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N. Stosic
University of Sarajevo

A. Kovacevic
University of Sarajevo

K. Hanjalic
University of Sarajevo

Lj. Milutinovic
University of Sarajevo

MATHEMATICAL MODELLING OF THE OIL INFLUENCE UPON THE WORKING CYCLE OF SCREW COMPRESSORS

N.Stošić, A. Kovačević, K. Hanjalić, Lj. Milutinović

SUMMARY

The paper presents some results of mathematical modelling of the oil injection into the screw compressor and its influence upon the thermodinamic process of the engine. The model was subsequently applied to analyse the effect of various parameters (oil droplet size, inlet port position, oil temperature, oil to gas mass ratio, oil viscosity) upon the compressor physical cycle. The paper demonstrates that the model could be employed for the optimisation of the oil injection parameters.

INTRODUCTION

Screw compressors belong to a relatively new class of engines whose wider commercial manufacturing began only few decades ago. As a rotary engine of a simple design (it does not contain any oscilating elements) it can develop a high rotating speed and consequently a high power and efficiency per unit weight as well as a wide range of operating pressure and delivery /12,13/. The reason for their late appearance may be located in the complexity of the design and manufacturing of compressor rotating elements and in a strict requirements for the accuracy of their matching /2/. Some reasons could be traced in difficulties in defining an accurate analytical model of the physical process which could be utilized for estimating the compressor performances.

The operating process of a screw compressor is usually described by a range of algebraic relations obtained by integration of the continuity and energy equation after some approximations and introduction of a number of experimentally determined coefficients. These relations are usually restricted in application and they are far from an exact approach to solve the screw compressor thermodynamic process. If it is known that kinematic relations of a screw compressor do not have a simple algebraic form either, then the application of simplified expressions can be justified only in the case of rough analyses of the thermodynamic process /3, 5/.

A generalised approach has been applied which implies the computation of screw compressor thermodynamic process by numerical solving /18, 8, 10/ the set of differential equations for energy and continuity with appropriate initial and boundar y conditions. The equations have a simple form for the case of screw compressor, and in our model they were on the basis of the following assumptions (which ensure suffic ient universality and wide application): a) working fluid encountered in the model could be any gas having known equation of state and relations for internal thermal energy and entalpy, i.e. any ideal or real gas or gas-oil mixture, b) heat transfer between gas and compressor screws or its casing was incorporated in the model, c) gas or gas-oil mixture inflow or outflow through the compressor suction or discharge openings was assumed isentropic, d) gas or gas-oil mixture gap leakages which can hapen in any process stage were assumed isenthalpic, e) oil was allowed to be injected during all of the compressor process stages: suction, compression or discharge and it affected all theese processes. The temperature of the oil droplets and the gas during all of the processes could be different according to the gas-oil heat transfer conditions. The addopted approach differs from the common one in chosing the internal energy as a dependadnt variable instead of pressure or enthalpy which was the usual practice before.

The extension of the model application presented here concerns the investigation of the oil influence on the screw compressor process. The injection of lubricating oil has a strong effect upon the total energy interchange between the fluid and the engine /15/. The oil temperatures, oil-to-air mass ratios, oil droplet sizes and locations of oil-injection ports were varied over a wide range to obtain the necessary informations about the influence of oil to the working porocess of the compressor. The history of presure and temperatures of gas and oil during the compressor cycle and the volumetric and power utilization efficiences, as well as specific powers were compared for various input data variations.

MATHEMATICAL MODELING OF THE OIL INFLUENCE ON THE SCREW COMPRESSOR WORKING PROCESS

The conservation equation for the internal energy in terms of angle of rotation for the control volume (compressor working volume) can be written as:

$$\frac{dU}{d\varphi} = \left(\frac{dI}{d\varphi}\right)_{in} - \left(\frac{dI}{d\varphi}\right)_{out} + \frac{dQ}{d\varphi} - \frac{dW}{d\varphi} \tag{1}$$

where I is fluid enthalpy, Q and W heat and work respectively, exchanged between the system and its surrounding through the control surface, and the subscripts "in" and "out" denote the control volume inlet and outlet. If the working fluid is a mixture of gas and oil droplets, the terms of the equation can be expressed in the form:

$$\left(\frac{dI}{d\varphi}\right)_{in} = i_{in}\left(\frac{dm}{d\varphi}\right)_{in} - i_{in}\left(\frac{dm}{d\varphi}\right)_{o}$$

