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# DYNAMIC RESPONSE OF COMPRESSOR VALVE SPRINGS TO IMPACT LOADING

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## ABSTRACT

Compression springs are as important to a self acting valve as a self acting valve is to a reciprocating compressor. Valve springs are not only subjected to dynamic loading but also to impact loading during the valve opening and closing events. Hence proper design and selection of helical springs should consider modeling the dynamic and impact response. This paper describes a computational method which was developed to simulate the behavior of helical springs subjected to impact loading and its application for predicting the dynamic stresses along the length of the spring. This study has underlined the importance of dynamic response analysis, and provided a tool for evaluation of various designs.

## 1. NOMENCLATURE

$r$  -- spring radius  
 $d$  -- diameter of spring wire  
 $p$  -- pitch angle  
 $\rho$  -- the density of spring material  
 $G$  -- shear modulus  
 $E$  -- Young's modulus  
 $J$  -- the moment of inertia of the spring  
 $I_m$  -- mass moment of inertia  
per unit length of spring wire  
 $s$  -- arc length  
 $L$  -- total helix length  
 $\psi$  -- rotation angle of spring wire  
 $\kappa$  -- curvature of spring  
 $\tau$  -- torsion of spring  
 $F, P$  -- force  
 $\sigma$  -- stress  
 $y$  -- axial displacement  
 $n$  -- number of active coils  
 $H_0$  -- spring free height  
 $H_s$  -- spring preset height  
 $a$  -- velocity of elastic wave propagation inside  
the spring  
 $\beta$  -- damping coefficient  
 $t$  -- time  
 $V_0$  -- valve impact velocity against guard

## 2. INTRODUCTION

Helical compression springs play a very important role in compressor valves. Springs not only control the dynamics of the valve and thus the performance of the compressor but also determine the reliability of its operation. Hence selection and design of valve springs is of utmost importance in the efficient and reliable operation of a compressor.

Valve springs are subjected to dynamic and impact loading during the valve opening and closing events. Such a loading results in a phenomenon called "surge", which is a response in the form of initiation and propagation of a wave similar to an acoustic wave. Therefore proper design and selection of helical springs should consider modeling this type of response. This study is focused on computational simulation of a compressor valve spring under impact loading and its application for predicting the dynamic stresses along the length of the spring. Both cases - one with a button inserted between the valve plate and the spring and the other without the button were considered. The main objective of a design would be to select a spring to transmit the expected impact load, without its maximum stress exceeding that allowable for the spring material. For this purpose, the maximum stresses were computed as a function of plate impact velocity and the results are presented for the two cases studied.

Thomson and Tait (1883) pioneered the study of helical springs followed by Love (1927). Their formulae expressing curvature and torsion in terms of pitch angle and helix radius are still widely used. Stokes (1974) used their relationships to study the impact response of springs. It was not until 115 years later, that Lin and Pisano (1988) extended the relationship between those important spring parameters to the more complex modern day spring designs involving variable pitch and helix radius. Wahl (1963) in his review of spring research noted that radial expansion could be analyzed only for helical springs with small constant pitch angle. This was later extended to the large pitch angle case by Wittrick (1966), but he neglected the effect of curvature of the coil and the deformation due to tension and shearing forces. Pearson (1982)

suggested a method of modeling the effect of variable pitch present in the ends of helical springs.

The present work provides a simple computational tool based on a finite difference solution of the equations derived using Hamilton Principle (Lin and Pisano, 1987) for analyzing the dynamic response of helical valve springs subjected to impact loading. Computations done for one particular case supported the observations of Swanobori et al (1985) that the dynamic stresses are larger than static stresses. This emphasizes the need for dynamic stress analysis for spring design and selection.

### 3. THEORETICAL ANALYSIS

The analysis of dynamic response involves the mathematical simulation of the transient event by applying fundamental physical laws, in this case - the principle of conservation of energy. The governing equations were derived using Hamilton Principle (Lin and Pisano, 1987). The equations were reduced to a finite difference form using an implicit method and solved by using appropriate boundary and initial conditions for the two cases - with and without the button.

