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Analysis of the Working-Cycle of Single Stage Refrigeration-Compressors Using Digital Computers

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ANALYSIS OF THE WORKING CYCLE OF SINGLE - STAGE REFRIGERATION COMPRESSORS USING DIGITAL COMPUTERS

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INTRODUCTION

After 1950 Costagliola [1] had reported first about the basic relations to calculate the dynamic working cycle of a reciprocating compressor in the Sixties because of the increasing application of digital computers, complex calculation models for compressor simulation have been developed, including the important dynamic valve behaviour of the compressor valves (for example: [2] [3]). In order to gain a better description of the whole compressor system, the basic models have been extended by integrating certain sub-systems in the last years. Till today the most important extension consists in incorporating the instationary gas pulsations in the valve chambers and the connected pipes of the compressor (for example [4]). Recently investigations have been made, treating further problems as for example heat transfer in a compressor [5] and its insertion in the calculation model [6]. Despite regarding of all these additional aspects most of the investigators apply perfect gas laws. The general validity of this assumption has to be proved, especially if a realistic p, V - diagram is needed for other calculations concerning compressor design, as for example the predetermination of the sliding bearings [7] and the lubrication conditions at the piston [8]. Comparing estimations show that deviations between corresponding computations using either real or ideal gas laws reach values up to 10% and more. More exact statements about their influence on the calculation of the working cycle of the compressor can only be made by applying a generally valid compressor model, which has to be established in a structure allowing the use of either the ideal or different real gas equations of state for the working fluid.

This given situation was the reason to develop at the Technical University of Hannover a compressor model, which is able to use up by choice either the ideal gas equation or some different complicated real equations of state. Further the model was constructed in such a manner that the heat transfer in the cylinder and also the instationary gas pulsations in the valve chambers can be included.

The aim of the project was, to clear up special problems of compressor modeling in order to see, how much it is necessary to take into account the mentioned model extensions. For this purpose the influence of several parameters on the compressor working cycle and its valuation factors as for instance the adiabatic and volumetric efficiency, the specific working power... etc. have to be stated by means of variations. It is necessary to study the possibility of simplifications.- Especially this question is very important concerning a practical economic application of the model for further development of compressors. Combined with these basic questions there is coupled a great complex of many individual problems which can only be solved by a direct application of the extentioned model. Some important questions shall be answered in the following chapters by a short description of the structure and the application of the mathematical model.

PROBLEMS IN MODEL DEVELOPING

The problem of model developing involves the general mathematical description of the working mechanism of a reciprocating compressor.
In order to use different equations of state it is necessary to develop a mathematical description which is independent from the working fluid itself and its special form of equation of state. In order to avoid unnecessary iterations, only has to be taken into account, which value is the dependent variable of the gas equation. Because most of the complicated equations of state express the pressure p in terms of temperature T and specific volume v, $p = f(T,v)$, it was demanded here to use this type of equation with the secondary condition to reduce time-consuming iterations.
In doing this, it is important to take care, that the describing differential equations are constructed in such a way that by numerical integration the independent variable T has suitable to be computed. The conservation theorem of energy applied to the compressor system (Fig.1) yields the following equation (see Nomenclature)

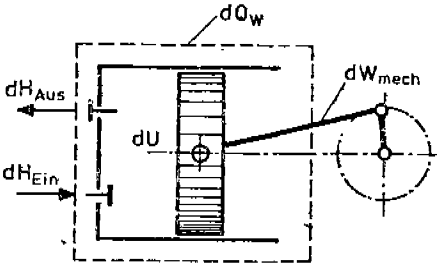

Fig.I

$$\frac{dT}{d\varphi} = \frac{1}{\left(\frac{\partial u}{\partial T}\right)_v} \left\{ \frac{1}{m}\frac{dQ_W}{d\varphi} - \frac{1}{m}\left[\left(\frac{\partial u}{\partial v}\right)_T + p\right]\frac{dV}{d\varphi} \right. \tag{1}$$

$$\left. + \frac{1}{m}\left\{\left[\left(\frac{\partial u}{\partial v}\right)_T + p\right]v + h_{EIN} - h\right\}\frac{dm_{EIN}}{d\varphi} - \frac{v}{m}\left[\left(\frac{\partial u}{\partial v}\right)_T + p\right]\frac{dm_{AUS}}{d\varphi} \right\}$$

This equation is valid for the whole woking process. For solving, it is necessary, to know the values of the terms $dV/d\varphi$, $dm/d\varphi$ and $dQw/d\varphi$. The change of the cylinder volume V in respect to the crankshaft angle φ can easily be destinated from the geometry of the crank assembly. For the suction and discharge process $dm/d\varphi$ can be computed by comparing the flow through the valves with an isentropic flow through a nozzle; so we gain

$$\frac{dm}{d\varphi} = A_{Qeff}(y)\frac{1}{\omega\, v_2}\sqrt{2\,(h_1 - h_2)} \tag{2}$$

