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ABSTRACT

This paper deals with the roots compressor, which is a rotary machine used for compressing air or refrigerant. It proposes an original way of describing the geometry of the three-lobe compressor in which the symmetries are exploited in order to establish the analytical expressions of volumes and leakages as a function of the orbiting angle. This approach allows the volumes and leakages to be described analytically without any special assumption. Based on the geometrical model, a thermodynamic model is built to predict pressures, temperatures of the different compressor chambers and massflows through the leakages between the chambers and at the compressor outlet and inlet. The simulation and experimentation results show that the model is able to predict these state variables but also the mechanical power and torque on the compressor shaft.

1. INTRODUCTION

This paper present a geometrical and thermodynamic model of the compressor three-lobe. The aim of the paper is to propose a new approach to describe the geometry of the three-lobe compressor where the expressions of the control volumes and leakage areas variations are described analytically as a function of the orbiting angle θ. This methodology has already been applied for the Scroll compressor and gave very good results (Blunier, et al., 2009).

A thermodynamic model of the control volumes and leakage areas is coupled to the geometrical one to predict the different state variables (*i.e.*, pressures, temperatures, massflows, power, torque). The control volumes and the leakage areas components are defined to implement the thermodynamic model in a multi-domains simulation language known as VHDL-AMS described in section 4. The section 5 describes in details the test bench and the experimental procedure for the model validation. Finally, the simulation results of the compressor model are presented together with experimental results. The comparison of several simulation and experimental maps obtained shows good agreements between simulations and experimentations. This study will permit the modeling of the air supply system for fuel cell (PEMFC), (Blunier, et al., 2010) to be done and also the control strategy to be built to satisfy the fuel cell systems requirements .

2. GEOMETRICAL MODEL OF THE THREE-LOBE COMPRESSOR

2.1 Description of the chambers

The aim of the geometrical description of the three-lobe compressor is to compute for any rotation angle θ, the control volumes and the leakage areas between these control volumes:

- The analytical expression of the control volumes: $V = f(\theta)$;
- The analytical expression of the leakage area: $S = f(\theta)$;

Where θ is the orbiting angle of the rotor. The geometrical model will be coupled together with a thermodynamic model (see section 3) to compute the pressure, the temperature inside the control volumes and massflows through the leakages.

For the three-lobe compressor four variables are used to describe the compressor geometry namely (see Figure 1):

- The radius of the lobe *r*;
- The diameter of suction and discharge sections d,
- The orbiting angle θ,

- The compressor height h.

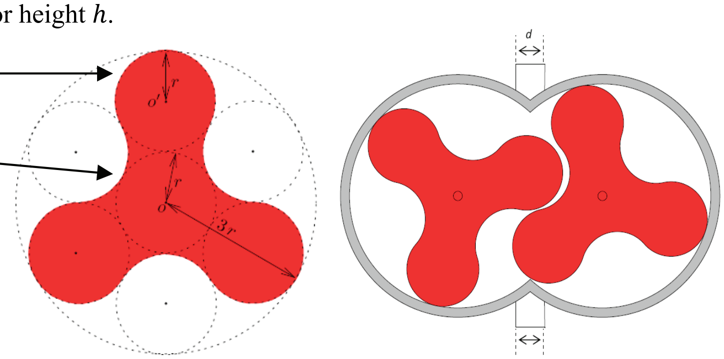

Figure 1 Schematic view of three-lobe compressor.

Several models of volumetric compressors are based on the geometrical description (Blunier et *al*., 2006) (Halm, 1997) which determines, according to the orbiting angle, the volumes and the leakages of the chambers.
The geometry, coupled with a thermodynamic model, allows the pressure, the temperature in each control volume of the compressor and the mass flow through the leakages to be computed.
A chamber is defined as a portion of the compressor included between two rotors or between the rotor and the compressor casing.
It is important to know that only the *chambers* are described in this section. The *control volumes,* based on the chambers description and which will be used to compute the state variables, will be presented later.
As it can be seen on Figure 2, each chamber has three steps:

- The air suction (*i.e.*, an increasing chamber volume at the compressor inlet);
- The air transfer (*i.e.*, constant chamber volume),
- The air discharge (*i.e.*, an decreasing chamber volume at the compressor outlet).

