

## Purdue University Purdue e-Pubs

International Compressor Engineering Conference

School of Mechanical Engineering

2010

# High Efficiency Development of a Reciprocating Compressor by Clarification of Loss Generation in Bearings

Masaru Matsui Panasonic Corporation

Yoko Kitsunai Panasonic Corporation

Ko Inagaki Panasonic Corporation

Follow this and additional works at: https://docs.lib.purdue.edu/icec

Matsui, Masaru; Kitsunai, Yoko; and Inagaki, Ko, "High Efficiency Development of a Reciprocating Compressor by Clarification of Loss Generation in Bearings" (2010). *International Compressor Engineering Conference*. Paper 2025. https://docs.lib.purdue.edu/icec/2025

Complete proceedings may be acquired in print and on CD-ROM directly from the Ray W. Herrick Laboratories at https://engineering.purdue.edu/ Herrick/Events/orderlit.html

This document has been made available through Purdue e-Pubs, a service of the Purdue University Libraries. Please contact epubs@purdue.edu for additional information.

## High Efficiency Development of a Reciprocating Compressor by Clarification of Loss Generation in Bearings

Masaru MATSUI<sup>1</sup>\*, Yoko KITSUNAI<sup>2</sup>, Ko INAGAKI<sup>1</sup>

<sup>1</sup>Appliances Development Center, Panasonic Corporation, 2-3-1-2 Noji-higashi, Kusatsu City, Shiga 525-8555, Japan Phone: +81-77-561-5664, Fax: +81-77-563-1967, E-mail:matsui.masaru@jp.panasonic.com

<sup>2</sup>Living Environment Development Center, Panasonic Corporation,
 3-1-1 Yagumo-naka-machi, Moriguchi City, Osaka, 570-8501, Japan
 Phone: +81-6-6906-4846, Fax: +81-6-6904-5163, E-mail:kitsunai.yoko@jp.panasonic.com

\* Corresponding Author

## ABSTRACT

An analytical model for mixed lubrication in bearings of reciprocating compressors for refrigerators has been developed and a new bearing which could decrease its friction losses by 20% has been designed. Friction losses, which are generated in the journal and thrust bearing of our reciprocating compressor, are estimated to be one-third or more of the all losses, so it is an essential issue to decrease bearing losses to design more efficient compressors. The developed analytical model can calculate the oil film pressure and solid contact pressure between bearing and shaft in each bearing simultaneously. And it can also specify the shaft posture which changes at each time step and calculate the loss generated by oil viscosity and that by contact pressure. By using this model, how and how much friction loss is generated in each bearing was clarified and specific approach to decrease bearing losses was also clarified.

## **1. INTRODUCTION**

A reciprocating compressor installed in our refrigerator is a machine which compresses working fluid (refrigerant) by reduction of cylinder volume by the reciprocation of a piston and has been produced for 80 years. Figure 1 shows a vertical cross section of a reciprocating compressor. The eccentric rotational motion of a crank pin, which is set up in the upper part of a shaft, is converted into the reciprocation of a piston through a connecting rod. Bearings are constructed by a journal bearing which supports a horizontal load from a connecting rod ( $F_C$ ) and a thrust bearing which supports a vertical load of shaft weight and so on ( $F_T$ ). For a journal and thrust bearing in our compressor, a plain bearing where a load is supported by lubricant pressure is adopted. Friction losses generated in these bearings are estimated to be one-third or more of all losses, so it is an essential issue to decrease bearing losses to make a reciprocating compressor more efficient. In this study, an analytical model for mixed lubrication in bearings was developed to reduce bearing losses and how and how much friction loss is generated in each bearing was clarified by using the model.

## 2. ANALYTICAL MODEL FOR MIXED LUBRICATION IN BEARINGS

The lubrication condition in a journal and thrust bearing of our reciprocating compressor is considered to be the mixed lubrication, because loads acting on a shaft are supported by oil film pressure of lubricant, which filled in clearance between shaft and bearing surface, and by solid contact pressure, which generated by contact of shaft and bearing surface. The developed model analyzes a shaft posture as loads acting on the shaft and support force generated by oil film pressure and solid contact pressure in each bearing are balanced at each time step, and then calculates bearing losses.

