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DERIVATION OF A GENERAL RELATION GOVERNING THE CONJUGACY OF SCROLL PROFILES

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ABSTRACT

A number of geometric curves have been used in the past to form the working surfaces of scroll machines. These include involutes of circles, segments of arcs, and modified Archimedes spirals. While these may work satisfactorily, no general rule exists in the literature governing what type of surfaces may be appropriate for this application. In this paper, the general requirement for conjugacy of scroll surfaces is defined. This is then used to derive a relation which may be used to generate conjugate surfaces of a general form. Subsidiary relations governing displacement volume are given and the conjugacy test is applied to historical scroll forms, including the standing vane rotary compressor.

INTRODUCTION

We typically credit Léon Creux with the invention of the scroll-type machine, based on his U.S. patent of 1905 [1]. This is perhaps the single most-cited reference in scroll literature. His choice of the involute of a circle for his scroll vane form has endured to the present. All commercially available scroll-type machines known to the authors use this type of scroll wrap or profile.

Some variants to this form have appeared over the years. They include modified Archimedes spirals [2], offset spliced involutes [3], spliced arcs of circles of varying radius [4] (deriving perhaps from Wankel's definition of an arcuate displacement device) [5], and spliced combinations of involute curves and circular arcs [6]. Nowhere, though, is any great thought given publicly to what inherent property makes these curves appropriate for scroll machines or to what method may be used to define curves of a more general nature with practical applications.

In this study, a general relationship is derived which governs conjugacy of scroll surfaces or profiles. This may be used to design scroll profiles with special properties unavailable to more conventional geometries.

DEFINITION OF CONJUGACY

What is it that makes a scroll work? A typical sort of explanation speaks of offsetting two intermeshed spiral-shaped wraps or vanes so that a series of crescent-shaped pockets are formed. These pockets are formed, sealed off, change in volume, are unsealed, and disappear as the two scroll elements are moved orbitally relative to each other. Some machines allow relative rotation of the scrolls [7] but will not be treated here.

As a pocket begins to form, the two vanes or wraps come into contact at an extreme end of the contacting or working surfaces. As the vanes move relative to one another, the point of contact moves continuously from the initial point towards the terminal point on the other extreme

end of the working surfaces. At the point of contact, the instantaneous action is sliding in nature, and any given point is in contact with a mating point for only an infinitesimal, theoretically zero, period of time. The first general condition for conjugacy is that for any given point on a working scroll surface, there is one and only one unique point on the other surface which is its conjugate.

Since a particular point of vane contact exists continually and moves along the entire length of the wrap (in other words the wraps are always in contact for any given phase or crankshaft angle), the orbit path is an inherent property of the scroll vanes themselves, not of any particular feature of the drive mechanism. We restrict this discussion to scrolls with circular orbit paths. The second general condition for conjugacy is that when any arbitrary conjugate pair of points are in contact, the centers of the two scroll elements are offset by a constant distance, the orbit radius (Ror). An inverse way of stating this is that when the two scroll elements are positioned so that they share a common center, any arbitrary pair of conjugate points are separated by a distance Ror directed normally to the respective surfaces.

When the two conjugate points are in contact, they represent a point of tangency of the two scroll surfaces. The relative motion is parallel to the direction of tangency and is perpendicular to the directed radius of orbital motion Ror, whose direction is the same as the direction of offset between the two scroll centers. The third general condition for conjugacy is that at the two conjugate points, vectors tangent to the two surfaces are parallel to each other and normal to the direction of offset of the two scroll centers.