$$\left(\frac{dI}{d\varphi}\right)_{out} = i_{out}\left(\frac{dm}{d\varphi}\right)_{mix}$$

$$\frac{dQ}{d\varphi} = \frac{A\alpha}{\omega}(T_s - T)$$

$$\frac{dW}{d\varphi} = -p\,\frac{dV}{d\varphi}$$

where "i" denotes enthalpy per unit mass, "p" pressure, "T" temperature, "V" compressor volume, "A" heat transfer area, "ω" rotating speed, with indices "in", "out", "g" - gas, "o" - oil, "mix" - mixture and "s" - surface.

Together with the continuity equation:

$$\frac{dm}{d\varphi} = \rho w A/\omega \tag{2}$$

where "A" means flow area, "w" fluid velocity and "ρ" denotes density, the equations (1)and (2) form a system of differential equations describing the screw compressor thermodynamic process. The right hand side of the energy equation consists of the following terms which have been apropriately modelled:

- The energy gain due to the gas inflow into the working volume is represented by the gas enthalpy and change of its mass ($dm/d\varphi$). During the suction gas enters the working volume bringing gas enthalpy which prevails in the suction chamber. However, during the time when the suction opening is closed, certain amount of gas leaks through clearances filling the working space of the compressor. The mass of this gas, as well as its enthalpy are determined on the basis of the gas leakage equations.

- The energy loss due to the gas outflow from the working volume is determined by its mass and outgoing gas enthalpy. During the compression this is the gas which leaks through the clearance from the working volume into the surrounding space.

- Mechanical work brought to the gas during the working process is positive if the volume of working space is decreased. The gradient $dV/d\varphi$ is obtained from the kinematic screw rotor relations.

- The effect of the fluid heat exchange between the fluid and screw rotors and casing due to the different temperature of the gas and the rotors and casing surfaces is accounted through the simple cooling law with heat transfer coefficient determined on the basis of the well known expression $Nu=0.023\,Re^{0.8}$. The characteristic length and velocity in Reynolds number are obtained from the geometric characteristics of the compressor.

If the oil is injected into the compressor volume, the oil mass and its enthalpy should be accounted for. The oil is injected into the working volume of a srew compressor for reason of sealing, cooling and lubrication purposes. It affects considerably the thermodynamic process of a screw compressor. Better sealing means less leakage of the gas and the improvement of the compression and delivery rates. In addition an increased mass rate of the working medium due to the oil injection, lowers the compression temperatures, allowing in such a way higher compression rates.

The heat transfer between the gas and oil droplets is defined by the simple cooling law $Q=\alpha\,A(T\ To)$ where "α" is heat transfer coefficient, "A" is a mean droplet surface and "T" and "To" are gas and oil temperatures. At the same time $Q = m_o c_o(To-Tob)$ where "m_o" is the oil mass, "c_o" is specific heat of oil, and Tob is the oil temperature in previous time step.

Heat transfer coefficient "α" at the droplet surface is obtained from the expression:

$$Nu = 2 + 0.6\ Re^{0.6}\ Pr^{0.33} \tag{3}$$

where the oil velocity in Reynolds number is determined from the oil mass rate injected through the orifice of diameter D. In order to define the Re and Nu numbers the mean droplet diameter d was used as suggested by Sauter. A range of droplet diameters from 1 to 1000 μm was covered in the mathematical model as to examine the influence of droplet size upon the working process of a screw compressor.

The oil temperature is directly obtained from heat transfered from gas to oil as:

$$T_o = \frac{T+kT_{ob}}{1+k} \tag{4}$$

where k=mc/ A =do c/6 is the time constant and o is the oil density.

In the continuity equation (2) the density "ρ" is obtained from the known mass and volume V as $\rho=m/V$, the suction and discharge opening cross section surface area, as well as the area of theclearance between the mobile parts from the known kinematic characteristics of the compressor, and the flow rate through these openings from the energy equation and the rate of change of momentum depending on the character on the flow. In this case, the flow through the suction and pressure opening is considered as adiabatic, and gas leakage through the clearances is considered as isenthalpic for which the corresponding differential relations exist.

