History-Preserving Bisimilarity for Higher-Dimensional Automata via Open Maps

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History-Preserving Bisimilarity for Higher-Dimensional Automata via Open Maps Extended Abstract

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One of the popular notions of equivalence for non-interleaving concurrent systems is *history-preserving bisimilarity* (*hp-bisimilarity*). *Higher-dimensional automata* (*HDA*) [6], [7] is a non-interleaving formalism for reasoning about behavior of concurrent systems, which provides a generalization (up to hp-bisimilarity) to "the main models of concurrency proposed in the literature" [8].

Using open maps [4], we can show that hp-bisimilarity for HDA has a characterization directly in terms of (higher-dimensional) *transitions* of the HDA, rather than in terms of runs as *e.g.* for Petri nets. Our results imply *decidability* of hp-bisimilarity for finite HDA. They also put hp-bisimilarity firmly into the open-maps framework of [4] and tighten the connections between bisimilarity and weak topological *fibrations* [1], [5].

A full version of this report is available as [3].

A *precubical set* is a graded set $X = \{X_n\}_{n\in\mathbb{N}}$ together with mappings $\delta_k^\nu : X_n \to X_{n-1}$, $k = 1, \dots, n$, $\nu = 0, 1$, satisfying the *precubical identity* $\delta_k^\nu \delta_\ell^\mu = \delta_{\ell-1}^\mu \delta_k^\nu$ for $k < \ell$. The mappings δ_k^ν are called *face maps*, and elements of X_n are called n-*cubes*. Faces $\delta_k^0 x$ of an element $x \in X$ are to be thought of as *lower faces*, $\delta_k^1 x$ as *upper faces*. *Morphisms* $f : X \to Y$ of precubical sets are graded mappings $f = \{f_n : X_n \to Y_n\}_{n\in\mathbb{N}}$ which commute with the face maps: $\delta_k^\nu \circ f_n = f_{n-1} \circ \delta_k^\nu$. This defines a category pCub of precubical sets and morphisms.

The category of *higher-dimensional automata* is the comma category $\mathsf{HDA} = * \downarrow \mathsf{pCub}$ of *pointed precubical sets* and with morphisms which respect the point.

We say that a precubical set X is a *path object* if there is a (necessarily unique) sequence $(x_1, \dots, x_m)$ of elements in X such that $x_i \neq x_j$ for $i \neq j$,

- for each $x \in X$ there is $j \in \{1, \dots, m\}$ for which $x = \delta_{k_1}^{\nu_1} \cdots \delta_{k_p}^{\nu_p} x_j$ for some indices $\nu_1, \dots, \nu_p$ and a *unique* sequence $k_1 < \cdots < k_p$, and
- for each $j = 1, \dots, m-1$, there is $k \in \mathbb{N}$ for which $x_j = \delta_k^0 x_{j+1}$ or $x_{j+1} = \delta_k^1 x_j$.

If X and Y are path objects with representations $(x_1, \dots, x_m)$, $(y_1, \dots, y_p)$, then a morphism $f : X \to Y$ is called a *path extension* if $x_j = y_j$ for all $j = 1, \dots, m$ (hence $m \leq p$). The category HDP of *higher-dimensional paths* (*HDP*) is the subcategory of HDA which as objects has pointed path objects, and whose morphisms are generated by isomorphisms and pointed path extensions.

Following [2], we say that a morphism in HDA is *open* if it has the right lifting property with respect to HDP, and that HDA X, Y are *bisimilar* if there is $Z \in \mathsf{HDA}$ and a span of open maps $X \leftarrow Z \to Y$ in HDA. It can be shown [2] that X and Y are bisimilar iff n-cubes with matching lower faces can be matched; this is a straight-forward generalization of ordinary bisimulation for transition systems and appears hence to be rather badly suited for concurrent systems. We can, however, show that this bisimilarity is precisely hp-bisimilarity.

A *cube path* in a precubical set X is a morphism $P \to X$ from a path object P. Using the notion of adjacency from [7], [8], we can define what it means for two cube paths to be *homotopic*, *i.e.* to represent the same execution up to concurrency. The *unfolding* $\tilde{X}$ of a HDA X is then defined to be the set of homotopy classes of pointed cube paths in X. With suitable structure maps, this becomes a precubical set, indeed, a *higher-dimensional tree*.

The category of *HDA up to homotopy* $\mathsf{HDA_h}$ has as objects HDA and as morphisms pointed precubical morphisms $f : \tilde{X} \to \tilde{Y}$ of unfoldings. Noting that any HDP is isomorphic to its own unfolding, we have an embedding $\mathsf{HDP} \hookrightarrow \mathsf{HDA_h}$. We can then say that a morphism in $\mathsf{HDA_h}$ is *homotopy open* if it has the right lifting property with respect to HDP and define *homotopy bisimilarity* accordingly.

Theorem: Two HDA are homotopy bisimilar iff they are hp-bisimilar [8], iff they are bisimilar.

Using an arrow category, we can easily extend the above considerations to the (more interesting) case of *labeled HDA*.

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