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Abstract

The paper derives the equations of motion of a valve plate in two dimensions, translation and rotation, with the objective of forecasting valve behaviour and valve life more realistically.

1 Introduction

To the knowledge of the author, all attempts to model the dynamic behaviour of automatic valves for reciprocating compressors published so far[1] consider the valve plate to be a point mass having only a one–dimensional motion perpendicular to the valve seat, they assume the impacts of the valve plate against its stops (seat or guard) to take neglegible periods of time and relate the velocities after impact to those before the impact by using coefficients of restitution.

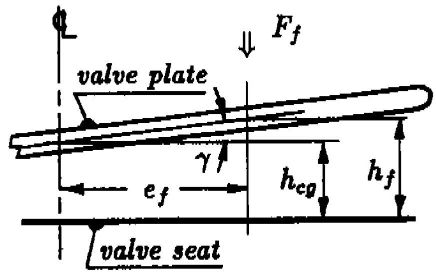
Reality however is different: Valve plates have finite dimensions, the resultants of the different forces (due to gas drag and impacts in particular) acting on them may not pass through the center of gravity, such that the total motion is composed of a translatory displacement perpendicular to the seat, and two rotatory displacements about two perpendicular axes parallel to it, resulting in a *tumbling motion*. Unfortunately it is difficult to quantify with reasonable precision the excentricities of forces causing this tumbling motion for a given application, and one might question the benefit of a more–than–one–dimensional approach. However, once the valve plate arrives at a stop having an oblique position, its first impact will be on its periphery, with the biggest possible and well known excentricity. The resulting angular acceleration will be high, the second impact will take place at a velocity considerably higher than the first, and higher than the impact velocities found with identical parameters using a one–dimensional model. In addition and unlike a one–dimensional model which assumes the valve to be tight whenever there is contact between plate and seat, a valve plate tumbling near the seat will allow gas to flow into the cylinder or out of it, such that the pressure gradients inside it will remain moderate and the unrealistic reopening of the valve often seen with simpler models is avoided.

Both effects, the one of increased impact velocity and the other one of only partial closure of a valve plate tumbling on the seat, are shown by a simpler two–dimensional model as well, which will be treated in the following paper.

[1]the two investigations [2] and [3] were available to a limited number of persons only

2 Tumbling motion in two dimensions

The translatory displacement or lift h_{cg} of the center of gravity and the rotatory displacement γ are the two dependant variables to be found. For small rotatory displacements, i.e. with $\sin\gamma \approx \gamma$, and $\cos\gamma \approx 1$, the lift h and the velocity $\frac{dh}{dt}$ of a point in a valve plate at the distance e from its center of gravity are given by



$$h = h_{cg} + e\,\gamma \qquad \text{and} \qquad \frac{dh}{dt} = \frac{dh_{cg}}{dt} + e\,\frac{d\gamma}{dt} \quad (2.1)$$

If $e_f \neq 0$ is the excentricity of the resultant force F_f of all closing springs with total translatory stiffness $C_{f,t}$, the spring force will change with h_f and not with h_{cg}. The apparant stiffness will therefore be $k_f\,C_{f,t}$ where k_f is a leverage factor to be found from equation 2.1 as follows:

$$C_{f,t} = \frac{dF_f}{dh_f} = \frac{dF_f}{dh_{cg}} \cdot \frac{1}{k_f} \quad \text{such that} \quad k_f = \frac{dh_f}{dh_{cg}} = 1 + e_f \frac{d\gamma}{d\vartheta} \cdot \frac{d\vartheta}{dh_{cg}} \qquad (2.2)$$

A similar expression has to be used for the force due to an elastic impact at the periphery, in this case $e_i = \pm\frac{D_o}{2}$. *Sign convention*: All excentricities e are positive when on the right hand side of the center.

