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SIMULATION STUDIES ON THE ROTARY TYPE
COMPRESSOR SYSTEMS

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ABSTRACT

This work is on the mathematical simulation of a sliding vane compressor with its piping system. Although the emphasis is on a two-vane compressor, the mathematical treatment and the computer program developed is capable of dealing with compressors of any number of vanes. Unsteady gas flow is considered in the pipes. Numerical treatment of flow in the pipes,through boundaries,and in compressor cells is described. Simulation of a sliding two-vane compressor with and without a delivery valve is presented. Leakage between the cells is considered in the analysis. The simulation computer program is explained. Cell pressure fluctiations with the vane position is presented at different operating conditions. The effect of vane thickness, discharge pressure, discharge valve,and leakage between the cells on the performance of the machine is given.

INTRODUCTION

Sliding vane rotary compressors are becoming increasingly important in engineering applications. It is claimed that they are best suited for delivery rates between 0-3000 lit/s and pressure ratios of 4-10. They permit high rotational speeds between 400-3600 rpm, and consequently small size. They are free from mechanical vibration however,gas flow is generally non-steady in the connected pipes. They have no suction valves, and usually no delivery valves. They also have small clearance volumes. Cells are formed by subdividing the crescent-shaped space between the casing and rotor by vanes. As the number of cells increase the strength of the unsteady flow decreases. Due to the compactness of sliding-vane compressors they are videly used in airconditioning and refrigeration systems. There has been work on the kinematics and simulation of these machines [1,2,4]. The lubrication aspects of these machines have also been investigated [3]. Research work on other types of the same family have been performed [6,7]. Continuing work on the thermo-fluid behavior would lead to better designs. It is believed that the interaction of the machine with its system should be considered in the modelling in order to predict its behavior with a better accuracy. This paper is a report of the preliminary work on the simulation of a sliding-vane type compressor system allowing for unsteady flow in the connected piping. It is intended to produce a working model (computer program) that could simulate a wide variety of sliding-vane machines with their systems.

KINEMATIC ANALYSIS

The main geometrical variables of a sliding vane compressor may be listed as follows: (See figure 1)

1. Casing diameter
2. Rotor diameter
3. Vane number (cell number)
4. Suction and delivery port angles (θ_s, θ_d)
5. Compressor length (L)
6. Vane thickness (b), and tip shape.

Any change in one of these variables tend to effect the performance of the machine to a great extend. The performance is also effected by the interaction of gas flow behavior in the pipes that are connected to the cells at one time of the operation. Vane thickness becomes important as the compressor becomes smaller in size, or as the number of vanes increase. In this work it is assumed that the vanes are placed in radial slots that are milled into the rotor. The vane tips are taken as circular arcs with radius R_V.

The thermodynamic properties of each cell volume is effected by the properties at the pipe end at the time when the cell is exposed to the connected piping. This makes it necessary to calculate the properties in each cell individually. In order to write the mathematical model of the sliding vane compressor one should calculate the change of cell volumes with displacement angle θ. The change of volume for a machine with vanes of no thickness may be written as

$$V(\theta) = \frac{L}{2}\,[(R_C{}^2-R_R{}^2)\theta + \frac{1}{2}\,\varepsilon^2\mathrm{Sin}2\theta - \varepsilon\mathrm{Sin}\theta\,\sqrt{R_C{}^2-\varepsilon\mathrm{Sin}^2\theta}$$

$$+ R_C{}^2\ \mathrm{Arc\ Sin}\,\frac{\varepsilon\ \mathrm{Sin}\theta}{R_C}\,] \qquad (1)$$

ε in the above equation is the eccentricity (R_C-R_R).

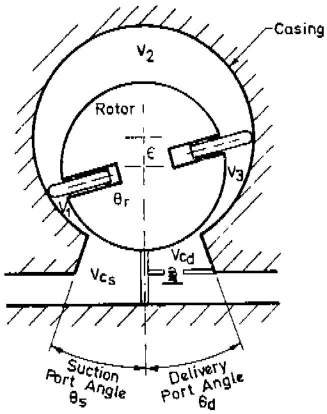

Figure 1. Geometry.

