Analysis of Cavity Resonance in 5 HP Hermetic Reciprocating Compressor with Elliptical Shell

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ABSTRACT

Hermetic reciprocating compressor with elliptical shell radiates noise whose frequency spectrum has a peak level at about 200 Hz and about 300 Hz. The 200 Hz and 300 Hz noise is mainly radiated from the top and the side of the compressor shell, respectively.

For clarifying the noise radiation mechanism, experimental and theoretical analyses were performed. A noise source simulation test was set up where a speaker (driver unit), connected to a compressor suction pipe, was driven by a white noise oscillator, and transfer function was measured by using four microphones. These tests clarified that the noise is cavity resonance in the compressor shell.

By simulating a compressor mechanism and a shell to an annular cavity, resonance frequency can be calculated and it was clarified that 200 Hz and 300 Hz are cavity resonant frequencies in vertical derection and radius direction of compressor,respectively. By investigating the source using a compressor with some pressure and vibration acceleration transducers, it was clarified that the source is suction pressure pulsation in the compressor shell.

INTRODUCTION

Noise suppression is one of the most important requirements in a refrigeration compressor, outside of its compression function. With increasing air conditioner diffusion, demand for noise reduction is increasing year by year. In Japan, it is a fact that the noise reduction ratio for residential room air conditioner is 2 dB(A) every year. As noise reduction for frequencies below 500 Hz by noise absorbing method is difficult, countermeasures on the compressor itself are necessary.

This paper describes experimentally and theoretically that the noise at about 200 Hz and about 300 Hz radiated from 5 HP hermetic reciprocating compressor with elliptical shell is cavity resonance in compressor shell, and describes the investigation method for determining the exiting source of cavity resonance too.

EXPERIMENTAL ANALYSIS

Compressor sound measurements were made on a compressor testing refrigeration system in a semi-anechoic chamber. (Fig. 1) Three condenser microphones were set at a distance of 100 mm from the compressor shell. The microphones are set on the upper direction, long axis radius and short axis radius directions with regard to the elliptical compressor shell.

Frequency spectrums were measured by one-third actave band frequency analyzer and narrow band frequency analyzer (Fourier analyzer). Typical one-third actave band noise spectrums for three directions are shown in Fig. 2. 200 Hz band on the upper direction and 315 Hz band on the radius direction show a peak amplitude. For clarifying the noise radiation mechanism, the compressor, on which was mounted three pressure and three vibration acceleration transducers as shown in Fig. 3, was connected to a compressor testing refrigeration system and operated.

When compressor revolution speed was changed by varying electric source frequency, the amplitude changes of 4th, 5th and 6th harmonics of compressor revolution are as shown in Fig. 4. Frequency spectrums of pressure pulsation in the compressor shell and in the motor cover used for suction gas entrance to the cylinder, are compared in Fig. 5. From these spectrums, the followings results are reported.

(1) The cavity resonant frequencies in compressor shell are 220 Hz and 275 ∿ 300 Hz.

(2) The compressor shell vibration amplitude shows peak level at 220 Hz in the

vertical direction and at 275 ∿ 330 Hz in the radius direction for the compressor shell.

(3) Exiting source of cavity resonance is suction gas pressure pulsation in the compressor shell.

As next step, noise source simulation tests were performed. By driving the speaker (driver unit) which was connected to a compressor suction pipe with a white noise oscillator, cavity resonant frequencies were measured. Four microphones were prepared, one was set in the suction pipe as the standard microphone for measuring the output sound pressure level from the driver unit. The others were set in the compressor shell for measuring the sound pressure level in the shell. (Fig. 6) As data for analysis, transfer function (T_{ri}) between the sound pressure level measured by standard microphone (P_{ref}) and the level in the shell (P_i, $i = 1,2,3$) was measured.

$$T_{ri} = P_i/P_{ref}, \quad (i = 1,2,3) \qquad (1)$$

where, i means microphone location, i = 1,2, 3 are, respectively, vertical direction, short axis radius direction and long axis radius direction. As this test was made in air, in which sound velocity is 340 m/s, and the velocity in refrigerant is 170 m/s, measuring data were shown in corrected frequency value.