The topology of the compressor allows to define six chambers instantaneously, which will be called respectively Ch_1, Ch_2, Ch_3, Ch_4, Ch_5, Ch_6. For example the evolution process of the chamber four Ch_4 is presented on Figure 2. All other chambers can be described according the same procedure.
The analytical expression of the volume is computed using the symmetry of the lobe. One rotor is built from six concave parts and six convex parts as shown in Figure 1. For each orbiting angle, the volume of air between the casing and one convex part, and the volume of air between the casing and one concave. Based on this procedure, the chamber volumes can be computed analytically as a function of the orbiting angle. The evolution of the chambers volumes as a function of the orbiting angle is given in Figure 3. The complete mathematical model can be found in (M'boua, et al., 2010 (submitted))

2.2 Description of the control volumes and the compression process

During a cycle, the orbiting angle θ (describing the position of two rotors) varies from 0 to 2π; it can be considered that exactly 6 *control volumes* (different from *chambers volumes*) exists (See Figure 4):

- One volume describing the air suction (V_2),
- One volume describing the air discharge (V_1),
- Two volumes describing the air transfer and discharge (V_4 and V_5),
- Two volumes describing the air suction and transfer (V_3 and V_6).

The number of control volumes is considered to be fixed and equal to exactly six during all the process, even if the pressures of the neighboring chambers are identical (in this case the two volumes could be considered as one single control volume). The compression process is considered as periodic, with a period of $2\pi/3$. One period can be described by the following steps:

1. In the first half period:
    a. At the beginning of the process, that is $\theta = 0$, the leakage $S_{1\text{-}3}$ between the control volumes V_1 and V_3 is at its maximal value, it is assumed that the pressure in these two volumes are identical, at this time the signal S_L (L stands for left part of the compressor) (see Figure **5**) is activated.
    b. When the signal S_L is activated, the control volume V_3 which is in a discharge process initializes its state variables (P_3; V_3; T_3) with those of the volume V_1, which is instantaneously connected to discharge pipe, and it initializes with those of the volume V_4. The volume V_4 initializes its state variables with the variables (P_2; V_2; T_2) of volume V_2 connected with suction pipe.

c. The control volume V_1 and V_2 are now the discharge and suction control volumes, respectively. The control volume V_3 starts the transfer process followed by the discharge process until the end of the first half period. The control volumes V_5 and V_6 which are respectively in the transfer and transfer-discharge process do not initialize during this period.

