



Performance analysis of DG and HDG methods for the simulation of seismic wave propagation in harmonic domain

M. Bonnasse-Gahot^{1,2}, H. Calandra³, J. Diaz¹ and S. Lanteri²

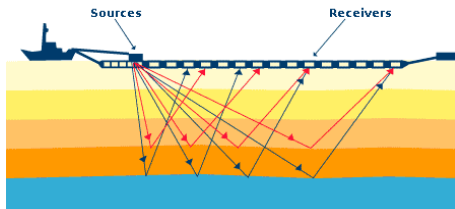
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³ TOTAL Exploration-Production

Motivation

Examples of seismic applications



Motivation

Imaging method : the full wave inversion

- ▶ Iterative procedure using the wavefield in order to obtain quantitative **high resolution** images of the subsurface physical parameters

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Seismic imaging : time-domain or harmonic-domain ?

- ▶ Time-domain : **imaging condition complicated** but **low computational cost**
- ▶ Harmonic-domain : **imaging condition simple** but **huge computational cost**

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Forward problem of the inversion process

- ▶ Elastic wave propagation in harmonic domain : **Helmholtz equation**
- ▶ Reduction of the size of the linear system

Motivation

Seismic imaging in heterogeneous complex media

- ▶ Complex topography
- ▶ High heterogeneities

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Use of unstructured meshes with FE methods

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Use of unstructured meshes with FE methods

DG method

- ▶ Flexible choice of interpolation orders (p – adaptativity)
- ▶ Highly parallelizable method
- ▶ Increased computational cost as compared to classical FEM

Motivation

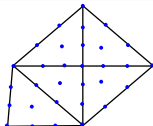
Seismic imaging in heterogeneous complex media

- ▶ Complex topography
- ▶ High heterogeneities

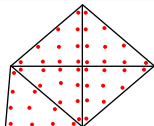
Use of unstructured meshes with FE methods

DG method

- ▶ Flexible choice of interpolation orders (p – adaptativity)
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- ▶ Increased computational cost as compared to classical FEM



DOF of classical FEM



DOF of DGM

Motivation

Objective of this work

- ▶ Development of an hybridizable DG (HDG) method
- ▶ Comparison with a reference method : a standard nodal DG method

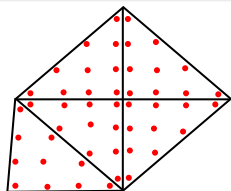


FIGURE : Degrees of freedom of DGM

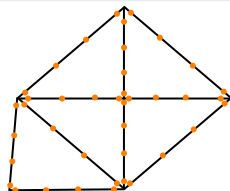


FIGURE : Degrees of freedom of HDGM

HDG methods

HDG methods

- ▶ **B. Cockburn, J. Gopalakrishnan, R. Lazarov** *Unified hybridization of discontinuous Galerkin, mixed and continuous Galerkin methods for second order elliptic problems*, SIAM Journal on Numerical Analysis, Vol. 47 (2009)
- ▶ **S. Lanteri, L. Li, R. Perrussel**, *Numerical investigation of a high order hybridizable discontinuous Galerkin method for 2d time-harmonic Maxwell's equations*, COMPEL, Vol. 32 (2013) (time-harmonic domain)
- ▶ **N.C. Nguyen, J. Peraire, B. Cockburn**, *High-order implicit hybridizable discontinuous Galerkin methods for acoustics and elastodynamics*, J. of Comput. Physics, Vol. 230 (2011) (time domain for seismic applications)

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2D Helmholtz elastic equations

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Hybridizable Discontinuous Galerkin method

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Conclusions-Perspectives

2D Helmholtz elastic equations

First order formulation of Helmholtz wave equations

$$\mathbf{x} = (x, y) \in \Omega \subset \mathbb{R}^2,$$

$$\begin{cases} i\omega\rho(\mathbf{x})\mathbf{v}(\mathbf{x}) = \nabla \cdot \underline{\underline{\sigma}}(\mathbf{x}) + \mathbf{f}_s(\mathbf{x}) \\ i\omega\underline{\underline{\sigma}}(\mathbf{x}) = \underline{\underline{C}}(\mathbf{x}) \underline{\underline{\varepsilon}}(\mathbf{v}(\mathbf{x})) \end{cases}$$

