CageIK: Dual-Laplacian Cage-Based Inverse Kinematics

Yann Savoye, Jean-Sébastien Franco

CageIK: Dual-Laplacian Cage-Based Inverse Kinematics

Yann Savoye and Jean-Sébastien Franco

LaBRI-INRIA Sud-Ouest, University of Bordeaux
{yann.savoye,jean-sebastien.franco}@inria.fr

Abstract. Cage-based deformation techniques are widely used to control the deformation of an enclosed fine-detail mesh. Achieving deformation based on vertex constraints has been extensively studied for the case of pure meshes, but few works specifically examine how such vertex constraints can be used to efficiently deform the template and estimate the corresponding cage pose. In this paper, we show that this can be achieved very efficiently with two contributions: (1) we provide a linear estimation framework for cage vertex coordinates; (2) the regularization of the deformation is expressed on the cage vertices rather than the enclosed mesh, yielding a computationally efficient solution which fully benefits from cage-based parameterizations. We demonstrate the practical use of this scheme for two applications: animation edition from sparse screen-space user-specified constraints, and automatic cage extraction from a sequence of meshes, for animation re-edition.

1 Introduction

Nowadays, mesh editing and animation techniques play an important role in Computer Graphics. This research domain has been intensively studied over the years. Nevertheless, the relentless increase in demand of industry has inspired researchers to exhibit new coordinate systems as well as new optimization frameworks. Building simple pipelines able to provide more flexible output for animation re-use is a challenging issue. Deformation techniques can be seen as an energy minimization process (defined locally or globally) that measures how much the object has been deformed from its initial pose given a support domain (for instance surface or volume). Approximating the global shape characteristics of the surface aims to produce specific surface resistance properties (like rigidity, flexibility or elasticity). One major challenge is to find a fast framework to achieve plausible boneless inverse kinematics that produce pleasing deformations and preserve the global appearance of the surface.

In this paper, we combine surface and volume deformation techniques. We focus on the estimation of desired enclosed models in a linear framework, which will allows artists to drag sparse surfel displacement constraints over the enclosed mesh surface itself or to fit a given cage across a mesh sequence. We explore a new approach, using a least-square cage as an intermediate and transparent tool, not directly edited by the user for the minimization process. The model is embedded in an adapted volumetric bounding cage using generalized barycentric coordinates having local properties. We take advantage of optimal reduced parameters offered by the given coarse cage surrounding the surface. To avoid artefacts induced by the large number of degrees of freedom, the cage layer is enhanced with laplacian regularization. The laplacian cage maintains a volumetric deformation of mesh vertices coordinates, more powerfully than applying separately surface-based or cage-based techniques.

The rest of the paper is organized as follows. After briefly reviewing some relevant works concerning surface and volume deformation and discussing in section 2, we give an overview of our system in section 3 and we present the key components of our method in section 4. Section 5 incorporates our novel deformation technique into our novel minimization framework to achieve cage estimation and extraction. We show the effectiveness of our method by both efficient applications in section 6. This paper is concluded and limitations are discussed in section 7.

2 Previous Work

In this section, we briefly overview the large body of relevant work on current techniques addressing the problem of interactive mesh deformation in recent years.

Intrinsic Surface Deformation Many efforts have been expanded on surface-based deformation. There are several types of approaches exploiting a differential descriptor of the edited surface in terms of laplacian and differential coordinates for mesh editing [1, 2]. Differential information as local intrinsic feature descriptors has been massively used for mesh editing in various frameworks over the decade. For instance, the proposed method in [3] allows the reconstruction of the edited surface by solving a linear system that satisfies the reconstruction of the local details in a least-squares sense.

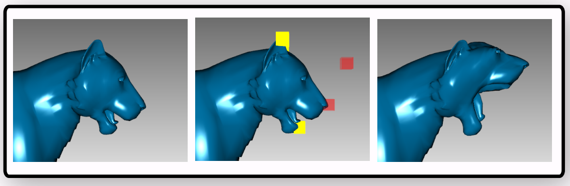

Fig. 1. Laplacian-based Deformation

Early approaches such as [4] motivated the use of Dual Laplacian system to reduce distortion in parametrization and geometry. Another mesh editing method working in the dual domain for regions of interest can be found in [5].

