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Abstract
This paper presents a saturated proportional controller that achieves depollution of wastewater in a continuous anaerobic digester. This goal is reached by defining a region of the state-space where the depollution is achieved and forcing attractivity and invariance of this region. The control variable is the dilution rate and the controlled variable is a linear combination (S_λ) of the substrates concentrations, that could be the Chemical Oxygen Demand (COD) or the Biological Oxygen Demand (BOD), depending on the value of λ. No measurement of the substrates concentrations in the input flow is required; the only necessary measurement is S_λ.

Keywords
Anaerobic digestion, bioreactor, COD, control, saturation.

INTRODUCTION

The control of bioreactors is a delicate problem since most of the time the available biological models are only rough approximations, the biological systems being known to be highly variable and difficult to measure. To circumvent this difficulty, Bastin and Dochain (1990) have introduced the mass balance based modelling. The main idea of this approach is to design estimators and controllers independently of any modelling of the biological kinetics.

Among the bioreactors, those dedicated to wastewater treatment especially suffer from the modelling uncertainties. A complex ecosystem composed by many different bacterial populations takes place in these processes, and the composition and concentration of the pollutant to degrade is not well known and evolves with respect to time. Moreover, most of the time no measurement of the involved chemical or biological species is available; this can be critical when the bioreactor is unstable as is the case for the anaerobic digestor. In these conditions, a control procedure that would guarantee the process stability should be as insensitive as possible to all these parameters.

In this paper, we will consider an anaerobic wastewater treatment process, that is a biological process in which biodegradable organic materials are decomposed in the absence of oxygen to produce methane. The underlying model assumes that two main bacterial populations are present (Bernard et al. 2001). The first one, the acidogenic bacteria X_1, consumes the organic substrate S_1 and produces through an acidogenesis step volatile fatty acids (VFA) S_2. The second population (methanogenic bacteria) X_2, uses the VFA in a methanogenesis step as substrate for growth and produces methane.

Despite its capacity to degrade difficult substrates, this process is known to become unstable under certain circumstances, like variations of the process operating conditions, and requires therefore a monitoring procedure to detect a destabilization. This must also be associated to a control action that can avoid the risk of acidification of the fermenter. Therefore, some control laws have recently been introduced for this process like the adaptive feedback of the gaseous flow-rate measurement (Perrier and Dochain 1993, Mailleret et al. 2003, Mailleret et al. 2004) or fuzzy control of the VFA concentration (Genovesi et al. 1999, Punal et al. 2000) to avoid acidification of the reactor. The controller that we have designed regulates a linear combination of the substrates concentrations, that we will denote

S_λ; depending on the value of the parameter λ, S_λ can represent the Biological Oxygen Demand (BOD) or the Chemical Oxygen Demand (COD), that is the standard measurement of the pollution level. Our controller requires the measurement, or the observation through software sensors (through the application of techniques similar to (Alcaraz-Gonzalez et al. 2002)), of S_λ and has a very simple structure that takes actuator limitations into account (as is also done in Antonelli et al. (2003)); it has the advantage of not requiring any measurement of the substrates concentrations in the input flow. The variable that is used for control is the dilution rate (D).

MODEL OF ANAEROBIC DIGESTION

In this paper, we will use the model AM1 of anaerobic digestion that was presented in (Bernard et al. 2001):

$$\begin{cases} \dot{X}_1 &= (\mu_1(S_1) - \alpha D)X_1 \\ \dot{X}_2 &= (\mu_2(S_2) - \alpha D)X_2 \\ \dot{S}_1 &= D(S_{1in} - S_1) - k_1\mu_1(S_1)X_1 \\ \dot{S}_2 &= D(S_{2in} - S_2) + k_2\mu_1(S_1)X_1 - k_3\mu_2(S_2)X_2 \end{cases} \tag{1}$$

with $X_1,\ X_2,\ S_1,\ S_2,\ D \in I\!R^+$, $\mu_1(S_1)$ a non-decreasing and bounded function such that

