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ABSTRACT

In this paper, a numerical model for hermetic compressors is presented. The compressor is modeled as a number of volumes, which transfer Mass and Heat through orifices, pipes and walls. Properties of the refrigerant in these volumes are uniform but depend upon time. Flow through the pipes is modeled as a one-dimensional unsteady compressible flow and a Lax-Wendroff numerical scheme is used to solve the conservation equations. Reed valves are modelled with a superposition of free vibration modes and the instantaneous mass flow rate through them is calculated by using the compressible flow-model and the corresponding discharge coefficient. The discharge coefficients have been measured in a steady flow rig for air. In order to reduce the calculation time, a method has been envisaged to uncouple the calculation of unsteady flow through the volumes and pipes from the calculation of the thermal problem, and then correct the properties of the fluid for the next calculation cycle

INTRODUCTION

The compressor, together with the evaporator, are the most important elements of a refrigeration unit. The behavior and performance of a compressor depend on a great number of design parameters and operating conditions. The calculation and also the analysis of this component is very complex, especially due to the unsteady flow of the vapor through it and also due to the complexity of the flow through the valves and pipes. The computer modeling of compressors should, in the future, have an important role in the development and optimization of their design, and also in the evolution and adaptation of new technologies to the future refrigeration systems.

Several levels of modeling techniques can be applied to the calculation of a compressor. The view of the authors, who have long experience in the field of internal combustion engine modeling, is that a combination of filling and emptying modeling for the volumes, including the cylinder, and a gas dynamic calculation in the pipes, combined with the calculation of the dynamics of the valves and the use of measured values of the effective section of the port passages, could be the way to lead to a balanced combination between accuracy and computing time.

This paper presents the details and first results of such a model, developed by the authors for the performance calculation of piston compressors, although the basis of the submodels can be applied to any other volumetric compressor.

NOMENCLATURE

A	Area
C	Damping coefficient
C_d	Discharge coefficient
c_p	Specific heat
E	Modulus of elasticity of the reed
G	Friction Force
h	Enthalpy
I	Inertia moment
$\dot{m}$	Mass flow rate
$m(x)$	Mass per length unit
m	Mass
P	Pressure
$\dot{Q}$	Heat rate
t	Time
T	Temperature
U	Internal energy
u	Specific internal energy
V	Volume
v	velocity
$\dot{W}$	Work rate
y	Reed deflection
α	Heat transfer coefficient
ρ	Density
γ	Ratio of specific heats
Index :	
i	Inlet
o	Outlet
p	Periodic
oo	Upstream stagnant conditions

Submodels description

The global model of the compressor is composed of several submodels which refer to the different elements which have an influence on its behavior: *volumes, walls, reed valves, pipes and orifices*. The scheme of a hermetic compressor following that categorization is presented in fig.1. In the following, a description of the main characteristics of the different submodels is given.

A1 Evaporator Atmosphere
O1 Inlet Orifice
C1 Suction Pipe
V1 Casing Volume
V2 Suction Volume
L1 Inlet Reed Valve
V3 Cylinder
O2 Leakage Orifice
L2 Outlet Reed Valve
V4 Discharge Volume
C2 Discharge Pipe
A2 Condenser Atmosphere

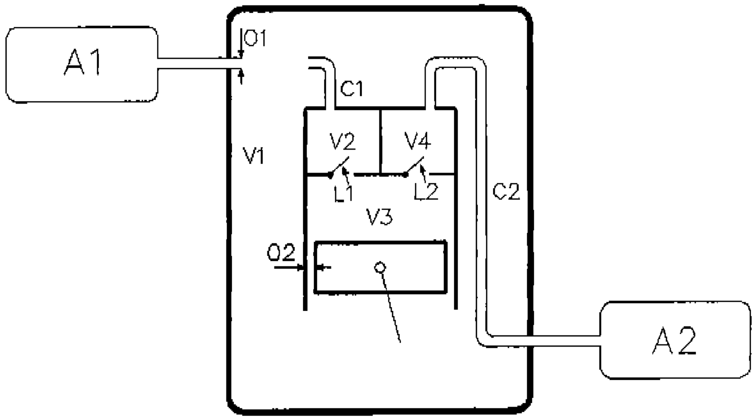

Figure 1 Scheme of elements

Volumes

In these elements a zero-dimensional model has been used to model the behavior of the refrigerant inside them, i.e. the thermodynamic properties of the gas are uniform throughout the volume but time-dependent [2]. Volumes can inter-exchange mass through orifices and pipes, and transfer heat to the walls. The inlet (suction) and discharge volumes have constant capacity, while the cylinder and the casing (if it is applicable) have time-dependent capacities following the kinematics of the reciprocating mechanism.