### 2.1 Analytical Object

Journal bearing and thrust bearing as the analytical object are explained simply here. On the shaft surface in a journal bearing, an oil groove to pump lubricant up (its depth is more than 1000 µm) and annular depression are formed. The average clearance between journal bearing and upper/lower rubbing part on the shaft surface, which are set facing each other across annular depression, is set to be about 10 µm. Loads acting on the shaft are supported by oil film pressure and solid contact pressure generated in these rubbing parts. Oil groove and annular depression on the shaft surface make clearance between journal bearing and rubbing parts discontinuous in places. On the other hand, the shape of a thrust bearing is an almost ring whose center is set at the center of the journal bearing, but a groove to discharge lubricant is formed on its surface partly. Surfaces of these bearings are ground smoothly.

Figure.2 shows an example of horizontal load ( $F_C$ ) variation acting on the crank pin, taking shaft rotation angle  $\theta_s$  in the horizontal axis. Rotational direction of  $\theta_s$  is same as shaft rotational one and  $\theta_s$  becomes zero when the eccentric direction of crank pin agrees with the center line of cylinder (xaxis). Horizontal load ( $F_C$ ) varies by ten times or more during one rotation of shaft. A shaft receives a load that changes greatly as shown in Figure 2 and rotates while tilting and coming near to a certain direction to a journal bearing.



Figure 1: Longitudinal cross section of reciprocating compressor



Figure 2: Horizontal load variations acting on crank pin

#### 2.2 Oil pressure analysis

In clearance between shaft and bearing surface, lubricant forms oil film and it flows with the rotation of the shaft. In this study, the lubricant filled in clearance is assumed to be incompressive fluid. When the shaft comes near to a certain direction to a journal bearing, oil pressure on the upstream side of the eccentric direction rises up and then oil pressure distribution is changed (wedge effect). To analyze oil pressure distribution in oil film, Patir-Cheng's modified Reynolds equation (Patir *et al.*, 1978 and Patir *et al.*, 1979), which bases on the average flow model that considers the influence of surface roughness of shaft and bearings, is used.

$$\frac{\partial}{r\partial\theta} \left( \phi_x \frac{h^3}{12\eta} \frac{\partial p}{r\partial\theta} \right) + \frac{\partial}{\partial z} \left( \phi_z \frac{h^3}{12\eta} \frac{\partial p}{\partial z} \right) = \frac{U}{2} \left( \frac{\partial \overline{h}_T}{r\partial\theta} + \sigma \frac{\partial \phi_s}{r\partial\theta} \right) + \frac{\partial \overline{h}_T}{\partial t}$$
(1)

In Equation (1), p is oil pressure, r is radius of shaft,  $\theta$  is angle to the circumference direction of shaft, h is clearance between shaft and bearing,  $\overline{h}_T$  is expected value of local clearance,  $\eta$  is oil viscosity, U is shaft rotation speed, and  $\phi_x$ ,  $\phi_z$ ,  $\phi_s$  are correction factors.

To solve Equation (1) numerically, divergence formulation (DF) method (Kawabata, 1987) is applied as descretization method. DF method can apply discontinuous oil film thickness in bearings because it puts computational grids to fit the discontinuous line of clearance.

#### 2.3 Solid contact pressure analysis

To calculate solid contact pressure, Greenwood's model (Greenwood et al., 1970 and Patir et al., 1978) is adopted. This model assumes that solid contact pressure is generated by elastic deformation caused by contact of asperities on shaft and bearing surfaces.

$$P_{c} = 4.4086 \times 10^{-5} \cdot k' \cdot E' \left( 4.0 - \frac{h}{\sigma'} \right)^{6.804} \qquad (h < 4\sigma')$$
<sup>(2)</sup>

In Equation (2),  $P_c$  is solid contact pressure, k' is a coefficient decided by the shape of surface roughness, E' is combined Young's modulus of shaft and bearing,  $\sigma'$  is combined surface roughness of shaft and bearing. E' is shown in Equation (3) and  $\sigma' = (\sigma_1^2 + \sigma_2^2)^{0.5}$ . E is Young's modulus, v is Poisson's ratio, and  $\sigma$  is surface roughness. Subscript 1 and 2 represent bearing's or shaft's property respectively.

$$E' = \frac{1}{2} \left\{ \frac{1 - v_1^2}{E_1} + \frac{1 - v_2^2}{E_2} \right\}$$
(3)

#### 2.4 Shaft posture analysis

Shaft posture analysis in the developed model makes it possible to clarify shaft rotational behavior. It analyzes shaft posture at each rotation angle  $\theta_s$  as loads acting on the shaft (horizontal load  $F_C$  and vertical load  $F_T$ ) and pressure field in journal and thrust bearing obtained by the oil pressure analysis and solid contact pressure analysis are balanced.