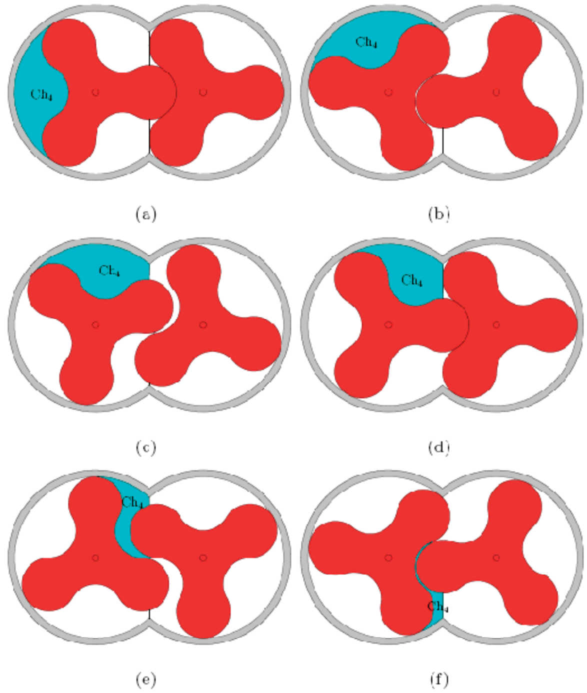

Figure 2: Evolution steps of the various chambers

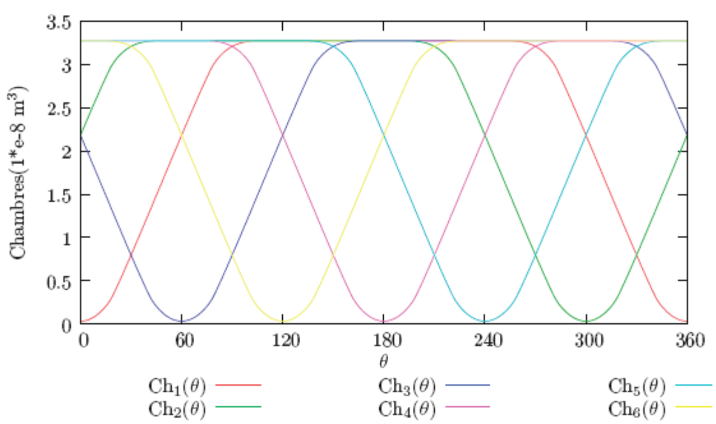

Figure 3: Chambers volumes as a function of the orbiting angle

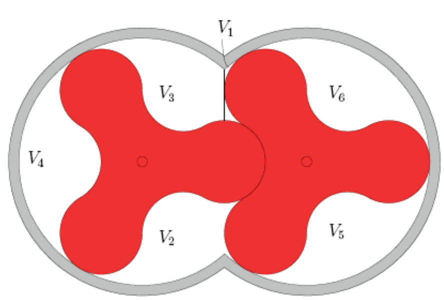

Figure 4: Presentation of control volumes

2. In the second half period:
    a. At this time $\theta = \pi/3$, the signal "S_R" is actived (R stands for left part of the compressor), the leakage $S_{1\text{-}6}$ is at its maximal value. The control volumes V_5 and V_6 have the same kind reinitialization that V_3 and V_4 in the first half period. The volume V_1 and V_2 are still in the suction and discharge process, respectively.

This representation will allow thereafter the dynamic behavior of the compressor to be simulated with a simulation language. As it has been previously explained, the topology of the circuit is fixed, that is the number of components and their interconnections do not change during the simulation. The compressor is represented by means of two elementary components (see Figure **5**):

- The control volumes represented by "rectangles" where the number of fluidic terminals can vary depending on their number of interconnection;
- The leakages are assumed to be isentropic. They are represented by two opposed arcs, with two fluidic and two thermal terminals.

The number of fluidic terminals can vary depending of their number of interconnection.
The leakages are assumed to be isentropic. Each control volume has two other signal inputs "left" and "right" which permit the variables P and T of each control volume to be initialized during the simulation.

2.3 Model of the leakages

In a real compressor the pressures in different control volumes differ from the ideal case because the controls volumes are not completely hermetic. The leakages, due to internal clearances, increase the energy consumption of the compressor. Indeed, the gas which flows in the volume where the pressure is the highest through a volume where pressure is lower will have to be compressed again.
According to the model, three kinds of leakage exist (See Figure **6**): (1) *Tangential leakages:* the gas flows through the clearance $\delta_T$which exists between the lobe and the casing because the contact point is not perfect; (2) *Leakages between two control volumes:* the leakage between the control volume "x" and the control volume "y" will be written S_{x-y}; (3) *Radial leakages:* the gas flows through the clearance δ_R between the joint and along the surface of the compressor rotor.

3. THERMODYNAMICAL MODEL

3.1 Model of control volume component

The model of a control volume is based on energy and mass conservation. If the kinetic energy is neglected, the internal energy in the control volume can be written, (Blunier et al., 2009):

$$U = Mc_vT$$

Where M is the mass of the gas, c_v its specific heat at constant volume and T, the temperature of the gas. The internal energy is changed due to (Gravensen, et al., 2001):

- The work done by changes in the volumes: $-p\ dv$
- The energy transported into and out of the chamber through its leakage areas. If it is assumed that the flow through the leakages is adiabatic, the energy inside the flow is qc_pT where c_pT represents the enthalpy, q the mass flow where c_p is the specific heat of the gas at constant pressure.
- Dissipation and energy losses due to the external environment, dQ.

The equation linking the pressure and the temperature of the control volume can be obtained from the following equation:

$$\frac{dT}{dt} = \frac{T}{p}\frac{dp}{dt} + \frac{T}{v} + \gamma p \frac{dv}{d\theta}\Omega - \frac{rT^2}{pv}\sum_{i=1}^{n} q_i$$

3.2 Model of the leakage component

The nozzle flow equation (isentropic assumption) is used to calculate the leakages between the control volumes. It has to be noted that the suction and discharge processes are considered as leakages in the model. The mass flow through a leakage area is (Yanagisawa, et al., 1990):

$$q = \begin{cases} C_d\ P\ S_{\text{leakage}} \sqrt{\dfrac{2kM_{\text{air}}}{(k-1)\ R\ T}\left[\pi^{\frac{2}{k}} - \pi^{\frac{k+1}{k}}\right]} & \text{if} \quad \pi > \pi_{\text{crit}} \\ C_d\ P\ S_{\text{leakage}} \sqrt{\dfrac{k}{R\ T}\left(\dfrac{2}{k+1}\right)^{\frac{k+1}{k-1}}} & \text{otherwise} \end{cases}$$

Where C_d is the flow coefficient, S_{leakage} the leakage area, π the pressure ratio. $\pi_{\text{crit}} = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}$, and P is the upstream pressure.

