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Leakage Effects on Indicator Diagrams of Reciprocating Compressors

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LEAKAGE EFFECTS ON INDICATOR DIAGRAMS AT STOPPING OF RECIPROCATING COMPRESSORS

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INTRODUCTION

When the motion of the piston-crank mechanism of reciprocating compressors is analyzed theoretically, one of the most important matters is how to evaluate the cylinder pressure exerted on the piston. In order to evaluate the cylinder pressure presisely, it is necessary to give the characteristics of leakage effects from the piston clearance and dynamic properties of a discharge reed valve. However, it is not always easy to theoretically evaluate them. Hence, when a steady state of operation of compressors is studied, a measured steady indicator diagram is given as the cylinder pressure to avoid the above problem. But, when the transient behavior of the piston-crank mechanism is studied to analyze transient vibrations of compressors arising after a source of electric power for them has been cut off, it is not sufficient even if a measured steady indicator diagram is given: when a source of electric power for compressors under a steady operation is cut off suddenly, the crank shaft starts to rotate in the opposite direction after a few revolutions. Indicator diagrams which are built up before this opposite revolution of the crank shaft change in similar figures as a steady one. Hence, these indicator diagrams can be given on the basis of a steady one if the similarity law is previously obtained in experiments. But Indicator diagrams which are built up during opposite revolutions of the crank shaft come to be quite different from a steady one since the discharge reed valve does not act and hence the refrigerant in the cylinder is not discharged.

The indicator diagrams built up during opposite revolutions of the crank shaft which are quite different from a steady one are considered in this study: it is firstly shown in experiments that the main factor on which the indicator diagrams depend is leakage effects from the piston clearance, and it is secondly shown that the indicator diagrams are theoretically derived with due regard to the leakage effects but the piston of a compressor chosen as the subject of the study is of a type without a piston ring. What is theoretically derived is a first order and nonlinear differential equation which determines the indicator diagrams. When the equation is solved actually, a question arises as to how to determine unknown quantities such as the specific heat ratio, the kinetic viscosity and the piston clearance. Then a method which enables it to ditermine these quantities on the basis of experimental results is shown. Theoretically derived results have a good agreement with experimental ones, and hence it is concluded that an approach shown in this study is of great utility to reveal the leakage effects from the piston clearance on the indicator diagrams built up during opposite revolutions of the crank shaft.

EXPERIMENTAL RESULTS SHOWING LEAKAGE EFFECTS

When the indicator diagrams built up after a source of electric power for compressors was cut off, are measured, leakage effects from the piston clearance are distinctly found. Measured indicator diagrams are shown in this section and leakage effects from the piston clearance are estimated on the basis of the data.

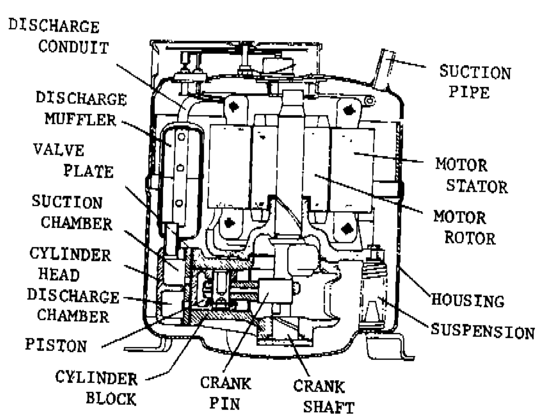


Fig.1 General view of a compressor

The construction of a single cylinder reciprocating compressor chosen as the subject of the study is shown in Fig.1. The specifications and the dimensions of the compressor are shown in Tab.1. The vertical crank shaft is secured at its upper portion to the motor rotor. The whole compressor is sealed completely in the housing. When the compressor is operated steadily, the pressure inside the housing is 0.39×10^6 Pa. The circulating refrigerant R22

