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# THE CONJUGACY ANALYSIS OF MODIFIED PART
# OF SCROLL PROFILES

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## ABSTRACT

A key and very important problem in the design of a scroll compressor is the wrap modification of the scroll profile. This problem has attracted more and more attention from researchers worldwide and several methods have been used in the past. In this paper a mathematical conjugacy relation derivation and analysis of the new wrap modification method are given, based on my previous paper presented at the 1992 International Compressor Engineering Conference at Purdu [1]. It is hoped that this analysis will make the new modification method of scroll profile more reliable.

## NOMENCLATURE

Ror Rotational radius of crank
a Radius of the base circle of involute
$\gamma$ Modified angle

## INTRODUCTION

The general requirement for conjugacy of scroll surfaces has been defined by several researchers [2]. The first condition for conjugacy is that for any given point on a working scroll surface, there is one and only one unique point on the other surface which is its conjugate. The second condition for conjugacy is that when any arbitrary conjugate pair of points is in

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contact, the centers of the two scroll elements are offset by a constant distance, the orbit radius ( Ror) . And the third condition for conjugacy is that at the two conjugate points, vectors tangent to the two surfaces are parallel to each other and normal to the direction of the offset of the two scroll centers. These rules can be used to govern what types of surfaces may be appropriate for scroll machines and to design scroll profiles with special properties.

In recent years, in the process of theoretical study and engineering design of the scroll compressor, the authors have developed a new wrap modification method [1].As a supplement, two kinds of mathematical derivation and analysis of the conjugacy relationship between the two modified parts of a pair of scroll profiles are presented in this paper. It is hoped that this work will make the method more reliable.

## 1.Graphic Analysis of Conjugate Relation of The Modified Part

Fig.1 shows the modified result of the involute of a circle by the new method, and it satisfies the following equations [3]:

$$R=r+Ror$$
$$r = \frac{a}{\sin 2\beta} - \frac{Ror}{2}$$
$$\text{ctg}\,\beta + 2\beta = \pi + \gamma$$

Assuming that the figure as shown in Fig. 1 represents the fixed scroll element, starting from this position and around point $O_1$, an identical scroll vane is turned through 180° , then an orbiting scroll member is found and shares a common center F with the fixed scroll.If the orbiting scroll moves toward the fixed scroll in any direction, an arbitrary conjugate pair of points is in contact, the centers of the two arcs are offset by a constant distance ——the orbit radius (Ror), as shown in Fig. 2. It is quite evident that the movement traces of all points in the circular arc B′ C′ ,the modified part of the orbiting scroll, are the circles with equal radius Ror as shown in Fig.2. Since R=Ror +r, the envelope of those movement traces of all points on the circular arc B′ C′ is none other than the arc AB. This implies that the circular arc B′ C′ , the modified part of orbiting

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scroll, and the circular arc AB, the modified part of the fixed scroll, are in conjugate relation and can mesh with each other correctly.

## 2. Mathematical Analysis of The Conjugate Relation of The Modified Part

The meshing engagement between the orbiting scroll and the fixed scroll in scroll compressors is a problem of contact transmission between scroll surfaces. By studying the plane meshing at the constant speed ratio based on the envelope theory for plane curves, we can obtain the conjugate profile of a given profile. This method can also be used in the case where the speed ratio is varying.

As shown in fig.3, $C_\alpha$ is a family of curves on a plane. L is such a curve, for its any point there is only one unique curve that belongs to the family of curves $C_\alpha$ and is tangential to L at this point. In addition, there are different points of tangency in curve L for different curves in the family. On the concept of envelope, the curve L is called the envelope of the family of curves $C_\alpha$.

The family of curves here includes any instantaneous position of the orbiting scroll profiles when it moves in the scroll compressor. If its envelope is just the profile of the inner surface of the fixed scroll AB, the meshing surfaces of orbiting scroll and the fixed scroll are in conjugate relation.