$$\mu_1(0) = 0 \text{ and } \mu_1(S_1) < \mu_{1max}\,\forall\, S_1 \geq 0$$

and $\mu_2(S_2)$ a function such that

$$\mu_2(0) = 0 \text{ and } \mu_2(S_2) \leq \mu_{2max} = \mu_2(S_2^*)\,\forall\, S_2 \geq 0$$

and $\mu_2(S_2)$ is non-decreasing from $S_2 = 0$ to $S_2 = S_2^*$ and non-increasing afterwards with

$$\lim_{S_2 \to +\infty} \mu_2(S_2) = 0$$

Classically, μ_1 is of the Monod type and μ_2 of the Haldane type. The terms S_{1in} and S_{2in} are the influent concentrations of S_1 and S_2 respectively. The k_i represent the yield coefficients associated with bacterial growth. The parameter $\alpha \in [0, 1]$ represents the proportion of bacteria that are not fixed on the bed, and therefore that are affected by the dilution effect: $\alpha = 0$ would correspond to an ideal fixed bed reactor, $\alpha = 1$ to an ideal continuous stirred tank reactor. This model has been built and validated with the spirit of finding a trade-off between model complexity and mathematical handling of the model for control purpose. It is not intended at giving an accurate view of all the phenomena that take place in the reactor as higher-dimensional models do (e.g.the IWA Anaerobic Digestion Model No.1 (Batstone et al. 2002)).

OBJECTIVE AND CONSTRAINTS

The original control objective for depollution is to regulate the output $S_\lambda = S_1 + \lambda S_2$ (with $\lambda \geq 0$ not always equal to 1 because S_1 and S_2 do not need to be expressed in the same units), which, depending on the chosen value for λ, can be the COD or BOD. The target value for S_λ is some $\bar{S}_\lambda \leq S_{\lambda max} \leq S_{\lambda in} = S_{1in} + \lambda S_{2in}$. In this paper, the objective is modified as follows

Objective 1 *Given $S_{\lambda min} \leq \bar{S}_\lambda \leq S_{\lambda max}$, steer all the solutions of the controlled system to a region where $S_{\lambda min} \leq S_\lambda \leq S_{\lambda max}$ is satisfied and stays valid for all future times*

Instead of achieving regulation, we will achieve attractivity and invariance of a security zone. In this formulation, $S_{\lambda max}$ is an unalterable data of the problem (fixed by depollution norms); on the other hand, $S_{\lambda min}$ can be chosen more freely: if it is taken close to $S_{\lambda max}$, the achievement of Objective 1 is almost equivalent to the regulation of the output S_λ; if $S_{\lambda min}$ is taken close to zero, there is a risk that the system settles at a small value of S_λ with a small value of the dilution rate. The pollutant concentrations in the input, S_{1in} and S_{2in}, are supposed to be constant. They do not need to be known for the application of the controller. However, in order to show stability of the controller, those values need to be known.

In order to design a controller, we first analyze the different parameters associated to the control objective. In the sequel, we will show that the following assumption needs to be imposed.

Assumption 1 *The parameters satisfy the following three inequalities*

$$\lambda < \frac{k_1}{k_2} \tag{2}$$

$$S_{\lambda max} < \min(S_{1in}, \lambda T_{2in}) = \min(S_{1in}, \frac{\lambda k_2}{k_1} S_{1in} + \lambda S_{2in}) \tag{3}$$

$$D_{max} < \frac{\min(\mu_1(S_{1in}), \mu_2(S_{2in}), \mu_2(T_{2in}))}{\alpha} \tag{4}$$

The parameter λ
The evolution of the pollution level follows the following equation:

$$\dot{S}_\lambda = D(S_{\lambda in} - S_\lambda) - (k_1 - \lambda k_2)\mu_1(S_1)X_1 - \lambda k_3 \mu_2(S_2)X_2 \tag{5}$$

Condition (2) imposes that the pollution level decreases when the flow rate is stopped (which is the intuitive behavior of a digester). This condition is met by the identified parameters of the experimental process (Bernard et al. 2001) when S_λ is the COD ($\lambda = 0.064\, g/mmol$ and $\frac{k_1}{k_2} = 0.368\, g/mmol$).