Tab.1 Specifications and dimensions of compressor

Motor power	1100	W
Average rotational frequency	3420	r/min.
Size of compressor	275×203×122	mm
Piston diameter	38.8	mm
Axial length of piston	36.4	mm
Length of crank arm	10.6	mm
Length of connecting rod	42.7	mm

sucked in the housing through the suction pipe is drawn into the cylinder via the suction muffler, the suction chamber in the cylinder head and the suction opening formed in the valve plate. The refrigerant of low pressure thus introduced into the cylinder is compressed and is discharged outside via the discharge opening, the discharge chamber, the discharge muffler and the discharge conduit. The valve plate is provided, at its suction and discharge openings, with reed suction and discharge valves respectively which open and close in response to the pressure of the refrigerant. The piston diameter is 38.8 mm, the axial length of the piston without a piston ring 36.4 mm and the top clearance 0.5 mm. The piston clearance is designed to the size of about from 16 μm to 20 μm.

The cylinder pressure was measured by a device shown in Fig.2: a pressure hole with a bore of 1 mm which leads to inside the cylinder is made in the valve plate. A short capillary tube of copper is attached to the end of the pressure hole and the cylinder pressure is led to a pressure pick-up (TOYODA PMS-5-50H) attached to the end of the capillary tube. The length of the pressure hole between the cylinder and the pick-up is about 28 mm. The angle of rotation of the crank shaft is measured by a device in which small stick magnets are fixed on the upper side of the motor rotor and its magnetism is detected by the tape head of a tape-recorder.

When a source of electric power for the compressor is cut off suddenly, the kinetic energy of the system of rotation comes to be zero after a few revolutions and the crank shaft starts to rotate in the opposite direction. This opposite revolution follows these two patterns: the first is the case where the crank shaft stops suddenly, almost at the same time with the opposite revolution. The second is the case where the crank shaft stops gradually after repeating a few opposite revolutions.

A measured indicator diagram in the first pattern is shown in Fig.3(a). The upper is the cylinder pressure. The lower shows the rotation of the crank shaft and the pulse interval corresponds to the rotation of 10 degrees. The abscissa shows the elapsed time after the sudden stop of the crank shaft. The piston suddenly stops at about 20 de-

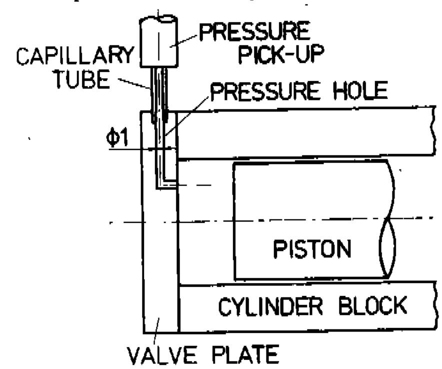
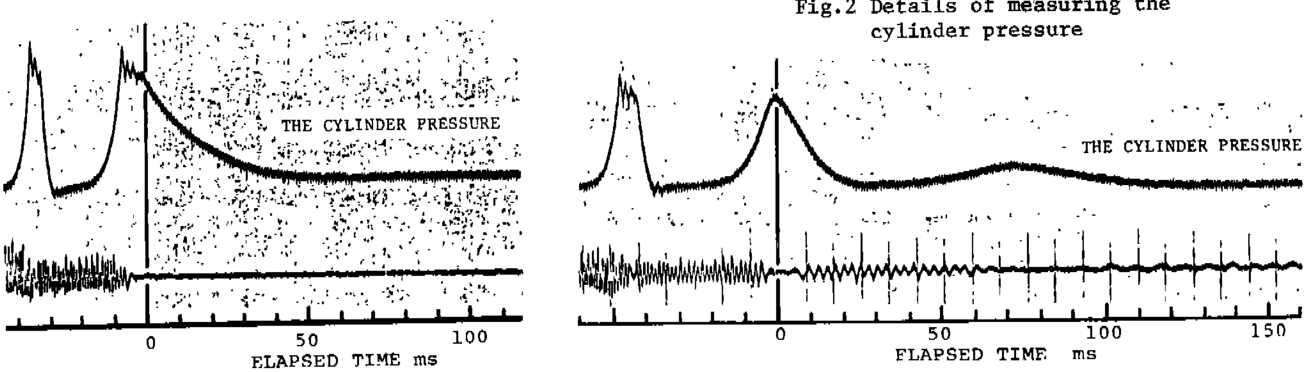