The mass and energy conservation equations in the volumes are solved using the real gas properties:

1) Mass conservation

$$\frac{dm}{dt}=\sum\left.\frac{dm}{dt}\right)_i-\sum\left.\frac{dm}{dt}\right)_o \tag{1}$$

2) Energy conservation

$$\frac{du}{dt}=\frac{1}{m}\left[\dot{Q}-P\frac{dV}{dt}+\sum\left.\frac{dm}{dt}\right)_i h_i-\sum\left.\frac{dm}{dt}\right)_o h_o-u\frac{dm}{dt}\right] \tag{2}$$

Pipes

In these elements a one-dimensional compressible model has been adopted. The 1-D Euler system of conservation laws is solved:

$$\frac{\partial W}{\partial t}+\frac{\partial F}{\partial x}=B+C \tag{3}$$

where

$$W=\begin{pmatrix}\rho A\\ \rho vA\\ \rho A(u+\frac{v^2}{2})\end{pmatrix}\quad F=\begin{pmatrix}\rho vA\\ \rho v^2A+pA\\ \rho vA(u+\frac{p}{\rho}+\frac{v^2}{2})\end{pmatrix}\quad B=\begin{pmatrix}0\\ p\frac{dA}{dx}\\ 0\end{pmatrix}\quad C=\begin{pmatrix}0\\ -G\rho A\\ \dot{q}\,\rho A\end{pmatrix} \tag{4}$$

The solution for this hyperbolic system of equations is obtained by the two-step Lax-Wendroff method [1], which is an explicit second order numerical scheme in space and time.

The flow at the pipe ends is calculated with the method of characteristics and the appropriate boundary condition depending on the type of end connection: pipe - volume or anechoic end

Orifices

An orifice inter-connects two volumes and allows enthalpy and mass transfer between them. The mass flow rate through an orifice is calculated by the compressible homentropic ideal gas model and a discharge coefficient. For non-choked flow:

$$\dot{m} = C_d\, A\, \rho_{\text{hom entropic}} \sqrt{\frac{2\gamma}{\gamma - 1}} \sqrt{\frac{P_{00}}{\rho_{00}}} \sqrt{1 - \left(\frac{P}{P_{00}}\right)^{\frac{\gamma-1}{\gamma}}} \tag{5}$$

For choked flow we substitute the pressure ratio by the critical pressure ratio

$$\frac{P}{P_{00}} = \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma}{\gamma-1}} \tag{6}$$

In these equations the ratio of specific heats is taken as the ideal ratio of the real gas.

Reed valves

The cylinder inlet and outlet flows are controlled by the reed valves motion. Their instantaneous position is determined by the integration of the dynamics of the valve, following the evolution of the instantaneous pressure difference existing across them.

The reed valve is modeled as a cantilever beam. In the model, the reed displacement is considered to be the superposition of the solution for the three first separate modes associated with the reed in free vibration. A solution based on the first five main modes has also been tested but no major difference was obtained in the results.

The dynamics of a beam of constant cross section loaded by a distributed force p(x,t) can be calculated by the following equation **[3]** :

$$\frac{\partial^2\left(EI\frac{\partial^2\left(y(x,t)\right)}{\partial x^2}\right)}{\partial x^2} + m(x)\frac{\partial^2 y(x,t)}{\partial t^2} - c\frac{\partial y(x,t)}{\partial t} = p(x,t) \tag{7}$$

This equation is solved by a fourth order Runge-Kutta method to find the displacement at all points along the petal.