As shown in fig.4, the modified part of fixed scroll wrap is the circular arc AB with F as its center and with the length of R as its radius. The trace of point E' which is the center of circular arc C'B', an arbitrary curve of the family of curves, forms a circle with the length of Ror as the radius when the orbiting scroll works. So we can get the equation of circular arc C'B' in rectangular coordinates system XOY:

$$C_{\alpha 0}: \quad x = Ror \cdot \cos\alpha_0 + r \cdot \cos\theta$$
$$y = Ror \cdot \sin\alpha_0 + r \cdot \sin\theta$$

Obviously, the equation of the family of curves in the

rectangular coordinates system XOY is:

$$C_\alpha: \qquad x = Ror \cdot \cos\alpha + r \cdot \cos\theta$$
$$y = Ror \cdot \sin\alpha + r \cdot \sin\theta$$

If L is the envelope of $C_\alpha$, the equation of L can be written as follows:

$$L: \qquad x = x(\theta, \alpha)$$
$$y = y(\theta, \alpha)$$

Where $\alpha$ is a variable, $\theta$ is varying with $\alpha$, and it can be written as: $\theta = f(\alpha)$.

Thus, the equation of the envelope L becomes:

$$L: \qquad x = x(f(\alpha), \alpha)$$
$$y = y(f(\alpha), \alpha)$$

Because L and $C_\alpha$ are tangent to each other at corresponding points, the gradient of L and $C_\alpha$ at these points must be equal. That is, $k = k_\alpha$, where $k$ and $k_\alpha$ are gradient of L and $C_\alpha$ on the points of tangency.

We can also obtain the following equation by derivating the equation of L and $C_\alpha$ separately, and using the chain rule.

$$\frac{\partial x}{\partial \alpha} \frac{\partial y}{\partial \theta} - \frac{\partial x}{\partial \theta} \frac{\partial y}{\partial \alpha} = 0$$

Thus by solving the equations as follows:

$$x = Ror \cdot \cos\alpha + r \cdot \cos\theta \qquad\qquad ①$$
$$y = Ror \cdot \sin\alpha + r \cdot \sin\theta \qquad\qquad ②$$
$$\frac{\partial y}{\partial \alpha} \frac{\partial x}{\partial \theta} - \frac{\partial y}{\partial \theta} \frac{\partial x}{\partial \alpha} = 0 \qquad\qquad ③$$

We can get:

$$\alpha = \theta$$

Substituting $\alpha = \theta$ into equations ① and ②, we obtain

$$x = (Ro + r) \cos \theta$$
$$y = (Ro + r) \sin \theta$$

Obviously, the above equations represent a circle whose center is point F, and radius is Ro+r. This circle is identical to the previous modified curve segment AB. That means the circular arc AB and C' B' are in conjugate relation.

## CONCLUSION

From the graphic and mathematical analysis of the conjugate relation of the modified part of the fixed and orbiting scroll wrap, we can draw the conclusion that the modified part of the fixed and obiting scroll wrap produced by the new modification method presented at the last conference is in conjugate relation, and the new method is correct and reliable.