As a supplement to the basic differential equations of the compressor thermodynamic process several constitutive algebraic relations are to be added. These include equation of state. If the gas can be treated as ideal, the internal thermal energy of the gas-oil mixture is given by:

$$U = m\,u + m_o u_o = \frac{mRT}{(\varkappa-1)} + m_o c_o T_o = \frac{pV}{(\varkappa-1)} + m_o c_o T_o \tag{5}$$

where "R" denotes gas constant. Hence, from the equation (5) either the pressure or temperature of the fluid in the compressor working space can be explicitely calculated with help of the equation for gas-oil heat transfer which gives the gas temperature:

$$T = (\varkappa-1)\,\frac{(1+k)U - m_o c_o T_o k}{(1+k)mR + m_o c_o} \tag{6}$$

It is obvious, that for k=0 (high heat transfer coeficients or small oil droplet size) the oil temperature approaches the gas temperature.

The situation is somewhat more complex if the equation of state of a real gas is considered: p=f1(T,V), for example in /11, 7/, which with the equation for internal enrgy of real gas: u=f2(T) and with equation for internal energy (1) form a system of equations. After the internal thermal energy U and mass of the gas in screw compressor working volume are determined, the resulting sistem of algebraic equations is usually decoupled, and can be conveniently solved by numerical method.

The modelled diferential equations are solved by means of Runge-Kutta four order procedure. Since the initial conditions are arbitrarly selected, the convergence of the solution is obtained after the difference between the two consecutive compressor cycles showed a sufficiently small value prescribed in advance.

PRESENTATION OF RESULTS

The investigation of the influence of the oil injection upon the screw compressor thermodynamic cycle was carried out through the analysis of the separate effects of the following oil parameters: the size of oil droplet, oil inlet temperature, position of oil inlet port and the oil viscosity. The considered compressor had the following characteristic dimensions: distance between the rotor pairs a=95 mm, the angle of helix 45 degres, the ca lculation clearance 0.09 mm (Fig.1). The compression process has started at 0 deg and finished at 298 deg of the male-rotor rotation angle. More information could be found in ref. /21/.

Table 1. Input conditions

Speed of rotation 4222 rpm	Tip speed 30 m/s
Suction ambient pressure 1 b	Suction temperature 288 K
Discharge pressure 6 b	Rotor/housing mean temperature 333 K
Air gas constant 287 J/kgK	Specific heat ratio 1.4

Droplet size varied over a wide range from 1 to 1000 μm as to cover all aticipated sizes. The oil inlet temperature varied from 293 to 373 K, while the oil-to-gas mass ratio varied from 0 (dry air) to 8. A range of position angles of the oil inlet port from 0-270 degrees was considered. In addition, five types of oil, each of different viscosity, were considered and for each a different clearance has to be selected. Volumetric efficiency (delivery flow rate normalized by the theoretical delivery), power efficiency (isothermal power normalized by indicated one) and specific power (indicated power normalized by delivery) were selected as output parameters. All considered input values together with volumetric and power efficiency and specific power, are listed in table 2.

Table 2. Volumetric and power efficiency and specific power

Oil in. temperat. (K)	Oil/gas mass ratio	Oil in. position (deg)	Oil type (viscos.) (-)	Volumet. effic. (-)	Power effic. (-)	Specific power (kW/m3/min)
293	4	150	3	0.6691	0.8261	3.614
313	4	150	3	0.6614	0.7812	3.823
333	4	150	3	0.6539	0.7480	4.033
353	4	150	3	0.6469	0.7037	4.244
373	4	150	3	0.6400	0.6710	4.456
333	0	150	3	0.6248	0.5436	5.493
333	2	150	3	0.6509	0.7218	4.137
333	4	150	3	0.6539	0.7405	4.033
333	6	150	3	0.6553	0.7480	3.992
333	8	150	3	0.6560	0.7520	3.971
333	4	0	3	0.6495	0.7482	3.991
333	4	90	3	0.6562	0.7449	4.008
333	4	150	3	0.6539	0.7480	4.033
333	4	210	3	0.6487	0.7262	4.112
333	4	270	3	0.6403	0.6887	4.336
333	4	150	1	0.6228	0.7331	4.074
333	4	150	2	0.6392	0.7371	4.051
333	4	150	3	0.6539	0.7480	4.033
333	4	150	4	0.6674	0.7435	4.016
333	4	150	5	0.6796	0.7461	4.002

Figs. 2 to 4 show the calculated temperature of gas and oil in function of the rotational angle during a compressor cycle for different droplet sizes. It is clearly visible that the oil and gas temperature have the same value for a wide range of droplet sizes, almost up to 500 m, and even for extremely large droplets of 1 mm, gas to oil temperature difference is still les than 1.5 deg. C. It confirms the assumption that the value of the oil temperature in screw compressors can be considered the same as the gas temperature. It may be different if oil droplets are alolowed to reach the housing surface due to inertial forces and to form oil film which would cause different heat transfer conditions.