Frequency spectrum on the long axis radius direction (i = 3) while closing some suction holes in the motor cover, is shown in Fig. 7. This data indicates that 170 Hz and 290 Hz are cavity resonant frequencies defined by the cavity between compressor mechanism and shell. Furthermore, for making clear cavity resonance phenomenon, can of which dimension was 210 mm H x 130 mm L x 80 mm D was assembled in the compressor shell, and cavity resonant frequency was measured. The same measurement was also made in the case of empty shell. Testing results are shown in Fig. 8. From these data, it is clarified that the larger the size of object assembled in the compressor shell, the lower the cavity resonant frequency becomes.

Phase angles, in case of compressor assembled shell, are compared in Fig. 9. 1st harmonic modes are the same phase at three points, but phase angle between point 1 and points 2 and 3 shifts 180 degrees in 2nd harmonic mode. Actual mode in the shell is presumed as shown in Fig. 10. From this phase shift, it is understood that the resonant frequencies of short axis and long axis radius direction are very close.

THEORETICAL ANALYSIS

(1) Cavity transfer function

Consider the propriety of sound source simulation test. A suction pipe and a compressor shell are assumed to be a simple one dimensional acoustical tube as shown in Fig. 11. Sound pressure P_1 at the tube entrance, P_2 in cavity, and volumetric velocities U_1 and U_2 satisfy the following equation.

$$\begin{bmatrix} P_1 \\ U_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} P_2 \\ U_2 \end{bmatrix} \qquad (2)$$

Boundary condition is $U_2 = 0$. Therefore,

$$P_1 = AP_2$$

Transfer function $H(f) = P_2/P_1 = 1/A$:

$$H(f) \cong -\frac{S_1}{S_2} \cdot \frac{1}{\sin k\ell_1 \cdot \sin k\ell_2} \qquad (3)$$

$$(\text{as} \quad S_2/S_1 \gg 1)$$

Equation (3) means that transfer function is a composition of resonant characteristics for pipe length ℓ_1 and ℓ_2. As an actual compressor is a three dimensional structure, the following equation is assumed from Eq. (3).

$$H(f) \cong \alpha \cdot \frac{1}{\sin k\ell_1} \cdot z \qquad (4)$$

where,
α is constant.
z is resonant characteristic in compressor shell.

Therefore, in order to accurately measure the resonant characteristics in the shell, pipe length ℓ_1 has to be as short as possible. As actual compressor suction pipe length is 60 mm,

$f_0 \cong 2800$ Hz (in air) is gained.

This frequency causes no trouble, because of being separated from 400 Hz and 600 Hz in air, corresponding to 200 Hz and 300 Hz in refrigerant.

(2) Cavity resonance in annular cavity

The cavity bounded by the outer surface of a compressor mechanism and the inner surface of its hermetic shell can be simulated as an annular cavity. In cylindrical coordinates, the acoustical wave equation is:

$$\frac{\partial^2 \Phi}{\partial r^2} + \frac{1}{r}\frac{\partial \Phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \Phi}{\partial \phi^2} + \frac{\partial^2 \Phi}{\partial z^2} = \frac{1}{C^2}\frac{\partial^2 \Phi}{\partial t^2} \qquad (5\text{-}1)$$

where,
c = sonic velocity
$\Phi(r,\phi,z,t)$ = velocity potential

If it is assumed:

$$\phi = R(r)\cdot\theta(\phi)\cdot H(z)e^{-jwt} \qquad (5\text{-}2)$$

and substitute into Eq. (5-1):

$$\frac{R''}{R} + \frac{1}{r}\cdot\frac{R'}{R} + \frac{1}{r^2}\cdot\frac{\theta''}{\theta} + \frac{H''}{H} + k^2 = 0 \qquad (5\text{-}3)$$

where, k is the wave number:

$$k^2 = w^2/c^2$$

As $\theta(\phi) = \theta(\phi + 2\pi)$:

$$\theta(\phi) = \sum_{n=0}^{\infty}(A_n \operatorname{Sin} n\phi + B_n \operatorname{Cos} n\phi) \qquad (5\text{-}4)$$