CONJUGACY RELATION

Figure 1 shows a segment of two conjugate surfaces which share a common center. When one of the surfaces is displaced toward the other in an arbitrary direction ψ a conjugate point pair will come into contact. A point directly between the two points (and which lies on the pitch or mid line between the two surfaces) is located by a pair of position vectors. The first, Rg, is parallel to the surface tangents and is tied to the coordinate origin. The second, Rs, is normal to the surfaces and to Rg and is parallel to the direction of offset ψ . Adding or subtracting a third vector parallel to Rs and of magnitude Ror/2 locates the concave (inner) or convex (outer) surface, respectively. The magnitudes of Rs and Rg are, for now, some arbitrary function of ψ . The position vector for the convex surface is given by

$$\vec{P} = R_S(\psi) e^{j\psi} + R_G(\psi) e^{j(\psi+\pi/2)} \quad (1)$$

in complex polar form. The tangent to a position vector is

$$\frac{d\vec{P}}{d\psi} = \vec{T} \quad (2)$$

Differentiating the polar position vector gives us

$$\vec{T} = \frac{dR_S}{d\psi} e^{j\psi} + jR_S e^{j\psi} + \frac{dR_G}{d\psi} e^{j(\psi+\pi/2)} + jR_G e^{j(\psi+\pi/2)} \quad (3)$$

Using the relation

$$e^{j(\psi+\pi/2)} = j e^{j\psi} \quad (4)$$

we get

$$\vec{T} = e^{j\psi} \left[\frac{dR_S}{d\psi} - R_S + j \left(R_S + \frac{dR_G}{d\psi} \right) \right] \quad (5)$$

Recall from the third general requirement for conjugate surfaces that the surface tangent vector

is normal to the direction of offset when the scrolls are in contact. The direction of offset is parallel to a unit vector in the direction of ψ . The dot product of the unit vector and the tangent vector will be zero.

$$e^{i\psi} \cdot \vec{T} = 0 \quad (6)$$

The dot product will be simply the component of the tangent vector parallel to the unit vector.

$$\frac{dR_s}{d\psi} - R_s = 0 \quad (7)$$

This relation between the position vectors R_s and R_g is the general relation governing conjugacy. As long as this rule is obeyed in generating the pitch line used to then generate the two surfaces in accordance with the second general requirement stated above, any general equation form may be used for R_s to define conjugate surfaces. Since the two surfaces are generated at the same time with the same value of ψ , conjugate points share the same "wrap angle." A single specification of wrap angle describes a unique point on each surface.

A little analysis will show that this relation describes a generalized involute. If, as the surfaces are generated, the tip of the R_g vector traces out a curve, the change in length of the R_s vector will be found to equal the line integral of that curve from the starting point to the point being generated. This is analogous to the illustration of an involute of a circle, in which the spiral is generated by unwinding a string from a circular spool. The amount of string unwound equates to the R_s vector and is equal to the distance around the spool it was formerly wrapped.

DISPLACEMENT VOLUME

Figure 2 shows a full wrap of a pair of conjugate surfaces which are positioned so that they share a common center (the "as-generated" position). The surfaces defining the area bounded by the two extreme pairs of conjugate points have been shaded. The value of R_g for the inner pair is shown less than for the outer pair. In Figure 3 one of the surfaces is displaced so that a sealed compression pocket is formed. To the inside (to the right) of the line defined by the inner conjugate point pair, displacement of the scroll wraps to form the pocket results in no net change of the bounded area. Outside that line and inside the line defined by the outer conjugate point pair, the area is reduced by the difference between the two values of R_g times the wrap displacement, R_{or} . If the inner R_g were larger, we would add a correction. The area of bounded portion less this correction equals the axially projected area of the sealed compression pocket. The bounded area is equal to the line integral of the polar position vector of the pitch line between the two point pairs times the distance R_{or} between the surfaces.

$$A_s = \int_{\psi-2\pi}^{\psi} R_{or} R_s(\psi) d\psi \quad (8)$$

The locating value of ψ for the pocket is determined by the outermost pair of conjugate points. The net area of the sealed pocket is given by

$$A_p = R_{or} \int_{\psi-2\pi}^{\psi} R_s(\psi) d\psi + R_s(\psi-2\pi) - R_s(\psi_2) \quad (9)$$

Multiplying this value by the wrap height will result in the displaced volume of the pocket.

When a pocket is sealed off at the outer periphery of the scroll set, it is defined by ψ_{end} , the ending value of wrap angle. When it just opens into the discharge zone at the center, it is defined by a wrap angle of 2π . The built-in volume ratio of the scroll set may be found by evaluating the area equation at these two extreme values and dividing the former by the latter.