A computer program was written in C, and it took approximately 20 minutes to obtain a converged solution on an IBM RISC 370 workstation. The optimum time step appeared to depend on the impact velocity with larger velocities requiring smaller time steps.

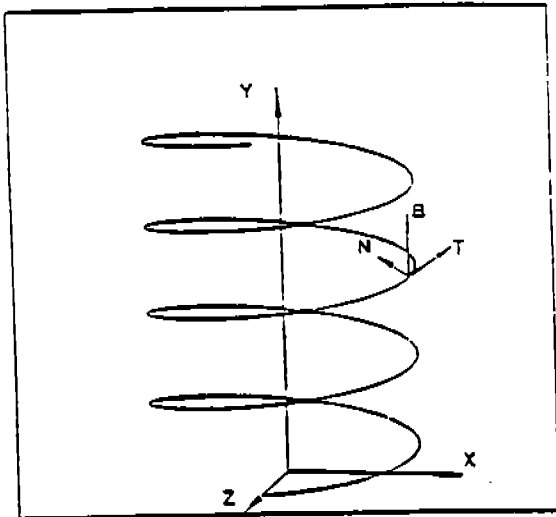


Fig. 1 Helix in local-global coordinate system

#### 3.1 Basic Equation

The equation of motion of a given helical spring along the spring axis is

$$\frac{\partial^2 y}{\partial t^2} + \beta \frac{\partial y}{\partial t} = \alpha^2 \frac{\partial^2 y}{\partial s^2} \quad (1)$$

with general boundary conditions:

$$y(0,t) = f(t) \quad (2)$$

$$y(L,t) = g(t) \quad (3)$$

and initial conditions:

$$y(s,0) = h(s) \quad (4)$$

$$\dot{y}(s,0) = 0 \quad (5)$$

where:  $f(t)$ ,  $g(t)$  and  $h(s)$  are known functions.

Using an implicit finite difference method, and letting;

$$y_{i,j} = y(i\Delta s, j\Delta t) \quad (6)$$

$$0 \leq i \leq M+1, \quad 0 \leq j \leq N$$

Eq. (1) can be transformed to

$$y_{i,j+1} - 2y_{i,j} + y_{i,j-1} = \alpha^2 m^2 / 2 [(y_{i+1,j+1} - 2y_{i,j+1} + y_{i-1,j+1}) + (y_{i+1,j-1} - 2y_{i,j-1} + y_{i-1,j-1})] - \beta \Delta t / 2 (y_{i,j+1} + y_{i,j-1}) \quad (7)$$

where  $m = \Delta t / \Delta s$ , and

$$y_{0,j} = f_j \quad (8)$$

$$y_{M+1,j} = g_j \quad (9)$$

$$y_{i,0} = h_i \quad (10)$$

$$(y_{i,1} - y_{i,-1}) / 2\Delta t = 0 \quad (11)$$

Eq. (7) along with the boundary and initial conditions was solved using a numerical technique.

Then, using the geometrical relationships (Wahl, 1963), the following parameters were obtained: pitch angle  $p$ :

$$\sin(p_{i,j}) = \frac{y_{i,j+1} - y_{i,j-1}}{2\Delta s} \quad (12)$$

dynamic radius:

$$r_{i,j} = \frac{r_0 \cos(p_{i,j})}{\cos(p_0)} \quad (13)$$

where subscript 0 represents the value at time  $t$  equal to zero.

curvature:

$$\Delta \kappa_{i,j} = \frac{\cos^2(p_{i,j})}{r_{i,j}} - \frac{\cos^2(p_0)}{r_0} \quad (14)$$

torsion:

$$\Delta\tau_{i,j} = \frac{\sin(p_{i,j}) \cos(p_{i,j})}{r_{i,j}} - \frac{\sin(p_0) \cos(p_0)}{r_0} \quad (15)$$

force:

$$F = \frac{GJ}{r_{i,j}} \cos(p_{i,j}) (\Delta\tau_{i,j}) - \frac{EI}{r_{i,j}} \sin(p_{i,j}) (\Delta\kappa_{i,j}) \quad (16)$$

and stress:

$$\sigma_{i,j} = \frac{16F_{i,j} r_{i,j}}{\pi d^3} K_c - \sigma_0 \quad (17)$$

#### 4. APPLICATION FOR WITH AND WITHOUT BUTTON CASE

The method was applied for a typical spring with the following specifications:

- $r_0 = 3.823$  mm ( free spring radius )
- $d = 1.0922$  mm ( wire diameter )
- $H_0 = 15.113$  mm ( free height )
- $L = 156.85$  mm ( spring length )
- $n = 6.5$  ( number of coils )
- $H_s = 10.3632$  mm ( preset height )
- $\beta = 100$
- $m_e = 1.362$  g
- ( the lumped spring and button mass )

##### 4.1 The Case without Button

The case without button is relatively simple, the only thing we need to describe is the motion of the moving end as an input boundary condition. This is shown in Fig. 2 for  $V_0 = 10$  m/sec.

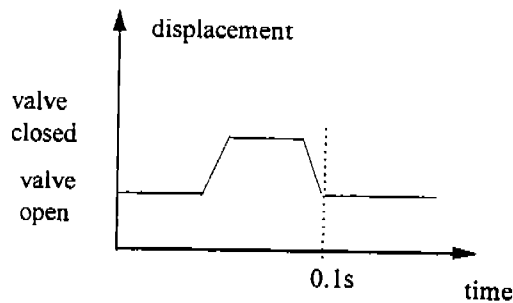


Fig. 2 Moving end displacement vs. time

Fig. 3 and 4 show the response of the spring at various instants of time close to the end of its compression (corresponding to the full open position of the valve). Fig. 3(a) and 4(a) show the spring displacement relative to neutral (steady state) point along its arc length and its propagation with time. Fig.

3(b) and 4(b) show the stress distribution at those instances.

During the initial phase of the spring motion ( $t = 0.0906$  sec), the spring remains close to its steady state position and stress is close to the static case. During the subsequent phase ( $t = 0.0908$  sec), the coil near moving end is squeezed, while the stress near fixed end has not changed yet. At the end of its motion ( $t=0.1$ sec), the fixed end gets squeezed, and shortly after this ( $t = 0.1002$  sec), the stress in the fixed end reaches a maximum value. After this, the "coil squeeze", as well as the compressive stress propagate back and forth before dying out gradually.

##### 4.2 The Case With Button

The case with the button is a little bit complicated than the case without button. The button separates the spring from the plate. Hence, even after the plate hits the guard and stops moving forward, the button continues its travel due to inertia. This button motion was modeled by making a quasi static assumption as follows:

The button - spring system is assumed as a simple one dimensional spring mass system shown as Fig. 5. After the plate stops moving, the button continues to move with the terminal velocity of the plate equal to  $V_0$ .

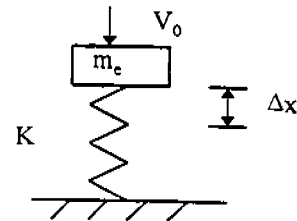


Fig.5 Illustration of button - spring motion after plate impacts the guard

From the principle of conservation of energy, we have

$$\frac{1}{2} m_e V_0^2 = \frac{1}{2} K \Delta x^2 \quad (18)$$

where,  $K$  is the stiffness of the spring

$$K = \frac{P}{\delta} = \frac{G d^4}{8 D^3 n} \quad (19)$$

$\Delta x$  is the distance where the button stops,

$$\Delta x = \sqrt{\frac{m_e}{K}} V_0 \quad (20)$$

The dynamic equation for the system of Fig. 5 can be written as

$$m_e \ddot{x} + Kx = 0. \quad (21)$$

with the initial conditions of

$$t=0, \dot{x}=V_0; \quad (22)$$

and

$$t=t_1, x=0, \dot{x}=\Delta v \quad (23)$$

the solution is

$$x = A \cos \sqrt{\frac{K}{m_e}} t + B \sin \sqrt{\frac{K}{m_e}} t \quad (24)$$

where

$$A = B c t g \sqrt{\frac{m_e}{K}} t_1 \quad (25)$$

$$B = \sqrt{\frac{m_e}{K}} V_0 \quad (26)$$

$$t_1 = \frac{\pi}{2} \sqrt{\frac{m_e}{K}} \quad (27)$$

The boundary condition for the moving end in the form of its displacement with time is shown in Fig. 6. Note the continued motion of the spring due to button inertia, even after the plate impacts the guard.

