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SOME ASPECTS OF DESCRIBING PROCESSES IN SLIDING-VANE ROTARY MACHINES

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Abstract

In the paper, various methods of geometrical description of sliding-vane rotational machines, as found in publications, are considered. Choice and modification of one of them has been proposed. With regard to precise analysis, formulae have been obtained to calculate the working chamber volume for any chamber position and for any type of sliding-vane rotational machine.

Nomenclature:
A - cross-sectional area
b - vane thickness
e - eccentricity
L - length of working chamber
r - rotor radius
R - cylinder radius
V - volume
y - radial clearance
λ - angle between successive vanes
ρ - radius vector
φ - amplitude; angular coordinate
ψ - vane/rotor radius inclination angle

1. Introduction

Many publications appeared in recent years discussing theoretical and experimental results of tests on sliding-vane rotary machines (see for example [1, 2, 3, 4]). A number of methods have been used in them to describe mechanical, thermodynamic and flow processes and to carry out the geometric, kinematic and dynamic analysis of sliding-vane machines; various coordinate systems are therein applied. This makes it difficult to compare the results obtained and leads often to misunderstandings. Besides, it makes superfluous variables appear sometimes in the mathematical model.

Geometric form of sliding-vane rotational machines implies, that values required to describe the machine operation (like cross-sectional area of the working chamber, working chamber volume, lengths of linear and circular segments) are calculated approximately, approximation degree being different for particular authors. With powerful microcomputers becoming widespread, design offices and research centres are looking for precise numeric algorithms, which do not necessarily assume the form of a simple equation.

With regard to the statement made above, the authors would like to put forward a convention concerning geometric description systematization of multisliding-vane rotational machines. In the opinion of the authors, this will provide a more readable description of geometric values, as well as of phenomena and processes relevant to these machines. We also intend to point out some properties of the $Z(\varphi)$ function, particularly its ability to evaluate working chamber volume at any chamber position and for various types of multisliding-vane machines.

2. Coordinate System

In almost all publications, cylindrical coordinate system is employed for describing multisliding-vane rotary machines (Fig. 1a). The system converts into polar coordinate system (Fig. 1b) when things are being considered within the plane perpendicular to the Z axis. The choice of the Z axis is still undetermined.

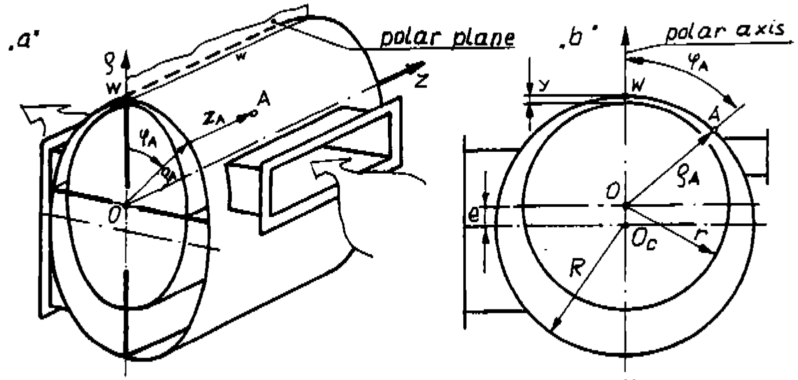

Fig. 1. Coordinate systems employed to describe multisliding-vane rotary machines
a - cylindrical, b - polar.

In some cases this is the cylinder axis [1, 2], in other the rotor axis [2]. For further analysis, the authors have adopted basically a coordinate system with the Z axis running through the rotor axis and the polar plane determined by the axis and its nearest cylinder generatrix. The z = 0 plane runs through the chamber side cover surface.

3. Arrangement of Points, Vanes and the Working Chamber in Multisliding-vane Rotary Machines

With the coordinate system adopted, the machines can be described from the geometrical point of view, i.e. position of arbitrary point, segment and figure, as well as that of the working chamber; length of a segment and arc, area of a figure and volume of a solid (working chamber, e.g.) can all be described. Also processes to which parts of the machine are exposed (movement, friction) can be specified in that way. This applies still more to the description of the thermodynamic state (p, T, i) and representation of processes, which the gas contained in the working chamber undergoes (i.e. heat exchange, flows, compression and decompression).