Dual laplacian are a class of approach where the laplacian is not directly expressed on mesh vertices. Unfortunately, laplacians cannot satisfy all natural properties and the differential coordinates are not invariant under rotation. Observing the local behavior of the surface has been proposed recently in [6], where as-rigid-as-possible surface modeling is performed by the minimization of the deformed surface under local rigidity transformation constraints.

Volumetric Space Deformation There has been a great deal of work done in the past on developing techniques for deforming a mesh with generalized barycentric coordinates. Inspired from the pioneering work presented in [7], caged-based methods are ideal for coherently deforming a surface by improving space deformation techniques. The cage parametrization allows model vertices to be expressed as a combinaition of cage vertices to generate realistic deformation. This family of methods has important properties: quasi-conformal mappings, shape preservation and smoothness. To animate the model, cage vertices are displaced and the vertices of the model move accordingly through a linear weighted combination of cage geometry parameters. An approach to generalize mean value coordinates is introduced in [8]. The problem of designing and controlling volume deformations used to articulated characters are treated in [9], where the introduction of harmonic coordinates significantly improves the deformation stability thanks to a volumetric heat diffusion process respecting the connectivity of mesh volume. This work has been extended in [10, 11] to realize spatial deformation transfer. A non linear coordinates system proposed in [12]

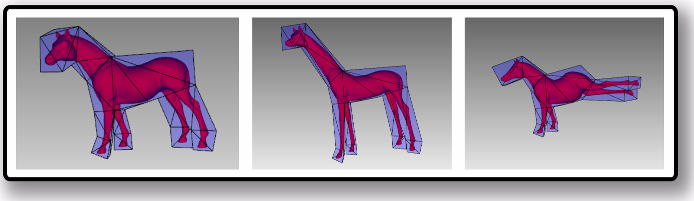

Fig. 2. Cage-Based Deformation

called Green Coordinates leads space deformations with a shape-preserving property. However such approaches require to obtain automatically a fairly coarse control mesh approximating enough a given surface [13, 14].

Boneless Inverse Kinematics Furthermore, there has been a great deal of work made feasible thanks to the work presented in [15–17], where the authors use an analogy to the traditional use of skeleton-based inverse kinematics. A volumetric laplacian approach to preserve the volumetric properties for large deformations has been studied in [18]. Volume preservation is addressed in [19] with a non-linear framework that projected the deformation energy onto the control mesh vertices.

3 Overview

The idea of combining space deformation techniques with surface based techniques proposed in [20] and the lack of reusable surface parameterization for non-rigid surface invited us to abandon the idea of requiring an underlying skeleton and to propose a novel approach called Indirect Cage-Based Dual-Laplacian Deformation. We aim to estimate a sequence of cage parameters expressing the mesh at each animation frame. To realize this cage-based inverse kinematics process we cast the problem as a minimization problem for cage retrieval. The main challenge is to deal with the high number of degree of freedom provided by the coarse cage. We express constraints directly over the enclosed surface and we transfer them to cage using its indirection. In our system, we employ laplacian on the cage to perform a volume deformation surfacically that allows us to obtain a coherent cage estimation.

Even if our work shares similarites with [21] on the idea of producing an hybrid mesh deformation and with [22, 10] on the idea of integrating the cage into a minimization framework, our work is novel for the presented optimization problem. However, the key contribution is to solve a sparse linear system to estimate the best cage parameters reproducing the desired deformation of the enclosed model. Besides, such constraints are expressed on the enclosed model and transfered to the subspace domain using the indirection of the bounding cage.

4 Energy formulation

This section presents the laplacian-based regularization applied on the cage structure only instead of the traditional used on the enclosed mesh. We introduce the association of harmonic subspace deformation with cage-based dual laplacian. In the rest of the paper, we use the following terminology. The coarse bounding mesh $\mathcal{C}$ and the enclosed mesh $\mathcal{M}$ are respectively called *the cage* and *the model*. We assume that both entities are 2-manifold triangular mesh fully-connected. The set of n cage vertices is denoted with $\mathcal{V}_{\mathcal{C}} = \{c_1, \cdots, c_n\}$ where c_i is the location of i^{th} cage vertex, and the set of m model vertices with $\mathcal{V}_{\mathcal{M}} = \{v_1, \cdots, v_m\}$ where v_i is the location of i^{th} model vertex. Vertex location are represented using absolute three dimensional cartesian coordinates.