Figure 2 shows a circular tipped vane displaced at an angle θ. It is clearly seen that the contact point of the vane with the casing wall depends on displacement angle θ. θ being measured in clockwise direction. As the contact point changes its location the angle δ between the vane centerline and the line which joins the rotor center to the contact point changes. The correction angle δ may be calculated from the geometry of the blower by using

$$\text{Sin } \alpha = \frac{\varepsilon}{E} \sin \theta \tag{2}$$

Noting that $\xi = \theta - \alpha$ for $0 \le \theta \le \pi$ and $\xi = 2\pi - \alpha - \theta$ for $\pi \le \theta \le 2\pi$, the correction angle may be written as

$$\text{Sin } \delta = \frac{R_V}{D} \sin \alpha \tag{3}$$

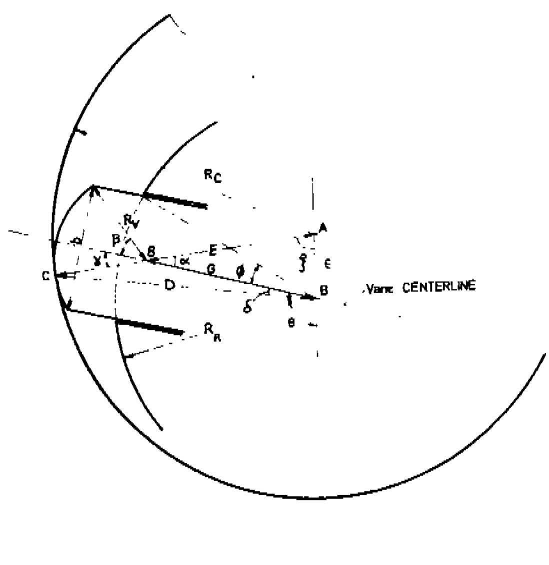

Figure 2. Vane Details.

where $D = \sqrt{R_C^2 + \varepsilon^2 - 2R_C\varepsilon \text{ Cos } \xi}$ (4)

Angles on the vane β and ϕ may be calculated from the vane geometry as

$$\beta = \text{Sin}^{-1} \frac{b}{2R_V} \quad \text{and} \quad \phi = \sin^{-1} \frac{b}{2R_R} \tag{5}$$

The volume of the vane that protrudes into the crescent shaped space between casing and rotor is calculated from

$$V_V(\theta) = L\ [R_V^2\beta - R_R\phi + \frac{b}{2}(R_V\text{Cos}\beta - R_R\text{Cos}\phi) + G] \tag{6}$$

where $G = \sqrt{E^2 + \varepsilon^2 - 2\varepsilon E \text{ Cos } \xi}$. The volume of the vane in the crescent shaped space above line CB shown in figure 1 may be calculated from

$$V_{V_1}(\theta) = \frac{1}{2} V_V(\theta) - [\frac{G}{4} \text{ Sin}2\delta + \frac{G}{2}(D - G\text{Cos}\delta) \text{ Sin}\delta + \frac{\alpha R_V^2}{2} - \frac{R_R^2\delta}{2}]\ L \tag{7}$$

The volume below line BC is $V_{V_2}(\theta) = V_V(\theta) - V_{V_1}(\theta)$.

The calculation of the different cell volumes of a sliding two vane compressor at any instant of time depends on the displacement angle. The volume of the cell at any instant is calculated from various combinations of the volumes that are calculated from Equations 1, 6, and 7.

THERMO-FLUID ANALYSIS

Cell Properties

It is assumed that no heat transfer occurs between the cells and to the surrounding. This implies that the process in each cell is of adiabatic type. However, the model may be modified for heat transfer effects with little effort. Time rate of change of properties in sliding-vane compressor cells is obtained by applying conservation of mass and energy to a control volume which surrounds the cell. The time rate of change of mass in the cell is obtained from

$$\frac{dm_c}{dt} = (\frac{dm}{dt})_{in} - (\frac{dm}{dt})_{out} \tag{8}$$

The incoming and outgoing mass rate may be due to leakage while the cell is not in connection with the suction and discharge ports. For the pressure change from energy equation,