The arising problem of calculating the isentropic change of state without iterations can be solved by integrating the following partial differential equations:

$$\left(\frac{\partial T}{\partial p}\right)_s = \left\{\left(\frac{\partial p}{\partial T}\right)_v - \frac{\left(\frac{\partial p}{\partial v}\right)_T\left(\frac{\partial u}{\partial T}\right)_v}{T\,\left(\frac{\partial p}{\partial T}\right)_v}\right\}^{-1} \tag{3a}$$

$$\left(\frac{\partial v}{\partial p}\right)_s = \left\{\left(\frac{\partial p}{\partial v}\right)_T - \frac{T\left[\left(\frac{\partial p}{\partial T}\right)_v\right]^2}{\left(\frac{\partial u}{\partial T}\right)_v}\right\}^{-1} \tag{3b}$$

The intention to compute a thermodynamic state 2 of which only the pressure P_2 and the entropy $s_2 = s_1$ are known so becomes an initial value problem. The numerical integration of the system of equations (3) leads after computation to a pair of thermodynamic values T_2, v_2, by which it is possible to calculate the enthalpy h_2 by means of the caloric equation of state and then finally by the equation 2 the wanted mass flow dm/dy. Because the effective flow area A_{Qeff} of the valve is an empirical function, of the relative valve displacement y the valve motion has to be included in the model. By comparing the valve system with a single-degree-of freedom vibrating system the equilibrium of forces leads to the "dynamic" equation

$$\omega^2 m_V H_V \frac{d^2y}{d\varphi^2} + \omega\, c_D H_V \frac{dy}{d\varphi} + \left(\frac{F}{F_{max}}\right)F_{max} = \left(\frac{A_{DReff}}{A_{DRmax}}\right)A_{DRmax}\cdot \Delta p \tag{4}$$

In order to complete the describing equation system the term for the heat transfer $dQw/d\varphi$ between gas and walls has to be added. This term can be calculated by the equation for the quasi stationary heat transfer

$$\frac{dQ_W}{d\varphi} = \frac{1}{\omega}\,\alpha\, A_W\,(T_W - T) \tag{5}$$

where α is the heat transfer coefficient which can be achieved from the exponential equation for the Nusselt - Number

$$Nu = C_1 \cdot Re^m \cdot Pr^n \tag{6}$$

The constants C_1, m and n have to be determined by experiments.
The equations (1)...(6) are the most important basic relations for the mathematical description of the mentioned compressor model.
An extension of the model by including the instationary gas pulsations in the valve chambers have been done by the method of acoustic impendance as described by Elson and Soedel [9] . The applicated acoustic model consists of a volume-element with a connected anechoic pipe.

APPLICATION OF THE MODEL

The first aim in applying the model was, to study the influence of the heat transfer in the cylinder and the gas pulsations in the valve chambers on the whole working cycle.
For this purpose different heat transfer coefficient correlations were used, as for instance the formulas of Nusselt [10], Eichelberg [11], Pflaum [12] , Woschni [13] ... in order to estimate the possible spectrum of the heat transfer influence on the working cycle. The principal interplay of the different variables influencing the heat transfer is shown in Fig. 2, qualitatively. The quantitative heat transfer effect on the compressor process is very small because of the small differences of temperatures between gas and walls in a refrigerating compressor. Variations of the volumetric efficiencies compared with corresponding adiabatic process calculations could not be stated and the p, V - diagrams did not show important differences - also no significant influences on the whole compressor process could be observed by including the computation of pressure pulsation by means of the acoustic theory in the model when using realistic sizes for the dimensions of the valve chambers and the diameters of the connected pipes. The reason for this result is the assumption of anechoic termination [14] of the lines, which has been made because they end in either the evaporator or the condenser where a phase-change of the refrigerant occurs.

When comparing the result of real gas simulation with ideal gas simulation important differences concerning the thermodynamic values could be stated. While only the plots of cylinder pressure, computed with both equations of state, show relative small deviations, there results more significant differences by regarding the trace of curves of temperature, density and mass of gas in the cylinder. Fig. 2 shows for example the plots of temperature either computed by ideal and real equation of state. Such differences influence the volumetric efficiency which is given for example in Fig. 3 as function of the initial spring force. Parallel simulation by using the ideal gas equation and a real gas equation by varying the description parameters of the valves represents a possibility to find out whether a model with real gas equations is necessary or not for valve optimization. By plotting the valuation variable, choiced for quality criterion, as function of the variation parameter, there is only interest to know the location of the extrem value and not its magnitude. The valve parameter have such a great influence on the closed compressor - working cycle, that the applied type of equation of state is of secondary importance. That's why for valve optimization the more simple and less computer-time consuming ideal gas - calculation is sufficient. For this, Fig. 4 illustrates, that the time depended valve displacements show - except a certain delay of phase - no significant deviations.