4.1 Harmonic Subspace Deformation

A cage is a coarse closed bounding polyhedral volume. This flexible low polygon-count polytope, topologically similar to the enclosed object, can efficiently control the deformation of enclosed object and produce realistic looking deformations. Model vertices are expressed as a linear combination of cage vertices. The weights are given by a set of generalized barycentric coordinates stored in a $m \times n$ deformation weights matrix denoted by $\mathcal{H}$. We also denote by $g_k(l)$ the normalized blend weights representing the deforming influence of the k^{th} cage vertex on the l^{th} model vertex. Furthermore it is also possible to deform an arbitrary point on the enclosed mesh written as a linear combination of the coarse mesh vertex position via a constant weight deformation. The forward kinematic a-like function is:

$$v'_i = \sum_{k=1}^{n} g_k(i) \cdot c'_k \tag{1}$$

where v'_i is the deformed cartesian coordinates according to a vector of cage geometry $\{c'_1, \cdots, c'_n\}$. In order to produce as-local-as possible topological changes on the enclosed surface, the model is rigged to the cage using harmonic coordinates. The harmonic rigging is the pre-computed solution of Laplace's equation with Dirichlet boundary condition obtained by a volumetric heat diffusion in the cage interior. The resulting matrix corresponds to the matrix $\mathcal{H}$. A more efficient technique is to compute harmonic coordinates in a closed-form manner using the BEM formulation, proposed in [23].

4.2 Cage-based Dual Laplacian

Given the fact that a fairly coarse cage preserves the mesh model structure, we prefer to define the Laplacian on the cage instead of the model to improve the computation process and to keep model detail properties good enough. Therefore expressing the Laplacian on the cage can be seen as expressing a model dual laplacian. Thus, this Dual Laplacian provides an external parameterization of the enclosed mesh ensuring its internal global characteristic thanks to an over determined linear system of equation. Let's denote the Dual Laplacian operator defined at each cage vertex domain by $\mathcal{L}_C(\cdot)$ by the weighted sum of the difference vectors between the vertex and its adjacent neighbors. We also denote the differential coordinates of the cage by $\hat{\delta}$. Encoding each control vertex relatively to its neighborhood preserves the local geometry using differential coordinates. Differential coordinates are obtained by computing the original difference between its absolute cartesian coordinates and the center of mass of its immediate neighbors in the mesh. We determine the internal energy functional $E_{int}(\mathrm{c}')$ that measure how smooth the cage is and how similar the deformed cage c' is to the original shape in term of local detail as follows:

$$E_{int}(\mathrm{c}') = \left\| \mathcal{L}_C(c') - \hat{\delta}' \right\|_2^2 \tag{2}$$

This functional guarantees smoothness on large deformation in order to preserve the subspace boundary intrisinc properties without rigidity assumption. Ensuring such a property leads to guarantee global characteristic of the model linearly.

4.3 Surface Constraints

Contrary to existing frameworks where positional constraints enforce vertices to move to a specific target 3D position, we prefer to enforce surface features that are not limited to the set of enclosed mesh vertices. In other to deform the bounding cage, positional constraints are defined on the model using barycentric anchor points. A barycentric anchor a on a piecewise linear surface can be evaluated and described using a linear combination of the barycentric coordinates $\{\gamma_1, \gamma_2, \gamma_3\}$ associated to three vertices $\{v_1, v_2, v_3\}$ of the surrounding triangle T that contains this anchor point as follows:

$$a = \sum_{v_i \in T} \gamma_i \cdot v_i \tag{3}$$

In the scenario where the user directly specifies source and target positions over the enclosed mesh surface in screen space, dragged-and-dropped barycentric anchors always offer suitable and precise sparse positionnal deformation constraints. To estimate the target point in world space coordinates, we compute the intersection point between the ray passing through the target screen point and the parallel plane to the screen plane defined by the source point. Barycentric informations are collected in a map computed on GPU.

Mixing Equation 2 with Equation 3 leads to a new formulation expression of the cartesian coordinates of a point q_α over the model in term of the cage parameters only:

$$q_\alpha = \sum_{v_i \in T^\alpha} \sum_{k=1}^{n} \gamma_i \cdot c_k \cdot g_k\left(i\right) \tag{4}$$

We denote by q'_α the cartesian coordinates position of the target point associated to q_α to form a positional constraint. The last equation is key component of the proposed method for the handling interaction. The transfer of surfacic contraints into the volumetric domain exploiting the cage indirection is expressed by this function. In other words, the last formulation permits to express surface constraints directly in terms of cage parameters linearly using a inverse quasi-conformal harmonic mapping, motivating the idea of boneless inverse kinematics.