Fig.2 Details of measuring the cylinder pressure



(a) (b)

Fig.3 Measured indicator diagrams at stopping of the compressor
(a): an indicator diagram which is built up after the sudden stop of the piston
(b): an indicator diagram which is built up during opposite revolutions of the crank shaft

Nomenclature

d =piston diameter
G =weight of refrigerant in cylinder
G_*=initial weight of refrigerant
l_p=axial length of piston
l_p'=ratio of l_p and d
M_a=Mach number
n =polytropic index
P =cylinder pressure
P'=P/P_*
P_0=pressure in housing
P_0'=P_0/P_*
P_*=initial cylinder pressure
P_{**}=initial cylinder gas force
R_e=Relnolds number
t =elapsed time
t'=dimensionless time
$\bar{v}$ =mean velocity of leakage flow
v_p=piston velocity
V =cylinder volume
V'=V/V_*
V_*=initial cylinder volume
γ =specific weight
δ =piston clearance
δ'=δ/d
θ =angle of rotation of crank shaft
$\dot{\theta}$ =speed of revolution of crank shaft
μ =viscosity
ν =kinetic viscosity
ρ =specific weight

grees before the upper dead point. Hence the cylinder pressure is very high and shows 1.90×10^6 Pa at that instant. Such a high pressure drops to 0.39×10^6 Pa which is the pressure inside the housing, just after 40 ms. This rapid pressure drop must be caused only by the leakage effects from the piston clearance. Hence it is known from this datum that the leakage effects on the indicator diagrams are unexpectedly large. A measured indicator diagram in the second pattern is shown in Fig.3(b). The angle of rotation of the crank shaft at the instant of the first opposite revolution is about 70 degrees before the upper dead point and the piston stops after repeating two opposite revolutions. The refrigerant is sucked into the cylinder but is not discharged during the opposite revolutions. Though this indicator diagram does not directly show the leakage effects from the piston clearance since the volume inside the cylinder changes, it is expected that such unexpectedly large leakage as shown in the first pattern has a greater or lesser effect upon this indicator diagram. When transient vibrations at the stopping of reciprocating compressors are studied, such a case where the crank shaft repeats the opposite revolutions becomes a serious problem. Hence a method which enables one to correctly evaluate the indicator diagrams in such a case should definitely be established.

A EQUATION DETERMINING INDICATOR DIAGRAMS

As mentioned in the previous section, the leakage from the piston clearance has a considerably large effect upon the indicator diagrams. In this section the leakage effect is theoretically evaluated on the basis of a simple consideration, and an equation which determines the indicator diagrams, depending on the leakage effect, is derived.

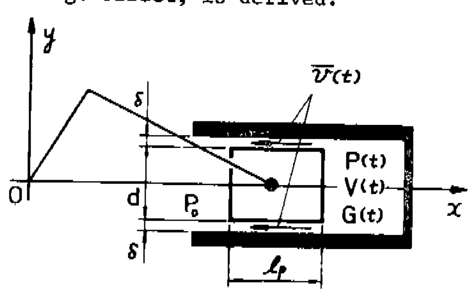


Fig.4 Piston-crank mechanism of compressor

Leakage Flow

The horizontal plane on which the piston-crank mechanism moves is represented by the x,y-coordinate system with the x-axis taken in the piston axis and the y-axis perpendicularly to the x-axis through the crank shaft, as shown in Fig.4. It is generally said in reciprocating compressors of a type without a piston ring that the clearance between the piston and the cylinder wall is very small and the axial length of the piston is very large compared with the clearance. Hence the velocity of the leakage flow is considered not to exceed that of sound. Then it is assumed that the leakage flow is incompressible. Moreover it is assumed that the piston is situated at the center of the cylinder and the leakage flow from the gap around the piston is steady and uniform in the direction of the piston. Since the clearance δ is very small compared with the piston diameter d, it is possible to replace the leakage flow around the piston by a flow between two parallel plates with the distance of δ, the width of $\pi/4\cdot d$ and the length l_p.