The analysis of the working cycle of reciprocating compressors is especially then very clear and evident, if the results can be given in a graphical manner. Studying for example the effect of compressor speed variations the results can be plastically demonstrated in a three dimensional p, V-diagram, plotted with the speed as third dimension (Fig.5). Contemplating this diagram great pulsations of the cylinder pressure can be registrated in the lower speed range. These pulsations are based on the behaviour of the dynamic self-acting compressor valves which are not designed for this range of compressor speed. The alternating intersection line of reexpansion-curves shows obviously, that the volumetric efficiency will alternate too in this speed range because of the corresponding shortening of the suction line. A continuous decrease occurs finally with higher speeds. This result can be seen in Fig.6 in which the graph for the volumetric efficiency is determined by a greater number of calculated points. Especially the volumetric efficiencies have a close correlation to the valve forces and displacements which may effect remarkable backflows through the valves at special speeds by an unfavourable interplay of the valve parameters. The valve displacement can be demonstrated clearly in Fig.7:
In the lower speed range the valves never reach the resting position of the full opened valve and the occuring valve flutter causes the mentioned pressure pulsations in the cylinder.
Normal valve displacement diagrams can only be stated at higher speeds. By plotting in an analog manner the relative pressure difference, defined as the relation of the absolute pressure difference to the actual stationary pressure at the valve, pressure plots as in Fig.8 can be gained: Those points where the pressure curves are cutting the zero-plane the backflow, caused by a negative pressure difference, begins and the backflow ends when the valve has closed finally. This is given by the projection of the final points of the pressure-curves on the zero-plane. In contemplating this diagram it can be seen, that, when increasing the speed, the time of the not closed valves increases too. At a medium speed a "cut" can be seen where practically no backflow occurs. Up to higher speeds a continuous increase of the time of backflow can be stated again. Besides of the volumetric efficiency in Fig.6 further valuating factors of the working cycle of a refrigerating compressor are plotted as a function of the speed. It can be seen that the average temperature behind the discharge valve increases with higher speeds, leading to a lower adiabatic efficiency. The cycle work, given by the included area of the corresponding p, V-diagrams shows a maximum value at a medium speed which results from the superposition of two influences:

1.) When increasing the speed work, losses are increasing too and with them the needed cycle work.
2.) Because of the increasing pressure drops, combined with increasing period of backflow, the periods of compression and re-expansion are starting with time delay which effects a smaller included work area.

At higher speeds the secondary influence is more important leading to a decrease of the cycle work. But with regard to the specific energy per unit of mass it can be seen clearly, that the smallest specific power-consumption occurs at lowest speed.

CONCLUSION

A complex calculation model for refrigerating compressors had been presented. By applying this it was shown, that heat transfer and instationary pressure pulsations with the chosen acoustic systems have only unimportant influences on the shape of the whole working process. The most important parts of the system are the dynamically working compressor valves, which influence the working cycle in such a great amount, that in order to optimize the valves it is allowable with good accuracy to use the simpler computation model with the ideal gas equation.
For the determination of the valuation factors of the working cycle as for instance the volumetric efficiency and the specific compression work the real gas behaviour should not be neglected. In order to solve special problems for compressor design, therefore, a using of simulation models including the real gas behaviour is favourable.

NOMENCLATURE

A_{DReff}	Effective force area (m^2)
A_{Qeff}	Effective flow area (m^2)
A_W	Area for heat transfer (m^2)
c_D	Damping coefficient ($N \cdot s \cdot m^{-1}$)
C_1	Constant value (-)
F	Spring force (N)

h	Enthalpy ($N \cdot m \cdot kg^{-1}$)
H_v	Valve lift (m)
m	Mass (kg)
m	Exponent (-)
m_v	Valve mass (kg)
n	Exponent (-)
Nu	Nusselt - number (-)
p	pressure ($N \cdot m^{-2}$)
Pr	Prandtl - number (-)
Q_w	Heat energy ($N \cdot m$)
Re	Reynolds number (-)
s	Entropy ($N \cdot m \cdot kg^{-1} \cdot K^{-1}$)
T	Temperature (K)
T_w	Temperature of walls (K)
u	Specific internal energy ($N\ m\ kg^{-1}$)
v	Specific volume ($m^3 \cdot kg^{-1}$)
V	Cylinder volume (m^3)
y	Relative valve displacement (-)
α	Heat transfer coefficient ($N \cdot m^{-1} \cdot s^{-1} \cdot K^{-1}$)
φ	crank angle (rad)
ω	crank speed (S^{-1})