3 Translatory motion of the valve plate in the n^th^ step

All equations, those for displacements of a valve plate as well as those for pressures in working cylinders, are differential equations and are usually solved in steps. At the beginning of the n^{th} step, during which time changes from t_n to $t_n+\Delta t$ and crank angle from θ_n to $\theta_n+\Delta\theta$, the position of the valve plate is given by its translatory displacement $h_{cg,n} = h_N\,Y_{1,n}$ (where $0 \leq Y_1 \leq 1$) and its rotatory displacement $\gamma_n = \gamma_N\,Y_{2,n}$ (where $-1 \leq Y_2 \leq +1$), with h_N and $\gamma_N = \frac{h_N}{D_o}$ as the maximum displacements the valve plate can take between its stops, in other words its *nominal translatory and rotatory lifts*. The starting point of every step is selected as origin of a new coordinate system with crank angle increment ϑ as independent variable, $y_1 = \frac{\Delta h_{cg}}{h_N}$ and $y_2 = \frac{\Delta\gamma}{\gamma_N}$ as the two dependent variables. During this n^{th} step the gas drag force $F_g = A_v\,C_w\,\Delta p$ changes with ϑ since Δp shall be given by the polynome $\Delta p = p_1\,(a_0 + a_1\vartheta + a_2\vartheta^2)$, while the spring force $F_f = F_{f,n} + k_f\,C_{f,t}\,\Delta h_{cg} = F_{f,n} + k_f\,C_{f,t}\,h_N\,y_1$ changes with y_1. The same holds for the impact force F_i = *local stiffness* $C_{i,t}\times$ *local deformation* of the valve plate in the point of impact. The drag coefficient C_w and the apparant spring rates $k_f\,C_{f,t}$ and $k_i\,C_{i,t}$ are assumed to be constant during that step. The following *sign convention* is used: A positive F_g shall open the valve, positive F_f and F_i shall close it. A viscous damping force $r_t\,\frac{dh}{dt}$ is also included. The translatory movement of a valve plate with mass m will be given by

$$m\,\frac{d^2(\Delta h_{cg})}{d(\Delta t)^2} = \underbrace{A_v\,C_w\,\Delta p}_{gas\ drag\ force} - \underbrace{(F_{f,n} + k_f\,C_{f,t}\,\Delta h_{cg})}_{spring\ force} - \underbrace{(F_{i,n} + k_i\,C_{i,t}\,\Delta h_{cg})}_{impact\ force} - \underbrace{r_t\,\frac{d(\Delta h_{cg})}{d(\Delta t)}}_{damping\ force} \tag{3.1}$$

Changing the independant variable to crank angle $\vartheta = \omega_{cs}\,\Delta t$, the dependant variable to a dimensionless $y_1 = \frac{\Delta h_{cg}}{h_N}$ using nominal lift h_N, equation 3.1 can be written as

$$m\,\omega_{cs}^2\,h_N\,\frac{d^2y_1}{d\vartheta^2} + r_t\,\omega_{cs}\,h_N\,\frac{dy_1}{d\vartheta} + \underbrace{(k_f\,C_{f,t} + k_i\,C_{i,t})}_{=\Sigma C_t}\,h_N\,y_1 = A_v\,C_w\,\Delta p - F_{f,n} - F_{i,n} \tag{3.2}$$

Rearranging and giving the names ν_1, λ_1, Π_1, χ_1 and ψ_1 to the constant coefficients leads to

$$\begin{aligned}\frac{d^2y_1}{d\vartheta^2} &+ \underbrace{\frac{r_t}{m\,\omega_{cs}}}_{=2\,\lambda_1\,\nu_1}\cdot\frac{dy_1}{d\vartheta} + \underbrace{\frac{\Sigma C_t}{m\,\omega_{cs}^2}}_{=\nu_1^2}\,y_1 = \underbrace{\frac{A_v\,C_w\,p_1}{\Sigma C_t\,h_N}}_{=\Pi_1}\cdot\underbrace{\frac{\Sigma C_t}{m\,\omega_{cs}^2}}_{=\nu_1^2}\,\underbrace{\left(a_0 + a_1\vartheta + a_2\vartheta^2\right)}_{=\frac{\Delta p}{p_1}} - \\ &- \underbrace{\frac{F_{f,n}}{h_N\,\Sigma C_t}}_{=\chi_1}\cdot\underbrace{\frac{\Sigma C_t}{m\,\omega_{cs}^2}}_{=\nu_1^2} - \underbrace{\frac{F_{i,n}}{h_N\,\Sigma C_t}}_{=\psi_1}\cdot\underbrace{\frac{\Sigma C_t}{m\,\omega_{cs}^2}}_{=\nu_1^2}\end{aligned} \tag{3.3}$$