OTHER RELATIONSHIPS

Forces. Force relations, such as axial, radial, and tangential forces and gas torque may be determined using more conventional or traditional methods. Note that with variable R_g , the radial gas force will vary with crank angle, unlike for scrolls of involutes of circles.

Length of Wrap. The length of the working surface of the scroll wrap is useful for manufacturing estimates. It may be determined by use of a modified line integral similar to that used for the area calculation. Add or subtract $1/2 R_{or}$ to or from R_s for inner or outer surfaces, respectively.

$$L_s = \int_0^{\psi_{end}} \left[R_s(\psi) \pm \frac{r_{or}}{2} \right] d\psi \quad (10)$$

Radius of Curvature. It is useful to know the inside radius of the inner wall to ensure that cutting or forming tools will fit within without undercutting the scroll surface. This is given by the magnitude of the tangent vector. Noting that only the complex, or imaginary, term is nonzero,

$$R_c = R_s + \frac{dR_s}{d\psi} = R_s + \frac{d^2 R_s}{d\psi^2} \quad (11)$$

COMPARISON TO HISTORICAL FORMS

For the involute of a circle, the generating radius is held constant. Generally, the governing relations become

$$R_s = C_0 + C_1 \psi; R_c = \frac{dR_s}{d\psi} = C_1 \quad (12)$$

The circular involute scroll is simply a special case where R_s is a linear first order polynomial.

Consider the case where

$$R_s = C_0; R_c = \frac{dR_s}{d\psi} = 0 \quad (13)$$

R_s has become a constant and the two working surfaces are nesting circular cylinders. This is the displacement mechanism used in the conventional standing vane rotary which may be considered a special case of the scroll. The rotary is analogous to a one-sided scroll wrap which needs the sliding vane or other sealing device to prevent the high to low leak which should exist where the other set of pockets is missing. Spiral-shaped scrolls which are made of spliced sections of circular arcs with alternately displaced centers also fall under this category. While it is clear the conjugacy relation holds for the arcs when written relative to their respective centers, a little analysis will show the relation to be true no matter where an arbitrary center of generation is chosen.

HYBRID WRAPS

In several instances a particular profile form is preferred to better meet a specific requirement. For example, arcs of circles are often used at the discharge region of a wrap to increase the compression ratio [8]. For reasons such as this it is desirable to be able splice different types of curves together to satisfy more than one design requirement.

To splice curves together, three general requirements of the end conditions must be met. First both curves must satisfy the general requirement governing conjugacy stated previously:

$$\frac{uR_s}{d\psi} = R_c \quad (14)$$

Second, both curve equations must generate the same R_s value at their intersection. Third, both equations must also generate the same R_g value. These last two conditions assure first and second order continuity.

An example of the use of generalized relations and hybrid wrap design is given in the following section.

DESIGN EXAMPLE

Figure 4 shows a conventional scroll wrap profile of an involute of a circle. It has a total number of $5\frac{1}{2}$ active wraps (measured on the working surfaces only) and a volume ratio of 5.6:1. Scrolls of similar volume ratios and higher may be used in refrigeration, air compression, vacuum pumps, and other high ratio applications. A drawback to the large volume ratio type of circular involute scroll is the long wrap length. The length of the active wrap shown in figure [4] is 3.831 meters. This length requires long machining times and a large surface area over which to control profile tolerances.

The scroll shown in Figure 5 is generated using a more general equation that conforms to the requirements governing conjugacy. This profile is generated using a pitch line equation that begins as a involute of a circle, deviates to a more general equation, and then returns to an involute of a circle to complete the wrap. This scroll has the same volume ratio of 5.6:1, occupies the same physical space, but only has $2\frac{1}{2}$ wraps and a resulting active wrap length of 1.738 meters, almost a 55 percent reduction.

It can be seen comparing figure 6 and 7 that the number of internal pockets is reduced from 8-10 to 2-4. The suction and discharge pockets of the hybrid scroll in figure 7 are formed by the portion of the wraps that remain an involute of a circle.