Indices

EIN	Massflow into the cylinder
AUS	Massflow out of the cylinder
1	Upstream condition
2	Downstream condition

REFERENCES

[1] Costagliola, M.: The Theory of Spring-Loaded Valves For Reciprocating Compressors, Journal of Applied Mechanics, Dec. 1950, 17,4, p. 415

[2] Mac Laren, J.F.T., Kerr, S.V.: Valve Behaviour in a Small Refrigerating Compressor Using a Digital Computer, Jr. Refrig. 1968, No. 6

[3] Wambsganss, M.W., Cohen, R.: Dynamic of a Reciprocating Compressor With Automatic Reed Valves, Part I, Part II, Proceedings of the XIIth International Congress of Refrigeration, Madrid, 1967, p. 779 - 799

[4] Brablik, J.: Computer Simulation of the Working Process in the Cylinder of a Reciprocating Compressor With Piping System, Purdue Compressor Technology Conference, July 1974, pp. 151 - 158

[5] Adair, R.P., Qvale, E.B., Pearson, J.T.: Instantaneous Heat Transfer to the Cylinder Wall in the Reciprocating Compressors, Proc. of the 1972 Purdue Compr. Techn. Conference, pp. 521 - 526

[6] Prakash, R., Singh, R.: Mathematical Modeling and Simulation of Refrigerating Compressors, Proc. of the 1974 Compr. Techn. Conf., pp. 274 - 285

[7] Kruse, H., Röttger, W, Vauth, R.: Calculation and Measurement of the Crankshaft-Motion in the Bearings of Refrigerating Compressors, Proc. of the 1974 Purdue Technology Conference

[8] Kruse, H., Wrede, F.: Computer Simulation of Compressor Lubrication Conditions of Trunk Piston in Refrigerating Compressors, Proc. of the 1976 Purdue Technology Conference

[9] Elson, J.P., Soedel, W.: Simulation of the Interaction of Compressor Valves with Accoustic Back Pressures in Long Distance Lines, Jr. of Sound and Vibration, 1974

[10] Nußelt, W.: Der Wärmeübergang in der Verbrennungskraftmaschine VDI-Forschungsheft 264 (1923)

[11] Eichelberg, G.: Some New Investigations on Old Combustion Engine Problems, Engine Problems, Engineering Vol. 148, 1939, S. 547 - 550

[12] Woschni, G.: Die Berechnung der Wandverluste und thermischen Belastung der Bauteile von Dieselmotoren, MTZ 31 (197o), Heft 12,

[13] Woschni, G.: Beitrag zum Problem des Wärmeüberganges im Verbrennungsmotor, MTZ 26 (1965), 4,S. 128 - 133

[14] Singh, R., Soedel, W.: A Review of Compressor lines Pulsation Analysis and Muffler Design Research, Part I, II, Purdue Compr. Techn.Confer. 1974, pp. 102

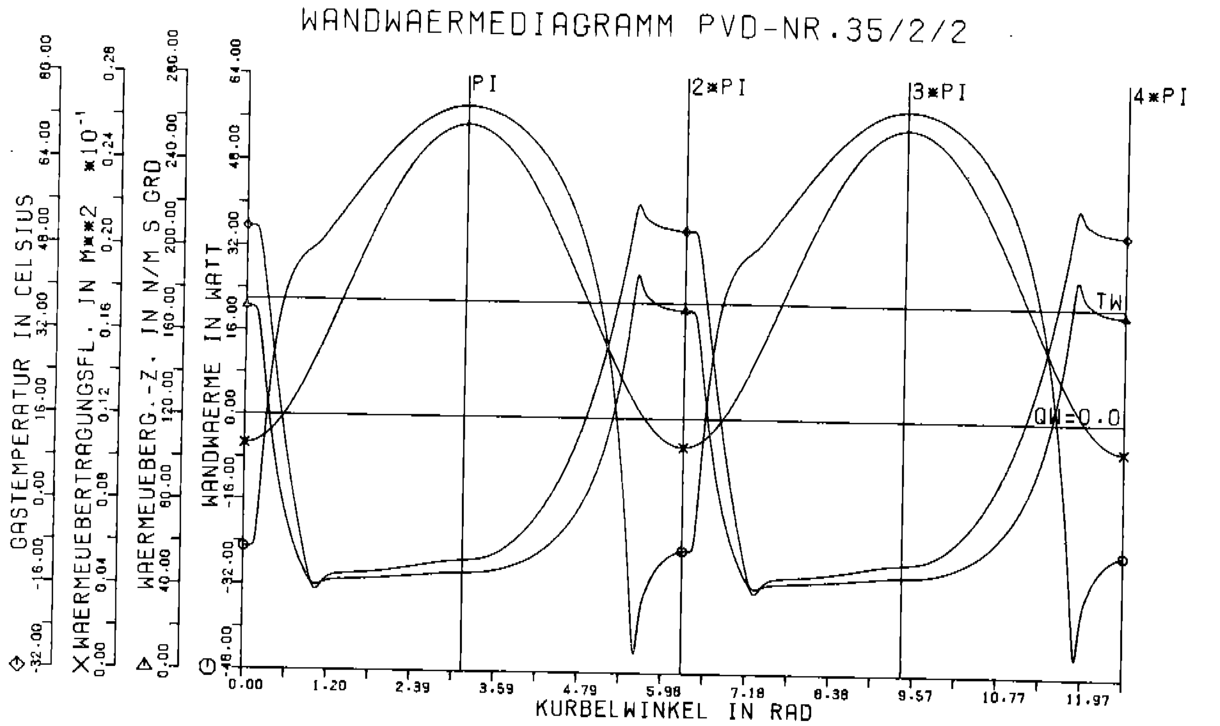
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Fig.1