$$\frac{dp_c}{dt} = -\frac{1}{V_c}\frac{de}{dt} - \frac{kP_c}{V_c}\frac{dV_c}{dt} - \frac{k-1}{V_c}\frac{dQ}{dt} \tag{9}$$

where e is the net energy convected into the cell during time interval dt. de/dt is the product of the mass flow rate and the stagnation speed of sound of the gas that crosses control volume boundary. Heat transfer may be introduced through the last term in Equation 9. The time rate of change of cell volume dV_c/dt should be calculated from the kinematics of the machine in order to find the properties in the cells. For constant rotational speed, time or displacement angle rate of change of cell volume for vanes of no thickness may be written as,

$$\frac{dV_c}{d\theta} = \frac{L}{2}\left[(R_C^2-R_R^2) + \varepsilon^2 \mathrm{Cos}2\theta - \varepsilon \mathrm{Cos}\theta \sqrt{R_C^2-\varepsilon^2\mathrm{Sin}^2\theta} + \frac{1}{2}\frac{\varepsilon^3 \mathrm{Sin}\theta\ \mathrm{Sin}2\theta}{\sqrt{R_C^2-\varepsilon^2\mathrm{Sin}^2\theta}} - \frac{R_C^2\ \varepsilon\mathrm{Cos}\theta}{\sqrt{R_C^2-\varepsilon^2\mathrm{Sin}^2\theta}}\right] \quad (10)$$

Equation 10 should be modified for the vane thickness by using equations 6 and 7. The rate of change of volume of vane above the rotor with displacement angle is obtained from

$$\frac{dV_V}{d\theta} = L\ b\ \frac{\varepsilon\ E\ \mathrm{Cos}\ \xi}{G}\ \frac{d\ \xi}{D\theta} \quad (11)$$

The portion of the vane above line BC (figure 2) is obtained from

$$\frac{dV_{V_1}}{d\theta} = \frac{1}{2}\left\{\frac{dV_V}{d\theta} - L\left[\varepsilon\mathrm{Cos}\ \xi\ \mathrm{Sin}\delta\ \left(\frac{DE}{G} - \frac{GR_C}{D}\right)\frac{d\xi}{d\theta} - (GD\ \mathrm{Cos}\delta + R_R^2)\frac{d\delta}{d\theta} + R_V^2\frac{d\alpha}{d\theta}\right]\right\} \quad (12)$$

where

$$\frac{d\xi}{d\theta} = -\frac{\varepsilon}{D}\frac{\mathrm{Cos}\theta}{\mathrm{Cos}\alpha}$$

$$\frac{d\alpha}{d\theta} = \frac{\varepsilon}{D}\ \frac{\mathrm{Cos}\theta}{\mathrm{Cos}\alpha} \quad (13)$$

$$\frac{d\delta}{d\theta} = \frac{R_V}{\mathrm{Cos}\delta\ D^2}\left(D\frac{d\alpha}{d\theta}\mathrm{Cos}\alpha - \frac{R_C\varepsilon}{D}\frac{d\xi}{d\theta}\mathrm{Cos}\alpha\ \mathrm{Cos}\ \xi\right)$$

The calculation of the properties in the time domain proceed using an explicit type of integration. However an iterative procedure is adopted to correct the initial quess [8]. The time step is calculated from the stability of the unsteady flow solution in the pipes.

Flow At the Boundaries of Cells

The flow at the boundaries of cells may be considered under three different items

1. Leakage flow between cells
2. Backflow between discharge port and discharging cell
3. Flow at the pipe boundaries

The leakage flow between the cells is calculated by using the known pressures at the two sides of the vane of the previous time step. The flow is assumed to be quasi-steady with isentropic flow to the throat and adiabatic thereafter. The constriction between two volumes is assumed as a converging nozzle with a throat area calculated from

$$F_t = C_V L + 2C_s\ (G + R_V - R_R\ \mathrm{Cos}\phi\ ) \quad (14)$$

where C_V is the equivalent clearance between vane slot, and the vane, C_s is the clearance between side plates and the vanes. The mass flow rate through the leakage paths may be written as

$$\frac{dm}{dt} = \sqrt{\frac{k}{RT_o}}\ P_o F_t \frac{M_t}{\left(1+\frac{k-1}{2}M_t^2\right)^{\frac{k+1}{2(k-1)}}} \quad (15)$$

Mach number at the throat is calculated from

$$M_t = \sqrt{\frac{2}{k-1}\left[\left(\frac{P_o}{P_t}\right)^{\frac{k-1}{k}} - 1\right]} \quad (16)$$

Both subsonic and sonic flow at the throat are considered during the calculation.