Fig. 2 outlines the cross-section of a multisliding-vane rotary machine. In the coordinate system adopted, the position of an arbitrary point (e.g. point C on the

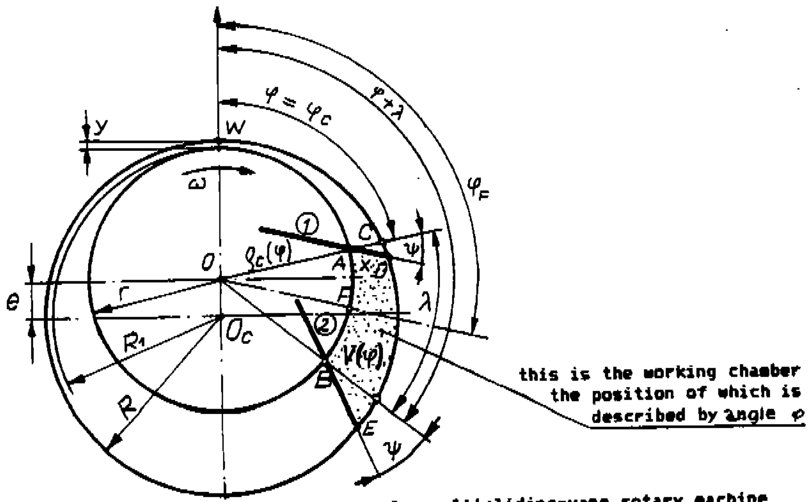

Fig. 2. Cross-section outline of a multisliding-vane rotary machine

cylinder surface) is determined by two coordinates ρ_c and φ_c. The vane position is given by specifying coordinates of either two points situated on the vane, or of vane axis/rotor surface intersection point and vane/rotor radius inclination angle. Coordinates of the (1) vane (Fig. 2) are equal $\rho = \rho_A = r$; $\varphi = \varphi_A$ and ψ. When considering a machine with constant r and ψ, then the only variable, which describes position of the vane is the vane amplitude φ. Values relating to the vane, which vary with the rotor rotation, will be thus functions of φ (e.g. $x(\varphi)$, $P_i(\varphi)$).

Position of the working chamber is usually determined in publications [1, 2] by specifying position of the bisector of the angle λ contained between vanes limiting the chamber (angle φ_F in Fig. 2). This however implies certain inconveniences, consisting in principle in necessity of two at least (sometimes three) position coordinates for one given chamber (i.e. position coordinates for the vanes and for the chamber itself). In order to avoid this complication, the author's proposal is to determine the working chamber position by means of coordinates of one of its limiting vanes and to adopt the convention that the vane in question will be thevane which "closes" the chamber in the sense of chamber movement direction (vane (2) in Fig. 2). Then the angle φ becomes the coordinate of the working chamber position and all quantities relating to the chamber and chamber medium. This convention is also useful for one- and two-sliding vane machines.

4. Working Chamber Volume

Mindful of the simplifications generally applied to calculate volume of the working chamber being in arbitrary position, the authors reconsidered the above relations. With no simplifications assumed, they derived formulae, whose applicability exceeds multisliding-vane rotary machines.