Figure 3 shows a shaft posture at a certain rotation angle  $\theta_s$ . The initial position of the center of gravity of the shaft is set as the origin, x axis is taken in the same direction as the center line of cylinder, y axis is taken in the direction that rotated by 90 degree from x axis, and z axis is taken in vertical upper direction. Variables of shaft posture are represented as follows:  $e_x$  and  $e_y$  are x and y components of eccentricity of the center of gravity of the shaft e,  $\gamma_x$  and  $\gamma_y$  are tilting angle around x and y axis of the shaft  $\gamma$ , and  $d_t$  is distance between the center of the ring of thrust bearing and that on the shaft, respectively. Clearance distribution between shaft and bearing surface is decided by these shaft posture parameters ( $e_x$ ,  $e_y$ ,  $\gamma_x$ ,  $\gamma_y$ ,  $d_t$ ) and pressure distribution in oil film can be calculated when it is inputted to the Equation (1) and (2). Then support forces of bearings  $F_B$  and moment around the center of gravity of the shaft  $M_B$  can be calculated by integrating pressure distribution on the shaft surface. When x, y, and z components of  $F_B$  are represented by  $F_{Bx}$ ,  $F_{By}$ ,  $F_{Bz}$  and x and y

components of  $M_B$  are represented by  $T_{Bx}$ ,  $T_{By}$ ,  $T_{Bz}$  and w and yrespectively, shaft posture parameters ( $e_x$ ,  $e_y$ ,  $\gamma_x$ ,  $\gamma_y$ ,  $d_t$ ) should be decided to meet Equation (4) and (5) simultaneously.

$$F_{Bx} = F_x, F_{By} = F_y, F_{Bz} = F_z$$
(4)

$$M_{Bx} = M_{x}, M_{By} = M_{y} \tag{5}$$

In these equations,  $F_x$  and  $F_y$  represent x and y components of horizontal load  $F_c$ ,  $M_x$  and  $M_y$  represent x and y components of moment around the center of gravity of the shaft  $M_B$ , respectively.

Shaft posture parameters ( $e_x$ ,  $e_y$ ,  $\gamma_x$ ,  $\gamma_y$ ,  $d_t$ ) cannot be decided directly by giving load condition ( $F_x$ ,  $F_y$ ,  $F_T$ ,  $M_x$ ,  $M_{yt}$ ). In this study, amounts of change of  $F_{Bx}$ ,  $F_{By}$ ,  $F_{Bz}$ ,  $M_{Bx}$ and  $M_{By}$  when shaft posture parameters change slightly are assumed to contribute to slight change of shaft posture parameters linearly. Shaft posture at each rotation angle  $\theta_s$ can be decided by changing shaft posture parameters little by little to meet Equation (4) and (5) approximately.



Figure 3: Shaft posture parameters

#### 2.5 Bearing loss analysis

When shaft posture is obtained, the distribution of clearance h between shaft and bearing surface is also decided. Then the distributions of oil film pressure and solid contact pressure are also analyzed. To calculate losses in journal and thrust bearings, loss generated by shear stress of oil viscosity and the one generated by friction force by solid contact are calculated separately in each bearing.

To calculate shear stress by oil viscosity, Patir-Cheng's equation derived from the average flow model is used.

$$\tau(\theta_s) = \frac{\eta U}{h} \left\{ \phi_f + (1 - 2V_{r2})\phi_{fs} \right\} + \left\{ h\left(V_{r2} - \frac{1}{2}\right)\phi_{fp} - V_{r2}\overline{h}_T \right\} \frac{\partial p}{r\partial\theta}$$
(6)

In Equation (6),  $\tau$  is viscous shear stress per unit area on shaft surface,  $\phi_f$ ,  $\phi_{fs}$ ,  $\phi_{fp}$  are shear stress factors in average flow model, and  $V_{r2} = (\sigma_2 / \sigma^2)^2$  is contribution ratio of shaft surface roughness.

