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A Simultaneous Solution for Transfer Matrix Acoustic Models

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ABSTRACT

The paper describes a method of carrying out an acoustic analysis using the transfer matrix method by assembling the equations which define the boundary conditions and the transfer function of sections of the system into one equation matrix. This equation matrix is then solved to obtain the pulsating pressure and flow components at all of the nodes defined in the system.

NOMENCLATURE

P_n = Single frequency pulsating pressure at node n

U_n = Single frequency pulsating volumetric flow at node n in direction of increasing node Nos.

Z_n = Acoustic impedance at terminal node n

r,s,t,v,w,x Node reference numbers

a,b,c,d,e Complex coeficients or variables

T = No. of end points in system

S_n = No. of sections in system with n nodes

N = Total No. of nodes in system

E = Total No. of equations

INTRODUCTION

The transfer matrix method for the acoustic simulation of compressor inlet and discharge systems to calculate the pulsating components of pressure is generally carried out by multiplication of the matricies representing the individual sections in the system followed by the solution of the resulting two equations using the known end conditions.[1] When the system includes a side branch the impedance of that branch must be determined before the matrix representing the branching section can be constructed. When the pulsating components at intermediate points of the system are required a transfer matrix for the system between one end and the intermediate point has to be obtained and the solution for the pulsating components of pressure and flow at the end substituted to determine the components at the intermediate point.

A solution for the system which determines the pressure and flow components at all intermediate and end points can be obtained by solving simultaneously all the equations which make up the individual transfer matrices of the sections. When a solution for all intermediate points in a system is required the simultaneous method can produce the result with less computation than by matrix multiplication providing the equations are handled in the appropriate sequence.

The simultaneous method will handle systems which include side branches,loops and multiple end points. A set of equations can be assembled to represent a manifold with volume and several nozzles. The end points can be defined by their acoustic impedance or as a source of flow to or from the system.This makes it suitable for acoustic simulation of single and multi stage reciprocating compressor systems with several cylinders driven from the same crankshaft.

EQUATIONS TO BE SOLVED

The equations to be solved are represented by the transfer matrix of each section of the acoustic system under analysis and by the boundary conditions at the end points of the system. The following description assumes that nodes have been numbered in ascending order along the system as connected except where a loop in the system prevents this convention being applied. Fig.1 shows a system layout numbered in this way.

a.Boundary conditions

The end points of the system are either open,closed or connected to another system whose acoustic impedance is known at the frequency being analysed. In each case the boundary condition can be defined by one equation containing complex variables. A particular form of a closed end is the connection to a compressor where the volumetric flow is given by a complex variable defining its magnitude and phase.

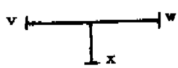
b.Sections with two nodes

For a section of the system such as a pipe,volume chamber or change of section a transfer matrix can be defined to relate the pulsating pressure and flow components at two nodes[2][3] which has the form:

$$\begin{bmatrix} P_s \\ U_s \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} P_t \\ U_t \end{bmatrix} \qquad s < t: \qquad \ldots\ldots\ldots\ldots\ldots(1).$$

c.Multi-node sections

Sections of the system with more than two nodes such as branches yeild the same number of equations as there are nodes.A branch with 3 nodes has 2 equations to define the pressure equality between nodes and a third equation to define the continuity of pulsating flows at the separate nodes.



$$P_v - P_w = 0 \;..(2a);\; P_v - P_x = 0 \;..(2b);\; U_v + U_w + U_x = 0 \;..(2c).$$

$$v < w;\; v < x:$$

The addition of an extra node at the junction will add one pressure equality equation and increase the number of terms in the flow equation by one.

When the multi-node section completes a loop in the system and the node numbering convention cannot be applied to the adjoining section a change of sign at the term in the equation representing the joining flow is required.

When the multiple junction occurs at a manifold which is to be treated as a volume with several nozzles the flow continuity equation requires a pressure term.

$$U_v + aP_w + U_w + U_x = 0 \qquad \text{........(3).}$$

d. All equations

The number of equations obtained is:

$$T + 2S_2 + 3S_3 + \text{.........} + nS_n = E \qquad \text{.......(4).}$$

Since each node on a section of the system is either connected to another section or is an end point the number of nodes in a system is:

$$T + 2S_2 + 3S_3 + \text{.........} + nS_n = 2N \qquad \text{.......(5).}$$

$$E = 2N \qquad \text{..............(6).}$$

In every system there is available for solution 2N equations which includes those equations which define a known condition at a node. The solution of this set of equations will determine the two variables of pulsating pressure and pulsating flow at each node.

ASSEMBLING THE EQUATIONS

The equations are assembled into a 2N by 2N+1 matrix for solution by Gaussian Elimination.