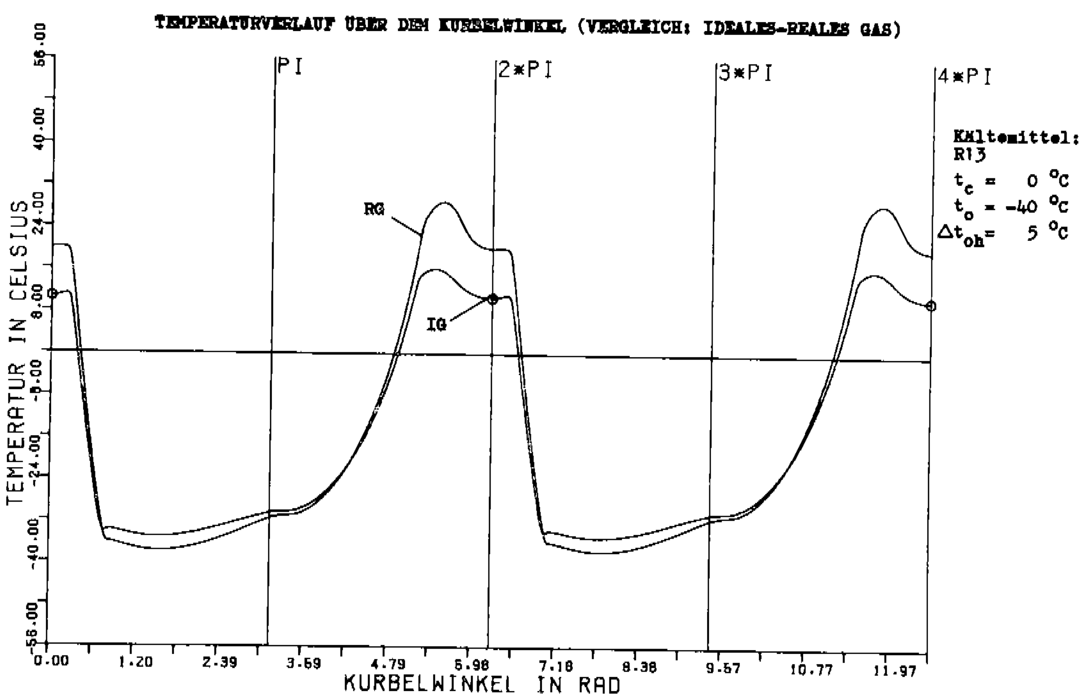
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Fig.2