- ▶ Free surface condition : $\underline{\underline{\sigma}}\mathbf{n} = 0$ on Γ_f
 - ▶ Absorbing boundary condition : $\underline{\underline{\sigma}}\mathbf{n} = v_p(\mathbf{v} \cdot \mathbf{n})\mathbf{n} + v_s(\mathbf{v} \cdot \mathbf{t})\mathbf{t}$ on Γ_a
-
- ▶ \mathbf{v} : velocity vector
 - ▶ $\underline{\underline{\sigma}}$: stress tensor
 - ▶ $\underline{\underline{\varepsilon}}$: strain tensor

2D Helmholtz elastic equations

First order formulation of Helmholtz wave equations

$$\mathbf{x} = (x, y) \in \Omega \subset \mathbb{R}^2,$$

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-
- ▶ ρ : mass density
 - ▶ $\underline{\underline{C}}$: tensor of elasticity coefficients
 - ▶ v_p : P-wave velocity
 - ▶ v_s : S-wave velocity
 - ▶ \mathbf{f}_s : source term, $\mathbf{f}_s \in L^2(\Omega)$

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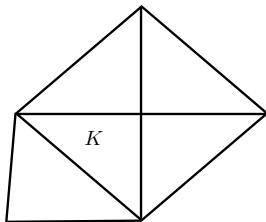
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Notations and definitions

Notations

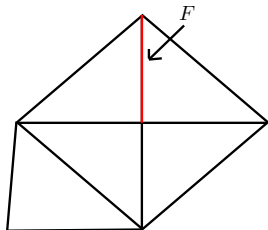
- ▶ \mathcal{T}_h mesh of Ω composed of triangles K



Notations and definitions

Notations

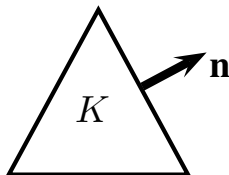
- ▶ \mathcal{T}_h mesh of Ω composed of triangles K
- ▶ \mathcal{F}_h set of all faces F of \mathcal{T}_h



Notations and definitions

Notations

- ▶ \mathcal{T}_h mesh of Ω composed of triangles K
- ▶ \mathcal{F}_h set of all faces F of \mathcal{T}_h
- ▶ \mathbf{n} the normal outward vector of an element K



Notations and definitions

Approximations spaces

- ▶ $P_p(K)$ set of polynomials of degree at most p on K

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- ▶ $\underline{\Sigma}_h^p = \{\underline{\underline{\sigma}} \in (L^2(\Omega))^3 : \underline{\underline{\sigma}}|_K \in \underline{\Sigma}^p(K) = (P_p(K))^3, \forall K \in \mathcal{T}_h\}$

Notations and definitions

Approximations spaces

- ▶ $P_p(K)$ set of polynomials of degree at most p on K
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- ▶ $\underline{\Sigma}_h^p = \{\underline{\sigma} \in (L^2(\Omega))^3 : \underline{\sigma}|_K \in \Sigma^p(K) = (P_p(K))^3, \forall K \in \mathcal{T}_h\}$
- ▶ $\mathbf{M}_h = \{\eta \in (L^2(\mathcal{F}_h))^2 : \eta|_F \in (P_p(F))^2, \forall F \in \mathcal{F}_h\}$

Notations and definitions

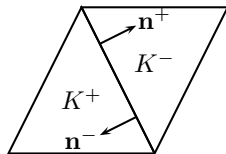
Definitions

- ▶ Jump $[[\cdot]]$ of a vector \mathbf{v} through F :

$$[[\mathbf{v}]] = \mathbf{v}^+ \cdot \mathbf{n}^+ + \mathbf{v}^- \cdot \mathbf{n}^- = \mathbf{v}^+ \cdot \mathbf{n}^+ - \mathbf{v}^- \cdot \mathbf{n}^+$$

- ▶ Jump of a tensor $\underline{\underline{\sigma}}$ through F :

$$[[\underline{\underline{\sigma}}]] = \underline{\underline{\sigma}}^+ \mathbf{n}^+ + \underline{\underline{\sigma}}^- \mathbf{n}^- = \underline{\underline{\sigma}}^+ \mathbf{n}^+ - \underline{\underline{\sigma}}^- \mathbf{n}^+$$



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HDG formulation of the equations

