar mechanisms with joint clearances

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Summary. This work deals with four-bar planar mechanisms with clearances at the joints, which induce unilateral constraints, impacts and friction, rendering the dynamics nonsmooth. The objective is to determine sets of parameters (clearance value, restitution coefficients, friction coefficients) such that the system's trajectories stay in a neighborhood of the trajectories of mechanism with no clearance. The analysis is based on numerical simulations obtained with the projected Moreau-Jean time-stepping scheme.

Introduction

In this work the non-smooth dynamic method is used to model the revolute joint with clearance through a set of unilateral constraints, as done in [3]. The projected Moreau-Jean event capturing (time-stepping) scheme is used to solve the contact-impact problem [1]. Finally numerical results with different clearance values and coefficients of restitution are compared for different configurations of the planar four-bar mechanisms.

Lagrangian formulation with unilateral constraints

The unilateral constraints between bodies can be considered in a Lagrangian formulation as follows [2],

$$\begin{cases} M(q(t))\ddot{q}(t) + F(t, q(t), \dot{q}(t)) = G_N^{\top}(q(t))\lambda_N + H_T^{\top}(q(t))\lambda_T, & \text{(1a)} \\ g_{(N,k)}(q(t)) = 0, \quad k \in \mathcal{E} & \text{(1b)} \\ g_{(N,k)}(q(t)) \geq 0, \quad \lambda_{(N,k)} \geq 0, \quad \lambda_{(N,k)} g_{N,k}(q(t)) = 0 \quad k \in \mathcal{I} & \text{(1c)} \\ \dot{q}^+(t) = -e_r \dot{q}^-(t), \text{ if } g_N(q(t)) = 0 & \text{(1d)} \\ (U_k, R_k) \in \mathcal{C}(n_k, \mu_k) & \text{(1e)} \end{cases}$$

where $q(t) \in \mathbb{R}^n$ is the generalized coordinates vector, $M(q(t)) \in \mathbb{R}^{nxn}$ is the mass matrix, $F(t, q(t), \dot{q}(t)) \in \mathbb{R}^n$ is the generalized forces, $g_N(q(t)) \in \mathbb{R}^m$ is the kinematic constraint, $G_N(q) \in \mathbb{R}^m$ is the jacobian matrix of g_N with respect to q, $H_T(q) \in \mathbb{R}^{n \times m}$ is the linear map of local tangent frame at contact point, $\lambda_N \in \mathbb{R}^m$ and $\lambda_T \in \mathbb{R}^m$(*for 2D case*) are the Lagrange multiplier vectors associated with the constraints in normal and tangential direction, e_r is the restitution coefficient, $\mathcal{E} \subset \mathbb{N}$ and $\mathcal{I} \subset \mathbb{N}$ represent the sets of indices of bilateral and unilateral constraints, $\dot{q}^+$ (respectively $\dot{q}^-$) is the function defined as the limit on the right (left) of bounded variation function $\dot{q}$. $R_k \in \mathbb{R}^2$ the contact force at the k^{th} contact, U_k local velocity at contact and μ_k is the coefficient of friction at the contact point.