In order to compute the mass flow rate through the reed valve, the effective section for every lift must be determined through experiments. These empirical values have been determined every 1 mm for an air flow in a steady flow rig, for both flow directions: normal and reversed.

Walls

The fluid in every volume and pipe is able to exchange heat with the walls through forced or natural convection formulae. Figure 2 shows the different heat paths for the compressor shown in figure 1. Wall temperatures are assumed constant through a calculation cycle (one crankshaft revolution) due to its great thermal inertia and then they are recalculated in order to balance the different heat flows as will be described below.

CALCULATION METHOD

An initial guess is needed for every unknown variable at the beginning of the calculations ; Properties in suction volumes and pipes are set to the evaporator atmosphere conditions and at the discharge parts are set to the condenser atmosphere conditions. Velocity is set to zero everywhere.

The time step is then calculated as the minimum value among those allowed by the CFL (Courant-Friederich-Levy) criterium at every point in the pipes. A maximum time step, corresponding to a half degree of crankshaft angle, has been used as the upper boundary.

Then, the flow inside the pipes is calculated, and also the boundary conditions at the end of the pipes assuming that the fluid properties at the connected volumes do not significantly vary during the time step (Euler 1st order approach). This calculation makes possible to determine the mass flow rate at the end of the pipes. The variation of the fluid properties at the volumes is calculated by the use of a Runge Kutta method.

Finally, the new position of the reed valves and the corresponding effective area are calculated, and a new iteration starts. This procedure is repeated until a periodic solution is obtained.

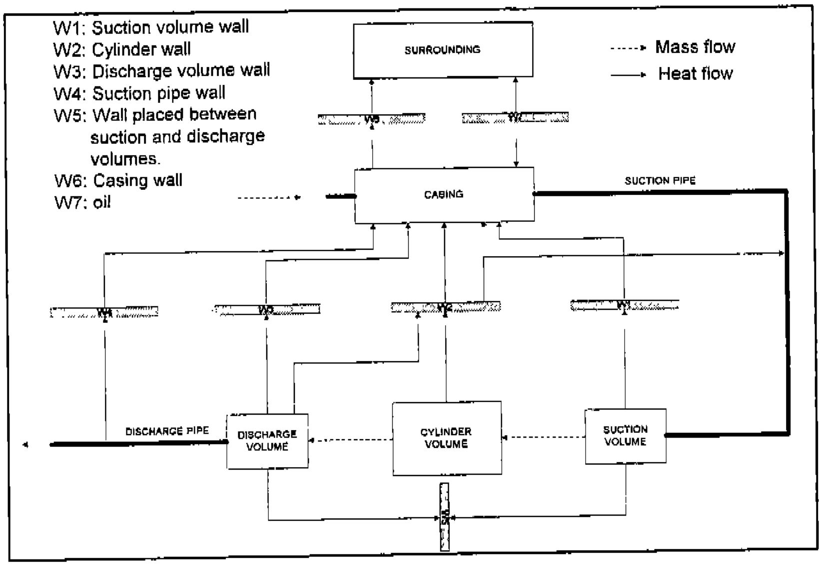

Figure 2 Heat and mass transfer paths

Acceleration of the convergence to the periodic solution

Another important aspect which must be considered in the modeling of a refrigeration compressor is the fact that two different phenomena take place at the same time: the unsteady gas dynamics of the flow produced by the displacement of the piston (hydraulic problem), and the heat transferred among the walls and the flow (thermal problem). The characteristic times of both phenomena are very different. Typically, the unsteady flow which is controlled by the piston motion and the action of the pressure waves is very fast; it takes approximately 5 cycles to get a more or less periodical flow pattern. However, thermal inertia of the fluid makes the thermal problem much slower and requires a much greater time to reach a steady solution. Therefore, the modeling of a compressor from the unsteady point of view requires the decoupling of the calculation of both phenomena and the progressive coupling of their solution in order to get reasonable calculation times.

Since the time step is strictly controlled by the hydraulic problem, the convergence of the calculations towards the periodical solution when both problems are solved simultaneously is very slow.