The bound $S_{\lambda max}$
In the rest of this section, we will replace S_2 with the coordinate $T_2 = S_2 + \frac{k_2}{k_1} S_1$. This results in the following system:

$$\left\{ \begin{array}{lcl} \dot{X}_1 & = & (\mu_1(S_1) - \alpha D)X_1 \\ \dot{X}_2 & = & (\mu_2(T_2 - \frac{k_2}{k_1}S_1) - \alpha D)X_2 \\ \dot{S}_1 & = & D(S_{1in} - S_1) - k_1\mu_1(S_1)X_1 \\ \dot{T}_2 & = & D(T_{2in} - T_2) - k_3\mu_2(T_2 - \frac{k_2}{k_1}S_1)X_2 \end{array} \right. \tag{6}$$

considered in the positively invariant set $\{(X_1, X_2, S_1, T_2) \in I\!R^4_+ | T_2 \geq \frac{k_2}{k_1} S_1\}$.

In these new variables, the measure S_λ is rewritten as $S_\lambda = S_1 + \lambda S_2 = (1 - \lambda\frac{k_2}{k_1})S_1 + \lambda T_2$. We will now impose a condition that we will call “regulability”: this condition makes sure that, whatever the level $\bar{S}_\lambda \leq S_{\lambda max}$ that is regulated, there corresponds a non trivial equilibrium for system (6). If S_λ is set at some prespecified value $\bar{S}_\lambda$, there should exist a constant dilution $\bar{D} > 0$ corresponding to an equilibrium. From the $\dot{X}_i = 0$ equations, we see that such an equilibrium should satisfy:

$$\mu_1(\bar{S}_1) = \mu_2(\frac{\bar{S}_\lambda - \bar{S}_1}{\lambda}) > 0$$

This potentially results in several values of $\bar{S}_1 > 0$ for our equilibrium, and corresponding values of $\bar{D}$. Introducing this into the $\dot{S}_1 = \dot{T}_2 = 0$ equations, we obtain

$$\begin{array}{lcl} 0 & = & (S_{1in} - \bar{S}_1) - k_1\alpha\bar{X}_1 \\ 0 & = & (T_{2in} - \bar{T}_2) - k_3\alpha\bar{X}_2 \end{array}$$

Isolating $\bar{X}_1$ and $\bar{X}_2$, we get $\bar{X}_1 = \frac{S_{1in}-\bar{S}_1}{k_1\alpha}$ and $\bar{X}_2 = \frac{T_{2in}-\bar{T}_2}{k_3\alpha}$. At the equilibrium, $\bar{X}_1$ and $\bar{X}_2$ should be positive. Noticing that $\bar{S}_1 \leq \bar{S}_\lambda < S_{\lambda max}$ and $\lambda\bar{T}_2 \leq \bar{S}_\lambda < S_{\lambda max}$, it suffices to impose (3) to have $\bar{X}_1$ and $\bar{X}_2$ positive at any equilibrium having $\bar{S}_\lambda < S_{\lambda max}$. This assumption also forces $S_{\lambda max} < S_{\lambda in}$; it is reasonable, as we want to bring pollution to a lower level than its influent value.

Bounded control
The control variable is the dilution rate, so that it must be non-negative, and it cannot be arbitrarily high. There is an a priori upper-bound on the maximal flow-rate D_{max} due to the physical constraint associated to the pumping mechanism. This bound can be seen as a given data, but it can also be seen as a design parameter (a different choice of input valve can give a different value of upper-bound for D_{max}). On the other hand, the minimal value of the flow-rate is, theoretically, zero; however, in the industrial environment, the output of the industrial plant that produces the waste cannot be totally stopped, it is lower-bounded by some $D_{min} > 0$. We will design a controller that satisfies both these bounds. Moreover, equation (4) is imposed to avoid a wash-out of the bacteria of the reactor (we do not prove this property due to space limitation).