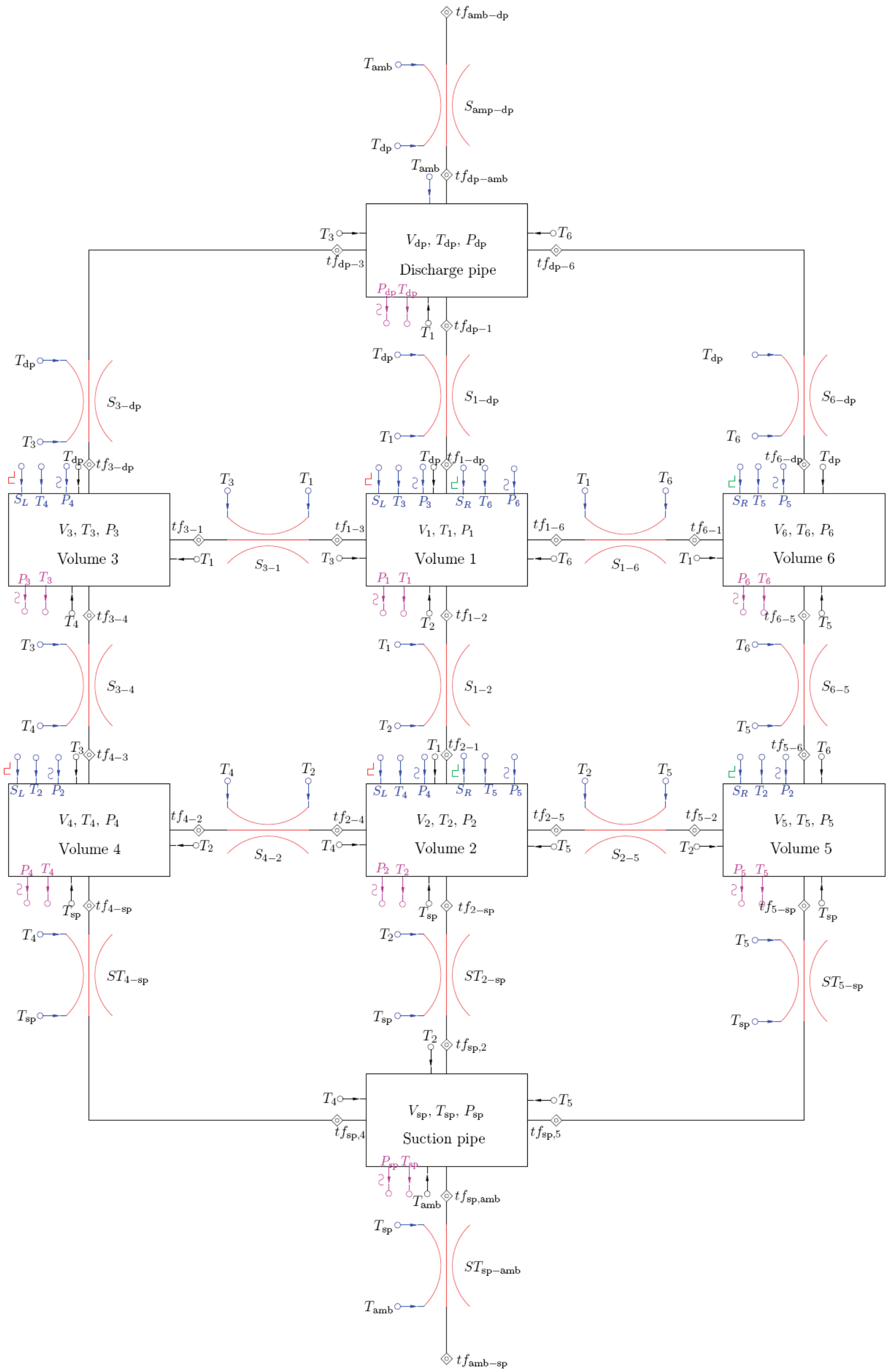

Figure 5: Schematic representation of the control volumes and fluidic terminals

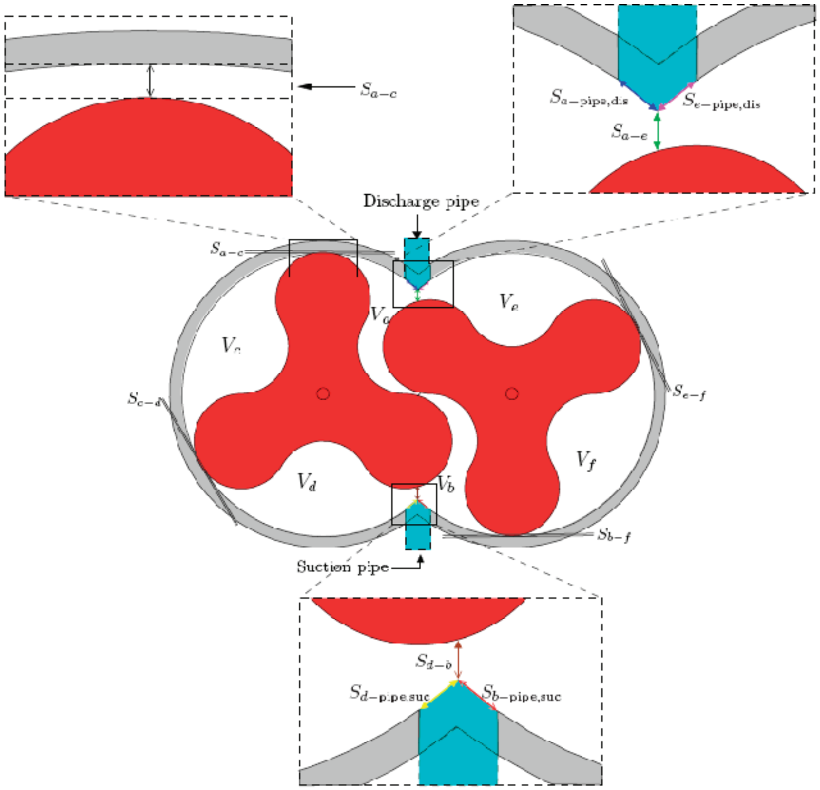

Figure 6 Different kinds of leakage existing in the model

3.3 Model of the heat exchange with the environment

The evolution of the thermodynamic states of the gas through the compressor can be divided into three steps: Isobaric heating up in the suction pipe, isentropic compression, and isobaric cooling down in the discharge pipe.
The shaft power P_{shaft} can be obtained using the power conservation equation:

$$P_{\text{shaft}} = P_{\text{isentropic}} + P_{\text{mecha}} + P_{\text{heat,leakage}}$$

Where $P_{\text{isentropic}}$ is the isentropic power, P_{mecha} is the mechanical losses of the compressor and $P_{\text{heat,leakage}}$, leakage represents the losses due to the air leakage and heat transfer between the air and the compressor walls.
The mechanical losses P_{mecha} can be defined from two terms, losses proportional to the isentropic compression power and hydrodynamic mechanical losses as shown in the following formula (Winandy, et al., 2002):

$$P_{\text{mecha}} = \alpha_0 P_{\text{isentropic}} + P_{\text{mecha,0}} \left(\frac{\Omega}{\Omega_0}\right)^2$$

Where $P_{\text{mecha,0}}$ and α_0 are positive parameters to be identified; Ω_0 is a reference (arbitrary) rotational speed.
In general, the different heat transfers in a compressor include the one of the suction gas $\dot{Q}_{\text{suc}}$, the heating-up due to mechanical losses P_{mecha}, the heat given by the high temperature discharge gas $\dot{Q}_{\text{dis}}$, and the heat flow to the environment $\dot{Q}_{\text{amb}}$. It is assumed that the definition of a fictitious wall uniform temperature T_W is sufficient to represent all the heat transfer modes mentioned above. The fictitious wall temperature T_W, has no influence on the mass flow rate so that its value can be assumed equal to the mean of compressor surface temperature during all test 47 °C. The equations for the fictitious supply heat exchanger are:

$$\dot{Q}_{\text{suc}} = q.\varepsilon_{\text{suc}} c_p (T_{\text{suc}} - T_W)$$
$$\dot{Q}_{\text{dis}} = q.\varepsilon_{\text{dis}} c_p (T_{\text{dis}} - T_W)$$