We determine the external energy functional $E_{ext}\left(\mathrm{c}'\right)$ that measure how smooth the cage enforces l positional constraints as follows:

$$E_{ext}\left(\mathrm{c}'\right) = \sum_{j=1}^{l} \left\| q'_j - q_j \right\|_2^2 \tag{5}$$

5 Indirect Dual-Laplacian Cage-Based Fitting

After having presented the key component of our method, we propose to develop in this section the core of our approach including the linear minimization process.

During the minimization process the cage is seen as a connectivity mesh and feature constraints are seen as external deformation. The surface-and-space based deformation technique preserves the mesh spatial coherence. The geometry of the cage can be reconstructed efficiently from their harmonic indirect coordinates and relative coordinates by solving a system of linear equations. We cast the problem of deformation as least-square laplacian cage reconstruction process using a consistent minimization approach of an objective function requiring linear constraints such as the positional edited constraints. Following the idea presented in [24], the cage parameters recover the sparse pose feature by minimizing an objective function in a least square sense in order to fit a continuous volume. Then the geometry of the desired model is simply obtained by generating its position vertex according to the reconstructed cage parameters obtained on the concept of Least-Square Cage.

Given the differential coordinates and laplacian operator of the default cage, and the harmonic weights $g_k(i)$ according the cage and the model at the default pose, and a several 2D sparse surface constraints the absolute coordinates of the model geometry can be reconstructed by estimating the absolute coordinates of the cage geometry. The combination of the differential coordinates and harmonic coordinates allows the reconstruction the edited surface by solving a linear system that satisfies the reconstruction of the local detail in least squares sense.

Since no exact solution generally exist, our linear least square system reconstructs the geometry of the coarse mesh that allows us to reconstruct the enclosed mesh using a linear caged-based deformation process. The key component of our inverse deformation algorithm is a least-squares minimization. We can formulate overall energy to lead an overdetermined linear system to extract the cage parameters as follows:

$$\min_{c'_i} \left(\alpha \sum_{i=1}^{n} \left\| \mathcal{L}_C \left(c'_i \right) - \hat{\delta'}_i \right\|_2^2 + \beta \sum_{j=1}^{l} \left\| q'_j - q_j \right\|_2^2 \right) \tag{6}$$

This least-squares minimization problem can be expressed exclusively in term of cage geometry from Equation 6 as follows:

$$\min_{c'_k} \left(\alpha \sum_{k=1}^{n} \left\| \mathcal{L}_C \left(c'_k \right) - \hat{\delta'}_k \right\|_2^2 + \beta \sum_{j=1}^{l} \left\| q'_j - \sum_{v_i \in T^j} \sum_{k=1}^{n} \gamma_i \cdot c'_k \cdot g_k(i) \right\|_2^2 \right)$$

Note that the first term of the energy preserves the global detail of the cage and ensure a pleasant deformation under sparse constraints. The second term of the energy enforces the position of vertices to fit the desired model defined by positional constraints. The system can be weighted by α and β to penalize or advantage both objectives. To our best knowledge, the simple global optimization component of our framework with such formulated constraints to minimize, do not already exist in the litterature. Overall energy performed by our technique reproduce harmonic space deformation recovery under indirected dual laplacian mesh editing. After the cage retrieval process, the geometry of the desired enclosed model is reconstructed in linear combination fonction of cage geometry parameters related to the new estimation, preserving the fix connectivity of the cage using Equation 1.

6 Results

This section describes our experiments using this system. Our framework proposes a robust mechanism to extract a cage for various applications. We demonstrate the feasibility and validity in practice with two experimental applications.

Cage-based Modeling For the user-driven approach, we apply our algorithm by specifing sparse screen-space positional constraints over the enclosed surface. We have developed an intuitive user interface that allows the user to modelize specific constraints by sketching them. The indirect cage estimation improves the computation of the modeling because of the small system size envolving the cage indirection. The example shown in Figure 3 was generated in 78 microseconds.

