Jitter-Adaptive Dictionary Learning - Application to Multi-Trial Neuroelectric Signals

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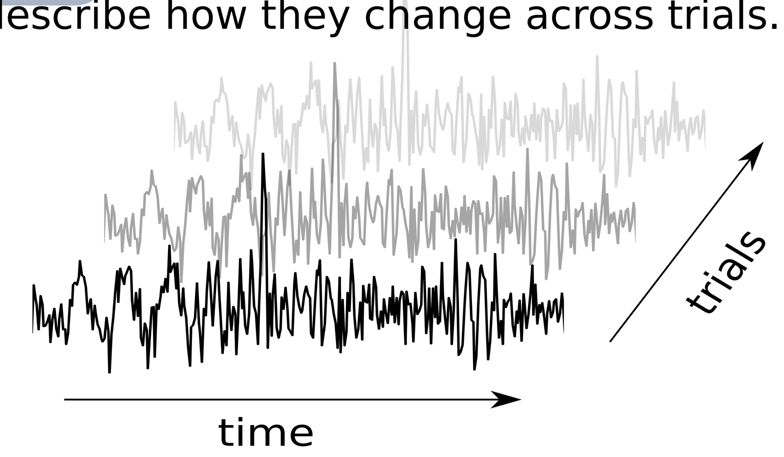
The simultaneous analysis of multiple recordings of neuronal electromagnetic activity is an important task requiring sophisticated and extremely noise robust techniques. A general goal is to find a representation of the similarities (e.g. repeating waveforms) as well as the differences (e.g. varying shape, latency, phase, or amplitude of waveforms) across the signals. Here, we present an extension to dictionary learning that explicitly accounts for small variations in latency and phase of atoms.

Multi-trial analysis in neuroscience

Trials: recordings of neuronal electro-magnetic activity under similar conditions.

Goal: detect similar waveforms and describe how they change across trials.

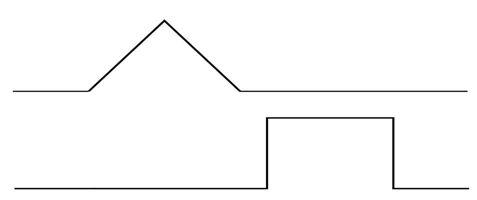


Existing approaches

- Averaging
  - Loses the information present in individual trials.
- Matrix factorization (PCA, ICA, dictionary learning [1])
  - Linear approach, does not account for temporal shifts [2].
- Variants of matching pursuit [3]
  - Do not learn waveforms but require predefined dictionary.

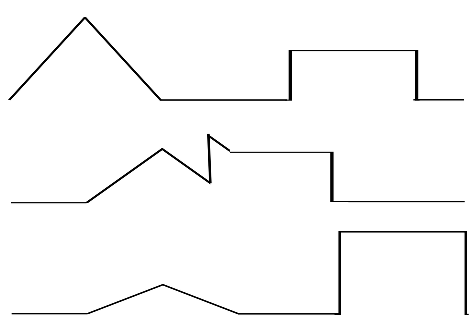
Model

Assumption: trials can be generated from a dictionary.



Dictionary $\mathbf{D} = \{\mathbf{d}_i\}_{i=1}^K$

scaling (a_{ij}) + shifting (δ_{ij})



Trials $\mathbf{X} = \{\mathbf{x}_j\}_{j=1}^M$

$$\mathbf{x}_j = \sum_{i=1}^{K} a_{ij}\delta_{ij}(\mathbf{d}_i) + \epsilon$$

ϵ: Gaussian noise

$\delta_{ij} \in \Delta$: finite set of allowed shifts, $|\Delta| = S$

Jitter-adaptive dictionary learning (JADL)

L_1-regularized optimization

$$\text{(1)} \quad \{\mathbf{d}_i, a_{ij}, \delta_{ij}\} \leftarrow \min \sum_{j=1}^{M} \left(\frac{1}{2} \left\| \mathbf{x}_j - \sum_{i=1}^{K} a_{ij}\delta_{ij}(\mathbf{d}_i) \right\|_2^2 + \lambda \|\mathbf{a}_j\|_1 \right) \quad \text{s.t.} \quad \|\mathbf{d}_i\|_2 = 1, \quad \delta_{ij} \in \Delta$$

Solve using alternating minimization

Algorithm 1

Require: trials $\mathbf{X}$, shifts $\Delta, K \in \mathbb{N}, \lambda \in \mathbb{R}$.
Initialize $\mathbf{D} = \{\mathbf{d}_i\}_{i=1}^K$ *(e.g. white noise)*
repeat
Sparse coding:
fix $\mathbf{D}$, solve (1) for $\{a_{ij}, \delta_{ij}\}$
Dictionary update:
fix $\{a_{ij}, \delta_{ij}\}$, solve (1) for $\{\mathbf{d}_i\}$
until convergence