Bounded state
Based on that assumption and for D_{min} and D_{max} fixed, it can be shown that the solutions are bounded: there exist $S_{1min}, T_{2min} > 0$ such that, for any controller $D_{min} \leq D(X_1, S_1, X_2, S_2) \leq D_{max}$ and for any initial condition in the positive orthant $(X_1(0), S_1(0), X_2(0), S_2(0)) \in I\!R_+^4$, there exists a finite time $T > 0$ after which the following four inequalities are valid for all $t \geq T$:

$$S_{1in} < k_1X_1(t) + S_1(t) < \frac{S_{1in}}{\alpha},\ T_{2in} < k_3X_2(t) + T_2(t) < \frac{T_{2in}}{\alpha} \tag{7}$$

$$S_{1min} < S_1(t) < S_{1in},\ T_{2min} < T_2(t) < T_{2in} \tag{8}$$

These inequalities are not proven here due to space limitation: they are a consequence of the differentiation of the quantities $k_1X_1 + S_1$, $k_3X_2 + T_2$, S_1 and T_2 and result in the following lemma

Lemma 1 Let $0 < D_{min} < D_{max}$ be fixed. Then, for any initial condition $(X_1(0), X_2(0), S_1(0), S_2(0))$ belonging to $I\!R_+^4$, and for given constants S_{1in}, S_{2in} such that Assumption 1 is satisfied, there exists a time $T > 0$ such that, for all $t \geq T$, we have

$$\begin{array}{lllll} X_{1min} & < & X_1(t) & < & \frac{S_{1in}}{k_1\alpha} \\ 0 & < & X_2(t) & < & \frac{T_{2in}}{k_3\alpha} \\ S_{1min} & < & S_1(t) & < & S_{1in} \\ T_{2min} & < & T_2(t) & < & T_{2in} \end{array}$$

along the solution of (1) for any choice of $D(t) \in [D_{min}, D_{max}]$.

CONTROL DESIGN
We choose a simple proportional controller in the form

$$D = \frac{D_{max} - D_{min}}{2}\left(1 + \text{ sat}\left(\frac{S_{\lambda max} + S_{\lambda min} - 2S_\lambda}{S_{\lambda max} - S_{\lambda min}}\right)\right) + D_{min} \tag{9}$$

where $\text{sat}(s) = \frac{s}{\max(|s|,1)}$ (the controller is illustrated on Figure 1). As stated in Objective 1, this controller is not designed to regulate S_λ at a prespecified value $\bar{S}_\lambda$, but rather to ensure attractivity and invariance of the region of the state space where S_λ belongs to an interval $[S_{\lambda min}, S_{\lambda max}]$. Such a controller should be more robust than a controller aimed at exactly regulating the output. The main

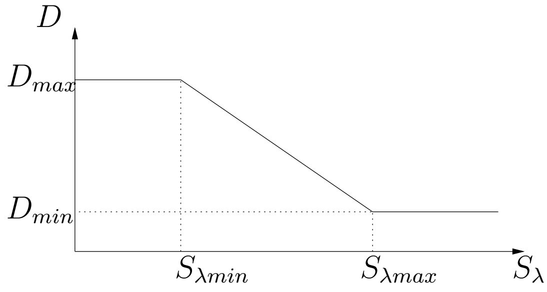

Figure 1: Form of the controller (9)

tuning parameters of this controller are the constants D_{max} and $S_{\lambda min}$ (though D_{max} might not be picked arbitrarily large in the actual plant due to physical constraints).

This controller is based on the following philosophy:

(i) if $S_\lambda \geq S_{\lambda max}$ then the flow is minimal: it prevents the pollution from leaving the plant in too large an amount; the pollution is lowered inside the plant and the bacteria grow in order to face the higher depollution requirement;

(ii) if $S_\lambda \leq S_{\lambda min}$ then the flow is allowed to be maximal because the pollution level is low enough to be certain that this maximal flow will not drive the system into the region where the pollution is too high;

(iii) if $S_{\lambda min} < S_\lambda < S_{\lambda max}$ then the controller is linear and built such that it is continuous at the boundaries of this region.