Local HDG formulation

$$\begin{cases} i\omega\rho\mathbf{v} - \nabla \cdot \underline{\underline{\sigma}} = 0 \\ i\omega\underline{\underline{\sigma}} - \underline{\underline{C}}\varepsilon(\mathbf{v}) = 0 \end{cases}$$

HDG formulation of the equations

Local HDG formulation

$$\left\{ \begin{array}{l} \int_K i\omega \rho^K \mathbf{v}^K \cdot \mathbf{w} + \int_K \underline{\underline{\sigma}}^K : \nabla \mathbf{w} - \int_{\partial K} \widehat{\underline{\underline{\sigma}}}^{\partial K} \cdot \mathbf{n} \cdot \mathbf{w} = 0 \\ \int_K i\omega \underline{\underline{\sigma}}^K : \underline{\underline{\xi}} + \int_K \mathbf{v}^K \cdot \nabla \cdot (\underline{\underline{C}}^K \underline{\underline{\xi}}) - \int_{\partial K} \widehat{\mathbf{v}}^{\partial K} \cdot \underline{\underline{C}}^K \underline{\underline{\xi}} \cdot \mathbf{n} = 0 \end{array} \right.$$

$\widehat{\underline{\underline{\sigma}}}^K$ and $\widehat{\mathbf{v}}^K$ are numerical traces of $\underline{\underline{\sigma}}^K$ and \mathbf{v}^K respectively on ∂K

HDG formulation of the equations

Local HDG formulation

$$\begin{cases} \int_K i\omega \rho^K \mathbf{v}^K \cdot \mathbf{w} + \int_K \underline{\underline{\sigma}}^K : \nabla \mathbf{w} - \int_{\partial K} \widehat{\underline{\underline{\sigma}}}^{\partial K} \cdot \mathbf{n} \cdot \mathbf{w} = 0 \\ \int_K i\omega \underline{\underline{\sigma}}^K : \underline{\underline{\xi}} + \int_K \mathbf{v}^K \cdot \nabla \cdot (\underline{\underline{C}}^K \underline{\underline{\xi}}) - \int_{\partial K} \widehat{\mathbf{v}}^{\partial K} \cdot \underline{\underline{C}}^K \underline{\underline{\xi}} \cdot \mathbf{n} = 0 \end{cases}$$

We define :

$$\begin{aligned} \widehat{\mathbf{v}}^F &= \lambda^F, & \forall F \in \mathcal{F}_h, \\ \widehat{\underline{\underline{\sigma}}}^{\partial K} \cdot \mathbf{n} &= \underline{\underline{\sigma}}^K \cdot \mathbf{n} - \tau \mathbf{l}(\mathbf{v}^K - \lambda^{\partial K}), & \text{on } \partial K \end{aligned}$$

where τ is the stabilization parameter ($\tau > 0$)

HDG formulation of the equations

Local HDG formulation

We replace $\widehat{\mathbf{v}}^K$ and $(\widehat{\underline{\underline{\sigma}}}^K \cdot \mathbf{n})$ by their definitions into the local equations

$$\left\{ \begin{array}{l} \int_K i\omega \rho^K \mathbf{v}^K \cdot \mathbf{w} + \int_K \underline{\underline{\sigma}}^K : \nabla \mathbf{w} - \int_{\partial K} \underline{\underline{\sigma}}^K \cdot \mathbf{n} \cdot \mathbf{w} \\ \quad + \int_{\partial K} \tau \mathbf{l} \left(\mathbf{v}^K - \lambda^{\partial K} \right) \cdot \mathbf{w} = 0 \\ \int_K i\omega \underline{\underline{\sigma}}^K : \underline{\underline{\xi}} + \int_K \mathbf{v}^K \cdot \nabla \cdot \left(\underline{\underline{C}}^K \underline{\underline{\xi}} \right) - \int_{\partial K} \lambda^{\partial K} \cdot \underline{\underline{C}}^K \underline{\underline{\xi}} \cdot \mathbf{n} = 0 \end{array} \right.$$