## ACKNOWLEDGEMENT

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3. Liu Zhenquan, Du Guirong, Yu Shicai, 'The Mathematical Derivation and Analysis of The Graphic Method For Wrap Modification of the Scroll Compressor' proc. of Xian International Compressor Engineeying Conference 1993. pp. 482~ 490
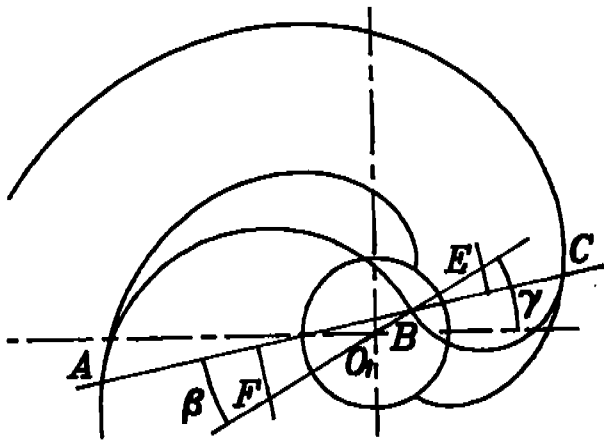
Fig.1 The illustration of the graphical
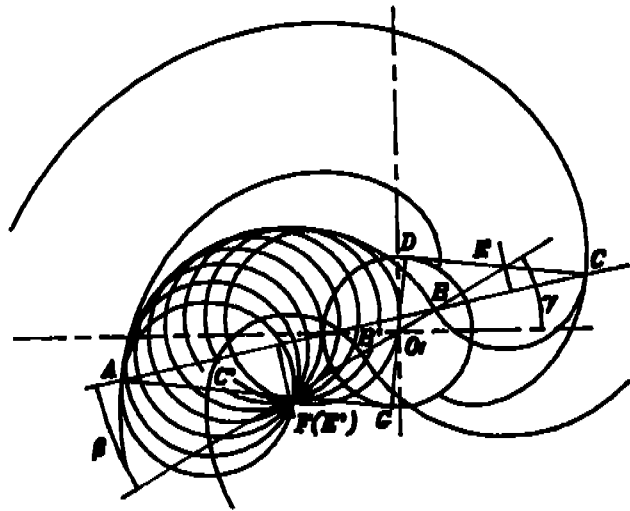method for modification of the
scroll wrap



Fig.2 The envelope of moving traces
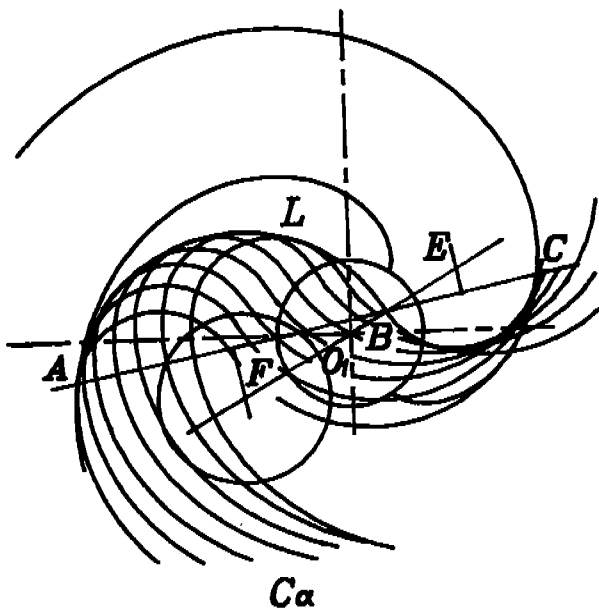of all points on the circular
arc B'C'



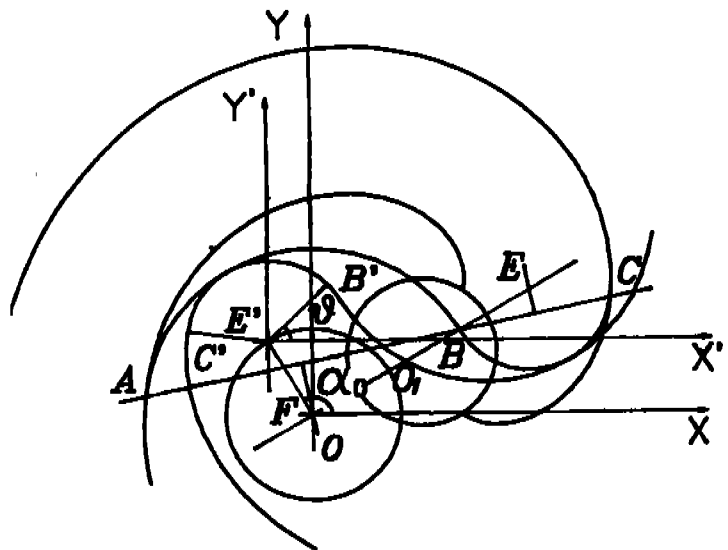Fig.3 The envelope of all
profiles of obiting scroll
when it works



Fig.4 The location of the points of
arc B'C' in XOY