The description of the controller as (i)-(ii)-(iii) allows for the separate description of the controlled system (1)-(9) in the three corresponding regions, that we will name Ω_1, Ω_2, and Ω_3, respectively:

Region Ω_1: $D = D_{min}$ The region Ω_1 is defined as

$$\Omega_1 = \{(X_1, X_2, S_1, S_2) \in (I\!R^+)^4 | S_1 + \lambda S_2 \geq S_{\lambda max}\}$$

In this region, where $S_\lambda \geq S_{\lambda max}$, the flow rate is rendered minimal to limit the outflow of pollutants. System (1) can be rewritten as

$$\left\{ \begin{array}{rcl} \dot{X}_1 & = & (\mu_1(S_1) - \alpha D_{min})X_1 \\ \dot{X}_2 & = & (\mu_2(S_2) - \alpha D_{min})X_2 \\ \dot{S}_1 & = & D_{min}(S_{1in} - S_1) - k_1\mu_1(S_1)X_1 \\ \dot{S}_2 & = & D_{min}(S_{2in} - S_2) + k_2\mu_1(S_1)X_1 - k_3\mu_2(S_2)X_2 \end{array} \right. \tag{10}$$

This system can be analyzed as a cascade system between the (X_1, S_1) subsystem and the (X_2, S_2) subsystem. For any constant $D_{min} < \frac{\mu_{1max}}{\alpha}$, the state of the (X_1, S_1) subsystem globally converges to the non-trivial equilibrium $(\bar{X}_1, \bar{S}_1) = \left(\frac{S_{1in} - \mu_1^{-1}(\alpha D_{min})}{k_1\alpha}, \mu_1^{-1}(\alpha D_{min})\right)$. Also, the smaller D_{min} is, the smaller $\bar{S}_1$ is. Because the solutions of the whole system are bounded, we know that the behavior of the whole system (10) can be deduced from the behavior of the (X_2, S_2) subsystem on the manifold $(X_1, S_1) = (\bar{X}_1, \bar{S}_1)$. This system is

$$\left\{ \begin{array}{rcl} \dot{X}_2 & = & (\mu_2(S_2) - \alpha D_{min})X_2 \\ \dot{S}_2 & = & D_{min}(\tilde{S}_{2in} - S_2) - k_3\mu_2(S_2)X_2 \end{array} \right. \tag{11}$$

Generically, this system has two non-trivial equilibria because μ_2 is similar to an Haldane function; the equilibria are characterized by the two values of S_2 that are such that $\mu_2(S_2) = \alpha D_{min}$ ($S_2^m < S_2^* < S_2^M$). It is straightforward to show that S_2^M is an unbounded increasing function D_{min}. Independently of the choice of D_{min}, Lemma 1 shows that $S_2 \leq T_2 \leq T_{2in}$ after a finite time. Also, if we take D_{min} small enough, we can have $S_2^M > T_{2in}$, so that no convergence to the equilibrium corresponding to $S_2 = S_2^M$ can take place and all solutions converge towards the equilibrium corresponding to $S_2 = S_2^m$. This equilibrium is characterized by $\bar{S}_\lambda = \bar{S}_1 + \lambda S_2^m = \mu_1^{-1}(\alpha D_{min}) + \lambda S_2^m$, which can be made as small as we want by reducing D_{min}. This ensures that system (10) has a single equilibrium, and that this equilibrium lies in the region where $S_\lambda < S_{\lambda max}$. We can show that this equilibrium is attractive for all initial conditions for system (10), so that we know that $S_\lambda = S_{\lambda max}$ is reached in finite time. We have then shown attractivity of $\Omega_2 \cup \Omega_3$ for D_{min} small enough. We now have to show invariance of this set. On its border, (5) becomes:

$$\dot{S}_\lambda = D_{min}(S_{\lambda in} - S_{\lambda max}) - (k_1 - \lambda k_2)\mu_1(S_1)X_1 - \lambda k_3 \mu_2(S_2)X_2$$

We can show that, in the region defined by the constraints (7)-(8), we have $(k_1 - \lambda k_2)\mu_1(S_1)X_1 + \lambda k_3 \mu_2(S_2)X_2 \geq M$ when $S_\lambda = S_{\lambda max}$ for some $M > 0$. This shows that, for $D_{min} > 0$ small enough $\dot{S}_\lambda < 0$ when $S_\lambda = S_{\lambda max}$. We then see that, as long as D_{min} is small enough, the region $\Omega_2 \cup \Omega_3$ is attractive and invariant. We then state the following assumption to deduce Lemma 2:

Assumption 2 *The minimal dilution rate $D_{min} > 0$ is taken small enough.*

where the exact extent of the "small enough" term is defined in the attractivity and invariance conditions stated before this assumption.