HDG formulation of the equations

Local HDG formulation

$$\left\{ \begin{array}{l} \int_K i\omega \rho^K \mathbf{v}^K \cdot \mathbf{w} - \int_K (\nabla \cdot \underline{\underline{\sigma}}^K) \cdot \mathbf{w} + \int_{\partial K} \tau \mathbf{l} (\mathbf{v}^K - \lambda^{\partial K}) \cdot \mathbf{w} = 0 \\ \int_K i\omega \underline{\underline{\sigma}}^K : \underline{\underline{\xi}} + \int_K \mathbf{v}^K \cdot \nabla \cdot (\underline{\underline{c}}^K \underline{\underline{\xi}}) - \int_{\partial K} \lambda^{\partial K} \cdot \underline{\underline{c}}^K \underline{\underline{\xi}} \cdot \mathbf{n} = 0 \end{array} \right.$$

HDG formulation of the equations

Transmission condition

In order to determine λ^K , the continuity of the normal component of $\underline{\hat{\sigma}}^K$ is weakly enforced, rendering this numerical trace conservative :

$$\int_F \llbracket \underline{\hat{\sigma}}^K \cdot \mathbf{n} \rrbracket \cdot \eta = 0$$

HDG formulation of the equations

Transmission condition

In order to determine λ^K , the continuity of the normal component of $\underline{\hat{\sigma}}^K$ is weakly enforced, rendering this numerical trace conservative :

$$\int_F \llbracket \underline{\hat{\sigma}}^K \cdot \mathbf{n} \rrbracket \cdot \eta = 0$$

Replacing $(\underline{\hat{\sigma}}^K \cdot \mathbf{n})$ and summing over all faces, the transmission condition becomes :

$$\sum_{K \in \mathcal{T}_h} \int_{\partial K} (\underline{\hat{\sigma}}^K \cdot \mathbf{n}) \cdot \eta - \sum_{K \in \mathcal{T}_h} \int_{\partial K} \tau \mathbf{l} (\mathbf{v}^K - \lambda^{\partial K}) \cdot \eta = 0$$

HDG formulation of the equations

Global HDG formulation

$$\left\{ \begin{array}{l} \int_K i\omega \rho^K \mathbf{v}^K \cdot \mathbf{w} - \int_K (\nabla \cdot \underline{\underline{\sigma}}^K) \cdot \mathbf{w} + \int_{\partial K} \tau \mathbf{l} (\mathbf{v}^K - \lambda^{\partial K}) \cdot \mathbf{w} = 0 \\ \int_K i\omega \underline{\underline{\sigma}}^K : \underline{\underline{\xi}} + \int_K \mathbf{v}^K \cdot \nabla \cdot (\underline{\underline{c}}_K \underline{\underline{\xi}}) - \int_{\partial K} \lambda^{\partial K} \cdot \underline{\underline{c}}_K \underline{\underline{\xi}} \cdot \mathbf{n} = 0 \\ \sum_{K \in \mathcal{T}_h} \int_{\partial K} (\underline{\underline{\sigma}}^K \cdot \mathbf{n}) \cdot \eta - \sum_{K \in \mathcal{T}_h} \int_{\partial K} \tau \mathbf{l} (\mathbf{v}^K - \lambda^{\partial K}) \cdot \eta = 0 \end{array} \right.$$

Discretization of the HDG formulation

Local HDG formulation

$$\begin{cases} \int_K i\omega \rho^K \mathbf{v}^K \cdot \mathbf{w} - \int_K (\nabla \cdot \underline{\underline{\sigma}}^K) \cdot \mathbf{w} + \int_{\partial K} \tau \mathbf{l} (\mathbf{v}^K - \lambda^{\partial K}) \cdot \mathbf{w} = 0 \\ \int_K i\omega \underline{\underline{\sigma}}^K : \underline{\underline{\xi}} + \int_K \mathbf{v}^K \cdot \nabla \cdot (\underline{\underline{C}}^K \underline{\underline{\xi}}) - \int_{\partial K} \lambda^{\partial K} \cdot \underline{\underline{C}}^K \underline{\underline{\xi}} \cdot \mathbf{n} = 0 \end{cases}$$

We define :

$$\underline{\underline{W}}^K = (\underline{v}_x^K, \underline{v}_z^K, \underline{\sigma}_{xx}^K, \underline{\sigma}_{zz}^K, \underline{\sigma}_{xz}^K)^T$$

$$\underline{\underline{\Lambda}} = (\underline{\Lambda}^{F_1}, \underline{\Lambda}^{F_2}, \dots, \underline{\Lambda}^{F_{n_f}})^T, \text{ where } n_f = \text{card}(\mathcal{F}_h)$$