In Figs. 5 to 8 the air/oil temperatures are plotted over the rotational angle for different oil inlet temperatures, oil inlet positions, oil to air mass ratio and for different oil type.

It is visible that oil temperature has a strong influence on the air exit temperature. In fact, it affects the air temperature over the whole cycle influencing in such a way the compressor power efficiency, but in the same time, exerting only a minor influence upon the volumetric efficiency. The situation is quite oposite if the oil type is considered, when both, the power and volumetric efficiences are considerably affected, but with only a slight slight influence on the air outlet temperature. For low oil to gas mass ratios the effects upon the compressor process are more visible than in the case of higher mass ratios. This indicates a possibility for an optimisation of the oil temperature and oil to gas mass ratio, especially if other equipement, like oil removers and oil coolers are considered. The position of the oil inlet point position has a strong influence on air temperature during the cycle, but it has no influence on the discharge temperature because the same oil quantity is injected at all oil inlet points. As the angle of the oil inlet point increases, the power efficiency descreases and so does the volumetric efficiency after passing through its maximum. Hence, it should be possible to find an optimum position of the oil injection port.

CONCLUSION

The following results calculated by means of the mathematical model of crew compressor working process have been obtained: the flows, pressures and temperatures of operating media in a function of rotational angle of the main rotor over a screw compressor cycle for differnt parameters representig the influence of the oil. These values provided the basis for determining the integral characteristics of screw compressors such as flow rate, delivery degrees, isothermal, adiabatic and indicated work and their corresponding powers, specific powers and power utilisation coefficients.

It can be concluded that the temperature of the oil folows the gas temperature during the compressor cycle closely, except for the extremely large oil droplet sizes (over 1 mm). An important influence of the oil temperature and oil injection location has been noticed, the oil to gas mass ratio, and the oil type have a surprisingly small influence on the compressor process.

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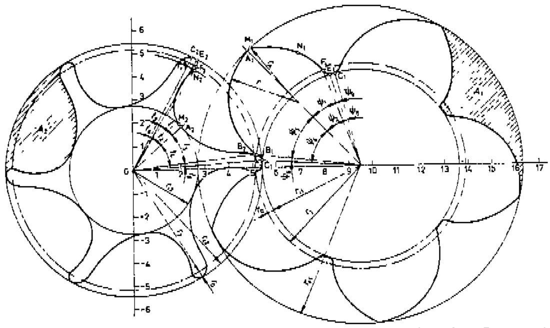

Figure 1. Screw compressor configuration as a basis for mathematical modelling

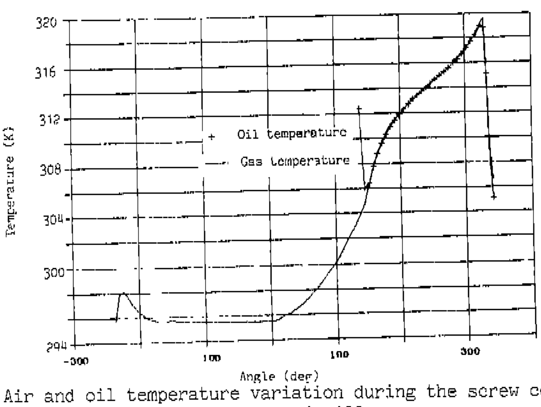

Figure 2. Air and oil temperature variation during the screw compressor cycle with Sauter mean oil droplet size d= 100 μm

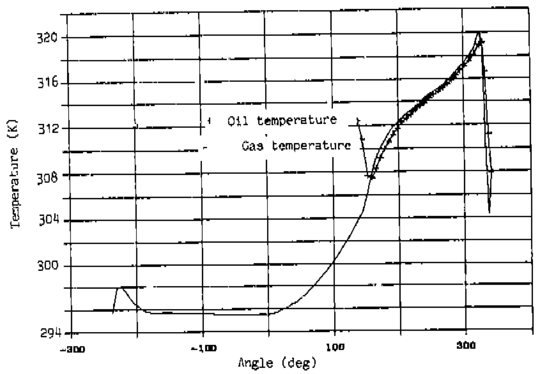

Figure 3. Air and oil temperature variation during the screw compressor cycle with Sauter mean oil droplet size d= 500 μm