If considering only the n_{th} term,
Eq. (5-3) becomes:

$$\frac{R''}{R} + \frac{1}{r}\frac{R'}{R} + (k^2 - \frac{n^2}{r^2}) + \frac{H''}{H} = 0 \qquad (5\text{-}5)$$

After some manipulation:

$$H = (C \operatorname{Sin} k_z\cdot Z + D \operatorname{Cos} k_z\cdot Z) \qquad (5\text{-}6)$$

$$R = \{E\, J_n(k\rho r) + F\, N_n(k\rho r)\} \qquad (5\text{-}7)$$

where, k_z is a separate constant.

$$k\rho^2 = k^2 - k_z^2$$

and $J_n(k\rho r)$ and $N_n(k\rho r)$ are BESSEL function and NEUMANN function, respectively.
The solution of Eq. (4-1) is:

$$\Phi = \sum_{n=0}^{\infty} \Phi n$$

$$\Phi_n = \{E_n J_n(k\rho r) + F_n N_r(k\rho r)\}\{A_n \operatorname{Sin} n\phi + B_n \operatorname{Cos} n\phi\}\{C_n \operatorname{Sin} k_z\cdot Z + D_n \operatorname{Cos} k_z\cdot Z\}e^{-jwt} \qquad (5\text{-}8)$$

Boundary conditions:

$$-\frac{\partial\Phi}{\partial r}\bigg|_{r=R_1} = 0 \qquad -\frac{\partial\Phi}{\partial r}\bigg|_{r=R_2} = 0 \qquad (5\text{-}9)$$

$$-\frac{\partial\Phi}{\partial z}\bigg|_{z=0} = 0 \qquad -\frac{\partial\Phi}{\partial z}\bigg|_{z=L_z} = 0 \qquad (5\text{-}10)$$

where, R_1 = Inner radius
R_2 = Outer radius
L_z = Height

Substituting Eq. (5-8) into Eq. (5-9)

$$E_n J_n'(k\rho R_1) + F_n N_n'(k\rho R_1) = 0$$
$$E_n J_n'(k\rho R_2) + F_n N_n'(k\rho R_2) = 0 \qquad (5\text{-}11)$$

For a solution to exist

$$J_n'(k\rho R_1)N_n'(k\rho R_2) - J_n'(k\rho R_2)N_n'(k\rho R_1) = 0 \qquad (5\text{-}12)$$

While, from Eq. (5-10) yields:

$$C_n = 0$$

$$\operatorname{Sin} k_z\cdot L_z = 0$$

Therefore,

$$k_z = \frac{s\pi}{L_z} \quad (s = 0,1,2,\cdots) \qquad (5\text{-}13)$$

$k\rho$ for satisfying Eq. (5-12):

$$k\rho = \sqrt{k^2 - k_z^2}$$

Therefore,

$$f = \frac{C}{2\pi}\sqrt{k\rho^2 + (\frac{s\pi}{L_z})^2} \qquad (5\text{-}14)$$

From $k\rho = U_{nm}$
$(n = 0,1,2,3,\cdots,\ m = 1,2,3,\cdots)$

$$f_{nms} = \frac{C}{2\pi}\sqrt{U_{nm}^2 + (\frac{s\pi}{L_z})^2} \qquad (5\text{-}15)$$

$(n = 0,1,2,3,\cdots,\ m = 1,2,3,\cdots,$
$s = 0,1,2,3,\cdots)$

and substituting ξ and λ into Eq. (5-12)

$$k\rho R_2 = \xi$$

$$R_1/R_2 = \lambda \quad (0 \leq \lambda \leq 1)$$

$$J_n'(\lambda\xi)N_n'(\xi) - J_n'(\xi)N_n'(\lambda\xi) = 0 \qquad (5\text{-}16)$$

The solution of Eq. (5-16) is shown in Fig. 12.[1] In a special case, that is, when there is no vibration in the transverse plane. Mode only in the vertical direction is generated, and the resonant frequency (f_s):

$$f_s = \frac{SC}{2L_z} \quad (s = 1, 2, \cdots) \qquad (5\text{-}17)$$

Calculating results for 5 HP compressor (L_z = 350 mm, R_2 = 120 mm) tested are shown in Fig. 13.