The mean velocity $\bar{v}(t)$ of the leakage flow derived on the basis of the above treatment is given by the form:

$$\bar{v}(t) = \frac{\delta^2}{12\mu l_p}\{P(t)-P_0\} - \frac{1}{2}v_p \tag{1}$$

in which $v_p(t)$ is the piston velocity which is defined as positive when the piston moves in the direction of the x-axis. The weight of the refrigerant in the cylinder decreases due to this leakage flow. When the time t has elapsed, the weight of the refrigerant G(t) is given by the following form:

$$G(t)=G_* - \pi d\delta\gamma\int_0^t\{\bar{v}+v_p(t)\}dt \tag{2}$$

hence,

$$G(t)=G_* - \frac{\pi}{12}d\delta g\int_0^t\Big[\frac{\delta^2}{\nu l_p}\{P(t)-P_0\}+6\rho v_p(t)\Big]dt \tag{3}$$

in which G_* is the weight of the refrigerant at the initial time (t=0) and the kinetic viscosity ν is assumed to be constant.

A First Nonlinear Differential Equation for the Cylinder Pressure

It is assumed that the state of the refrigerant in the cylinder is subjected to the polytropic change. Accordingly, the cylinder pressure P(t) is given by the form:

$$P(t) = P_*\Big\{\frac{V_*}{G_*}\cdot\frac{G(t)}{V(t)}\Big\}^n \tag{4}$$

in which n is an arbitrary constant which may be called the polytropic index. On substitution of (3) into (4), a equation determining the cylinder pressure is obtained as follows:

$$P(t)V^n(t)=P_*\Big(\frac{V_*}{G_*}\Big)^n\Big[G_* - \int_0^t\Big\{\frac{\delta^2}{\nu l_p}(P(t)-P_0)+6\rho v_p(t)\Big\}dt\Big]^n \tag{5}$$

Transforming the above integral equation into a differential form, a first nonlinear differential equation for the cylinder pressure P(t) is obtained:

$$\frac{dP(t)}{dt}+\frac{n}{V(t)}\cdot\frac{dV(t)}{dt}P(t) + \frac{\pi}{12}n\,gd\delta\,\frac{P_*^{\frac{1}{n}}}{G_*}\cdot\frac{V_*}{V(t)}\Big[\frac{\delta^2}{\nu l_p}\{P(t)-P_0\}+6\rho v_p(t)\Big]\{P(t)\}^{\frac{n-1}{n}} = 0 \tag{6}$$

The following dimensionless variables are introduced:

$$P'=\frac{P}{P_*},\quad P_0'=\frac{P}{P_*},\quad V'=\frac{V}{V_*},$$
$$t'=\frac{gd}{\nu}\cdot\frac{\delta^3}{l_p}\cdot\frac{P_{**}}{G_*}t,\quad M_a=\frac{v_p}{\sqrt{\frac{P_*}{\rho}}},\quad R_e=\frac{\sqrt{\frac{P_*}{\rho}}\,\delta}{\nu} \tag{7}$$

where,

$$l_p' = \frac{l_p}{d}\ , \quad \delta' = \frac{\delta}{d}\ , \quad P_{**} = \frac{\pi}{4} d^2 P_* \qquad (8)$$

in which P_0' is the initial pressure ratio of the pressure P_0 in the housing and the initial pressure P_* in the cylinder, M_a the Mach number of the piston velocity and R_e the Reynolds number based on the velocity of sound $\sqrt{P_*/\rho}$, the piston clearance δ and the kinetic viscosity ν. Then the equation (6) is transformed into the following dimensionless form:

$$\frac{dP'(t')}{dt'} + \frac{n}{V(t)}\frac{dV'(t')}{dt'}P'(t') + \frac{n}{3}\left[\{P'(t')-P_0'\}+6\frac{l_p'}{\delta'}\cdot\frac{M_a}{R_e}\right]\frac{\{P'(t')\}^{\frac{n-1}{n}}}{V'(t')} = 0 \qquad (9)$$

Hence the cylinder pressure is obtained from the above equation, provided that the cylinder volume is given.

THEORETICAL SOLUTIONS OF INDICATOR DIAGRAMS AND ITS VERIFICATION

The value of the polytropic index should be given in order to actually derive solutions from the dimensionless equation derived in the previous section, and the values of the piston clearance and the kinetic viscosity should be given in order to compare theoretical results with experimental ones. It is shown in this section that these quantities are actually determined on the basis of two kinds of experimental results and theoretical results have a good agreement with experimental ones.