Therefore, the unsteady calculation at volumes and pipes is performed for every crankshaft revolution considering that wall temperatures do not vary. Then, once a cycle is completed, a steady energy balance is applied to every element (volumes, pipes and walls) thus permitting the calculation of the new wall and fluid temperatures, assuming for the mass and enthalpy flow rates the corresponding last cycle averaged values. Finally, the density is recalculated in order to match with the averaged pressure and the new value for the temperature, and the procedure is repeated until convergence is attained.

An example of how to combine both calculations is shown below for the case in which the wall temperature is considered constant and the properties at the volume are corrected in order to accelerate the convergence.

The integration of equation (2) for the volume over a cycle leads to :

$$\int_{Cycle} \frac{dU}{dt} dt = \int_{Cycle} \dot{Q}\, dt - \int_{Cycle} \dot{W}\, dt + \int_{Cycle} \sum \dot{m} h_i - \int_{Cycle} \sum \dot{m} h_o \tag{8}$$

When the periodical solution has been reached, the result of the first integral over a cycle must be zero, therefore the following equation should be satisfied:

$$\int_{Cycle} \dot{Q}\, dt - \int_{Cycle} \dot{W}\, dt + \int_{Cycle} \sum \dot{m} h_i - \int_{Cycle} \sum \dot{m} h_o = 0 \tag{9}$$

or, in mean values

$$\overline{\dot{Q}_p} - \overline{\dot{W}_p} + \overline{\sum \dot{m} h)_p} = 0 \tag{10}$$

The variables calculated over a cycle do satisfy equation (8) but not equation (9) till the periodical solution is attained. The averaged temperature corresponding to the periodical solution can be considered as a correction of the averaged temperature obtained from the last calculated cycle :

$$\bar{T}_p = \bar{T} + \Delta T \tag{11}$$

then, considering that the wall temperature is constant, the periodical solution for the heat transferred to the wall should differ from the cycle averaged value in the following amount :

$$\bar{\dot{Q}}_p = \bar{\dot{Q}} - \alpha A_w \Delta T \tag{12}$$

On the other hand, the enthalpy correction for the outlet flow (inlet flow would not be affected by the correction of the inner volume temperature) could be estimated by a first Taylor series expansion :

$$h_p = h(T_p) = h(T + \Delta T) + \frac{\partial h}{\partial T}\Delta T + O(\Delta T^2) \tag{13}$$

Therefore, the enthalpy flow rate at the outlet should be modified in the following way :

$$\left.\overline{\dot{m}h)_p}\right]_0 = \frac{1}{\tau_{cycle}} \int (h(T) + C_p \Delta T)\dot{m}_0 \, dt = \left.\overline{\dot{m}h}\right]_0 + c_p \Delta T \left.\overline{\dot{m}}\right]_0 \tag{14}$$

Then, if the amount of work $\dot{W}$ and the enthalpy inlet flow rate are supposed to be close to their final solution, equation (10) leads to :

$$\bar{\dot{Q}} - \alpha A_w \Delta T - \bar{\dot{W}} + \sum \left.\overline{\dot{m}h} - c_p \Delta T \overline{\dot{m}}\right]_0 = 0 \tag{15}$$

Finally, the following expression for the temperature correction ΔT is obtained:

$$\Delta T = \frac{\bar{\dot{Q}} - \bar{\dot{W}} + \sum \overline{\dot{m}h}}{c_p \left.\overline{\dot{m}}\right]_0 + \alpha A_w} \tag{16}$$

As can be observed, this condition tends to cancel when the periodical solution is reached.

This strategy of combining the calculation of the hydraulic problem with the calculation of the thermal problem has lead to a very important reduction on the number of cycles required to reach the periodical solution : typically of more than an order of magnitude. In figure 3 the evolution of the solution following the described strategy is compared with the simultaneous calculation of both phenomena. The variation of the temperature shows the typical asymptotic behavior of the temperature in a thermal problem.