Friction force by solid contact pressure is assumed to be generated based on Coulomb's friction low. When the coefficient of friction is represented by  $\mu$ , friction stress by solid contact per unit area can be calculated as follows.

$$\tau_c(\theta_s) = \mu P_c \tag{7}$$

Viscous friction loss  $W_{Jl}$ ,  $W_{Tl}$  and solid contact friction loss  $W_{Jc}$ ,  $W_{Tc}$  are calculated as follows. At first, by integrating  $\tau(\theta_s)$  and  $\tau_c(\theta_s)$  on whole shaft surface, friction force acting on the shaft surface at rotation angle  $\theta_s$  can be calculated. Then, by integrating the value which multiply the friction force by the shaft radius at all rotation angle  $\theta_s$ ,  $W_{Jl}$ ,  $W_{Tl}$ ,  $W_{Jc}$  and  $W_{Tc}$ , can be obtained. In characters  $W_{Jl}$ ,  $W_{Tl}$ ,  $W_{Jc}$  and  $W_{Tc}$ , the first subscript represents journal (J) or thrust (T) bearing and the second one represents viscous (l) or solid contact (c) friction loss, respectively.

#### 2.6 Procedure of lubrication analysis

Figure 4 shows a flow chart of developed lubrication analysis. At first, bearing specifications, horizontal load  $F_C$ , etc are inputted, and shaft posture parameters ( $e_x$ ,  $e_y$ ,  $\gamma_x$ ,  $\gamma_y$ ,  $d_t$ ) are decided at each rotation angle  $\theta_s$ , then losses in journal and thrust bearing are calculated. After rotation angle reaches  $2\pi$ , shaft posture parameters are compared with these calculated at  $\theta_s - 2\pi$  at every calculation step. If both parameter sets agree with each other approximately, the analysis will finish.



Figure 4: Flow chart of analytical model for mixed lubrication

## 3. AN EXAMPLE OF ANALYSIS RESULT

The developed model makes it possible to clarify the shaft rotational behavior and quantify bearing losses. An example of analysis result for our current reciprocating compressor is shown as follows. In the calculation, the value of shaft radius *r*, rotation speed, oil viscosity  $\eta$ , the coefficient of friction  $\mu$ , and the vertical load  $F_T$  were set as 8 mm, 50 Hz, 2.7 mPa·s, 0.1, 14 N (constant), respectively. The data of horizontal load  $F_C$  were given by that shown in Figure 2.

#### 3.1 Locus of center of shaft rotational axis

Figure 5 shows loci of center of shaft rotational axis. In the figure, the thick line shows the locus at the top of bearing and the thin line shows that at the bottom of bearing. In Figure 5, the center and radius of the circle represent the center of journal bearing and average clearance ( $8\mu$ m), respectively. Each number which is written near a circled

mark on the locus shows shaft rotation angle  $\theta_s$ . The locus is very complicated and eccentric direction at the top and bottom of bearing at each rotation angle  $\theta_s$  is almost opposite across the center of the circle. This indicates that the shaft whirls in journal bearing. At the top of bearing, shaft rotational axis approaches the journal bearing most to the third quadrant at around  $\theta_s = 330^\circ$ . This suggests that the surface of shaft and moment could come in contact with each other at this direction. This direction is almost same as that of rubbing traces in the journal bearing which can be observed after operating. Thus it can be considered that the locus obtained by the developed model has validity.

#### **3.2 Bearing losses**

In Table 1, viscous friction loss  $W_{Jl}$  and solid contact friction loss  $W_{Jc}$  in journal bearing, and viscous friction loss  $W_{Tl}$  and solid contact friction loss  $W_{Tc}$  in thrust bearing, are shown. Each value is normalized by total bearing loss. From this table, it can be seen that solid contact friction loss  $W_{Tc}$  in thrust bearing cannot be ignored and loss generated in thrust bearing is more than



Figure 5: Locus of center of shaft rotational axis

that in journal bearing. The amount of vertical load  $F_T$  (=14 N) is only about one third of the average of horizontal load  $F_C$  (=41 N). This suggests that design of the thrust bearing in our compressor should be improved.

Table 1: Friction loss in bearings of reciprocating compressor

$W_{Jl}$	$W_{Jc}$	$W_{Tl}$	$W_{Tl}$	Total
0.46	0.00	0.45	0.09	1.00

## 4. LOAD SUPPORT MECHANIZM IN BEARINGS

#### 4.1 Journal Bearing

## Relation between whirl and the balance of forces and moments acting on the shaft:

In sub-section 3.1, it is shown that the shaft whirls in journal bearing, and the shaft behavior can be also explained macroscopically. When the average pressure generated in upper and lower rubbing part are represented by  $P_u$  and  $P_l$  respectively, the force and moment acting on the shaft are balanced only when the condition of the force balance becomes that shown in Figure 6. If the contribution of support force for  $F_c$  in thrust bearing is ignored, the balance can be expressed by the following two equations simply.