CONSIDERATION

Experimental values by noise source simulation test and theoretical values by annular cavity simulation are compared in Table I. The experimental values for the 2nd harmonic resonant frequency are in close agreement with the theoretical values, but, there are differences between experimental and theoretical value for the 1st harmonic resonant frequency, and the more complicated the object assembled in a compressor shell is, the lower the cavity resonant

Table-1. Experimental and theoretical values for compressor cavity resonant frequency

	Radius Ratio	1st Harmonic		2nd Harmonic	
		Exp.V	Th.V	Exp.V	Th.V
Empty shell	0.0	250Hz	243Hz	380Hz	415Hz
Can assembled	0.5	220	243	315	305
Compressor assembled	0.7	170	243	290	270

Exp.V: Experimental value
Th.V : Theoretical value

frequency becomes in the shell. This is the reason why 1st harmonic resonant frequency was calculated by Eq. (5-17), assuming the mode shown in Fig. 14. Therefore, differences in case of an empty shell are few. As the actual compressor is not a simple annulus, some differences are naturally produced. In order to determine an accurate frequency, some one-dimensional acoustical tubes connected in series have to be considered. For example, in case of compressor mechanism shown in Fig. 15. As it is considered that five different sectional tubes (I, II, ··· V) are connected in series, the following equation can be yielded.

$$\begin{bmatrix} P_1 \\ U_1 \end{bmatrix} = \begin{bmatrix} A_I & B_I \\ C_I & D_I \end{bmatrix} \begin{bmatrix} A_{II} & B_{II} \\ C_{II} & D_{II} \end{bmatrix} \cdots \begin{bmatrix} A_V & B_V \\ C_V & D_V \end{bmatrix} \begin{bmatrix} P_2 \\ U_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} P_2 \\ U_2 \end{bmatrix} \quad (5\text{-}18)$$

and

Boundary conditions:

$$U_1 = U_2 = 0 \quad (5\text{-}19)$$

Substituting Eq. (5-19) into Eq. (5-18):

$$C = 0 \quad (5\text{-}20)$$

By solving $C = 0$, the resonant frequency can be calculated.

CONCLUSION

(1) The cavity resonant frequency can be easily measured by sound source simulation test, that is, driving a speaker connected to a compressor suction pipe by a white noise oscillator.

(2) 1st and 2nd harmonic resonant frequency is the vibration mode in the compressor shell vertical direction and the long axis or the short axis direction of an elliptical section, respectively.

(3) The theoretical cavity resonant frequency value, calculated by simulating compressor mechanism and shell to annular cavity, coincides comparatively with the experimental value.

(4) The exiting source of cavity resonance was clarified to be suction gas pressure pulsation in the compressor shell.

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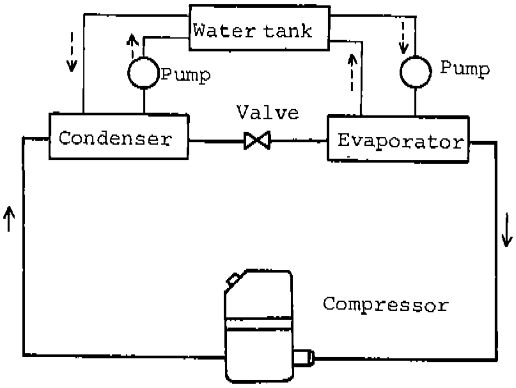