Lemma 2 Under Assumptions 1 and 2, there exists a finite time T after which the region $\Omega_2 \cup \Omega_3$ is attractive and invariant for system (1) with the controller (9).

This lemma ensures that the depollution objective is achieved by the controller; the pollution level will always be kept below $S_{\lambda max}$ once the controller has forced the system into that region. We will now study the behavior of the system in Ω_2 and check if Objective 1 is achieved.

Region Ω_2: $D = D_{max}$ The region Ω_2 is defined as

$$\Omega_2 = \{(X_1, X_2, S_1, S_2) \in (I\!R^+)^4 | S_1 + \lambda S_2 \leq S_{\lambda min}\}$$

In this region, where $S_\lambda \leq S_{\lambda min}$, system (1) can be rewritten as:

$$\left\{\begin{array}{rcl} \dot{X}_1 &=& (\mu_1(S_1) - \alpha D_{max})X_1 \\ \dot{X}_2 &=& (\mu_2(S_2) - \alpha D_{max})X_2 \\ \dot{S}_1 &=& D_{max}(S_{1in} - S_1) - k_1\mu_1(S_1)X_1 \\ \dot{S}_2 &=& D_{max}(S_{2in} - S_2) + k_2\mu_1(S_1)X_1 - k_3\mu_2(S_2)X_2 \end{array}\right. \tag{12}$$

In Ω_2, we only need to check the evolution of $S_\lambda(t)$, which follows the equation (5):

$$\begin{array}{rcl} \dot{S}_\lambda &=& D_{max}(S_{\lambda in} - S_\lambda) - (k_1 - \lambda k_2)\mu_1(S_1)X_1 - \lambda k_3\mu_2(S_2)X_2 \\ &\geq& D_{max}(S_{\lambda in} - S_{\lambda min}) - (k_1 - \lambda k_2)\mu_1(S_1)X_1 - \lambda k_3\mu_2(S_2)X_2 \end{array}$$

From Lemma 1, we know that there exists a finite time $T > 0$ after which $X_1(t) \leq \frac{S_{1in}}{k_1\alpha}$ and $X_2(t) \leq \frac{T_{2in}}{k_3\alpha}$. We then have

$$\dot{S}_\lambda \geq D_{max}(S_{\lambda in} - S_{\lambda min}) - (k_1 - \lambda k_2)\left[\max_{S_1 \leq S_{\lambda min}} \mu_1(S_1)\right]\frac{S_{1in}}{k_1\alpha} - \lambda k_3\left[\max_{S_2 \leq \frac{S_{\lambda min}}{\lambda}} \mu_2(S_2)\right]\frac{T_{2in}}{k_3\alpha}$$

for all $S_\lambda \leq S_{\lambda min}$. In order to have $\dot{S}_\lambda$ always positive, we impose the following assumption

Assumption 3 *Suppose that*

$$D_{max} > \frac{(k_1 - \lambda k_2)\mu_1(S_{\lambda min})\frac{S_{1in}}{k_1\alpha} + \lambda k_3 \left[\max_{S_2 \leq \frac{S_{\lambda min}}{\lambda}} \mu_2(S_2)\right] \frac{T_{2in}}{k_3\alpha}}{S_{\lambda in} - S_{\lambda min}} \tag{13}$$

As D_{max} is upper-bounded because of equation (4), this assumption can be satisfied by picking the free parameter $S_{\lambda min}$ small enough. From this expression, we deduce the following lemma:

Theorem 1 *Assumptions 1, 2, and 3 ensure that there exists a finite time after which Objective 1 is satisfied by system (1) with controller (9).*

This theorem is a consequence of the observations made prior to its statement, which show that all solutions have to leave Ω_2 after a finite time, and of Lemma 2 which shows the same thing for Ω_1. All solutions then converge to the invariant set Ω_3 inside which the depollution objective is achieved. Note that attractivity and invariance of the region of interest is not directly ensured: the solutions first have to converge to the region where (7)-(8) is satisfied (and we have shown that this takes place in finite time), and then we know that Ω_3 is attractive and invariant.