Dictionary update

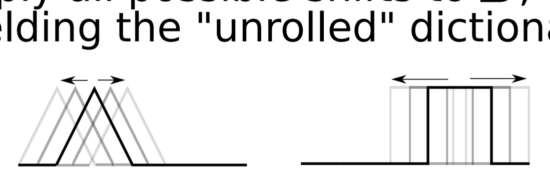
- Atoms can be updated iteratively using block coordinate descent

$$\widetilde{\mathbf{d}}_k = \sum_{j=1}^{M} a_{kj}\delta_{kj}^{-1} \left(\mathbf{x}_j - \sum_{i \neq k} a_{ij}\delta_{ij}(\mathbf{d}_i) \right)$$

where the inverse shift δ_{kj}^{-1} functions as a realignment operator. This is followed by normalization.

Sparse coding: update $\{a_{ij}, \delta_{ij}\}$

- Idea: as $S = |\Delta|$ is finite, we can first apply all possible shifts to $\mathbf{D}$, yielding the "unrolled" dictionary $\mathbf{D}^S$.



- Sparse coding can now be performed over $\mathbf{D}^S$, the non-zero coefficients show which shifts are used.
- A uniqueness constraint on the coefficients ensures, that at most one shifted version of each atom is used; the LARS algorithm [4] can be modified to guarantee this constraint.

Relations to other DL approaches

Dictionary learning (DL)

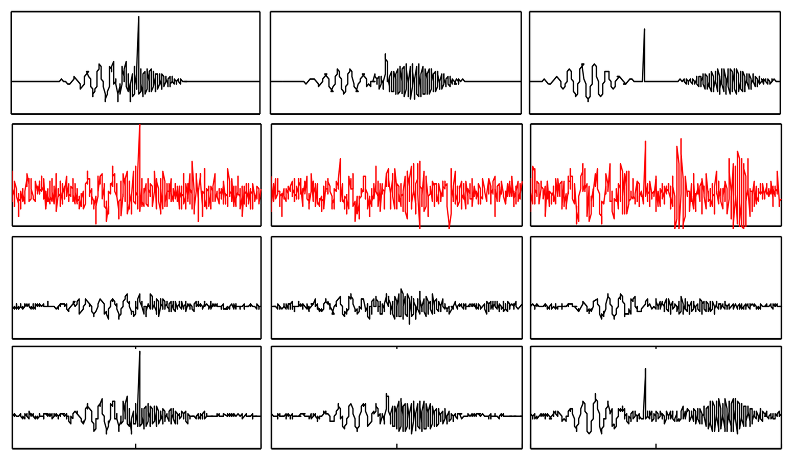
- Minimization (1) becomes DL for $\Delta = \{\mathbf{I}\}$.
- Structure of algorithms of JADL and DL are similar.

Shift-invariant sparse coding (SISC) [5]

- Atoms can shift arbitrarily.
- Atoms typically shorter than signal.
- No uniqueness constraint as in JADL: multiple shifts per atom allowed in each signal.

Synthetic data

200 signals were generated from a synthetic dictionary of $K = 3$ atoms. Random events and Gaussian noise were added. Dictionaries with different numbers of atoms (see table below) were learned for JADL and DL on the noisy signals (200 iterations each). Denoising was then performed by sparse coding over the learned dictionaries. The plots show the denoised signals for best performing K.



Error for different numbers K of reconstructed atoms*

	K=1	K=2	K=3	K=4	K=5	K=6	K=8	K=10	K=12
DL	0.869	0.747	0.635	0.535	0.515	0.498	**0.487**	0.505	0.539
JADL	0.505	0.283	**0.214**	0.230	0.277	0.284	0.317	0.325	0.330

*For each K, we selected the parameter λ that gave the smallest error. This was $\lambda = 0.1$ for DL and $K = 8$ and $\lambda = 0.1$ for JADL and $K = 3$.