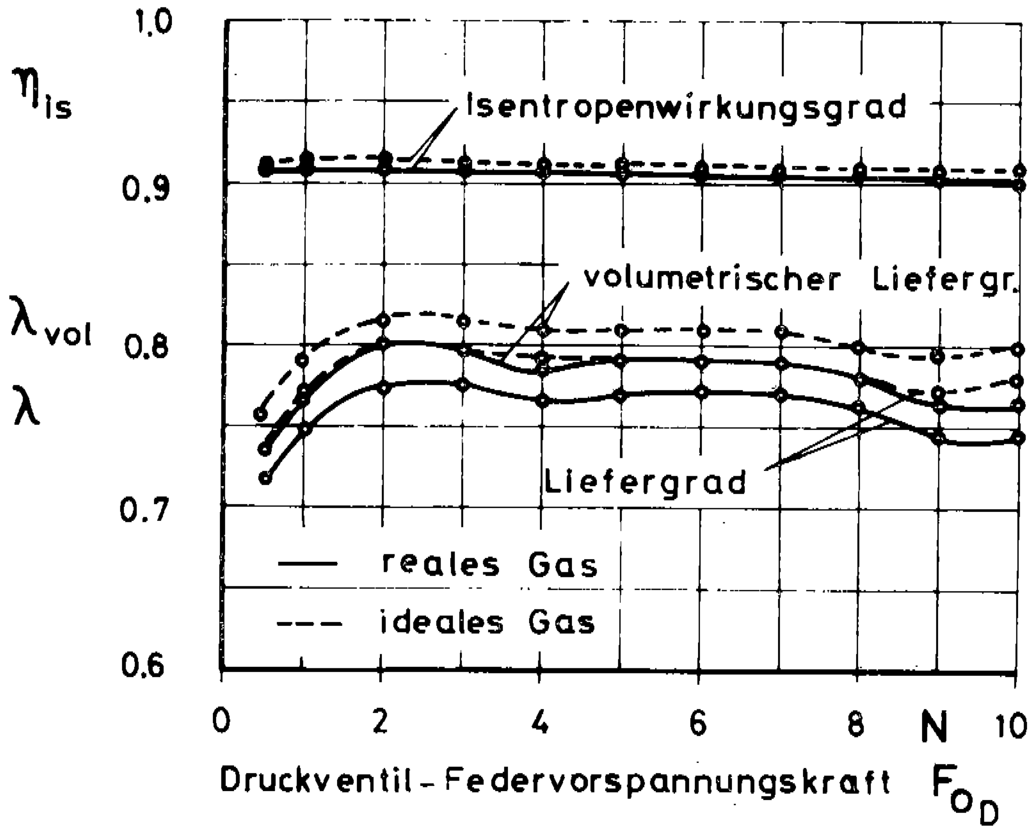

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Fig. 3

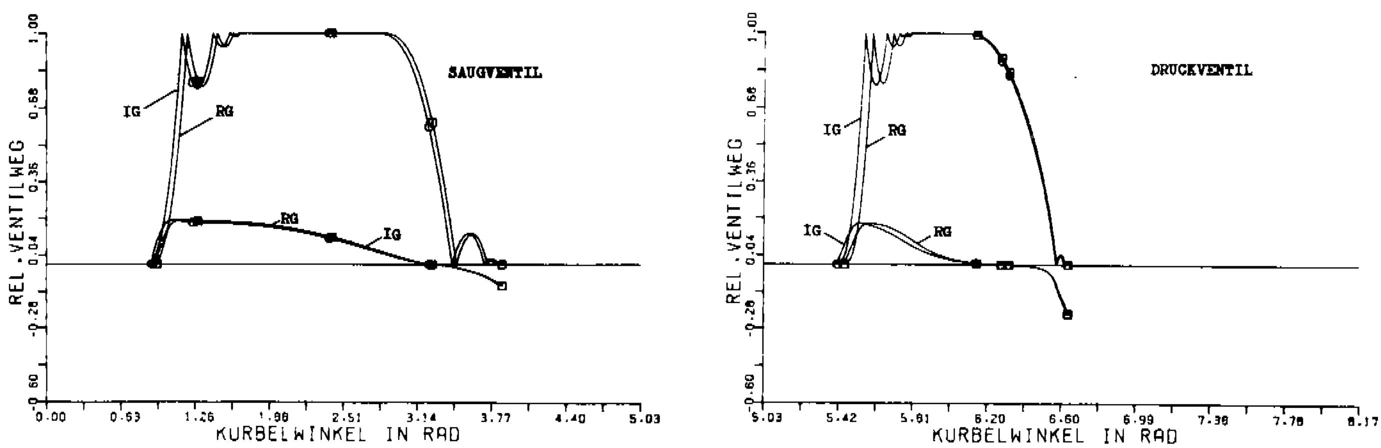