An Indicator Diagram Built up after A Sudden Stop of the Piston Motion

Since the piston-crank mechanism suddenly stops almost at the same time of the first opposite revolution in the case of the measured indicator diagram shown in Fig.3(a), the treatment for the equation (9) becomes simple-as follows: since the cylinder volume is constant, that is, $V'(t')=1$ and the piston velocity is zero, that is, $M_a=0$, (9) results in the following simple form:

$$\frac{dP'(t')}{dt'} + \frac{n}{3}\{P'(t')-P_0'\}\cdot\{P'(t')\}^{\frac{n-1}{n}} = 0 \qquad (10)$$

The charcteristics of the pressure drop in the cylinder due only to the leakage is obtained from the above equation, provided that the polytropic index n is given. The solutions of the above equation for n=1.23 are shown in Fig.5 in which the time t=0 corresponds to the instant of the sudden stop of the piston motion and the parameter is the initial pressure ratio P_0' at that time. Since there exist no standards to determine exactly the polytropic index, the value 1.23 of the specific ratio of the superheated refrigerant corresponding to the mean state of the suction and the discharge in which the pressure is 1.27×10^6 Pa and the temparature is 100 °C is approximateky used for n in this part. However, the polytropic index has not such a large effect upon the characteristics of the pressure drop only due to the leakage: since the specific heat ratio of the superheated refrigerant varies about from 1.2 to 1.3 when the refrigerant is compressed from the suction to the discharge, the characteristics of the pressure drop for n=1.2 and 1.3 are shown in Fig.6 in which P_0=0.2.

The initial pressure P_* of the measured indicator diagram shown in Fig.3(a) is 1.90×10^6 Pa. Since the pressure in the housing P_0 is 0.39×10^6 Pa, the initial pressure ratio P_0' is 0.205. The theoretical solution for this initial pressure ratio is shown by the solid line in Fig.7. In order to compare the experimental results shown in Fig.3(a) with the theoretical solution, it should be transformed into a dimensionless form according to (7). The piston diameter d, the ratio l_p' of the axial length of the piston and its diameter, the initial gas force P_{**} and the initial gas weight G_* among the quantities necessary for this transformation is easily obtained: d=38.8 mm, l_p'=0.94, P_{**}=2250 N and $G_*=1.23\times10^{-3}$ N. Here arises the problem of how to determine the kinetic viscosity ν and the piston clearance δ. Since there exist no definite standards to determine the kinetic viscosity, the value 0.418×10^{-6} m²/s of the kinetic viscosity of the superheated refrigerant corresponding to the mean state of the suction and the discharge is used in the same manner as the polytropic index. A very precise value should be given for the piston clearance since it has an effect of the third order upon the time dependent term as known from (7). However, it is very difficult to know the precise value of the piston clearance since it depends also on the mechanical wearing-out. Then the piston clearance δ is evaluated on the basis of the experimental result, that is, its value is so determined that an experimental result transformed into a dimensionless form has the best agreement with the above theoretical solution. In this way, the value of δ is determined as 14.3 µm and it almost agrees with the designed value (from 8 µm to 10 µm). The experimental result transformed into a dimensionless form by this value of δ is shown by the dotted line in Fig.7 and it closely agrees with the theoretical one.