4 Angular displacement of the valve plate

Besides the moment produced by any force acting excentrically, closing springs in addition exert a moment due to the elastic support they provide. In fact, when a valve plate tilts, the closing springs on one side will be compressed, those on the other side released, thus establishing a moment M_{el} tending to restore the horizontal position. For a guard with a total of k spring hole pitch circles, the j^{th} pitch circle having diameter D_j and $n_{j,f}$ spring holes uniformly distributed, all holes having identical springs with translatory stiffness $C_{f,t\,per\,spring}$ each, the total closing spring rates for translation $C_{f,t}$ and rotation C_r as well as the elastic moment M_{el} will be

$$C_{f,t} = \sum_{j=1}^{k} n_{j,f}\,C_{f,t\,per\,spring} \qquad C_r = C_{f,t}\,\frac{\sum_{j=1}^{k} n_{j,f}\,D_{j,f}^2}{8\sum_{j=1}^{k} n_{j,f}} \qquad M_{el} = C_r\gamma = C_r\,(\gamma_n + \Delta\gamma) \tag{4.1}$$

With forces acting excentrically, the valve plate will rotate about an instantaneous center of motion situated at a distance e_{cm} from its center of gravity, where e_{cm} is given by

$$e_{cm} = \frac{-I}{m}\cdot\frac{F_g - F_f - F_i}{F_g\,e_g - F_f\,e_f - F_i\,e_i - C_r\,\gamma} \tag{4.2}$$

To make the following equation 4.3 linear, the variation of e_{cm} during a step of integration shall be neglected, the resulting error decreases as $\Delta\theta$ decreases. $\Delta\theta$ can be chosen arbitrarily. The equation of rotation about an axis through e_{cm} will then be given by

$$
\begin{aligned}
I\,\frac{d^2(\Delta\gamma)}{d(\Delta t)^2} &= M_g - M_f - M_i - M_{el} - r_r\frac{d(\Delta\gamma)}{d(\Delta t)} = \\
&= (e_g - e_{cm})\,A_v C_w p_1\,\frac{\Delta p}{p_1} - (e_f - e_{cm})\,F_{f,n} - (e_i - e_{cm})\,F_{i,n} - C_r\,\gamma_n - \\
&- \underbrace{\left[k_f\,C_{f,t}\,(e_f - e_{cm})^2 + k_i\,C_{i,t}\,(e_i - e_{cm})^2 + C_r\right]}_{=\Sigma C_r}\Delta\gamma - r_r\frac{d(\Delta\gamma)}{d(\Delta t)}
\end{aligned}
\tag{4.3}
$$

Changing again variables from Δt to $\vartheta = \omega_{cs}\Delta t$ and from $\Delta\gamma$ to $y_2 = \frac{\Delta\gamma}{\gamma_N}$ will give

$$
\begin{aligned}
\frac{d^2 y_2}{d\vartheta^2} &+ \underbrace{\frac{r_r}{I\,\omega_{cs}^2}}_{=2\lambda_2\nu_2}\cdot\frac{dy_2}{d\vartheta} + \underbrace{\frac{\Sigma C_r}{I\,\omega_{cs}^2}}_{=\nu_2^2}\,y_2 = \underbrace{\frac{A_v\,C_w\,p_1\,(e_g - e_{cm})}{\Sigma C_r\,\gamma_N}}_{=\Pi_2}\cdot\underbrace{\frac{\Sigma C_r}{I\,\omega_{cs}^2}}_{=\nu_2^2}\cdot\left(a_0 + a_1\,\vartheta + a_2\,\vartheta^2\right) - \\
&- \underbrace{\frac{F_{f,n}\,(e_f - e_{cm}) + C_r\,\gamma_n}{\Sigma C_r\,\gamma_N}}_{=\chi_2}\cdot\underbrace{\frac{\Sigma C_r}{I\,\omega_{cs}^2}}_{=\nu_2^2} - \underbrace{\frac{F_{i,n}\,(e_i - e_{cm})}{\Sigma C_r\,\gamma_N}}_{=\psi_2}\cdot\underbrace{\frac{\Sigma C_r}{I\,\omega_{cs}^2}}_{=\nu_2^2}
\end{aligned}
\tag{4.4}
$$