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Fig.4

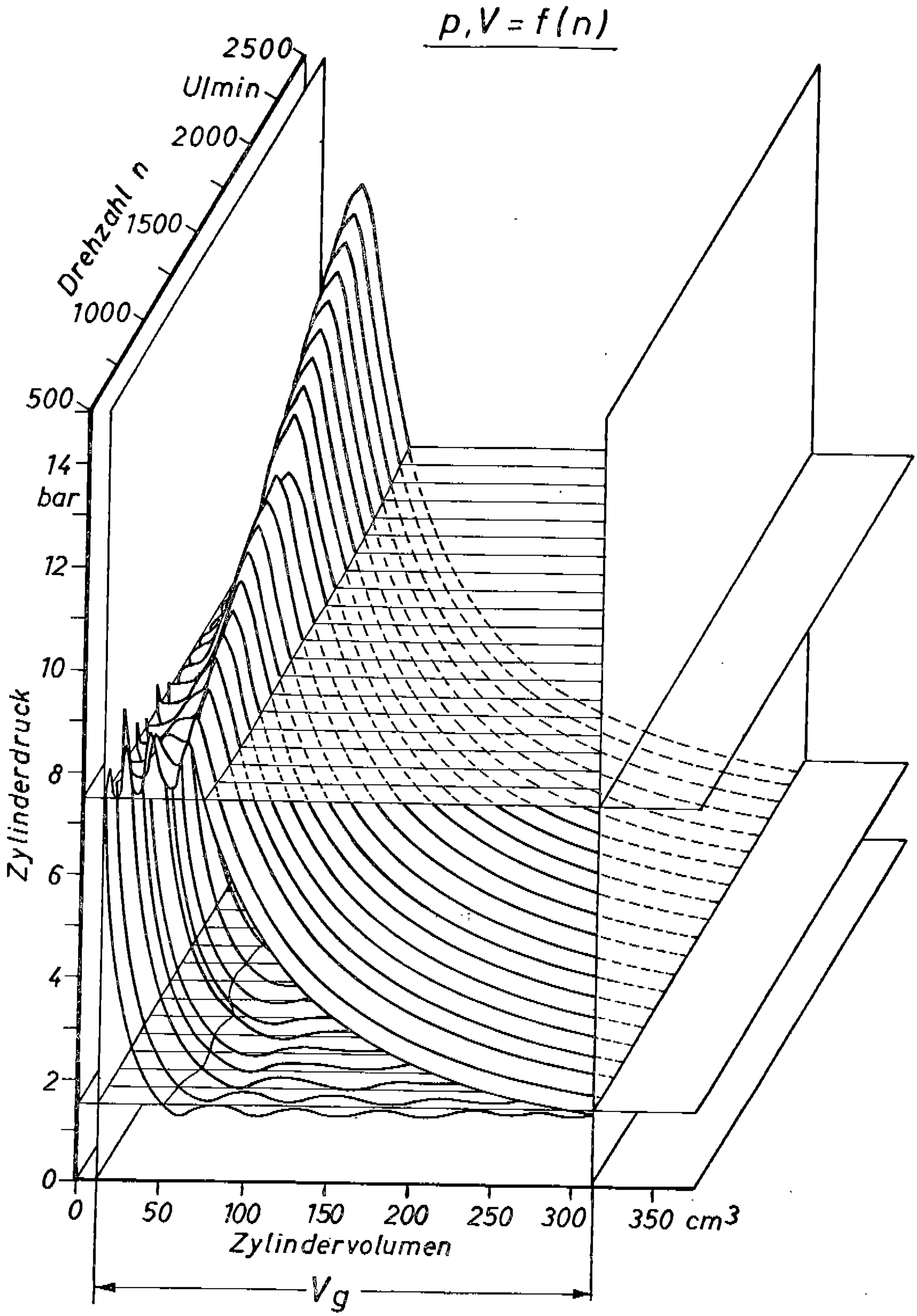
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Fig. 5