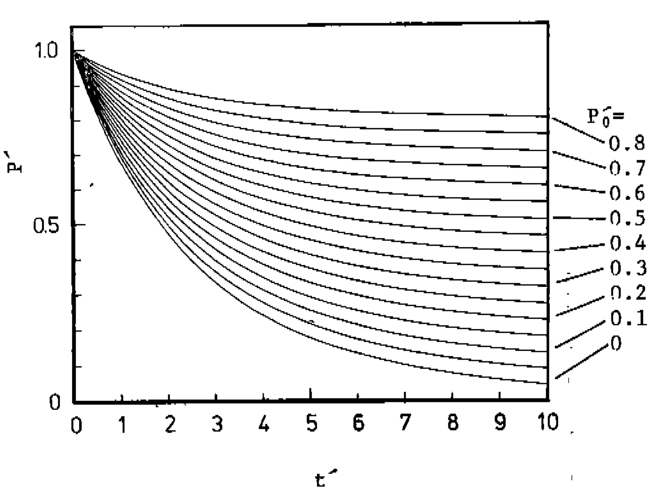


Fig.5 Characteristics of pressure drop in the cylinder: n=1.23

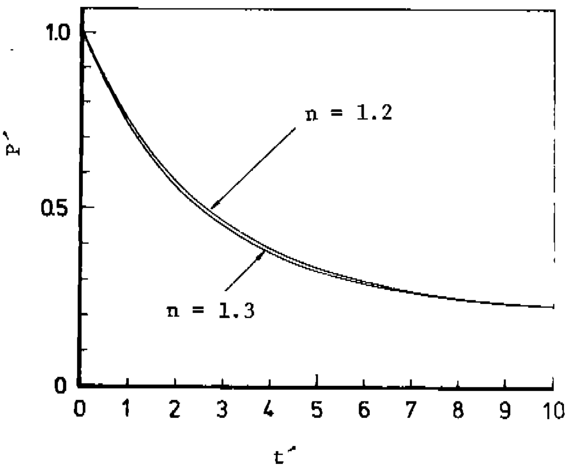


Fig.6 Characteristics of pressure drop for n=1.2 and 1.3.

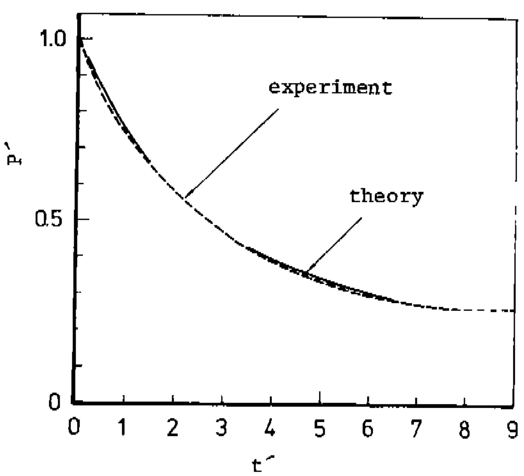


Fig.7 Characteristics of pressure drop in experiment and theory

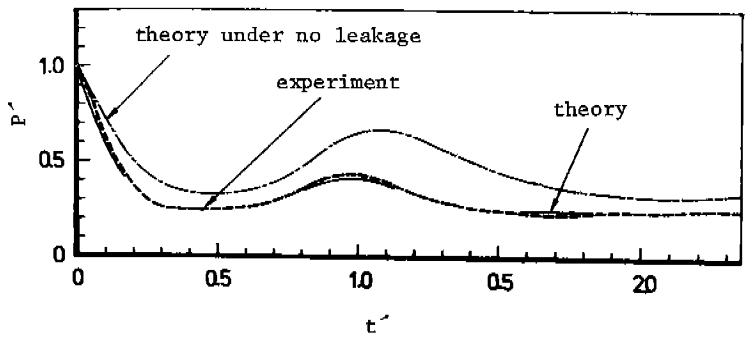


Fig.8 Cylinder pressure built up during opposite revolutions of the crank shaft

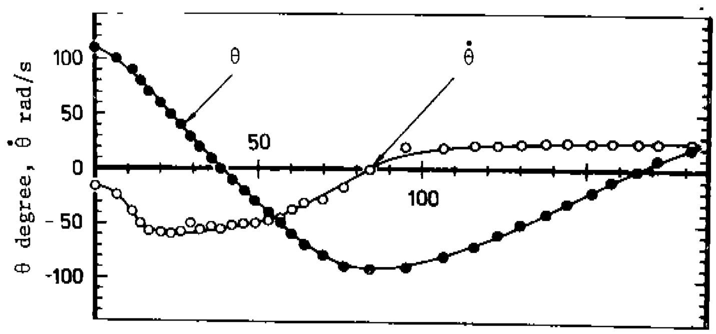


Fig.9 Angle of rotation $\theta(t)$ and speed of revolution $\dot{\theta}(t)$ built up during opposite revolutions of the crank shaft

An Indicator Diagram Built up during Opposite Revolutions of the Crank Shaft

The initial state quantities of the measured indicator diagram shown in Fig.3(b) are as follows: $P_*=1.56\times10^6$ Pa, $V_*=1.02\times10^{-5}$ m and $G_*=7.04\times10^{-3}$ N. The measured indicator diagram is transformed into a dimensionless form according to (7). Then the values of the kinetic viscosity ν and the piston clearance δ are taken to be the same as those determined in the previous section. The experimental result transformed into a dimensionless form is shown by the dotted line in Fig.8.