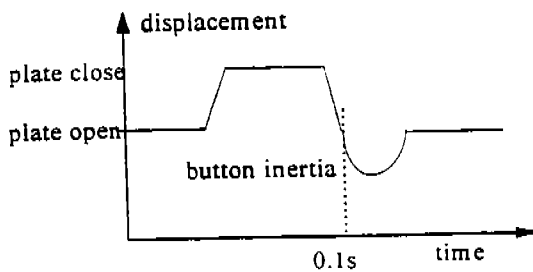


Fig. 6 Moving end displacement vs. time

Fig. 7 - 9 show the spring displacement relative to its neutral position and the stress distribution along its entire length at several instants close to the time when the plate impacts the guard. The motion of the spring in terms of the coil squeezing and its propagation towards the fixed end and the subsequent rebound and gradual decay, as well as the stress propagation and its decay appear to closely resemble the case without button.

## 5. PREDICTION OF MAXIMUM STRESS

The maximum stress over the entire length of the spring during the complete valve event was predicted for several values of plate impact velocity and the

results are shown in Fig. 10 for the two cases studied. Below 1 m/sec, the maximum dynamic stress for both cases appear to be close to the static case, but as the velocity increases above this value, the presence of the button appears to make an increasing impact on the magnitude of the maximum stress. It is also interesting to note that for impact velocities above 9 m/sec (in the case without button), neighboring segments of the coil started clashing, and the number of clashing segments increased with increase in velocity. Also, for the case with button, the clashing started occurring at a much lower velocity, equal to 6 m/sec.

## 6. COMMENTS AND CONCLUSIONS

The following important conclusions emerge from this study:

- i). The maximum stress obtained from dynamic analysis is higher than that obtained from static analysis and the difference appears to increase with impact velocity.
- ii). The maximum stress occurs at the fixed end shortly after the coils get squeezed near that location.
- iii). Segments of neighboring coil start clashing beyond a threshold velocity.
- iv). The button appears to increase the maximum stress by approximately 20% - 30%.
- v). Damping appears to reduce the maximum stress.

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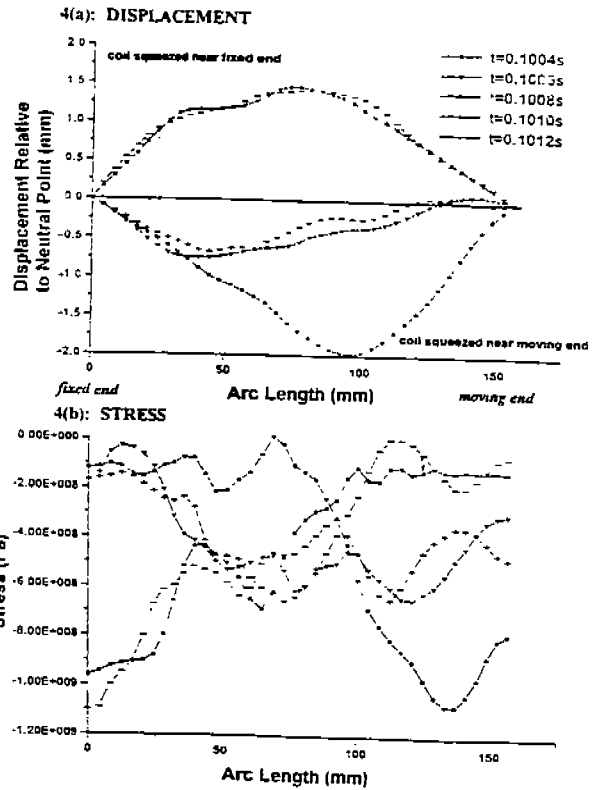