Equations derived from the end points are the first to be entered and are of the form:

$P_r = e$ ..(7a); $P_r - Z_rU_r = 0$ ..(7b); or $U_r = e$ ..(7c);

Equations defining pressure (7a) or impedance (7b) are entered as equation No. 2r-1 and an equation defining flow is entered as equation No. 2r into the matrix illustrated in Fig 2.

The equations for sections with 2 nodes are derived from (1):

$$P_s - aP_t - bU_t = 0 \qquad \text{.........(8a).}$$

$$U_s - cP_t - dU_t = 0 \qquad \text{.........(8b).}$$

$s < t$;

The two equations 8a & 8b are entered as equations Nos 2s-1 and 2s providing those positions in the equation matrix are not already occupied. Equations which cannot be entered into these positions are assigned to the spare equation matrix.

Entering the equations for end points and 2 node sections in this way with $s < t$ does not introduce any elements into the lower triangle of the main equation matrix.

The sections of the system with 3 or more nodes produce 2 equations which define P_v and U_v and can be entered in the rows 2v-1 and 2v subject to the same conditions as for the 2 node sections. The additional equations from multi-node sections are assigned to the spare equation matrix.

When the first allocation of equations is completed there will be an equal number of unused rows in the main equation matrix as there are equations in the spare equation matrix. A simple transfer operation will transfer equations from the spare equation matrix into these vacant rows of the main equation matrix.

The result is an equation matrix which is largely upper triangular and with most of the elements on the diagonal filled. The number of rows with elements in the lower triangle is equal to the sum of the number of end points and branches in the system.

SOLVING THE EQUATIONS

The matrix of equations assembled are solved by Gaussian Elimination. This method is described in mathematics texts [4] for real variables and also applies to complex variables which occur in the case of acoustic analysis. The method entails a forward substitution from equations in lower numbered rows to eliminate elements in the lower triangle. The presence of only a few elements in the lower triangle has reduced the computation at this phase of the equation solution. Before forward substitution the elements on the main diagonal are unity unless filled by the transfer of a spare equation and partial pivoting is not used during the solution. Trivial pivoting by exchanging rows during forward substitution is carried out only if the pivotal element is zero.

Experience in using this algorithm has shown there is a negligable loss of accuracy with double precision computation when using trivial pivoting compared with partial pivoting.

APPLICATIONS

The simultaneous method of solving the equations derived from the individual transfer matricies and the boundary conditions of the acoustic model is applicable to the pulsation analysis of compressor intake and delivery systems. A programme has been written which assembles the equations from input data describing the geometry and the interconnection of each section to other sections. The programme then proceeds to solve the equations determining pressure and flow pulsations at each node. This process is repeated for successive harmonics. The application of this algorithm to a compressor system analysis programme is shown in the flow diagram Fig 3.

It has particular advantages compared with methods using successive multiplication of transfer matricies when more than one compressor cylinder is connected in parallel. These cylinders are connected to a common crankshaft and hence the phase relationship of the pulsating flows are known and expressed together with the magnitude of the flows by the complex variables in the equations 7c. All the cylinder flows are concidered simultaneously and their combined influence on the pulsation at each node is determined.

CONCLUSIONS

The algorithm described allows sections of a system with two nodes to be described by the equations contained witin a 4 pole transfer matrix and for these sections to be assembled into the overall system by equations representing branches or manifolds. There is no restriction to the complexity of the system layout. The algorithm can be easily applied to compressor inlet and delivery systems with more than one cylinder per stage.

REFERENCES

[1] Sakai T.and Mitsuhashi K. 'Study on Pulsations of Reciprocating Compressor Piping Systems',Bulletin of the JSME Vol 16 No 101,1973.

[2] Abe T.,Fujikawa T.and Ito S. 'A Calculating Method of Pulsation in a Piping System',Bulletin of the JSME Vol 13 No 59,1970.

[3] Kammin H. 'The Development and Experimental Verification of an Acoustic Damping Model for a Frequency Domain Digital Pulsation Simulation',1988 International Compressor Engineering Conference at Purdue.

[4] Mathews J.H. 'Numerical Methods',Prentice Hall International,Inc. 1987.