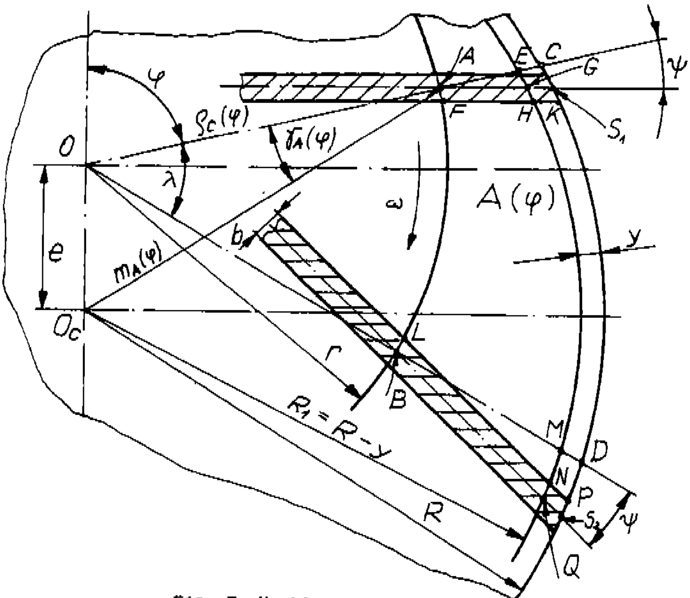

Fig. 3. Working chamber outline

Fig. 3 shows schematically a working chamber in position characterized by angle φ and specifies its dimensions necessary to calculate the chamber volume.

The working chamber volume equals to:

$$V(\varphi) = A(\varphi) \cdot L \tag{1}$$

where $A(\varphi)$ - cross-sectional area of the chamber.

According to designations of Fig. 3:

$$A(\varphi) = A_{AEMB} + A_{BMQ} - A_{AEG} - A_{AGHF} - A_{BQNL} - A_{HKPN} \tag{2}$$

In the above formula the following members can be distinguished:

$A_{AEMB}(\varphi)$ - cross-sectional area of the working chamber for a machine with $\psi = 0$; $y = 0$; $b = 0$.

$$A_{AEMB} = R_1^2 \cdot Z_1(\varphi) \tag{3}$$

$$Z_1(\varphi) = \frac{e}{R_1}\left\{\lambda - \frac{1}{2}\left[\sin(\varphi + \lambda)\sqrt{1 - \left(\frac{e}{R_1}\right)^2 \sin^2(\varphi + \lambda)} + \right.\right.$$

$$+ \frac{1}{\frac{e}{R_1}} \cdot \arcsin \frac{e}{R_1}\sin(\varphi + \lambda) - \sin\varphi \cdot \sqrt{1 - \left(\frac{e}{R_1}\right)^2 \sin^2\varphi} +$$

$$\left. - \frac{1}{\frac{e}{R_1}} \cdot \arcsin \frac{e}{R_1}\sin\varphi\right] +$$

$$\left. - \frac{1}{2}\left(\frac{e}{R_1}\right)\left[\lambda - \sin(\varphi + \lambda)\cos(\varphi + \lambda) + \sin\varphi\cos\varphi\right]\right\} \tag{4}$$

$A_{BMQ}(\varphi)$; $A_{AEG}(\varphi)$ - areas of figures arising by inclining the vanes by an angle of φ towards the rotor radius.

$$A_{AEG} = R_1^2 \cdot P_1(\varphi) \tag{5}$$

The $P_1(\varphi)$ function is represented by the following formula:

$$P_1(\varphi) = \int_0^{\psi} \frac{1}{2}\left\{\sqrt{1 - \left[\frac{m_A(\varphi)}{R_1}\right]^2 \sin^2\left[\psi + \gamma_A(\varphi)\right]} - \cos\left[\psi + \gamma_A(\varphi)\right]\right\} d\varphi \tag{6}$$

Functions $m_A(\varphi)$ and $\gamma_A(\varphi)$ in the above formula are defined as follows:

$$m_A(\varphi) = R_1\sqrt{1 - 2\left(\frac{e}{R_1}\right)\left(1 - \frac{e}{R_1}\right)(1 - \cos\varphi)} \tag{7}$$

$$\gamma_A(\varphi) = \arcsin \frac{e}{R_1}\,\frac{R_1 \sin\varphi}{m_A(\varphi)} \tag{8}$$