Fig. (4): Spring Dynamic Response for the Case Without Button;  $V_0 = 10$  m/sec

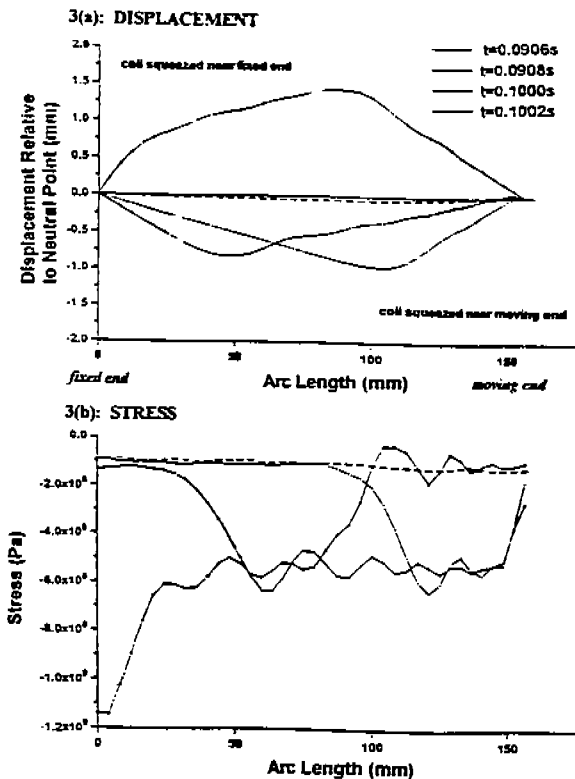


Fig. (3): Spring Dynamic Response for the Case Without Button;  $V_0 = 10$  m/sec