SIMULATIONS

We have implemented controller (9) on model (1). For the simulations, we have used the parameters of the model that were given in (Bernard et al. 2001). We then fixed the following "free" parameters as follows:

$$S_{\lambda max} = 1.5;\ S_{\lambda min} = 1.3;\ S_{1in} = 15;\ S_{2in} = 15;\ \lambda = 0.0064;\ D_{max} = 0.5;\ D_{min} = 0.05.$$

As can be seen from these parameters, the purpose of the control design is here to steer S_λ into the interval $[1.3, 1.5]$ with a control effort lying in the interval $[0.05, 0.5]$. We have considered

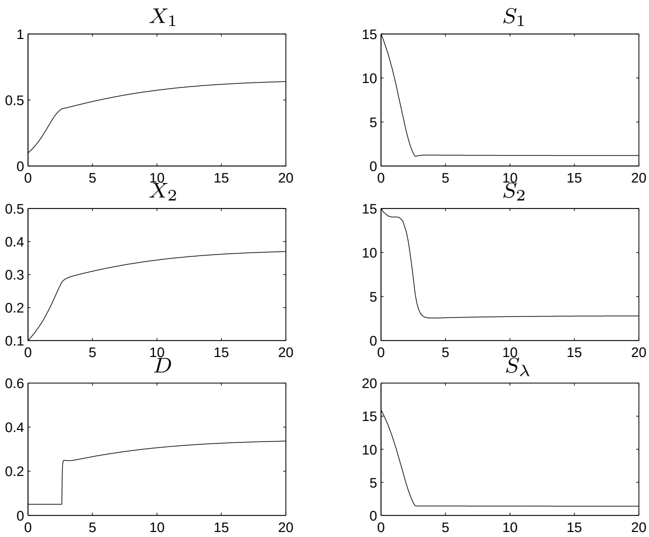

Figure 2: Time evolution of the states, control D, and output S_λ for the control system

$(S_1, S_2, X_1, X_2)(0) = (15, 15, 0.1, 0.1)$ as initial conditions. This set is characterized by a low biomass at the start and a high pollution level in the reactor ($S_\lambda(0) = 15.96$), coming from the large amount of S_1 in the reactor. The dilution rate is then set at the minimal level during the first two and half days. As can be seen on Figure 2, this forces a decrease of the pollution level S_λ, S_1 and S_2. Simultaneously, the biomasses X_1 and X_2 quickly increase. After 3 days, the pollution level settles at the desired value, between $S_{\lambda min}$ and $S_{\lambda max}$. However, it is interesting to notice that this does not mean that the solution has reached its equilibrium: between day 3 and day 20, we observe a continuing increase of X_1 and X_2, coupled with an increase of D; indeed, after three days, the reactor is able to treat the wastewater, but the dilution must stay moderate; the subsequent increase of biomass ensures that the plant can handle a higher dilution rate. After that, the equilibrium is reached.

CONCLUSION

In this paper, we have given a control law for the regulation of a model of anaerobic digestion with two bacteria. We have presented a control that regulates the pollution level: it ensures that the pollution level stays between a minimal and a maximal value while the dilution rate is also fixed between a minimal and a maximal value. No analysis of the actual behavior of the system inside the region where S_λ belongs to the desired interval has been presented here, but a condition can be given to ensure that the system has a single equilibrium.

Our controller requires that a measure of the pollution level is available online. If it is not the case, we will need to design an observer that will help reconstruct the value of S_λ from the available observations, namely the methane gaseous flow rate= $k_6\mu_2(S_2)X_2$, and some measures of S_λ (made with large time intervals in between them). No influent concentration knowledge is required.

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