When one of the vanes of a sliding vane compressor passes the discharge port angle θ_d, the outlet port or the leading cell would be in contact with the trailing cell and a back flow would occur. Flow takes place until the conditions at both cells become the same. At this instant the leading cell is assumed to be integrated with the trailing cell. Equations (15,16) are used to calculate the mass flow rate between the cells. The throat area being calculated from

$$F_t = C_c R_C L\ \ (\theta - \theta_d) \quad (17)$$

where C_c is the contraction coefficient.

Flow at the pipe boundaries occur between suction and discharge ports and the pipes. Mass flow into the port is calculated from the instantaneous properties at the end of the pipes. For flow into the pipe an iterative technique is employed which corrects the pipe end conditions. Two different models have been used at the discharge side of the compressor. A disc type reciprocating compressor valve is fitted in the first model. In the second model discharge is made from the discharge clearance volume straight into the pipe. The discharge valve dynamics is simulated solving the damped oscillation equation. The procedure explained in reference [5] is used in this calculation. In case of the flow from or to the clearance volume of the machine, open end boundary condition, which has been well established in wave action calculations, is used.

Flow In the Pipes

The unsteadiness of the flow in the connected pipes is taken into account by solving the one dimensional unsteady flow equations in the pipe. Method of characteristic is used for the solution and the boundary conditions are treated as reported earlier [8,9]. The entropy charge with time and space is taken into account in the calculations.

COMPUTER PROGRAM

The simulation computer program is based on the previous work which has been made for reciprocating compressors. In the present form the computer program is capable of simulating sliding vane compressors with a single discharge and suction pipe. The maximum number of vanes that is allowed by the program is 10. A block diagram of the computer program can be seen in figure 3.

The necessary data that one should supply the program are: Casing radius, rotor radius, vane tip radius, compressor length, vane thickness, angular rotar speed,pipe lengths and diameters, delivery and suction tank properties, initial conditions, discharge valve data, and wall temperatures. The program is capable of calculating the property