Example of Four-Bar Mechanism with Clearance Revolute Joint

We have considered crank–rocker four–bar mechanism (Grashof's class with $l_1 + l_3 \leq l_2 + l_4$). In this class the input link (crank) makes complete rotation and the output link (rocker) oscillates, i.e. it does not make complete 360° rotation (see Figure 1).
Let us consider a four-bar mechanism with mass of links m_i, length of links l_i, inertia of links I_i, $1 \leq i \leq 3$. Imperfect joint is defined by unilateral constraint $g_{Nj} = (c_{rj} - \overrightarrow{C_{j-1}C_j}\vec{n}) \geq 0$, $2 \leq j \leq 4$. C_{j-1} and C_j are the contact points (see Figure 2) on bearing and journal respectively, c_{rj} is the radial clearance at imperfect joints.
A four-bar mechanism with perfect revolute joint is described by one generalized coordinate q=$[\theta_1]$ same as the number of degrees of freedom (DOF) of the system. Each imperfect joint adds two extra DOF to the system, for two imperfect joints we select $q = [\theta_1, \theta_2, \theta_3, X_2, Y_2, X_3, Y_3]^T$. A four-bar mechanism is actuated with torque applied at the joint 1 (J_1) in counter-clockwise direction. We consider joints J_1 and J_4 to be perfect revolute joints while joints $J_2,$ and J_3 may be imperfect joints with radial clearances c_{r2} and c_{r3}. Crank–rocker four-bar mechanism is treated with two clearance revolute joints with different values of input torque $\tau_1 = 6.0Nm$, $\tau_2 = 6.0\sin(3\pi t)Nm$. The influence of different clearance sizes, coefficients of restitution and coefficients of friction are studied. The simulation environment SICONOS[1] is used to solve the problem and the results are compared with the case with no clearance. In the simulated example we have chosen l_1=1.0 m, l_2=4.0 m, l_3=2.5 m, l_0=3.0 m, m_1=1.0 kg, m_2=1.0 kg, m_3=1.0 kg, I_1=$8.33 \cdot 10^{-2}$ kgm^2, I_2=1.33 kgm^2, I_3=$5.21 \cdot 10^{-1}$ kgm^2 and the initial conditions are θ_1 = 1.5708 rad, θ_2 = 0.3533 rad, θ_3 = 1.2649 rad, ω_1 = 0.0 rad/s, ω_2 = 0.0 rad/s, ω_3 = 0.0 rad/s and the coordinates of the center of gravity of link 2 are X_2=1.8764m and Y_2=1.6919m. The simulations are run over time interval $[0, 10]s$, with time step $10^{-6}s$.
In Figure 3(b), $g_T dot$ is the relative tangential velocity and $g_N dot$ is the relative normal velocity at the clearance joint J_2 and J_3.