Discretization of the local HDG formulation

$$\underline{\underline{A}}^K \underline{\underline{W}}^K + \underline{\underline{C}}^K \underline{\underline{\Lambda}} = 0$$

Discretization of the HDG formulation

Transmission condition

$$\sum_{K \in \mathcal{T}_h} \int_{\partial K} (\underline{\underline{\sigma}}^K \cdot \mathbf{n}) \cdot \eta - \sum_{K \in \mathcal{T}_h} \int_{\partial K} \mathbf{s} (\mathbf{v}^K - \lambda^{\partial K}) \cdot \eta = 0$$

Discretization of the transmission condition

$$\sum_{K \in \mathcal{T}_h} [\mathbb{B}^K \underline{w}^K + \mathbb{L}^K \underline{\Lambda}] = 0$$

Discretization of the HDG formulation

Transmission condition

$$\sum_{K \in \mathcal{T}_h} \left[\mathbb{B}^K \underline{W}^K + \mathbb{L}^K \underline{\Lambda} \right] = 0$$

Local HDG scheme

$$\mathbb{A}^K \underline{W}^K + \mathbb{C}^K \underline{\Lambda} = 0$$

Discretization of the HDG formulation

Transmission condition

$$\sum_{K \in \mathcal{T}_h} \left[\mathbf{B}^K \underline{W}^K + \mathbf{L}^K \underline{\Lambda} \right] = 0$$

Expression of \underline{W}^K in terms of $\underline{\Lambda}$

$$\underline{W}^K = -(\mathbf{A}^K)^{-1} \mathbf{C}^K \underline{\Lambda}$$

Discretization of the HDG formulation

Transmission condition

$$\sum_{K \in \mathcal{T}_h} [\mathbf{B}^K \underline{W}^K + \mathbf{L}^K \underline{\Lambda}] = 0$$

Expression of \underline{W}^K in terms of $\underline{\Lambda}$

$$\underline{W}^K = -(\mathbf{A}^K)^{-1} \mathbf{C}^K \underline{\Lambda}$$

Global HDG system

$$\mathbf{K} \underline{\Lambda} = 0$$

$$\text{with } \mathbf{K} = \sum_{K \in \mathcal{T}_h} \left[-\mathbf{B}^K (\mathbf{A}^K)^{-1} \mathbf{C}^K + \mathbf{L}^K \right]$$

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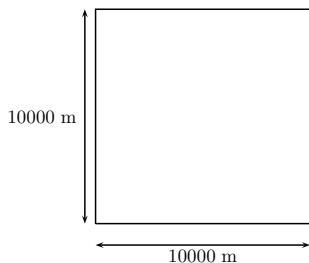
Plane wave in an homogeneous medium

Disk-shaped scatterer problem

Marmousi test-case

Conclusions-Perspectives

Plane wave



Computational domain Ω
setting

► Physical parameters :

► $\rho = 2000 \text{ kg.m}^{-3}$

► $\lambda = 16 \text{ GPa}$

► $\mu = 8 \text{ GPa}$

► Plane wave :

$$u = \nabla e^{i(k \cos \theta x + k \sin \theta y)}$$

where $k = \frac{\omega}{v_p}$

► $\theta = 0$

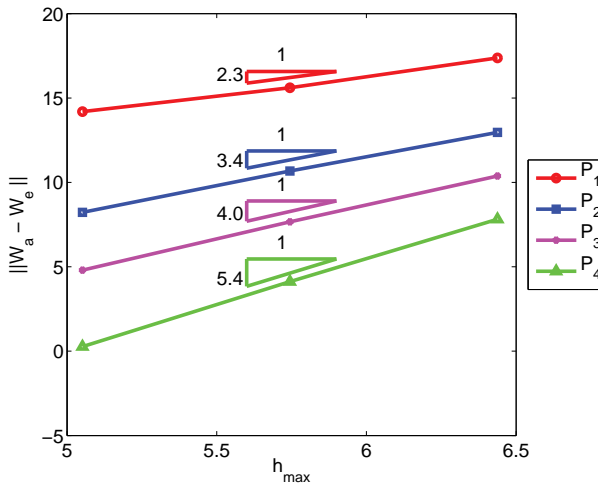
► Three meshes :