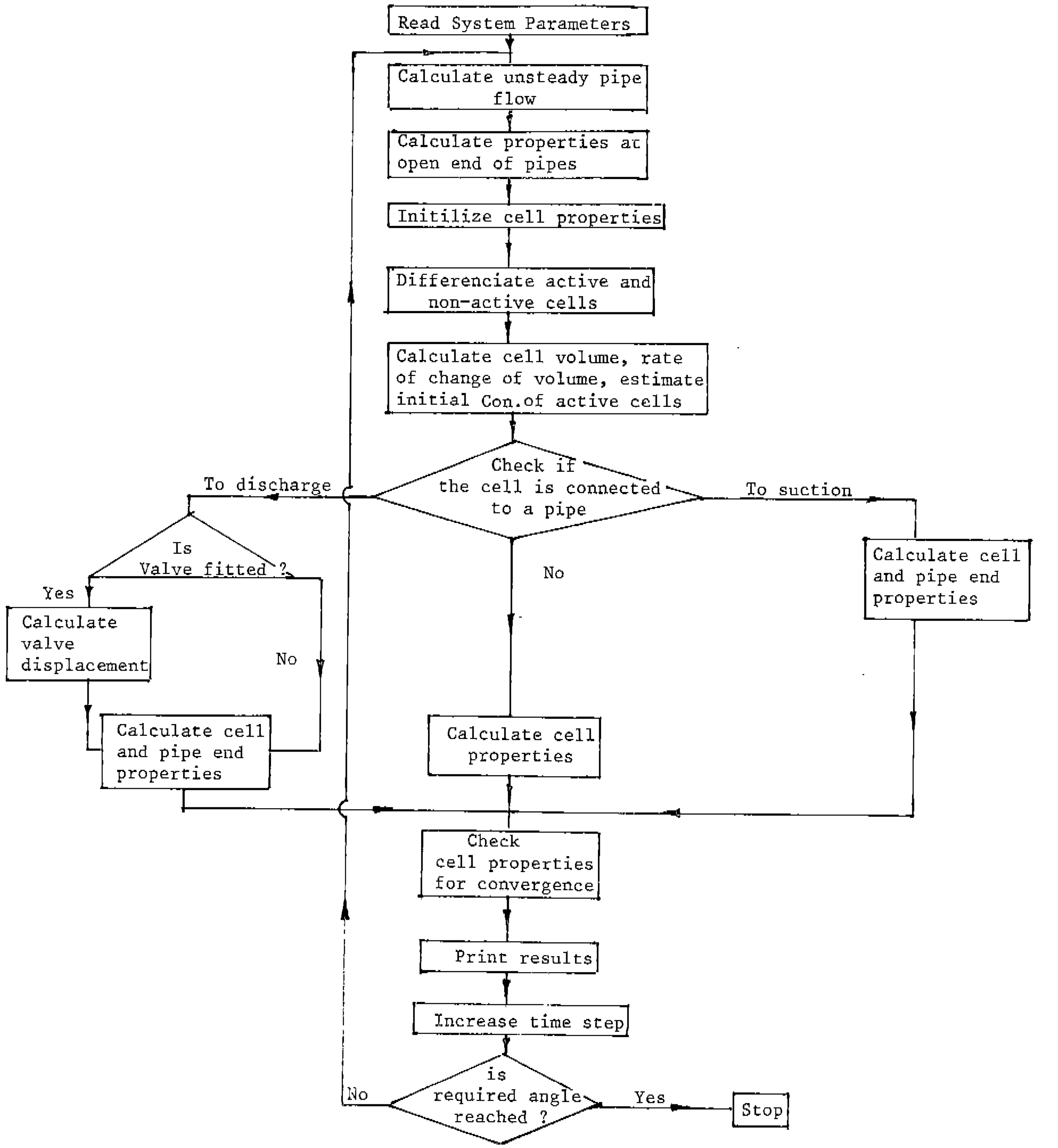

Figure 3. Block Diagram of the Simulation Computer Program.

changes at all points of the system, and performance variables.

The general treatment of the system for this time dependent solution is of classical type. Sliding-vane compressor is regarded as a boundary condition of the unsteady flow problem. The sliding-vane compressor kinematics and thermo-fluid treatment of the machine is introduced by seven subroutines. In order to trace the suction and delivery periods of each cell during the operation, a numbering sequence is developed. Energy and mass exchange between the cells due to leakage is coupled with this algoritm. Details of the simutation program is a given in reference [5].

The initial conditions of the problem at the suction side, are the suction tank properties. It is assumed that tank conditions prevail in the suction pipe and in the cell that is open to the inlet pipe. The discharge tank properties are used in the discharge pipe and in the discharging cell for the case when no discharge valve exists. The initial properties in the intermediate cells are estimated from the suction conditions assuming isentropic compression to the volume that they possess at the angle of calculation. The solution is weakly dependent on the initial conditions due to the cyclic nature of the process.

The simulations on the IBM 370/145 computer system takes in the order of 180 seconds for a complete cycle of a sliding two-vane compressor, which is 540 degrees. Calculations that consider the vane thickness take 10% more computing time.

RESULTS, DISCUSSION AND COMMENTS

A sliding vane compressor of the data given in Table 1 is simulated at different conditions. The effect of discharge valve, leakage, vane thickness, and discharge pressure on the performance of the machine is investigated.

Table 1. Geometrical Data of the Test Compressor.