$$2rL_uP_u - 2rL_lP_l = F_C \tag{8}$$

$$(2rL_uP_u)l_u + (2rL_lP_l)l_l = F_c l_c$$
(9)

In Equations (8) and (9),  $L_u$ ,  $L_l$  represent the length of upper and lower rubbing part,  $l_u$  and  $l_l$  represent the distance between the center of upper and lower bearing and the center of gravity of the shaft, and  $l_c$  represents the distance between the acting point of load  $F_c$  and the



Figure 6: The balance between forces and moments on the shaft

center of gravity of the shaft. It is suggested the shaft rotation while tilting because pressure  $P_u$  and  $P_l$  would generates in opposite direction (see Figure 6).

#### Ideal lengths of upper and lower rubbing parts:

When  $F_C$  is eliminated from Equation (8) and (9), the ratio of  $P_u$  and  $P_l$  can be obtained.

$$\frac{P_{l}}{P_{u}} = \frac{L_{u}}{L_{l}} \cdot \frac{l_{c} - l_{u}}{l_{c} + l_{l}}$$
(10)

The theoretical pressure ratio  $P_l / P_u$  is equal to 1.0, it is considered that lubrication condition in each rubbing part is same. Actually, the value of right side of Equation (10) when lubrication condition in each rubbing part is same is different from 1.0 because of the influence of shaft tilt. But, it would be an ideal condition when  $P_l / P_u$  is equal to 1.0, i.e. upper and lower bearing share horizontal load  $F_c$  equally.

Although only balance between  $P_u$  and  $P_l$  is considered in Equation (10), each rubbing part (especially upper rubbing part) needs enough length to bear horizontal load  $F_c$  as shown in Equation (8). If the length of each upper rubbing part ( $L_u$  or  $L_l$ ) is not enough, there is no method except raising the pressure on the surface in each rubbing part ( $P_u$  or  $P_l$ ) to bear  $F_c$ . As a result, clearance between surface of shaft and bearing becomes smaller, the possibility that both surfaces come in contact becomes stronger. In such a situation, much bearing loss would be generated and solid contact would exert a bad influence on operating reliability. Therefore, it is considered to be important that theoretical pressure ratio  $P_l / P_u$  is kept appropriately and each rubbing part is designed to have enough length.

#### 4.2 Thrust Bearing

The developed model clarified that, in thrust bearing, the tilt of the shaft generates oil film pressure and solid contact pressure to bear the vertical force  $F_T$ . This phenomenon can be explained by using Figure 7. When the tilt of the shaft is large, the sectional area along flowing route of lubricant in the thrust bearing changes greatly. So, on the upstream side of the place where the surface of the shaft approaches that of the bearing most, more lubricant is dragged into the flowing route and oil film pressure rises up greatly. As a result, high oil film pressure would be generated to bear  $F_T$ , even though the shaft does not approach the bearing so much.

On the other hand, when the tilt of the shaft is small, the ring on the shaft is nearly parallel to that on the bearing and the sectional area along flowing route of lubricant in the thrust bearing changes slightly. So, on the upstream side of the place where the surface of the shaft approaches that of the bearing most, oil film pressure rises up slightly. As a result, the surface of the shaft must approach that of the bearing more and then relative low pressure should be generated in wider area to bear  $F_T$ .

It is suggested that shaft rotation with appropriate tilt is effective to reduce the loss in thrust bearing, because the vertical load is borne in the condition that the shaft does not approach the bearing so much and solid contact of shaft and bearing is decreased.

## 5. APPROACH TO REDUCE BEARING LOSSES

When  $\tau(\theta_s)$  in Equation (6) is integrated on whole shaft surface, the term of oil film pressure gradient (the second term) becomes small than that of viscosity (the first term) relatively. This is because oil film pressure in journal and



Figure 7: The mechanism for thrust bearing to support load

thrust bearing is calculated in periodic boundary condition. Thus viscous friction loss  $W_{JI}$  and  $W_{TI}$  is almost proportional to the value which multiplies  $\eta U/h$  by area of upper/lower rubbing part A.

$$W_{JI}, W_{TI} \propto \eta U A / h \tag{11}$$

Only area A can be changed in variables in right side of Equation (11) in the design process. In this study, the approach to reduce of bearing loss has been taken, only by changing the length of rubbing parts  $(L_u + L_l)$  in the value of area of upper/lower rubbing part  $A=2\pi r(L_u + L_l)$ .