The major fabrication benefit of this 55 percent shorter wrap is the reduced machining time and the reduction in active wrap length over which to control tolerances. Other benefits may exist in the performance of the compression process. A shorter wrap has less overall tip leakage area, and the thicker wrap sections will increase the resistance to tip leakage flow. Another performance benefit may be associated with a shorter residence time of the gas in the compressor, $2\frac{1}{2}$ revolutions rather than $5\frac{1}{2}$. This allows less time, per unit mass of gas passing through the compressor, for tip leakage to occur and for detrimental heat transfer to take place.

Some factors to consider are the incrementally more complex machining and inspection of the wrap profile and greater degree of sub-revolution variations in the driving torque and gas forces in the compression process. However, even in extreme cases, these variations would still be substantially less than for a more conventional compressor such as single cylinder reciprocating or standing vane rotary.

CONCLUSION

A fairly simple geometric relation exists which allows generation of conjugate scroll profiles of almost any form. This can be used to design scroll machines with properties or characteristics not easily available with more conventional circular involute forms. The general relation describes a general sort of involute of a higher order surface than, for example, a circle. Some advantages, described here, include reduction of manufacturing requirements and tailoring of the compression process. The prior use of circular arcs in extending compression ratios suggests other possible areas of refinement as well.

REFERENCES

1. Creux, Léon, "ROTARY ENGINE," U.S. Patent 801,182, October 3, 1905.
2. Montelius, Carl Oscar Joseph, "ROTARY COMPRESSOR OR MOTOR," U.S. Patent 2,324,168, July 13, 1943.
3. Güttinger, Heinrich, "DISPLACEMENT MACHINE FOR COMPRESSIBLE MEDIA," U.S. Patent 3,989,422, November 2, 1976.
4. Vulliez, P., "VOLUMETRIC DEVICE SUCH AS A VACUUM PUMP OR THE LIKE, HAVING AN EXACT CIRCULAR TRANSLATION CYCLE," U.S. Patent 3,473,728, October 21, 1969.
5. Wankel, F., Rotary Piston Machines, London Iliffe Books, London, 1963.
6. Young, Niels O. & McCullough, John E., "SCROLL-TYPE POSITIVE FLUID DISPLACEMENT APPARATUS," U.S. Patent 3,884,599, May 20, 1975.
7. Muir, Earl B., Griffith, Russel W., & Lilienthal, Gerald W., "SCROLL-TYPE MACHINE WITH ROTATION CONTROLLING MEANS AND SPECIFIC WRAP SHAPE," U.S. Patent 4,609,334, September 2, 1986.
8. Morishita, Etsuo; Kitora, Yoshihisa; Suganami, Takuya; Yamamoto, Sakuei; & Nishida, Mitsuhiro; "Rotating Scroll Vacuum Pump," Proceedings Purdue Compressor Conference, 1988, pp. 198-205.

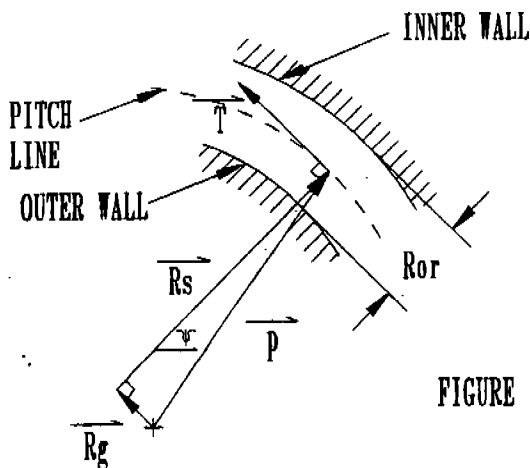


FIGURE (1)