Fig. 1 Compressor testing refrigeration system

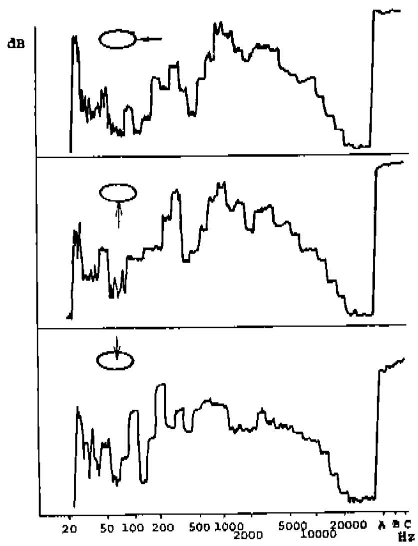

Fig.2 Typical one-third octave band spectrums for 5 HP compressor

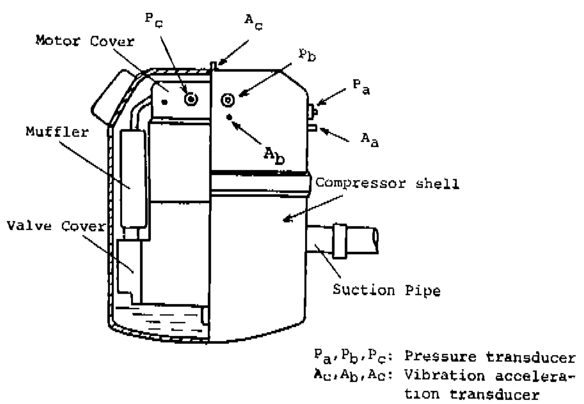

Fig. 3 Compressor with transducers

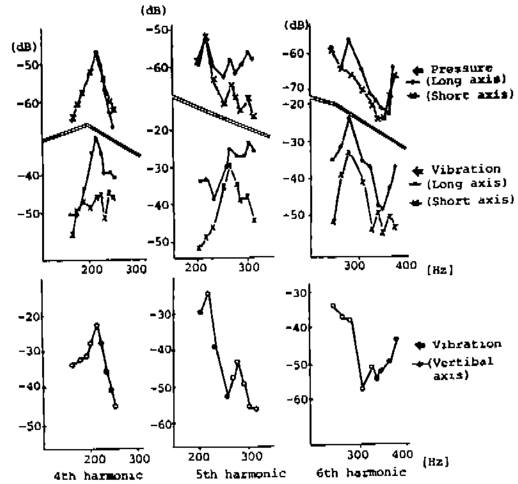

Fig.4 Amplitude changes in pressure and vibration 4th, 5th and 6th harmonics

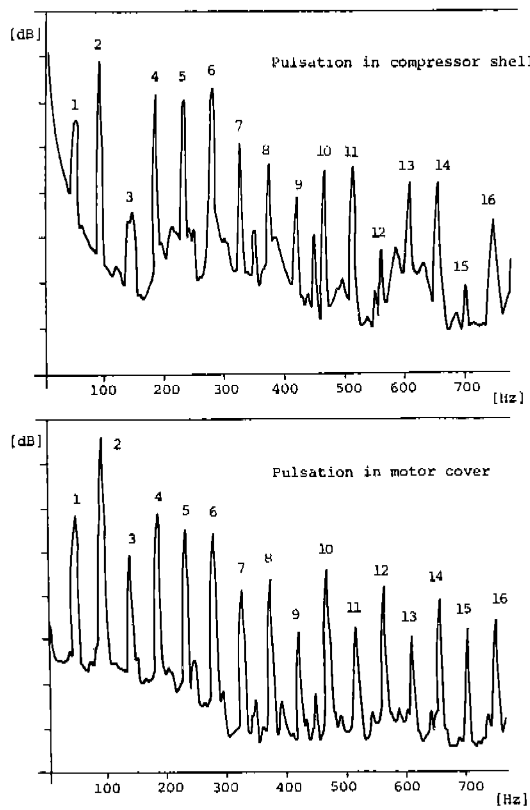

Fig.5 Pressure pulsation frequency spectrums

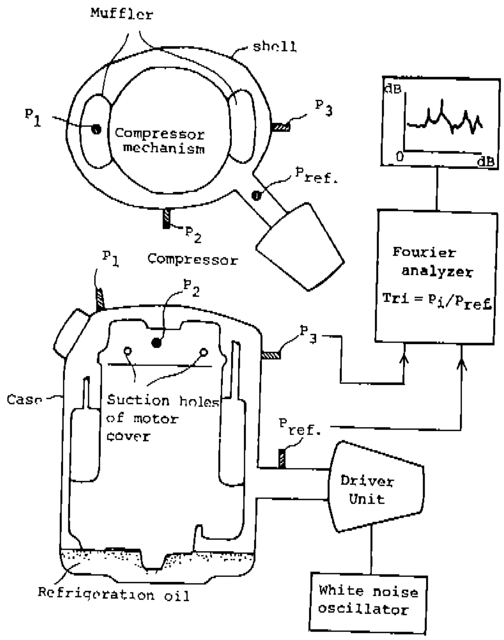

Fig. 6 Noise Source Simulation System

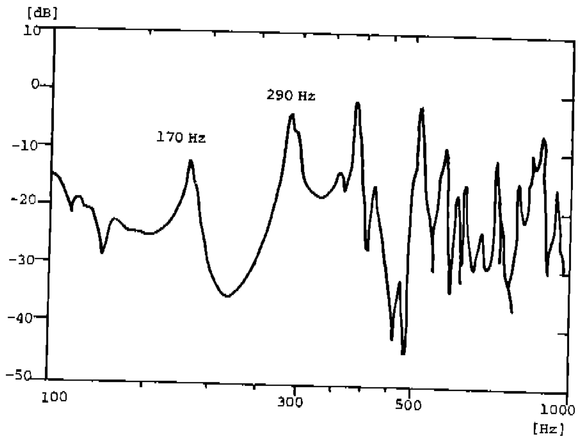

Fig. 7 Frequency spectrum on the long axis radius direction (i = 3)