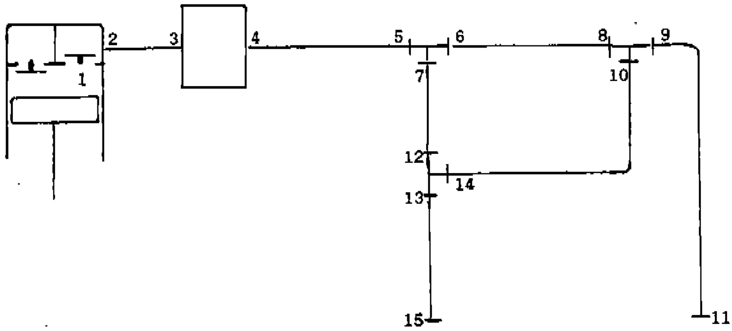


Fig.1 Typical Node Numbering

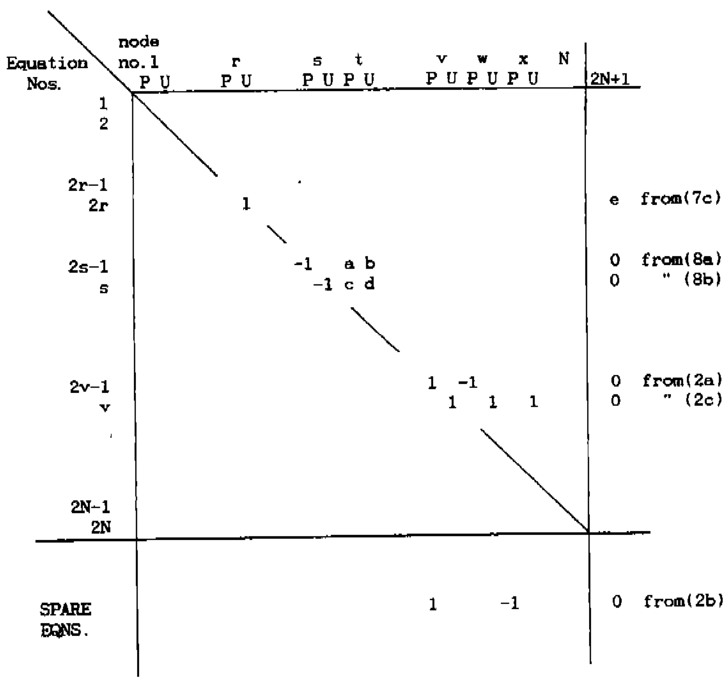


Fig.2 Equation Matrix

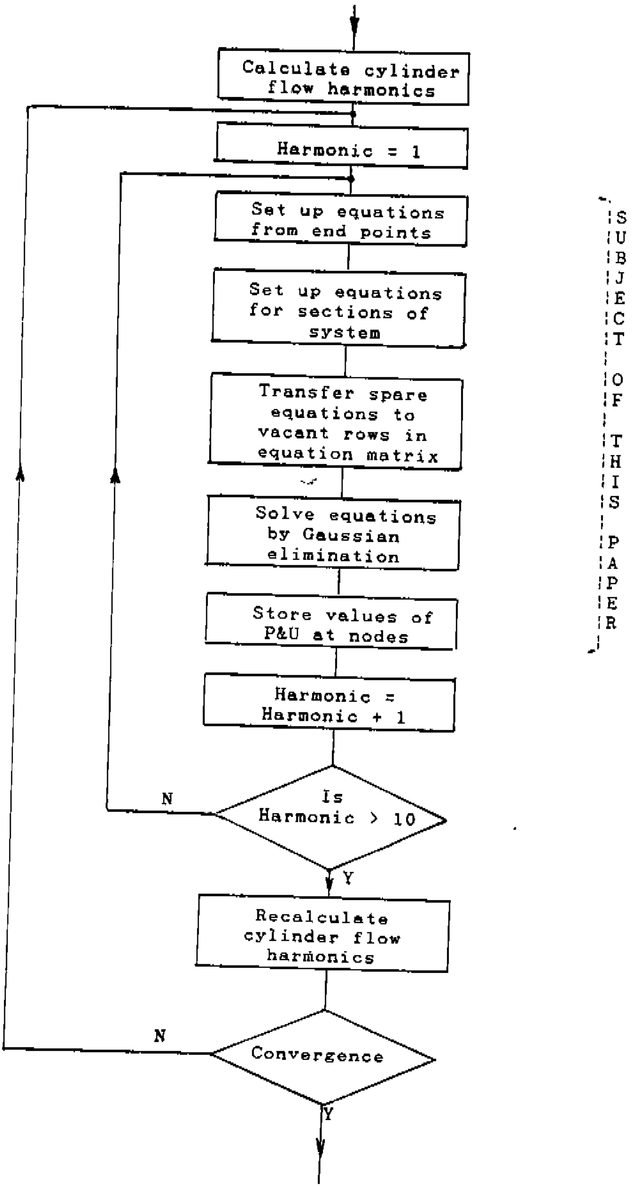


Fig.3 The Simultaneous Algorithm in a system analysis programme.