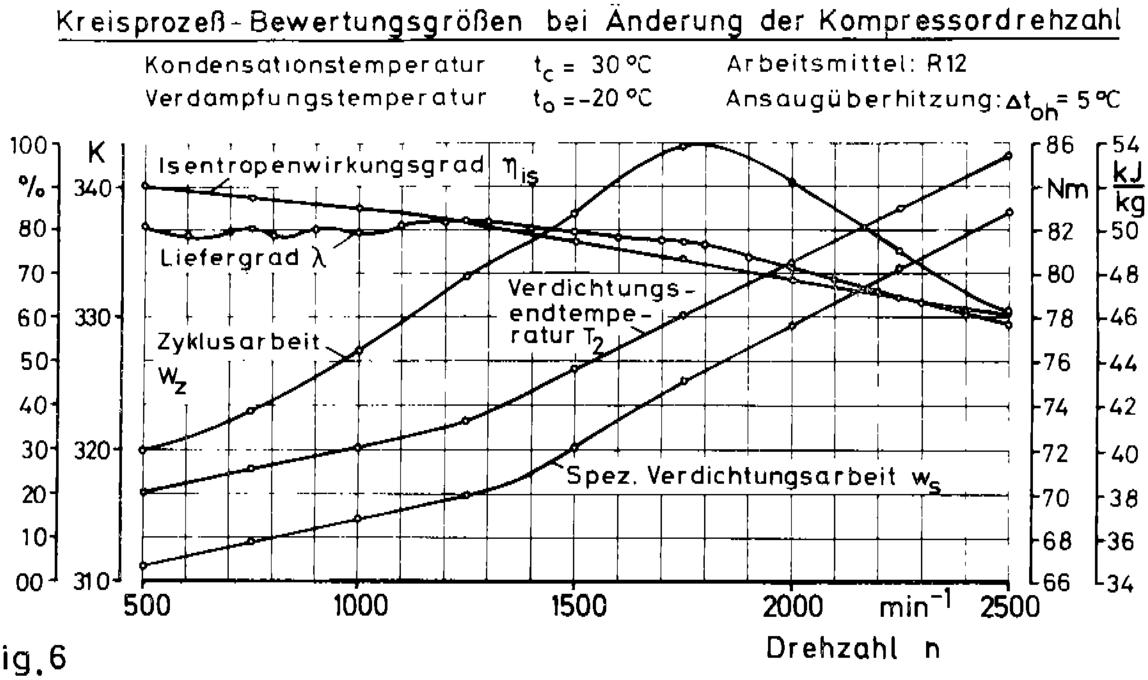

Fig. 6

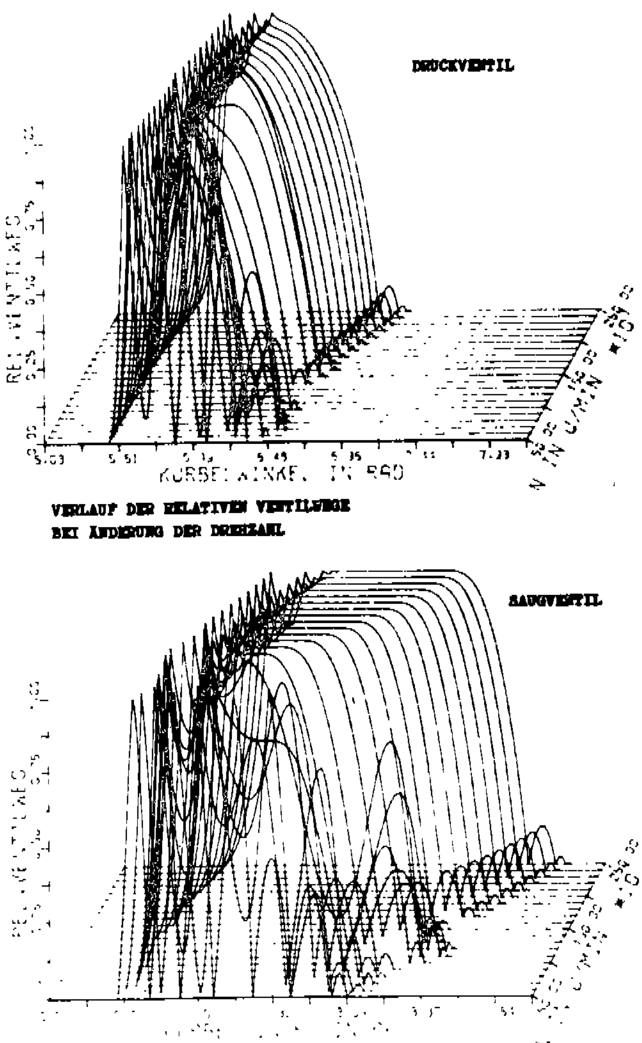

Fig. 7

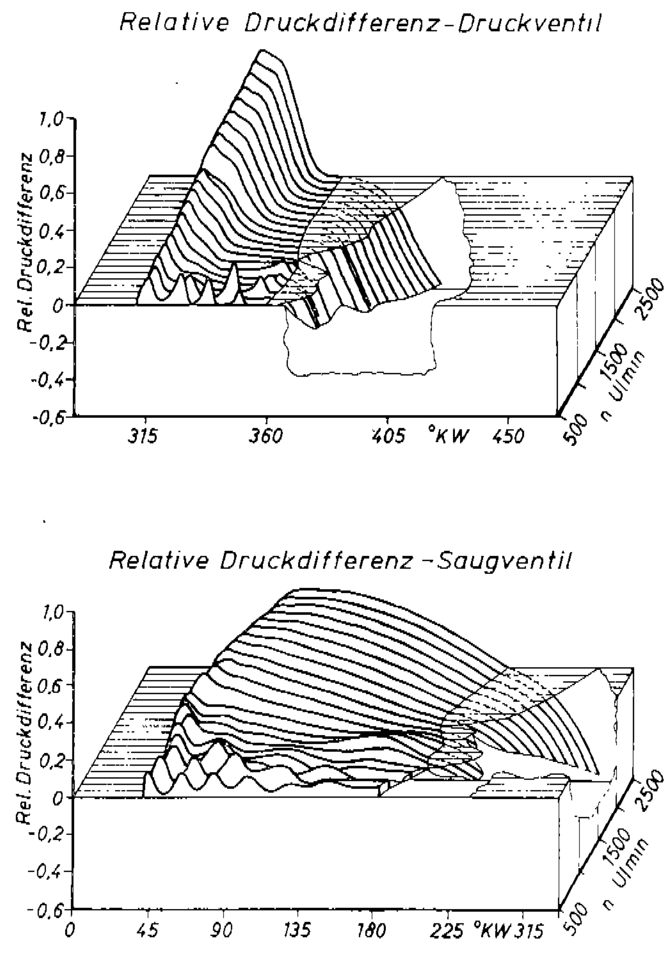

Fig. 8