Value of $A_{BMQ}(\varphi)$ can be evaluated from equation (4) for a vane in a position described by the angle of $\varphi + \lambda$:

$$A_{BMQ} = R_1^2 \cdot P_1(\varphi + \lambda) \tag{9}$$

Vane thickness is allowed for in the formulae for $A_{AGHF}(\varphi)$ and $A_{BQNL}(\varphi)$. The sum of the areas is equal to:

$$A_{AGHF}(\varphi) + A_{BONL}(\varphi) = R_1^2 \cdot P_2(\varphi) \tag{10}$$

where the $P_2(\varphi)$ function is given by:

$$P_2(\varphi) = \frac{1}{2} \frac{b}{R_1} \left\{ \sqrt{1 - \left[\frac{e_A(\varphi)}{R_1}\right]^2 \sin^2\left[\psi + \gamma_A(\varphi)\right]} + \right.$$

$$+ \sqrt{1 - \left[\frac{e_A(\varphi + \lambda)}{R_1}\right]^2 \sin^2\left[\psi + \gamma_A(\varphi + \lambda)\right]} +$$

$$\left. - \cos\left[\psi + \gamma_A(\varphi)\right] - \cos\left[\psi + \gamma_A(\varphi + \lambda)\right] \right\} \tag{11}$$

If the rotor does not adhere closely to the cylinder, i.e. if $y \neq 0$, then the chamber cross-sectional area should be increased by $A_{HKPN}(\varphi)$.

$$A_{HKPN} = R_1^2 \cdot P_3(\varphi) \tag{12}$$

where $P_3(\varphi)$ is given by the formula:

$$P_3(\varphi) = \left\{ \lambda - 2 \frac{e}{R_1} \cos \frac{2\varphi + \lambda}{2} \sin \frac{\lambda}{2} + 2 \frac{e}{R_1} \operatorname{tg} \psi \sin \frac{2\varphi + \lambda}{2} \sin \frac{\lambda}{2} + \right.$$

$$\left. - \operatorname{tg} \psi \left[\sqrt{1 - \left(\frac{e}{R_1}\right)^2 \sin^2 \varphi} - \sqrt{1 - \left(\frac{e}{R_1}\right)^2 \sin^2 (\varphi + \lambda)} \right] \right\} \frac{y}{R_1} +$$

$$- \frac{y}{R_1} \frac{b}{R_1} \tag{13}$$

Having allowed for (2), (4), (6), (11) and (13), equation (1) assumes the following form:

$$V(\varphi) = R^2 L \cdot \left(1 - \frac{y}{R}\right)^2 \left[Z_1(\varphi) - P_1(\varphi) + P_1(\varphi + \lambda) - P_2(\varphi) + P_3(\varphi)\right] =$$

$$= R^2 L \cdot Z(\varphi) \tag{14}$$

where:

$$Z(\varphi) = \left(1 - \frac{y}{R}\right)^2 \left[Z_1(\varphi) - P_1(\varphi) + P_1(\varphi + \lambda) - P_2(\varphi) + P_3(\varphi)\right] \tag{15}$$

In the above formula, $Z(\varphi)$ represents relative cross-sectional area of the working chamber of an expansion-type machine (motor, compressor). The form of the equation (15) emphasizes the fact that variables λ, e/R, y/R, b/R and ψ are regarded as parameters, and the variable φ as an argument.

Evaluation of the working chamber volume in an arbitrary chamber position is thus reduced to determining the $Z(\varphi)$ value for a given angle φ from formula (15). The chamber position, when the chamber volume is already known, is found by solving a simple nonlinear equation, which can be done easily by using standard numerical procedures and microcomputers.

5. Application of $Z(\varphi)$ Function to Other Machines

The $Z(\varphi)$ function can be useful when evaluating other types of machines. For compression multisliding-vane rotary machines (compressors, vacuum pumps), the polar coordinate system is produced from the system presented in paragraph 2 by rotating the polar axis by an angle of $\Delta\varphi = \pi$. The $Z(\varphi)$ function can then be employed, if the expression $\varphi + \pi$ is assumed as an argument.

$$V_K(\varphi) = R^2L \cdot Z_k(\varphi) \tag{16}$$

where: $V_K(\varphi)$ - the working chamber volume of that compression-type vane rotary machine, wherein the angle φ is used to describe the chamber position.
$Z_k(\varphi)$ - relative cross-sectional area of the working chamber for this machine.

$$Z_K(\varphi) = Z\ (\varphi + \pi) \tag{17}$$

Solving numerous technical problems requires evaluation of area for a segment of a figure formed by two non-concentric circles (Fig. 4). If figure ABCD is assumed to be the segment in question, then its area equals to the working chamber area in the chamber position of φ'; λ amounts then to $\Delta\varphi'$. Then

$$A_{ABCD} = R^2 \cdot Z(\varphi') \tag{18}$$

The value of $Z(\varphi')$ has been determined for a machine of a conventional number of vanes $z = 2\pi/\Delta\varphi'$. The number can assume values from within the range of $z \geq 2$.