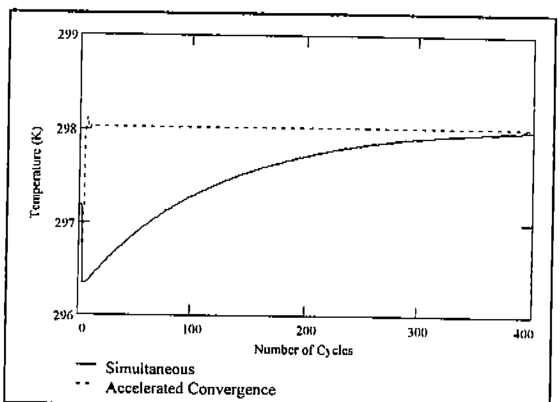

Figure 3 Evolution of the calculated mean temperature of the suction volume with the described methodologies

RESULTS

A compressor test rig is currently under construction at the authors' laboratory, and a full comparison between measurements and the calculated results obtained with the described model is scheduled for the near future, including global performance, operating temperatures and instantaneous pressure.

Figure 4 shows a sample of the calculated results for a small 1 cylinder hermetic compressor. Instantaneous pressure at the cylinder and at the suction and discharge volumes are presented

Figure 5 shows the influence on the pressure at the discharge volume, of the calculation of the gas dynamics phenomena in the pipes. As can be observed, the evolution of the pressure is very different, mainly provoking an increase in the back-pressure and forcing a delay in the discharge process of the volume following the cylinder release. The calculated instantaneous pressure at the middle of the discharge pipe is also included in figure 5, showing a strong peak-to-peak variation.

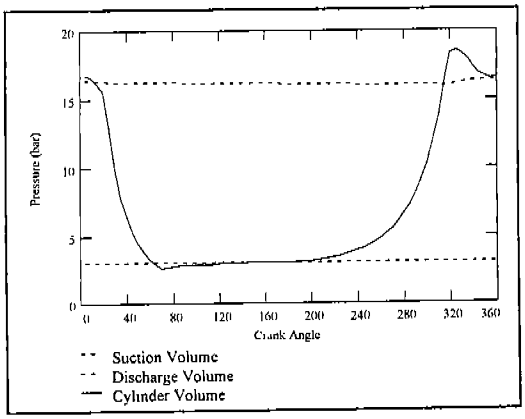

Figure 4 Instantaneous pressure results for the cylinder and the suction and discharge volume

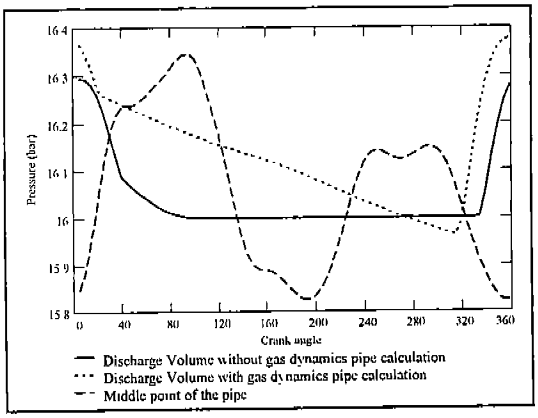

Figure 5 Instantaneous pressure of the discharge volume and discharge pipe with gas dynamics calculations

CONCLUSIONS

- A global model for displacement compressors has been developed which is able to take simultaneously into account the dynamics of the reed valves and the gas-dynamics phenomena in the pipes.
- The calculation of the volumes is based on the zero-dimensional approach, so that the mass and energy equations are transformed in a system of 1^{st} order differential equations. The Runge-Kutta method is used for its integration.
- The calculation in the pipes is based on the one-dimensional approach. The two step Lax & Wendroff finite difference scheme is used to integrate the corresponding 1D Euler system of equations.
- The reed valves motion is calculated as the superposition of the motion of the three main vibration modes. The Runge-Kutta method is also used to solve the corresponding equations.
- A special strategy has been conceived in order to accelerate the convergence of the calculations to the final periodical solution. The key-point has been to de-couple the solution of the flow ("hydraulic problem") from the thermal problem and to introduce a correction step for the temperature after every cycle of unsteady calculation. The improvement in the convergence velocity has been very important, reaching the periodical solution of the problem in just 6-7 revolutions.

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