Cylinder length	0.2484 m
Cylinder radius	0.0872 m
Rotor radius	0.0726 m
Vane tip radius	0.0065 m
Vane thickness	0.0065 m
Suction clearance volume	0.0001285 m^3
Discharge clearance volume	0.0001285 m^3
Number of vanes	2
Suction port angle	5^o
Discharge port angle	355^o
Vane leakage clearance	0.0001 m
Side clearance	0.0001 m
Suction pipe length	1.04 m
Discharge pipe length	2.7 m

The angular speed of the compressor is kept constant at 700 rpm in all test runs. The four different cases that has been studied are tabulated in Table 2. The effect of discharge pressure on the operation is investigated in each case study.

Some sample results showing the cell pressure variation of a sliding vane compressor is given in Figures 4 and 5. The pressure diagrams of a single cell are only plotted. No quantitative comparison was made due to the lack of well-documented test case at the time when the investigation is carried out. Only quantitative comparisons are made with regard to performance characteristics and indicator diagrams.

Table 2. Test cases.

TEST CASE	Discharge Valve	Leakage	Vane Thickness
A	Yes	No	No
B	Yes	No	Yes
C	No	No	No
D	Yes	Yes	No

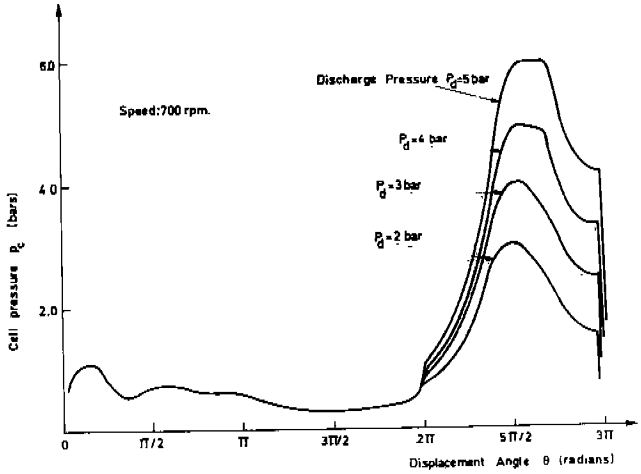

Figure 4. Cell Pressure Variation at Different Discharge Pressures (Case A).

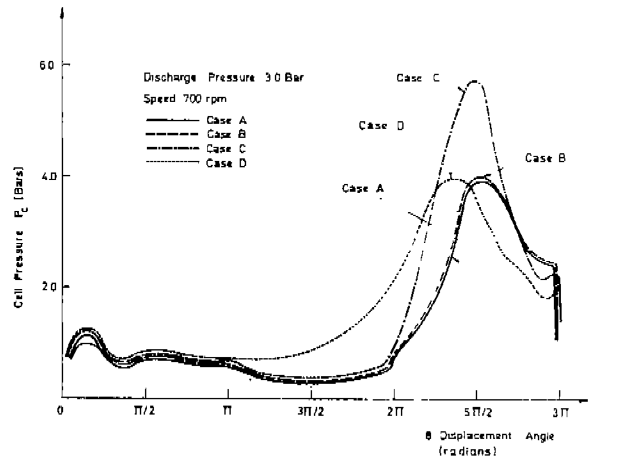

Figure 5. Comparison of Cell Pressure Variations at 3.0 bar Discharge Pressure.

The variation of pressure with the displacement angle for test case A is shown in Figure 4. The effect of discharge pressure on the indicator diagram can be seen in the figure. Between 0 and 2π radians the indicator diagram is not effectcd by the discharge pressure. The fast increase of cell pressure at the vicinity of 2π radians is due to the back flow from the leading cell through the delivery port just after the vane passes the discharge port opening angle. The sudden drop of pressure at the vicinity of 3π radians is because of the flow to the trailing cell through the port. At the vicinity of 3π radians, the cell becomes very small and it is considered as a part of trailing cell and discharge port combination. The fluctuating pressure between 0 and π radians is due to interaction of the unsteady flow in the suction pipe and the expantion process in the cell. The drop of pressure between angles π and 2π is due to the expansion of the gas in the cell. Case study B is devised in order to investigate the effect of vane thickness on the performance. It has been seen that vane thickness decreases both mass flow and power requirement slightly. However, its effect on the pressure changes in the cell is small.