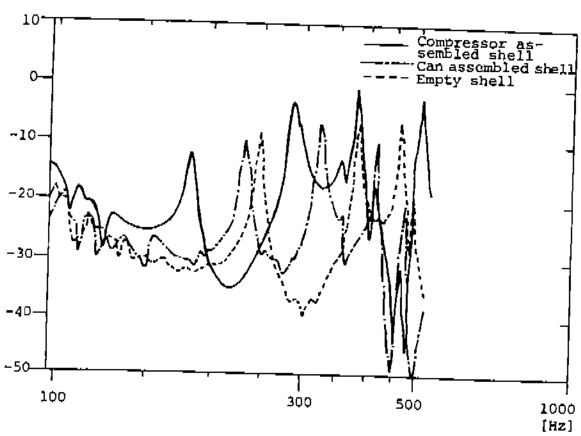

Fig. 8 Cavity resonant frequency spectrums

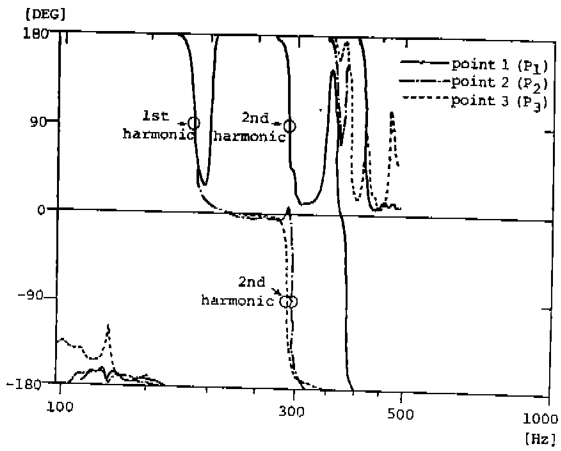

Fig. 9 Phase at the measuring point

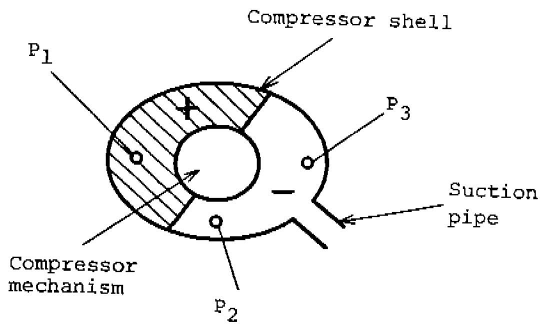

Fig.10 Transverse resonant mode in actual compressor shell

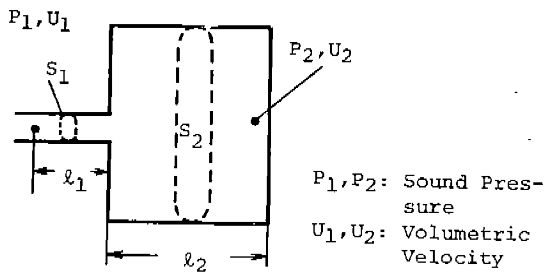

Fig.11 One-dimensional acoustic tube

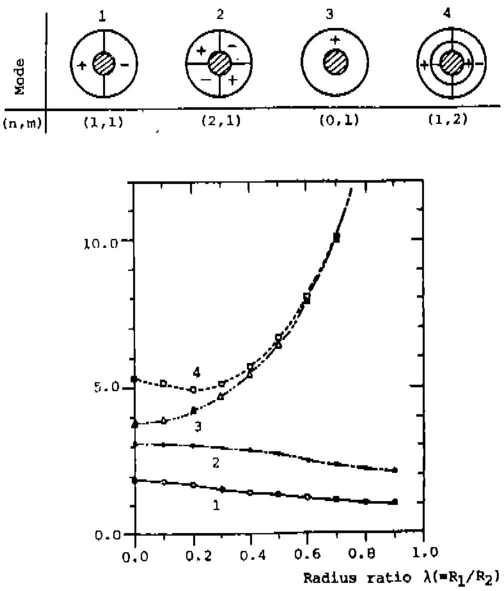