At first, the losses of journal and thrust bearing were calculated when changing the length of  $L_u + L_l$  by using the developed model. In this calculation, length of  $L_u$  and  $L_l$  were changed respectively to keep theoretical pressure ratio  $P_l / P_u$  almost constant. Figure 8 shows the calculation result. The value of horizontal axis is normalized by total bearing loss of current compressor. This figure shows that total bearing loss takes minimum value by 0.77 at around  $L_u + L_l = 16$ mm, but it should be careful to this length is proper to be adopted. So the loss generated in each bearing is considered furthermore.

When the length of  $L_u + L_l$  is reduced from 26mm of our current compressor, the loss of journal bearing decreases gradually, and takes its minimum value at around 17mm, and then increases oppositely. It is suggested that our current compressor has enough rubbing part in journal bearing, and viscous friction loss  $W_{Jl}$  could be decreased with keeping the situation that solid contact loss  $W_{Jc}$  is tiny even if the rubbing part is reduced by 7-8 mm. But If the length of  $L_u + L_l$  is reduced from 18mm furthermore, solid contact loss increases gradually and then the increase  $W_{Jc}$  of exceeds the decrease of  $W_{Jl}$ .

On the other hand, the loss of thrust bearing decreases monotonously according as the length of  $L_u + L_l$  decreases. As mentioned in section four, when the length of  $L_u + L_l$  is 26mm of our current compressor, the tilt of the shaft is small and the shaft rotation is stable. Thus the surface of shaft approaches that of bearing and both  $W_{Tl}$  and  $W_{Tc}$  are large. When the length of  $L_u + L_l$  is shorter, the tilt of the shaft would be larger and the shaft rotation would become more unstable. Thus the clearance between the surface of shaft and bearing increases and both  $W_{Tl}$  and  $W_{Tc}$  become small.

By deliberating both above mentioned consideration and the reliability of journal bearing which receive large load, the optimal length of  $L_u + L_l$  should be taken as 18mm which is the minimum length that solid contact is enough tiny to be ignored in journal bearing. The developed model estimates that total bearing loss could be reduced by 20% when the length of  $L_u + L_l$  is taken as 18mm.



Figure 8: The relation between  $L_u + L_l$  and bearing losses

### 6. CONCLUSIONS

An analytical mode for mixed lubrication analysis in bearings of reciprocating compressor has been developed. By using this model, the losses was quantified, mechanism of loss generation was clarified, and means of loss reduction was examined in journal and thrust bearing. As a result, these conclusions are obtained.

- In our current reciprocating compressor, it is shown that the loss at thrust bearing is larger than that at journal bearing and to reduce the loss of thrust bearing is essential issue for making the compressor more efficient.
- It also shown that the optimal length of  $L_u + L_l$  exists, which can minimize total losses of bearings while keeping theoretical pressure ratio appropriately. By using the developed model, it is estimated that the optimal length of  $L_u + L_l$  would reduce total losses of bearings by 20%.
- The validity of the developed model was verified by experiments, because a compressor which was designed to reduce the length of  $L_u + L_l$  decreased its input power and the amount of the decrease is almost same as the loss reduction calculated by the model.

## REFERENCES

- Patir, N., Cheng, H. S., 1978, Application of Average Flow Model for Determining Effects of Three-Dimensional Roughness on Partial Hydrodynamic Lubrication, *Trans.ASME, Journal of Lubrication Technology*, vol. 100, No. 1: p. 12-17.
- Patir, N., Cheng, H. S., 1979, Application of Average Flow Model to Lubrication between Rough Sliding Surfaces, *Trans.ASME, Journal of Lubrication Technology*, vol. 101, No. 4: p. 220-230.
- Kawabata, N., 1987, A Study on the Numerical Analysis of Fluid Film Lubrication by the Boundary-fitted Coordinates System : 1st Report, Fundamental Equations of DF Method and the Case of Incompressible Lubrication, *Transactions of the Japan Society of Mechanical Engineers*. C, vol. 53, No. 494: p. 2155-2160.
- Greenwood, J.A., Tripp, J. H., 1970, The Contact of Two Nominally Flat Rough Surfaces, *Proceedings of the institution of mechanical engineers*, vol. 185, No. 48: p. 625-633.
- Patir, N., Cheng, H. S., 1978, Effect of Surface Roughness Orientation on the Central Film Thickness in E.H.D. Contacts, *Proceedings of the fifth Leeds–Lyon symposium on Tribology*: p. 15-21.