[1] `http://siconos.gforge.inria.fr/`

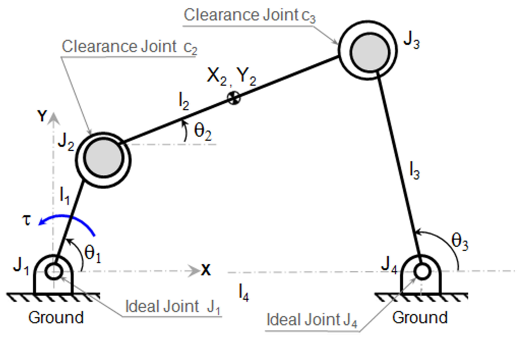


Figure 1: Four-bar mechanism with two imperfect revolute joints

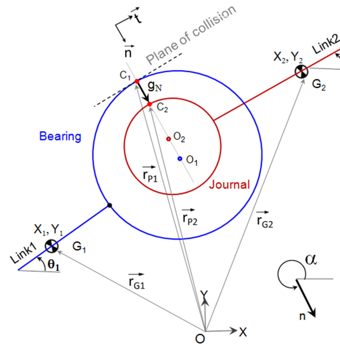
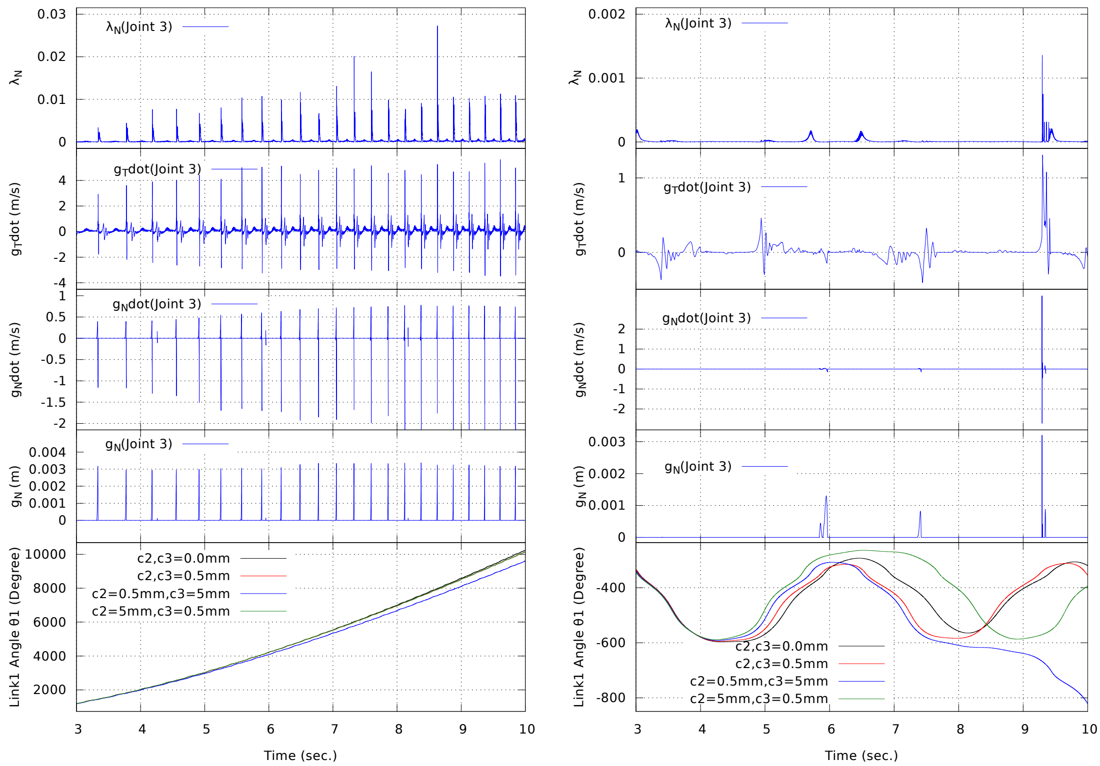


Figure 2: Revolute joint with clearance



(a) Comparison of θ_1, g_N, $g_N dot$, $g_T dot$ and λ_N vs Time, with τ_1

(b) Comparison of θ_1, g_N, $g_N dot$, $g_T dot$ and λ_N vs Time, with τ_2

Figure 3: Comparison of system's trajectories and behaviour of clearance joint (c_3), with $\mu = 0.1, e = 0.0$

Conclusion

It can be concluded that the system's trajectories substantially change when the joint clearance with friction (c_2 and c_3) are considered. For the constant torque τ_1, stick–slip regions are negligible and observed only during the initial phase of motion. The maximum energy loss is 19.6% for the case with c_2=0.5mm and c_3=5.0mm when compared to other cases (see Figure 3a:θ_1). The stick–slip regions are more dominant for sinusoidal torque τ_2. For the case with clearance c_2=0.5mm and c_3=5.0mm, it accounts for 33.7% of the time. Also the overall energy loss of the system is less by 30.2% when compared to the case with no clearance. We can conclude that the stick–slip transitions play significant role for the divergence of the angular positions (see Figure 3b:θ_1) for the case with sinusoidal torque. We can also conclude from Fig.3(b) that the differently located imperfect joints with radial clearances may have different influence on system's behaviour. No spurious numerical oscillations are observed during the sticking phases (see Figure 3b:$g_N dot$).

References

[1] V. Acary. Projected event-capturing time-stepping schemes for nonsmooth mechanical systems with unilateral contact and Coulomb's friction. *Computer Methods in Applied Mechanics and Engineering*, 256:224–250, 2013.

[2] V. Acary and B. Brogliato. *Numerical methods for nonsmooth dynamical systems. Applications in mechanics and electronics.* Lecture Notes in Applied and Computational Mechanics 35. Berlin: Springer. xxi, 525 p. , 2008.

[3] P. Flores, R.Leine, and C. Glocker. Modeling and analysis of planar rigid multibody systems with translational clearance joints based on the non-smooth dynamics approach. *Multibody System Dynamics*, 23:165–190, 2010.