Where,

$$\varepsilon_{\text{dis}} = \varepsilon_{\text{suc}} = 1 - e^{-\frac{AU_{\text{suc}}}{q.c_p}} = 1 - e^{-\frac{AU_{\text{dis}}}{q.c_p}}$$

The parameters AU_{suc} and AU_{dis} may be considered equals for similar gas from the point of view of thermo-physical properties (Winandy, et al., 2002).The ambient losses are given by:

$$\dot{Q}_{amb} = AU_{amb}(T_{amb} - T_W)$$

The experimental test results are used to identify the parameters related to the heat power losses and transfers. (Duprez, et al., 2007).

4. SIMULATION AND EXPERIMENTATION

The compressor model has been implemented in simulation software which supports the VHDL-AMS simulation language. The IEEE 1076.1 language, informally known as VHDL-AMS, is a superset of IEEE Std 1076-1993 (VHDL) that provides capabilities for describing and simulating continuous time and mixed-signal systems with conservative and non conservative semantics for the continuous time portion of the system. The language supports many abstraction levels in electrical and nonelectrical energy domains.
The test bench has been used to validate the scroll compressor model presented in (Blunier, et al., 2009). It has been adapted to validate the three-lobes compressor (see Figure **7**). The test bench has been automated with a Dspace system using Matlab/Simulink and a Python script. The tests have been done for several compressor speeds and valve positions, recording for each operating point in steady state, the average pipe pressure and temperature, the mass flow and the compressor mechanical (torque en speed) and electrical powers.

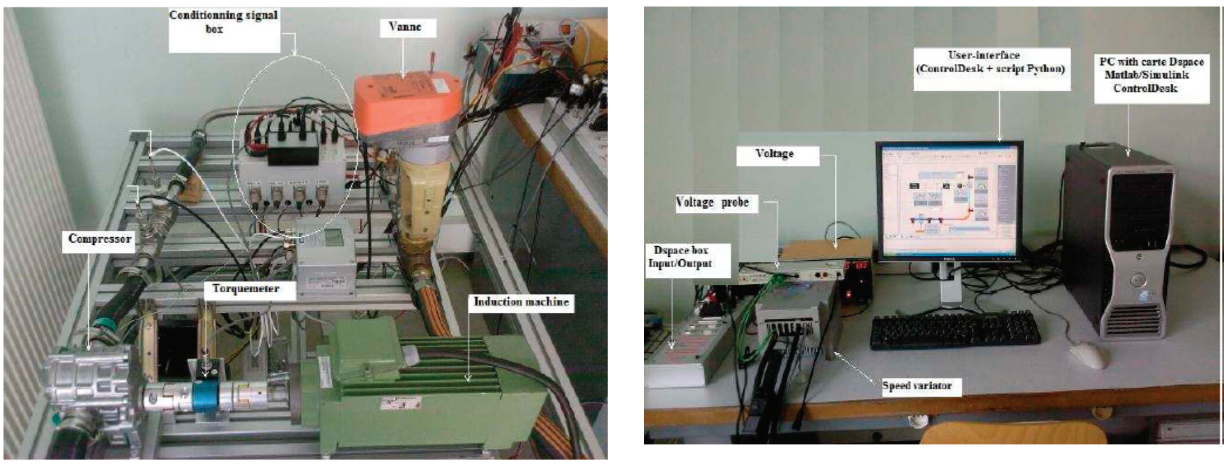

Figure 7 Photo of test bench

5. RESULTS

Figure 9 and Figure 10 show respectively the comparison between the experimental and the simulated mass flows as a function of the rotational speed for all operating points (pressures) and isentropic efficiency with speed. The simulation predicts relatively well the mass flow up to a speed of 13,000 rpm. It can see that it is possible to predict the behavior of the compressor by using the model. The model demonstrated thus good agreements with experimental profiles.