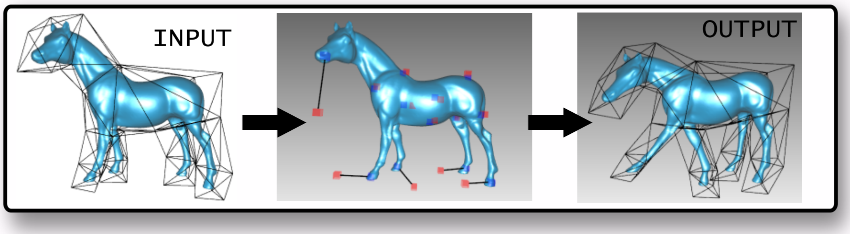

Fig. 3. surface-regularized volume-based deformation under sparse constraints.

Cage Recovery Our framework can also extracts a sequence of cages from a sequence of meshes. For the data-driven approach, we give a sequence of meshes sharing the same connectivity and a default cage. As a result, the system retrieves the corresponding sequence of cage expressing the given animation as output. The system generated automatically the positional constraints using a dense per-vertex mesh displacement mapping from one frame to another to ensure the volume preservation itself. We have processed more than 2000 frames with success. The RMS error is shown in green and the volume change in blue in Figure 4. Outputs are reusable to reedit the animation and to re-skin the model.

7 Conclusion

In this paper, we have presented a unified deformation framework based on a new hybrid surfacially-constrained volume deformation system. A mix of generalized barycentric coordinates and laplacian coordinates are used inside a linear minimization framework, to reconstruct an enclosed mesh. This indirect dual-laplacian caged-based mesh editing technique allows users to produce visually pleasing deformations. The linearity of the underlying objective functional makes

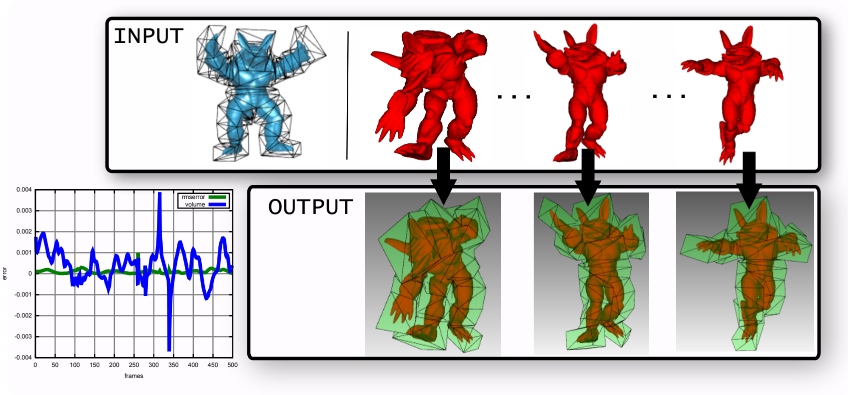

Fig. 4. Cage extraction from mesh sequence

the processing very efficient and improves the effectiveness of deformable surface computation. Our method offers the possibility to encode global topological changes of the shape with respect of local influence and allows animators to re-use the estimation paraterization. Our framework is not restricted to harmonic coordinates as far as the cage-based coordinate system is linear and local preserving.

A limitation is the cage design, because it is very tedious to define a default cage able to express every animation pose correctly according to a binding process. Last but not least, because of the reduction of parameter induced by the cage, the estimation of cage is sensitive to local variation of surface. The main benefit of our method is that the minimization framework is fully independent of the model resolution. We also observe that the connectivity and positional information of the default cage encode non-trivial soft kinemantic constraints as well as motion signal. We believe this novel approach will offer promising new directions because of the strong interest in hybrid deformation and boneless inverse kinematics.

8 Acknowledgments

We thanks Scott Schaefer and Qian-Yi Zhou for providing useful datasets.

References

1. Lipman, Y., Sorkine, O., Cohen-Or, D., Levin, D., Rössl, C., Seidel, H.P.: Differential coordinates for interactive mesh editing. In: Proceedings of Shape Modeling International, IEEE Computer Society Press (2004) 181–190
2. Sorkine, O.: Differential representations for mesh processing. Computer Graphics Forum **25** (2006) 789–807