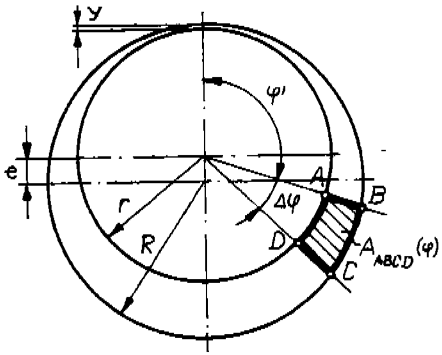

Fig. 4. Areas of figures contained between non-concentric circles

The $Z(\varphi)$ function can also be used for rotary piston machines (one-sliding vane rotational machines). To describe these machines, a polar coordinate system is usually used related to the cylinder centre (Fig. 5). If the rotor position is determined by the coordinate φ in this coordinate system, then the working chamber cross-section is represented by figure ABCD.

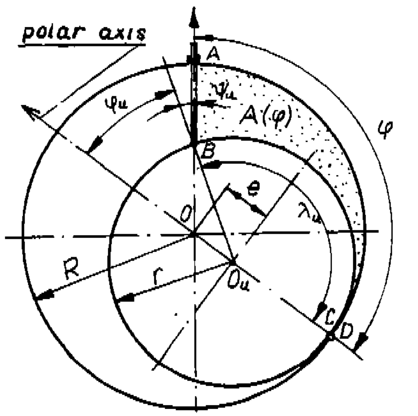

Fig. 5. Area of the figure which the forms in a rotary-piston machine.

To evaluate area of the figure, no more than a conventional compression machine should be analyzed with the momentary rotation axis in point Ou. The point acts also as a momentary pole, the conventional polar axis being the half-line O_uO. Then position of the conventional working chamber is described by the angle $\varphi_u = \pi - \varphi$. The vane is inclined to the rotor radius at the angle ψ_u. The other vane is almost completely inserted into the groove (point C). The angle between the vanes equals $\lambda_u = \varphi$. The following is true for such a chamber:

$$A_{ABCD} = A(\varphi) = R^2 \cdot Z_k(\varphi_u, \psi_u, \lambda_u, e/R, b/R, y/R) \tag{18}$$

where $Z_k(\varphi_u, \psi_u, \lambda_u, e/R, b/R, y/R)$ is relative cross-sectional area of the working chamber of a conventional vane rotational machine. Parameters φ_u, ψ_u and λ_u are functionally dependent on φ.

The $Z(\varphi)$ function would also appear in those thermodynamic relations for vane rotary machines, where working chamber volume or volume ratio is involved.

6. Final Remarks

The presented methods of describing vane rotational machines and evaluating working chamber volume have been applied by the authors in design and research studies over the machines. The microcomputer software package worked out for the purpose and the $Z(\varphi)$ function tables are intended for universal and extensive application by engineers working in the area of vane rotational machines.

7. References

[1]. Fischer R.D. "Feasibility study and hardware design of a pivoting-tip rotary-vane compressor and expander applied to a solar-driven heat pump". Proc. of the 1978 Purdue Compressor Technol. Conf. West Lafayette 1978.

[2]. Golovincov A.G. "Rotacjonnyje kompressory". Masinostrojenie, Moskva 1964r.

[3]. Gnutek Z. "Analiza procesow termodynamicznych w łopatowych maszynach rotacyjnych", Raport Polit. Wr. I-20, SPR 24/91; 29/91; 31/91; Wrocław 1991.

[4]. Gnutek Z. "Geometryczny opis i analiza procesów tarcia, przepływów i sprężania w jedno- i dwułopatkowych maszynach rotacyjnych". Raport Polit. Wr. I-20, SPR 3/92. Wrocław 1992 r.