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Z. Gnutek Wroclaw University of Technology

E. Kalinowski Wroclaw University of Technology

S. Pietrowicz Wroclaw University of Technology

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# ANALYSIS OF THERMODYNAMIC PROCESSES IN THE WORK CHAMBER OF A SPIRAL MACHINE IN THE FUNCTION OF THE ROTATION ANGLE

Zbigniew GNUTEK, Eugeniusz KALINOWSKI, Sławomir PIETROWICZ Institute of Heat Engineering and Fluid Mechanics Wrocław University of Technology 27 Wybrzeze Wyspianskiego Str. 50 – 370 Wrocław, POLAND

#### ABSTRACT

The paper describes the dependences of thermodynamic parameters in the work chamber upon the position of the rotor. It discusses the process of compression in individual working spaces. An analysis is made of the influence of friction onto the process of the adiabatic expansion (the influence of using different oils). It considers the possibility of applying a spiral expander in the ORC power stations.

Further research is being made for considering the influence of friction on the process of the adiabatic expansion (compression) and the expander's efficiency.

#### NOMENCLATURE

 $r_b$  – parameter of the spiral;

 $r_e$  - eccentricity;

 $\phi$  - forming angle;

 $p_s$  – suction pressure,

 $V_s = B \cdot S_s$  – volume of the chamber during the suction;

V – volume of the chamber in the function of the rotation angle  $\theta$ ;

 $S_s$  – cross section area of the chamber being filled,

 $\kappa$  - adiabatic exponent,

B – height of the spiral.

#### **1. INTRODUCTION**

One of the basic constructional elements of the ORC power station is the motor. It seems, the spiral expander can fulfil this role.

The main properties of the spiral engines are:

- a simple mechanism – only two elements, an active and a passive spiral are required to compress gas. (In piston compressors around fifteen elements cooperate with one another in order to perform the same work.)

- low level of noise,

- simplicity and self-balance of power, which also means a reduction of the machine's free vibration to minimum.

When compared to the piston compressors, the spiral compressors have three basic features which make them different:

1. The processes of suction and discharge in the spiral compressors are physically separate, which reduces the heat exchange between the input and output gases. In the piston compressors this process takes place in one working chamber-the heat flow between the input and output gas is significant and so the compressor's efficiency decreases.

2. The process of compression and discharge is more balanced. The processes of suction and discharge recur approximately every 30 - 60°.

3. In the spiral compressors there are no valves. Thanks to this leakages are eliminated in the set of valve - valve seat.

The further part of the article analyses the spiral machine, treating it as an expansive machine. The idea of applying the spiral machine as an expander appeared when in the Institute of Heat Engineering and Fluid Mechanics of the Technical University of Wrocław a C-R cycle station was designed and constructed, in which the working medium was a low-boiling liquid. The greatest problem was to select a suitable machine for expanding the low-boiling medium [3], [4].

### 2. THE SPIRAL OF ARCHIMEDES AS A BASIS OF CONSTRUCTION OF A SPIRAL EXPANDER

This is and analysis of work of the spiral expander on the basis of the spiral of Archimedes. In this spiral the course radius  $r = |\vec{OM}|$  is proportional to the angle of inclination  $\theta$  against

the polar semi-axis:

#### $r = a \cdot \theta$

(1)

where: a - is called the parameter of the spiral of Archimedes. Equation 1 describes the spiral of Archimedes in polar coordinates. While equation 2 provides a "general" equation of the spiral of Archimedes in coordinates x-y.

$$\sqrt{x^2 + y^2} = a \cdot arctg(y/x)$$
<sup>(2)</sup>



In the further part of the work we shall be using parametric equations of the *sloping line developing a circle*.

$$x = O\overline{M}_{x} + \overline{M}_{x}M_{x} = a(\cos t + t\sin t)$$
(2)

$$y = MM_x - MB = a(\sin t - t\cos t) \quad (3)$$

Fig. 1 Coordinates set used to define the parametric equation of the spiral

Figure 2. shows the active spiral with all markings. The passive spiral is defined by the set of equations:

- inner curve

$$x_{fw} = r_b (\cos(\phi) + \phi \sin(\phi))$$
(4a)

$$y_{fw} = r_b (\sin(\phi) - \phi \cos(\phi))$$
 (4b)

- outer curve

$$x_{fo} = r_b (\cos(\phi) + \phi \sin(\phi)) + b \sin(\phi)$$
(5a)  
$$y_{fo} = r_b (\sin(\phi) - \phi \cos(\phi)) - b \cos(\phi)$$
(5b)





Fig. 2 Active spiral

Fig. 3 A spiral expander system

The active spiral is defined by the following relationship:

$$-x_{ow} = r_b(-\cos(\phi) - \phi\sin(\phi)) + r_e\cos(\theta) \quad (6a)$$
$$y_{ow} = r_b(-\sin(\phi) + \phi\cos(\phi)) + r_e\sin(\theta) \quad (6b)$$

- outer curve -

$$x_{oz} = r_b(-\cos(\phi) - \phi\sin(\phi)) - b\sin(\phi) + r_e\cos(\theta) \quad (7a)$$
  
$$y_{oz} = r_b(-\sin(\phi) + \phi\cos(\phi)) + b\cos(\phi) + r_e\sin(\theta) \quad (7b)$$

Width of spiral *b* can be calculated by:

$$b = r_b(\phi_{io} - \phi_{oo}) \tag{8}$$

#### 3. A DUAL SPIRAL AS THE BASIS OF EXPANDER CONSTRUCTION

In the spiral expander the gas decompression is realised between two identical spirals (fig.3). One spiral – the passive one remains standstill to the still set of coordinates x - y with the middle point O. The second spiral – the active one moves eccentrically to point O. The coordinates of point  $O_m$  (x<sub>e</sub>, y<sub>e</sub>) can be calculated from:

$$x_e = r_e \cos(\theta) \tag{9c}$$

$$y_e = r_e \sin(\theta) \tag{9b}$$

Radius of rotation  $r_e$  can be calculated from the equation:

$$r_e = r_b \pi - t \tag{10}$$

The two spirals have points of contact. These points lie on the inner  $B_{ik}(x_{ow}, y_{ow})$  and outer  $B_{ok}(x_{oz}, y_{oz})$  side of the spiral. These points  $B_{ik}$  and  $B_{ok}$  can be found by defining the angle of the contact points. In this case such an angle can be evaluated as:

$$\phi_i(k) = 2\pi(k_1 + \delta) + \frac{3}{2}\pi + \theta \qquad k_1 = -1..1 \tag{11}$$

$$\phi_0(k) = 2\pi(k_2 + \delta) + \frac{1}{2}\pi + \theta; \quad k_2 = 0..2$$
(12)

where:  $\delta = 0$  for  $0 < \theta < \frac{1}{2}\pi - \phi_{os}$ 

 $\delta = 1$  for  $\frac{1}{2}\pi - \phi_{os} < \theta < 2\pi$ 

The coordinates of the contact points can be calculated: - on the inner curve  $B_{ik}\xspace$  -

$$x_{ow} = r_b(-\cos(\phi_o(k)) - \phi_o(k)\sin(\phi_o(k))) + r_e\cos(\theta)$$
(13a)

$$y_{ow} = r_b(-\sin(\phi_o(k)) + \phi_o(k)\cos(\phi_o(k))) + r_e\sin(\theta)$$
(13b)

- on the outer curve 
$$T_{oi}$$
 -  

$$x_{oz} = r_b (-\cos(\phi_i(k)) - \phi_i(k)\sin(\phi_i(k))) - b\sin(\phi_i(k)) + r_e \cos(\theta)$$
(14a)  

$$(14a)$$

$$y_{oz} = r_b(-\sin(\phi_i(k)) + \phi_i(k)\cos(\phi_i(k))) + b\cos(\phi_i(k)) + r_e\sin(\theta)$$
(14b)

The figures below present several reciprocal positions of the active and passive spirals:



Fig. 4 A system of spirals in an expander

## 4. PRESSURE DEPENDING ON THE ANGLE OF ROTATION $\phi$ IN AN EXPANDER

The cross-section of a spiral machine is specified by the formula (15):

$$S = \int_{0}^{t} \sqrt{x'^{2} + {y'}^{2}} dt$$
 (15)

In the considered case the area between the spirals can be calculated as:

$$S = \int \left( r_b^2 \phi^2 - r_b r_0 \phi \sin(\phi - k) \right) d\phi - \int \left( r_b \phi + b \right)^2 d\phi$$
(16)



Fig. 5 Area of the cross-cut in the function of

#### the rotation angle

The gas pressure present in individual chambers of the considered unit was calculated based on the assumption that the decompression would be adiabatic. Such pressure can be calculated from the formula:

$$p = p_s \left(\frac{V_s}{V}\right)^{\kappa} \tag{17}$$



Fig. 6 Pressure in the work chamber depending on the rotation angle

## 5. COMPONENTS OF FORCE ACTING ON EXPANDER'S ROTOR

Forces acting on the active spiral can be distributed on two  $F_t$  and  $F_r$  and moment  $M_0$  (Fig. 3)

- tangent force F<sub>t</sub>,

- radial force F<sub>r</sub>,

The tangent force can be deducted from:

$$F_{i} = Br_{b} \left( \int_{\phi_{o}(0)-\pi}^{\phi_{o}(K_{1})} p_{j}\phi \cos(\phi - \theta)d\phi - \int_{\phi_{i}(-1)}^{\phi_{i}(K_{2})+\pi} p_{j} \left(\phi + \frac{b}{r_{b}}\right) \cos(\phi - \theta)d\phi \right) + bB(p_{s} + p_{d}) \quad (18)$$

and the radial force can be deducted from:

$$F_r = -Br_b \left( \int_{\phi_o(0)-\pi}^{\phi_0(K_1)} p_j \phi \sin(\phi - \theta) d\phi - \int_{\phi_i(-1)}^{\phi_i(K_2)+\pi} p_j \left(\phi + \frac{b}{r_b}\right) \sin(\phi - \theta) d\phi \right)$$
(19)

The Moment is defined by:

$$M_{0} = Br_{b}^{2} \left( -\int_{\phi_{0}(0)-\pi}^{\phi_{0}(K_{1})} p_{j} \phi d\phi + \int_{\phi_{i}(-1)}^{\phi_{i}(K_{2})+\pi} p_{j} \left(\phi + \frac{b}{r_{b}}\right) d\phi \right) + bB \left( -\left(r_{b}(\phi_{0}(0) - b/2)p_{s} + \left(r_{b}(\phi_{i}(K) + b/2)p_{d}\right)\right)$$
(20)

In these formulas  $K_1$  and  $K_2$  is the highest number of the values  $k_1$  or  $k_2$ .

The force of the radial friction can be calculated from:

$$F_{fl} = F_l \cdot \mu \tag{21}$$

where:

 $\mu$  - friction factor - in this case mixed friction,  $\mu$  = 0.1

For the accepted data of the forces  $F_t$  and  $F_{ft}$  are presented in Fig. 7





Radial force is reduced to the formula:

$$F_r = 2Br_e(p_s - p_d) \tag{22}$$

In our concrete case this force equals  $F_r = 172.465$  N and the friction force  $F_{tr} = 17.24$  N

Pressure  $p_j$  is the pressure existing along the curve of the spiral. The example of distribution of pressure along the spiral is illustrated by Fig. 8.



**Fig. 8** Distribution of pressure along spiral (according to Fig. 2) for angle  $\theta = 0.3 \pi$ .

#### 6. CONCLUSION

Presented in this paper, the analysis of a spiral machine shows that it is possible to use it for decompression of a medium in an ORC micro power station. Such decompression takes place more evenly than in piston machines, which makes one expect that a spiral machine will better perform the function of an expansion machine in the ORC systems.

The influence of individual constructional parameters on the expander's efficiency and on its power will be the subject of further research.

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