3. Lipman, Y., Sorkine, O., Alexa, M., Cohen-Or, D., Levin, D., Rössl, C., Seidel, H.P.: Laplacian framework for interactive mesh editing. International Journal of Shape Modeling (IJSM) **11** (2005) 43–61
4. Au, O.K.C., Tai, C.L., Liu, L., Fu, H.: Dual laplacian editing for meshes. IEEE Transactions on Visualization and Computer Graphics **12** (2006) 386–395
5. Luo, Q., Liu, B., Ma, Z.g., Zhang, H.b.: Mesh editing in roi with dual laplacian. In: CGIV '07: Proceedings of the Computer Graphics, Imaging and Visualisation, Washington, DC, USA, IEEE Computer Society (2007) 195–199
6. Sorkine, O., Alexa, M.: As-rigid-as-possible surface modeling. In: Proceedings of Eurographics/ACM SIGGRAPH Symposium on Geometry Processing. (2007) 109–116
7. Meyer, M., Lee, H., Barr, A., Desbrun, M.: Generalized barycentric coordinates on irregular polygons. Journal of Graphics Tools **7** (2002) 13–22
8. Ju, T., Schaefer, S., Warren, J.: Mean value coordinates for closed triangular meshes. In: SIGGRAPH '05: ACM SIGGRAPH 2005 Papers, New York, NY, USA, ACM (2005) 561–566
9. Joshi, P., Meyer, M., DeRose, T., Green, B., Sanocki, T.: Harmonic coordinates for character articulation. ACM Trans. Graph. **26** (2007) 71
10. Ben-Chen, M., Weber, O., Gotsman, C.: Variational harmonic maps for space deformation. In: SIGGRAPH '09: ACM SIGGRAPH 2009 papers, New York, NY, USA, ACM (2009) 1–11
11. Ben-Chen, M., Weber, O., Gotsman, C.: Spatial deformation transfer. In: SCA '09: Proceedings of the 2009 ACM SIGGRAPH/Eurographics Symposium on Computer Animation, New York, NY, USA, ACM (2009) 67–74
12. Lipman, Y., Levin, D., Cohen-Or, D.: Green coordinates. In: ACM SIGGRAPH 2008 papers, New York, NY, USA, ACM (2008) 78:1–78:10
13. Ju, T., Zhou, Q.Y., van de Panne, M., Cohen-Or, D., Neumann, U.: Reusable skinning templates using cage-based deformations. ACM Trans. Graph. **27** (2008) 1–10
14. Chuhua Xian, Hongwei Lin, S.G.: Automatic generation of coarse bounding cages from dense meshes. In: IEEE International Conference on Shape Modeling and Applications (Shape Modeling International 2009). (2009)
15. Sumner, R.W., Zwicker, M., Gotsman, C., Popović, J.: Mesh-based inverse kinematics. ACM Trans. Graph. **24** (2005) 488–495
16. Der, K.G., Sumner, R.W., Popović, J.: Inverse kinematics for reduced deformable models. In: SIGGRAPH '06: ACM SIGGRAPH 2006 Papers, New York, NY, USA, ACM (2006) 1174–1179
17. Shi, X., Zhou, K., Tong, Y., Desbrun, M., Bao, H., Guo, B.: Mesh puppetry: cascading optimization of mesh deformation with inverse kinematics. ACM Trans. Graph. **26** (2007) 81
18. Zhou, K., Huang, J., Snyder, J., Liu, X., Bao, H., Guo, B., Shum, H.Y.: Large mesh deformation using the volumetric graph laplacian. ACM Trans. Graph. **24** (2005) 496–503
19. Huang, J., Shi, X., Liu, X., Zhou, K., Wei, L.Y., Teng, S.H., Bao, H., Guo, B., Shum, H.Y.: Subspace gradient domain mesh deformation. ACM Trans. Graph. **25** (2006) 1126–1134
20. Cohen-Or, D.: Space deformations, surface deformations and the opportunities in-between. J. Comput. Sci. Technol. **24** (2009) 2–5
21. P. Borosan, R. Howard, S.Z., Nealen, A.: Hybrid mesh editing. to appear in Proc. of Eurographics 2010 (short papers), Norrkoping, Sweden (2010)

22. Chen, L., Huang, J., Sun, H., Bao, H.: Cage-based deformation transfer. Computers & Graphics (2010)
23. Rustamov, R.M.: M.: Boundary element formulation of harmonic coordinates. Technical report (2008)
24. Sorkine, O., Cohen-Or, D.: Least-squares meshes. In: Proceedings of Shape Modeling International, IEEE Computer Society Press (2004) 191–199