5 Total displacement within the n$^{\text{th}}$ step

Equations 3.3 and 4.4 for translatory and rotatory displacements (subscripts $i = 1$ and $i = 2$) are identical in structure, their coefficients very similar, they can be written as

$$
\frac{d^2 y_i}{d\vartheta^2} + 2\,\lambda_i\,\nu_i\,\frac{dy_i}{d\vartheta} + \nu_i^2\,y_i = \nu_i^2\,\left[\Pi_i\,\left(a_0 + a_1\,\vartheta + a_2\,\vartheta^2\right) - \chi_i - \psi_i\right]
\tag{5.1}
$$

having the following general solution and constants of integration U_i and V_i

$$
\begin{aligned}
y_i &= e^{-\lambda_i\nu_i\vartheta}\,(U_i\,\sin\mu_i\vartheta + V_i\,\cos\mu_i\vartheta) - \chi_i - \psi_i + \\
&+ \Pi_i\,\left[a_0 + a_1\,\vartheta + a_2\,\vartheta^2 - \frac{2\,\lambda_i}{\nu_i}\,(a_1 + 2\,a_2\,\vartheta) - 2\,a_2\,\frac{1 - 4\,\lambda_i^2}{\nu_i^2}\right]
\end{aligned}
\tag{5.2}
$$

$$
V_i = \chi_i + \psi_i - \Pi_i\,\left(a_0 - a_1\,\frac{2\,\lambda_i}{\nu_i} - 2\,a_2\,\frac{1 - 4\,\lambda_i^2}{\nu_i^2}\right)
\tag{5.3}
$$

$$
U_i = \frac{1}{\mu_i}\left[\left(\frac{dy_i}{d\vartheta}\right)_{\vartheta=0} + \lambda_i\,\nu_i\,V_i - \Pi_i\,\left(a_1 - 4\,\frac{a_2\,\lambda_i}{\nu_i}\right)\right]
\tag{5.4}
$$

where $\mu_i = \nu_i\sqrt{1 - \lambda_i^2}$. The constant coefficients ν_i, λ_i, Π_i, χ_i and ψ_i used in differential equations 3.3, 4.4 and 5.1 are given in Table 1. Instantaneous displacements and velocities for any point on the valve plate at the distance e from its center can be found from equation 2.1.