The angle of rotation $\theta(t)$ of the crank shaft and the speed of revolution $\dot{\theta}(t)$ are derived from the pulse signals in Fig.3(b). These results are shown in Fig.9 in which the signs (•) show $\theta(t)$, and the signs (○) show $\dot{\theta}(t)$. The cylinder volume $V(t)$ and the piston velocity $v_p(t)$ are derived from these data. These results are transformed into a dimensionless form according to (7). On substitution of the results into the equation (9), an indicator diagram is theoretically derived, but when the calculated pressure becomes lower than that in the housing, the suction reed valve opens automatically and the refrigerant is sucked in such a way that the cylinder pressure comes to be equal to that in the housing. The indicator diagram theoretically derived from (9) is shown by the solid line in Fig.8, in which $P_0'=0.252$, the polytropic index n=1.20, the Reynolds number $R_e=8.9\times10^3$ and the initial velocity of sound $\sqrt{P_*/\rho}=185$ m/s. Since the polytropic index has a great effect upon the result in this case, it is not appropriate that the value is taken to be the same as that given in the previous section. Then the polytropic index n is evaluated on the basis of the experimental result, that is, its value is so determined that a theoretical solution derived from (9) has the best agreement with the above experimental result transformed into a dimensionless form. This determined value of n should be considered to include all errors caused by the assumptions in the theory. However, the value is not so unreasonable since it almost agrees with the expected values of the specific heat ratio of the superheated refrigerant. The theoretically derived indicator diagram has a good agreement with the experimental one.

The chain line shown in Fig.8 is the indicator diagram which is theoretically derived under the assumption that there exists no leakage. This result is very different from that which is derived from taking into account the leakage effects. Accordingly, it is concluded that the leakage effects on the indicator diagrams are also very large in the case where the piston moves and hence it is indispensable to take into account the leakage effects in deriving theoretically the indicator diagrams.

CONCLUSIONS

When transient vibrations at the stopping of reciprocating compressors are simulated numerically, it is necessary to establish previously a method for evaluating correctly the transient indicator diagrams. In this study, the method is shown where the indicator diagrams built up especially during the opposite revolutions of the crank shaft are precisely determined with due regard to the leakage effects from the piston clearance, and the method is verified on the basis of the experimental results. The method for analyzing is based on a very simple assumption that the leakage flow is considered to be incompressible. However, the derived theoretical results have a good agreement with the experimental ones, unexpectedly. The first reason why such a good result is obtained is as follows: since the piston clearance is very small and the axial length of the piston without a piston ring is very long in the compressor chosen as the subject of the study, the mean velocity of the leakage flow is at most 35 m/s. This value is adequately smaller than the the velocity of sound of the superheated refrigerant R 22, that is, 170 m/s$\sim$200 m/s, and hence it was not necessary to take into account the effect of compressibility. Such a condition will be satisfied in general compressors, provided that the piston of compressors is of a type without a piston ring. The second is as follows: it is very difficult to know the precise values of the polytropic index and the piston clearance, which has a great effect upon the leakage flow. Then the values of the above quantities were evaluated on the basis of the experimental results. Such a method for evaluating is very practical since it includes all errors caused by the assumptions in the theoretical treatment. However, it is considered that the assumptions in the theory are not so unreasonable since the polytropic index and the piston clearance determined by the above treatment closely agree with their expected values.

It is concluded that the method for analyzing shown in this study is adequately precise and practical enough to evaluate the leakage effects on the indicator diagrams and hence it is very applicable for a numerical simulation of transient vibrations at the stopping of reciprocating compressors.