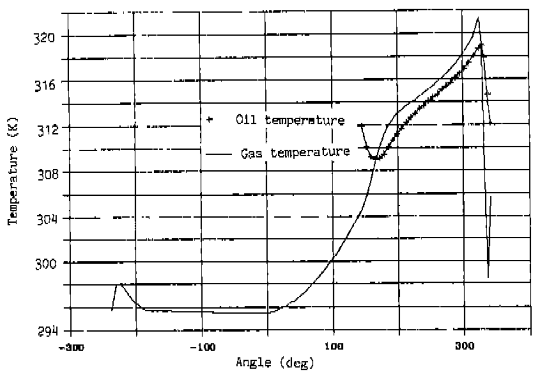

Figure 4. Air and oil temperature variation during the screw compressor cycle with Sauter mean oil droplet size d= 1000 μm

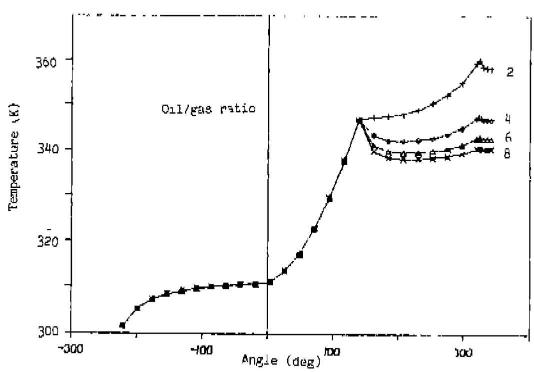

Figure 5. Influence of oil to gas mass ratio on air temperature during the compressor cycle

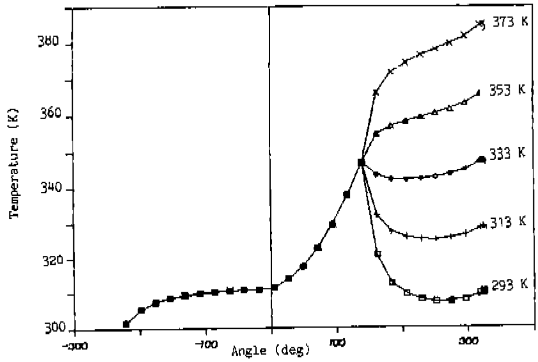

Figure 6. Influence of oil inlet temperature on air temperature during the compressor cycle

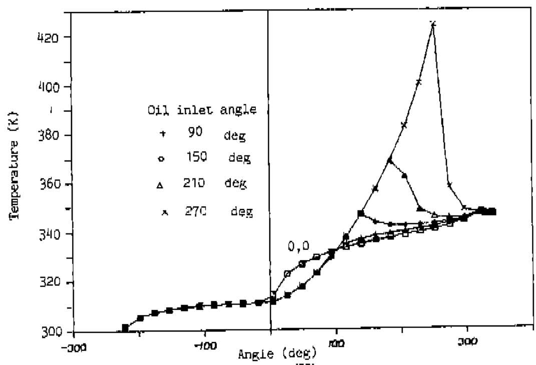

Figure 7. Influence of oil injection position on air temperature during the compressor cycle

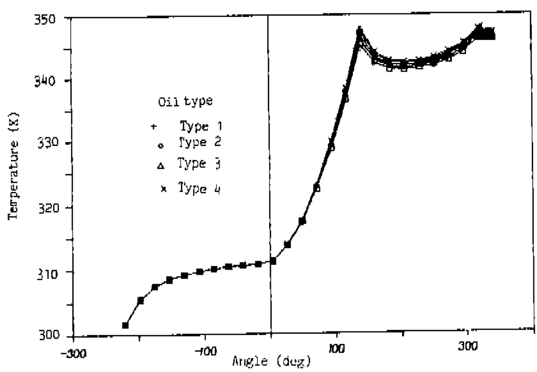

Figure 8. Influence of oil type (viscosity) on air temperature during the compressor cycle