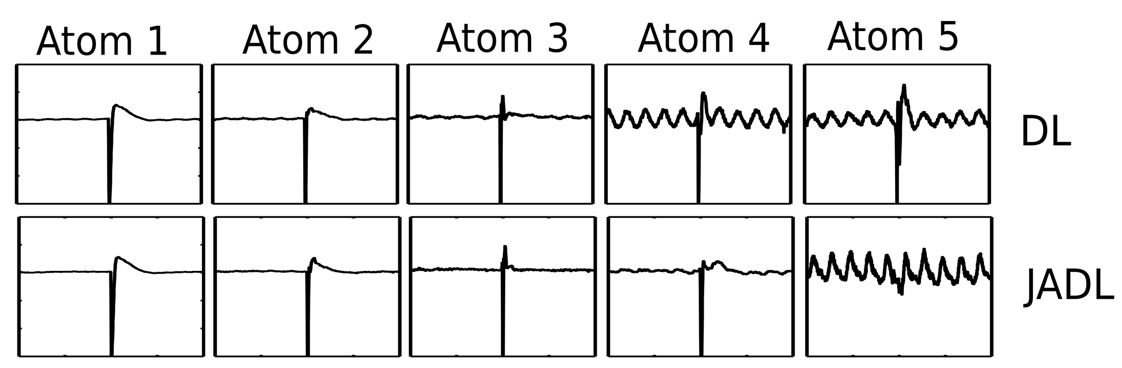
Real data

In an animal model of epilepsy, local field potentials were recorded during one hour with an intra-cranial electrode in a Winster-Han rat. Biccuculine (a blocker of inhibition) was injected in the cortex to elicit epileptic-like discharges. 169 of these spikes were then selected visually and segmented into epochs of 10 seconds.

Epoch 41 Epoch 111 Epoch 161 Average

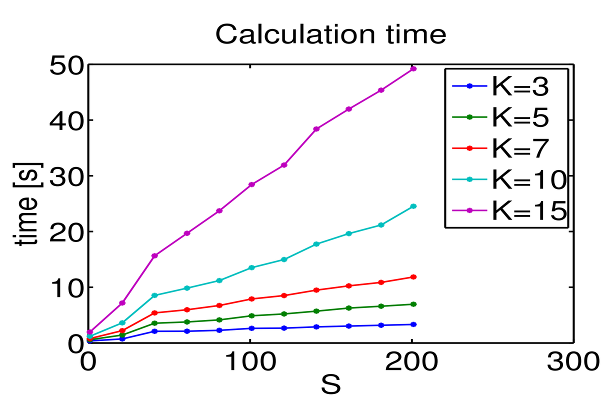
Learned dictionaries

On the 169 epochs, dictionaries were learned using DL and JADL, each algorithm performing 200 iterations.



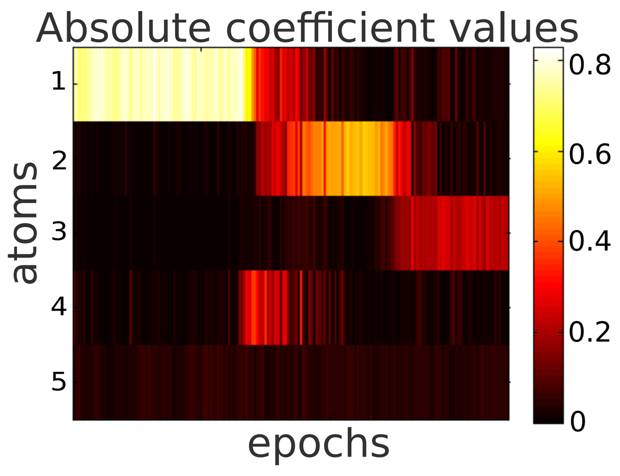
- Only JADL clearly separates spikes from oscillations.
- DL smoothes the periodic wave, as its atoms do not adapt to phase shifts.

Performance

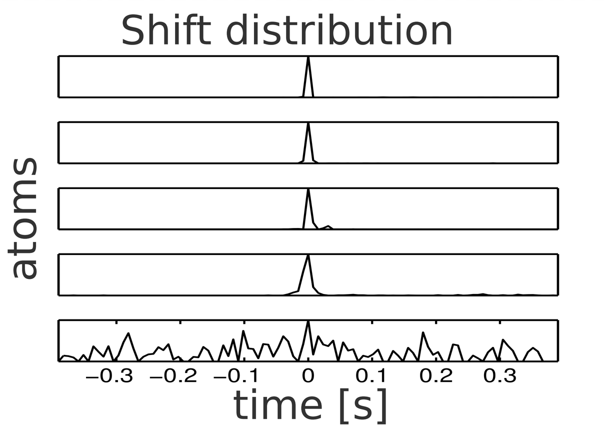


While both S (the number of allowed shifts) and K influence the size of the "unrolled" dictionary, the increase in computation time due to S is only linear. This comes from the uniqueness constraint which bounds the complexity as well as the use of fft-based convolution for large S.

Learned coefficients and shifts provide insight into data



- Epochs can be divided into segments with one dominating spike each.



- The spikes (first four atoms) are well aligned across trials.
- The periodic atom (five) occurs with uniformly distributed phase.

Conclusion

We presented a new method (JADL) which is an extension to dictionary learning and designed to analyze multi-trial neuroelectric datasets. We evaluated JADL on synthetic and real data and showed its superiority to common dictionary learning. In particular, JADL showed the following qualities:

- Ability to learn main waveforms in trials and to separate them.
- Learned shifts and coefficients give insight into the changes of waveforms (phase, latency, amplitude).
- Computational efficiency, even for high shift-tolerance.
- Robustness and denoising qualities.

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