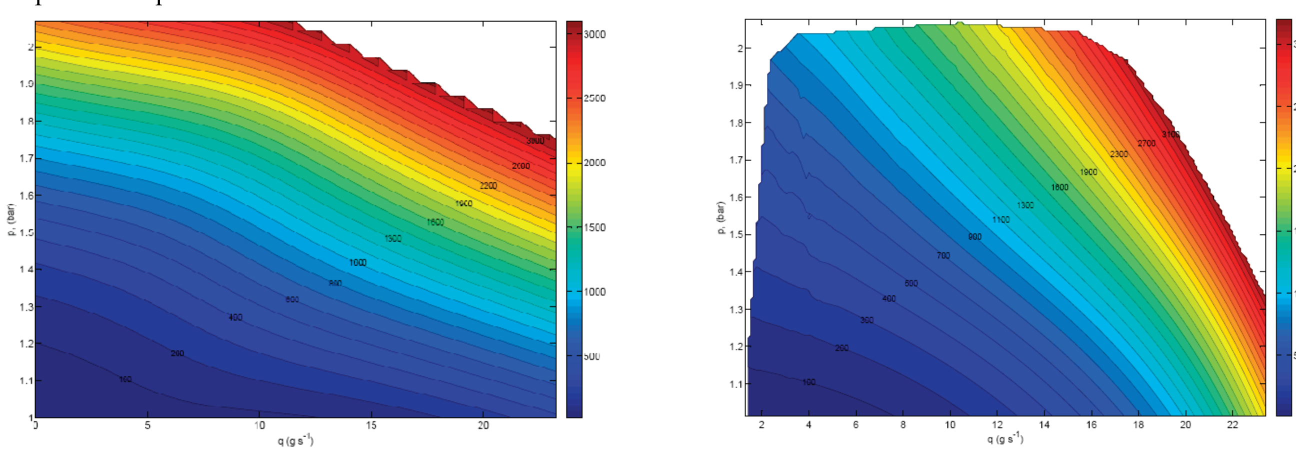

Figure 8 Experiment and Simulated iso-power maps

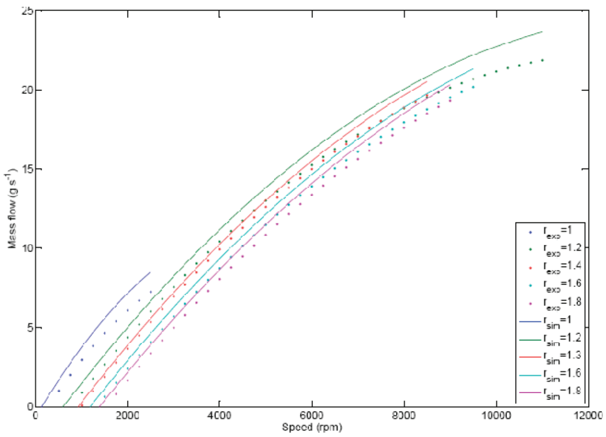

Figure 9 Comparison between experimental and simulated mass flow

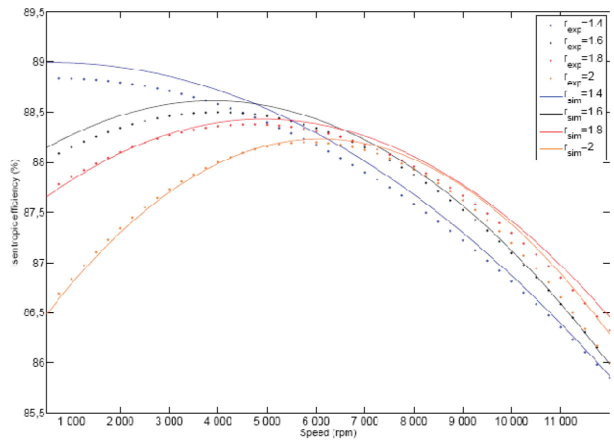

Figure 10 Comparison between experimental and simulated variation of isentropic efficiency with speed.

6. CONCLUSION

The model makes possible the investigation of the influence of leakages and compressor geometrical parameters on the compressor performance and can be used to improve the compressor's design. The comparison between the simulation and the experimentation maps shows good agreements. The analysis of the simulation (or experimentation) results allows the design of an adapted motor to be done. The motor will be coupled to the compressor together with its associated power converter electronic systems.

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