► 3000 elements

► 10000 elements

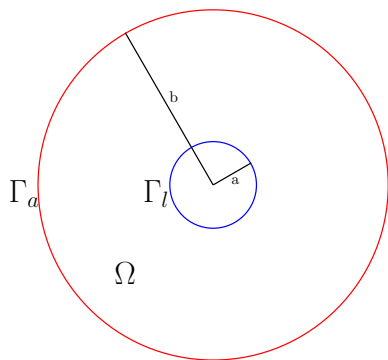
► 45000 elements

Plane wave



Convergence order of the HDG scheme

Disk-shaped scatterer problem



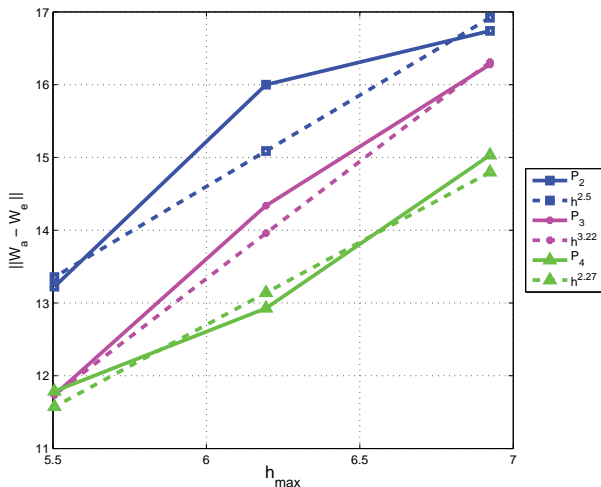
Computational domain Ω
setting

- ▶ $a = 2000.0m$ and $b = 8000.0m$
- ▶ Physical parameters in Ω :
 - ▶ $\rho = 1kg.m^{-3}$
 - ▶ $\lambda = 8GPa$
 - ▶ $\mu = 4GPa$
- ▶ Γ_l free surface boundary :

$$\underline{\underline{\sigma}}\mathbf{n} = 0$$
- ▶ Γ_a absorbing boundary :

$$\underline{\underline{\sigma}}\mathbf{n} = v_p(\mathbf{v} \cdot \mathbf{n})\mathbf{n} + v_s(\mathbf{v} \cdot \mathbf{t})\mathbf{t}$$
- ▶ Three meshes :
 - ▶ 1200 elements
 - ▶ 5400 elements
 - ▶ 22000 elements

Disk-shaped scatterer problem



Convergence order of the HDG scheme

Disk-shaped scatterer problem

Elements	Order	CPU Time (s)			Memory (MB)		
		HDG	UDG	IPDG	HDG	UDG	IPDG
1200	2	0.7			32		
5100	2	3.0			161		
21000	2	14.0			728		
1200	3	1.7			57		
5100	3	7.6			283		
21000	3	34.8			1284		
1200	4	3.9			86		
5100	4	17.7			430		
21000	4	79.1			1953		

Disk-shaped scatterer problem

Elements	Order	CPU Time (s)			Memory (MB)		
		HDG	UDG	IPDG	HDG	UDG	IPDG
1200	2	0.7	2.6	2.4	32	269	70
5100	2	3.0	15.0	11.9	161	1360	369
21000	2	14.0	94.8	58.0	728	6578	1857
1200	3	1.7	5.4	6.8	57	525	190
5100	3	7.6	38.8	35.9	283	2921	1017
21000	3	34.8	252.0	197.8	1284	14131	5126
1200	4	3.9	10.5	15.7	86	895	428
5100	4	17.7	67.0	87.9	430	4537	2279
21000	4	79.1	452.8	520.7	1953	21186	11503