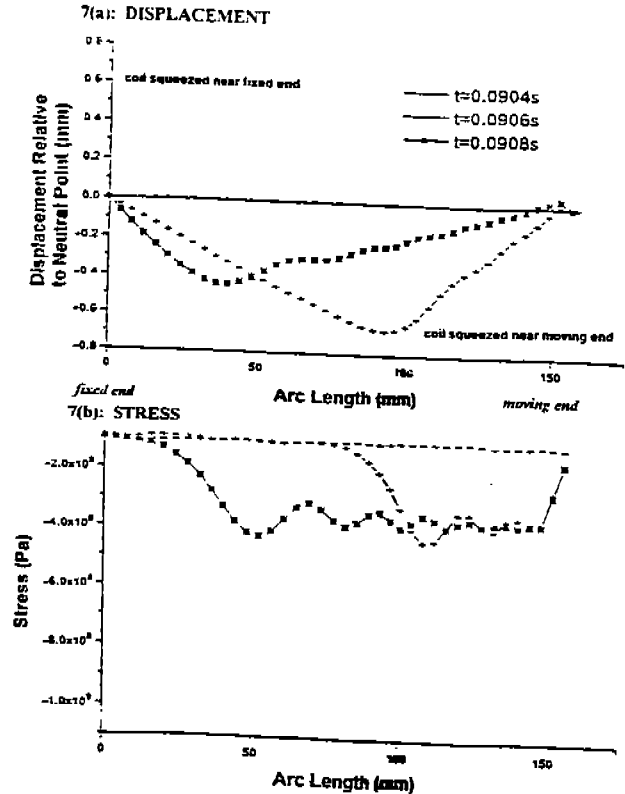


Fig. (7): Spring Dynamic Response for the Case With Button;  $V_0 = 6$  m/sec

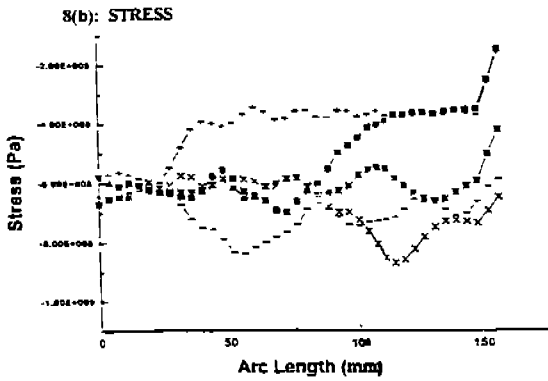
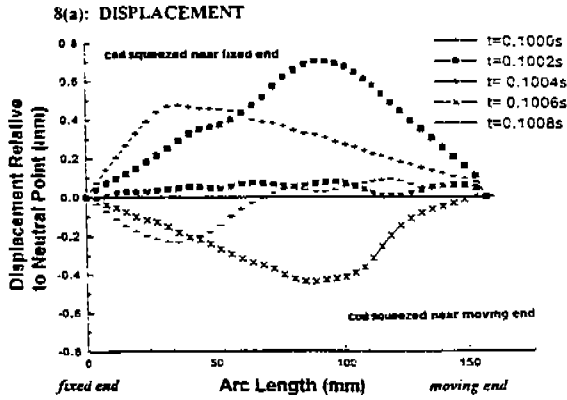


Fig. (8): Spring Dynamic Response for the Case With Button;  $V_0 = 6 \text{ m/sec}$

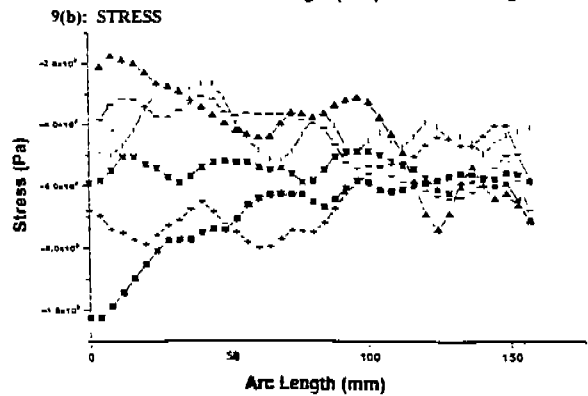
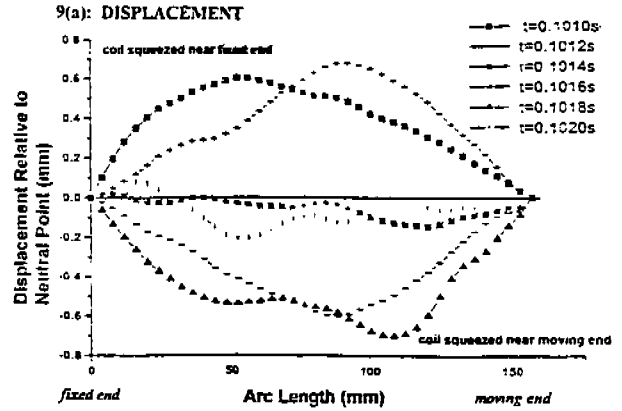


Fig. (9): Spring Dynamic Response for the Case With Button;  $V_0 = 6 \text{ m/sec}$

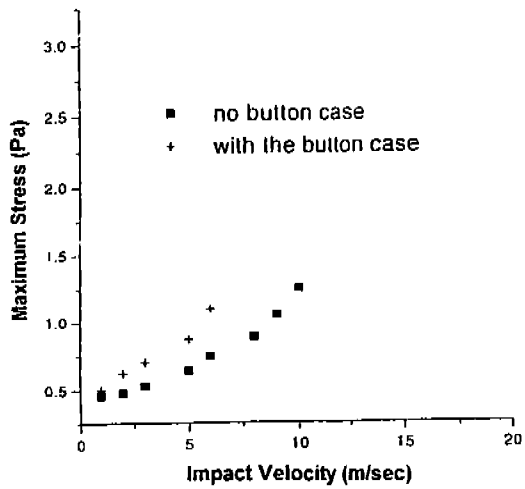


FIG. (10): Comparison of the Variation of Maximum Stress with Impact Velocity for the Two Cases