Fig.12 Solution for $J_n'(\lambda\xi)N_n'(\xi)-J_n'(\xi)N_n'(\lambda\xi)=0$

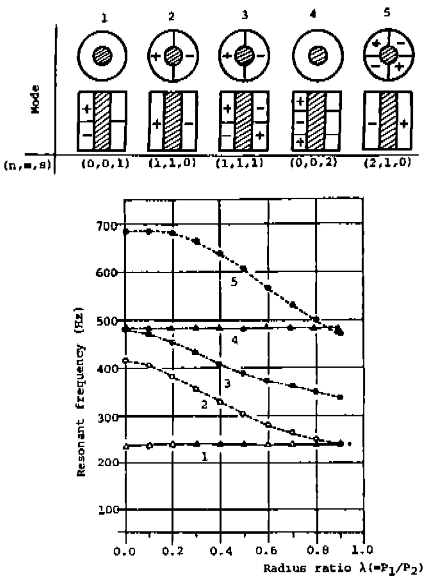

Fig.13

Calculated cavity resonant frequency in 5 HP compressor shell

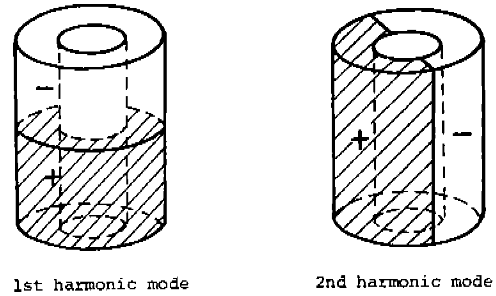

Fig. 14 Cavity resonant mode in compressor shell

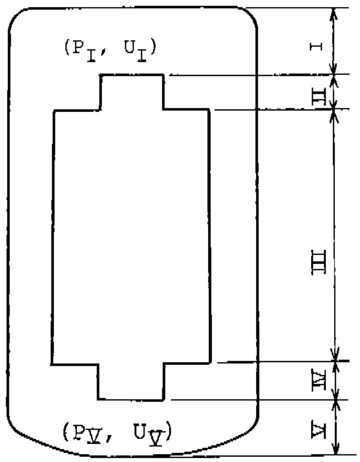

Fig.15 Ist harmonic resonant frequency calculation model for actual compressor