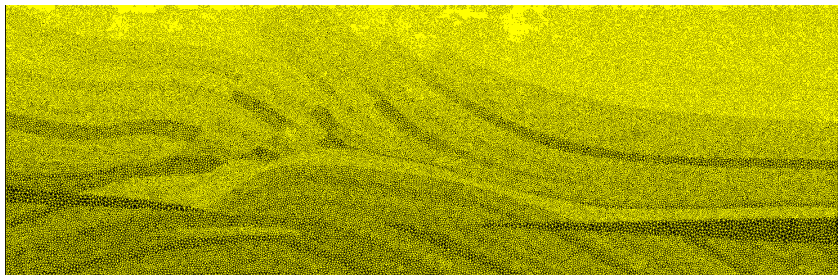
Disk-shaped scatterer problem

Elements	Order	CPU Time			Memory		
		HDG	UDG	IPDG	HDG	UDG	IPDG
1200	2	1	3.7	3.4	1	8.4	2.2
5100	2	1	5.0	4.0	1	8.4	2.3
21000	2	1	6.8	4.1	1	9.0	2.6
1200	3	1	3.1	4.0	1	9.2	3.3
5100	3	1	5.1	4.7	1	10.3	3.6
21000	3	1	7.2	5.7	1	11.0	4.0
1200	4	1	2.7	4.0	1	10.4	5.0
5100	4	1	3.8	5.0	1	10.5	5.3
21000	4	1	5.7	6.6	1	10.8	5.9

Disk-shaped scatterer problem

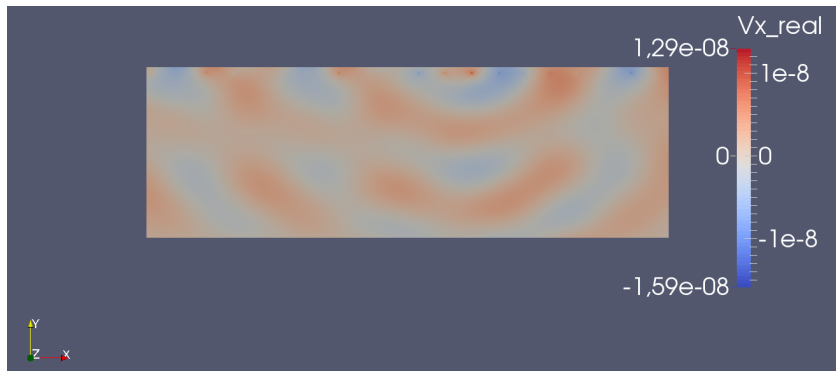
Elements	Order	CPU Time			Memory		
		HDG	UDG	IPDG	HDG	UDG	IPDG
1200	2	1	3.7	3.4	1	8.4	2.2
5100	2	1	5.0	4.0	1	8.4	2.3
21000	2	1	6.8	4.1	1	9.0	2.6
1200	3	1	3.1	4.0	1	9.2	3.3
5100	3	1	5.1	4.7	1	10.3	3.6
21000	3	1	7.2	5.7	1	11.0	4.0
1200	4	1	2.7	4.0	1	10.4	5.0
5100	4	1	3.8	5.0	1	10.5	5.3
21000	4	1	5.7	6.6	1	10.8	5.9

Marmousi test-case



Computational domain Ω composed of 235000 triangles

Parallel results for the Marmousi test-case with the HDG-P2 scheme



Parallel results for the Marmousi test-case with the HDG-P2 scheme

	CPU Time construction (s)	CPU Time resolution. (s)	Maximum Memory (MB)
sequential	67	133	9927
2 proc. (2/1)	32	93	5892
4 proc. (2/2)	15	56	3340
8 proc. (4/2)	8	38	2092
16 proc. (4/4)	4	39	3695
32 proc. (4/8)	2	21	1312
64 proc. (8/8)	1	19	893

Parallel results for the Marmousi test-case with the HDG-P3 scheme

	CPU Time construction (s)	CPU Time resolution (s)	Maximum Memory (MB)
sequential	207	321	17631
2 proc. (2/1)	96	196	10048
4 proc. (2/2)	47	116	5849
8 proc. (4/2)	23	75	5205
16 proc. (4/4)	12	72	4628
32 proc. (4/8)	6	42	1689
64 proc. (8/8)	3	33	1229

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Conclusions-Perspectives

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Perspectives

- ▶ Develop 3D Upwind flux DG and HDG formulations for Helmholtz equations
- ▶ Solution strategy for the HDG linear system

Thank you !

The logo for Inria, featuring the word "inria" in a stylized, cursive font with a color gradient from red to orange. Above the "ria" part, the words "informatiques" and "mathématiques" are written in a smaller, black, sans-serif font, separated by a small red dot.

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