Figure 5 shows the comparisons between the indicator diagrams at the condition when the discharge pressure is 3 bar. For the test case C the variation of pressure in the cell between 0 and 2π radians is similar to the pressure variation that is obtained in cases A and B. However, after 2π radians the pressure and temperature increases to large values due to back flow from the discharge pipe. This back flow occurs due to the pipe end instantaneous pressures that are higher than the cell pressures. Cell conditions are influenced by the unsteady pipe flow after the cell gets into contact with port-pipe combination. The case which takes the leakage flow into account is shown in figure 5 as well. The variation of pressure in the cell exibits some distinct changes from the other cases. The increase of cell pressure starts from 2π radians due to the leakage flow from the high pressure leading cell. The increase of pressure at the vicinity of 3π radians is due to the recompression after the valve closure.

Figures 6 and 7 show the variation of power required and mass flow rate with delivery pressure. Linear variation with pressure is observed in both performance variables. The horse power requirement for case A and B is approximately same and increase with pressure linearly. The rate of increase of power requirement with pressure is more in case studies C and D compared to A and B. The delivered mass per unit compression cycle do not change with discharge pressure for cases A, B and C. However, when leakage is considered between the cells the mass delivered decreases with the increase of delivery pressure as expected.

The qualitative comparisons of the existing simulation computer program looks satisfactory. Due to the versatile nature of the program it may used to simulate actual systems without much effort. However, the model has to be improved by introducing the heat transfer from the cells,and between the cells. Accurate simulation of oil flooded sliding vane compressors can only be achived if the amount of heat absorbed by the oil is modelled accurately.

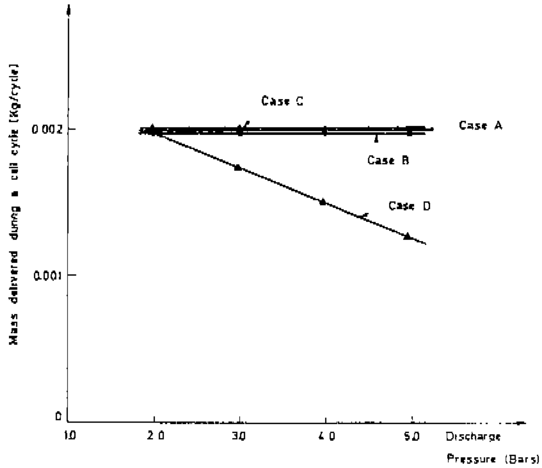

Figure 6. Change of Delivery Mass per Cycle with Discharge Pressure.

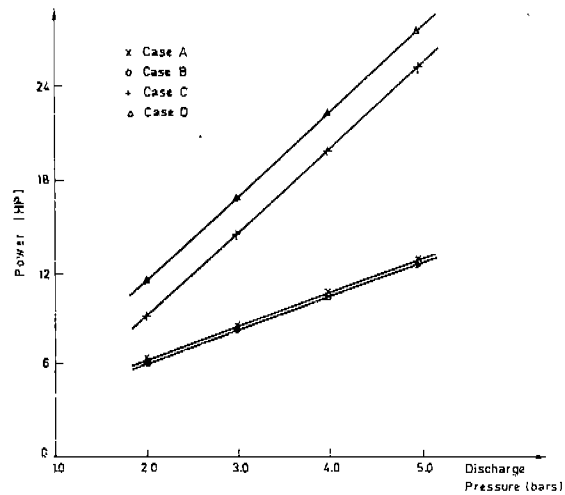

Figure 7. Change of Power Required with Discharge Pressure.

NOTATION

b	Vane thickness
C_s	Clearance between the side plates and vanes
C_v	Clearance between the vane slot and vanes
k	Ratio of specific beats
m	Mass
M	Mach Number

Q Heat transfer per unit mass

p Pressure

R Radius, Gas constant

t Time

T Temperature

V Volume

ϕ, α, γ, ξ, ε, β Angles of vane geometry (see figure 2)

θ Displacement angle

ρ Density

δ Correction angle

Subscripts

C Casing

c Compressor cell

d Discharge port opening

o Stagnation

R Rotor

s Suction

t Throat

V Vane

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