Table 1: Coefficients used in equations 3.3, 4.4 and 5.2

comment	translation	rotation
total spring stiffness	$\Sigma C_t = k_f\, C_{f,t} + k_i\, C_{i,t}$	$\Sigma C_r = k_f\, C_{f,t}\, (e_f - e_{cm})^2 + k_i\, C_{i,t}\, (e_i - e_{cm})^2 + C_r$
natural frequency	$\omega_1 = \sqrt{\frac{\Sigma C_t}{m}}$	$\omega_2 = \sqrt{\frac{\Sigma C_r}{I}}$
frequency ratio	$\nu_1 = \frac{1}{\omega_{cs}} \sqrt{\frac{\Sigma C_t}{m}} = \frac{\omega_1}{\omega_{cs}}$	$\nu_2 = \frac{1}{\omega_{cs}} \sqrt{\frac{\Sigma C_r}{I}} = \frac{\omega_2}{\omega_{cs}}$
gas drag force ratio	$\Pi_1 = \frac{A_v\, C_w\, p_1}{\Sigma C_t\, h_N}$	$\Pi_2 = \frac{A_v\, C_w\, p_1\, (e_g - e_{cm})}{\Sigma C_r\, \gamma_N}$
spring force ratio	$\chi_1 = \frac{F_{f,n}}{\Sigma C_t\, h_N}$	$\chi_2 = \frac{F_{f,n}\left(e_f - e_{cm}\right) + C_r\, \gamma_n}{\Sigma C_r\, \gamma_N}$
impact force ratio	$\psi_1 = \frac{F_{i,n}}{\Sigma C_t\, h_N}$	$\psi_2 = \frac{F_{i,n}\, (e_i - e_{cm})}{\Sigma C_r\, \gamma_N}$
damping coefficient	$\lambda_1 = \frac{r_t}{2\, m\, \omega_{cs}} \cdot \frac{1}{\nu_1}$	$\lambda_2 = \frac{r_r}{2\, I\, \omega_{cs}} \cdot \frac{1}{\nu_2}$

6 Conclusion

With a more–than–one–dimensional simulation of valve plate motion, results – more details will be given at the conference — closer match measured valve plate motion, in particular:

- A valve plate closing in time before dead center position and tumbling near the valve seat is not tight, hence pressure gradients in the cylinder remain small and the valve shows little tendency to reopen.
- Impact velocities and impact forces are often higher than those found with a one–dimensional model, which, in addition, can only determine impact velocities but no forces. The reason is that the excentric impact force acting during the first impact of a tumbling valve plate can produce a high angular acceleration.
- Small tumbling amplitudes will result in small impact velocities and are advantageous. As with any oscillator, they will be obtained with a high natural frequency ω_2 and therefore a high frequency ratio ν_2, i.e. with a high rotatory stiffness C_r.

References

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[2] F. Bauer, *Stresses in valve plates due to their impacts*, HOERBIGER–internal communication, 22 pages, May 1966.

[3] E. H. Machu, *Valve dynamics in three dimensions*, HOERBIGER–internal communication, 66 pages, Feb. 1978.

[4] E. H. Machu, *Valve dynamics in a pulsating environment*, paper presented at the International Reciprocating Machinery Conference at Denver/USA, Sept. 1992, organized by the PCRC, Dallas.

List of Subscripts

subscript	comment
$0,1,2,\ldots,j$	number of a term
1	nominal suction condition
$1\ldots2$	dimensions of motion
cg	center of gravity
cm	inst. center of motion
cs	crank shaft
el	elastic support
f	spring (force)
g	gas (drag force)
i	impact (force)
n	at beginning of step n
N	nominal (valve lift=h_N)
o	outer (diameter)
r	rotatotary
t	translatory
v	valve or valve plate
w	drag force coefficient

Nomenclature

symbol	unit	comment
a	–	polynomial coefficient
A	m^2	area
C_r	$N\,m/rad$	rotatory stiffness
C_t	N/m	translatory stiffness
C_w	–	drag coefficient
d	–	differential operator
D	m	diameter (of a plate)
e	m	excentricity
F	N	force
h	m	valve lift
I	$kg\,m^2$	moment of inertia
k	–	leverage factor
m	kg	mass
M	$N\,m$	moment
n	–	number (of springs)
p	Pa	pressure
r_r	$N\,m/rad/s$	rotatory damp. coeff.
r_t	$N/m/s$	translat. damp. coeff.
t	s	time
U, V	–	const. of integration
y	–	$= \Delta Y$ within step
Y	–	rel. valve lift $= \frac{h}{h_N}$
γ	rad	angular displacement
Δ	–	small increment
θ	rad	crank angle
ϑ	rad	$= \Delta\theta$ within step
λ	–	damping force ratio
μ	–	damped frequ. ratio
ν	–	frequency ratio
Π	–	drag force ratio
Σ	–	summation symbol
χ	–	spring force ratio
ψ	–	impact force ratio
ω	rad/s	angular velocity $\frac{d\theta}{dt}$
ω	rad/s	natural frequency