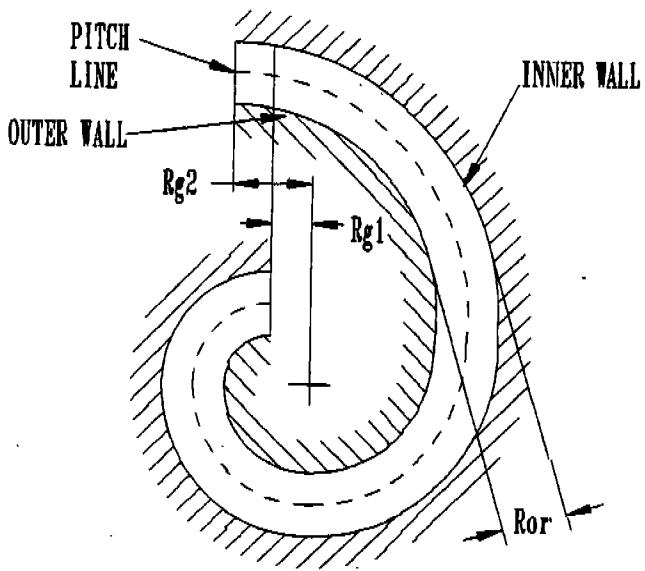


FIGURE [2]

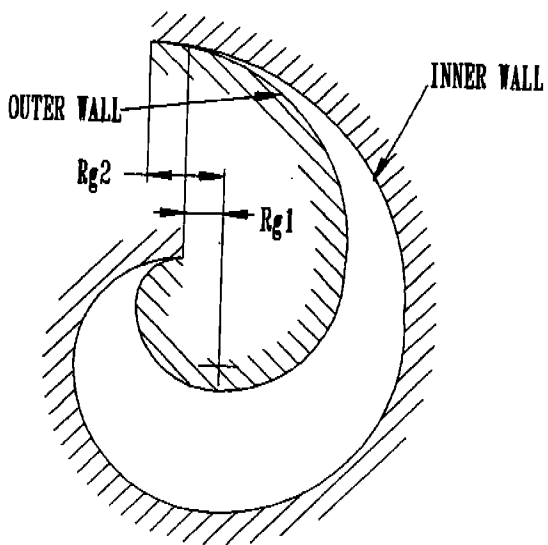


FIGURE [3]

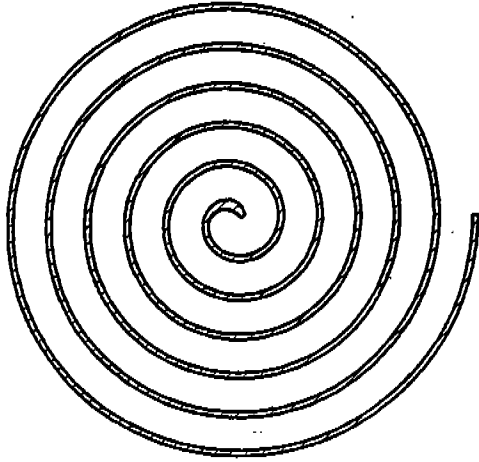


Figure 4

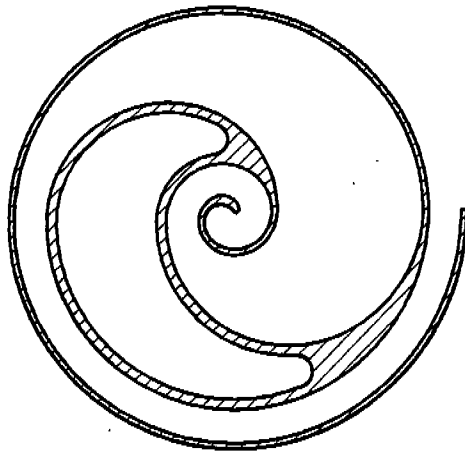


Figure 5



Figure 6

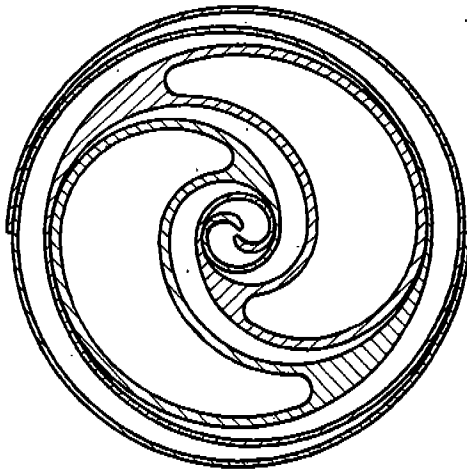


Figure 7