1974

Optimal Muffler Design

E. Sandgren
Purdue University

K. M. Ragsdell
Purdue University

OPTIMAL MUFFLER DESIGN

Eric Sandgren
Graduate Research Assistant

Kenneth M. Ragsdell
Assistant Professor,
Mechanical Engineering

Ray W. Herrick Laboratories
School of Mechanical Engineering
Purdue University
West Lafayette, Indiana 47907

The application of nonlinear programming methods to the design of compressor muffler systems is demonstrated. A simplified muffler geometry is assumed and a standard impedance analysis approach is employed. A square wave volume velocity muffler input is assumed, and muffler design parameters are selected using modern optimization techniques. The resulting optimal design most closely approximates specifications within the muffler class considered.

INTRODUCTION

Presently one of the greatest problems in the design of mufflers for compressors is in the reduction of the noise level. Usually the noise problem is limited to a frequency band and if the sound pressure level of the frequency components within this band are reduced the overall noise level is reduced. The frequency attenuation characteristics of a given muffler design can be calculated by any of a number of techniques such as the acoustical impedance method, the method of characteristics, or by the shock tube method. It is not, however, usually obvious how a given muffler should be changed to bring about the desired attenuation characteristics. This is an excellent example of where modern optimization theory can be applied. Since the frequency attenuation characteristics of a given muffler design can be calculated analytically, optimization theory can be applied to determine what changes need be made to provide the desired characteristics.

Alfredson [1] did much of the initial work in this field. In his work the sum of the squares of all frequency attenuations less than a specified minimum attenuation was used as a performance criteria, or as it is called in optimization theory, the objective function. This criteria, however, is just one of many possible objective functions. In this work a general design procedure is presented which allows the designer to pick his own desired frequency attenuation function. This method is then demonstrated on a simple muffler configuration, and a comparative study of some of the currently popular optimization algorithms is conducted to see which, if any, of the methods handles the problem most effectively.

ACOUSTICAL IMPEDANCE APPROACH

The acoustical impedance is defined as the complex ratio of the acoustic pressure to the volume velocity. Here acoustic pressure refers only to the fluctuations from the mean pressure and not the total pressure. If it is assumed that the acoustic pressure is small when compared to the mean pressure, that the flow can be considered to be one dimensional, and that the diameter of any muffler element is much smaller than the wavelength of sound in the acoustic medium, then the acoustic pressure can be given as:

$$p = A_1 \; e^{2(\omega t - kx)} + B_1 \; e^{2(\omega t + kx)} \qquad (1)$$

where A_1 and B_1 are the magnitudes of the positive and negative traveling waves. The volume velocity is related to the acoustic pressure by:

$$u = p / \bar{\rho} c \qquad (2)$$

Now by applying equations (1) and (2) to a given muffler configuration and applying the boundary conditions that both the acoustic pressure and volume velocity are continuous at a discontinuity, the impedance can be described by the mean gas properties of the acoustical medium and by the geometry of the muffler elements. A more complete discussion of acoustical impedance is given in the references [2, 3, 4].

MATHEMATICAL PROGRAMMING MODEL

The standard form for the mathematical programming problem is as follows: Given an objective function $F(\bar{X})$ of n variables, $\bar{X} = (X_1, X_2, X_3, \ldots, X_n)^T$, and a feasible starting position $\bar{X}_o$, find $\bar{X}_m$

such that $F(\overline{X}m)$ approaches with prescribed closeness a minimum satisfying the constraints $\phi_i(\overline{X}) \geq 0$, $i = 1, 2, 3, \ldots k$ and $\psi_j(\overline{X}) = 0$, $j = 1, 2, 3, \ldots l$.

For muffler optimization the $\overline{X}$ vector would contain the geometrical properties of the muffler elements which are allowed to be changed. Common examples of dimensions which could be used would be crossectional areas or diameters and lengths of the individual muffler elements. The objective function F $(\overline{X})$ must contain a method of calculating the frequency spectrum of a given muffler, whether the spectrum be the volume velocity, pressure, or sound pressure level frequency spectrum. The objective function must also contain the desired frequency spectrum which the designer has specified. The output then is a single number which provides an indication of how close a muffler design approaches the desired frequency spectrum. One simple way of obtaining an indication of the performance is the following:

$$F(\overline{X}) = \sum_{i=1}^{N} (fdes_i - fact_i)^2 \qquad (3)$$

where $fdes_i$ is the desired frequency response of the ith harmonic, $fact_i$ is the actual frequency response of the ith harmonic and N is the highest harmonic being considered. The common muffler constraints would basically be the geometrical constraints which are needed to make sure that the final design is physically reasonable and that the muffler will fit into the given space allowed. Also special constraints may be needed in order that the approximations made in calculating the frequency response of the muffler are valid.

Once the problem has been put into this standard form any of a number of popular optimization algorithms may be used to find the optimal solution. It must be realized, however, that the muffler class under consideration may not be able to provide the desired frequency response, but the resulting optimal design will be the design which comes the closest to the desired frequency response within the muffler class under consideration.

An example problem will now be used to further clarify this procedure.

SAMPLE PROBLEM

In order to demonstrate the solution procedure the simple muffler configuration shown in figure 1 was used. The $\overline{X}$ vector for this case would contain the diameter of each element and the length of the first element. So $\overline{X} = (D_1, D_2, L_1)$.

The desired output for this example will be the output volume velocity, since for this simple muffler configuration the impedance in the discharge element is a constant and thus frequency independent, and from the volume velocity the pressure or sound pressure level could easily be calculated. The acoustical impedance approach was used to calculate the output volume velocity u_2. For this case the input impedance is given as [5]

$$Z_I = \frac{\overline{\rho}c}{S1} \frac{S1/S2 \cos k L_1 + j \sin k L_1}{\cos k L_1 + j\, S1/S2 \sin k L_1} = \frac{pI}{UI} \qquad (4)$$

where S_1 and S_2 are the crossectional areas of the first and second sections. The transfer impedance is given by

$$Z_T = \overline{\rho}c \quad \frac{1}{S2} \cos k L_1 + j \frac{1}{S1} \sin k L_1 = \frac{PI}{U2} \qquad (5)$$

The input volume velocity u_I is assumed to be a square wave pulse. The input pressure p_I, can be calculated from equation [4] and the output volume velocity U2 can then be calculated from equation [5].

For this test case the desired volume velocity is to have only low frequency components so a muffler is being sought which will act as a low pass filter. The three frequency functions which were used are shown in figure 2. The objective function for each case then was put in the form of equation [3] or

$$F(\overline{X}) = \sum_{i=1}^{20} (U_{des\,i} - U_{2i})^2$$

where now the first twenty harmonics are being considered.

The constraints used were as follows:

$$1'' \leq D_1 \leq 5''$$

$$.5'' \leq D_2 \leq 5''$$

$$.5'' \leq L_1 \leq 5''$$

These constraints were chosen to limit the design physically and to assure that the assumptions made in the acoustic impedance approach are applicable.

The problem in this form was then solved using seven different optimization algorithms available in an interactive version of Optisep [6] available on Purdue's computer system. A listing of these seven different methods and a brief description of each is given below.

1. David - Davidon-Fletcher-Powell gradient search method with penalty function.

2. Memgrd - Miele's memory gradient search method with penalty function.

3. Simplex - The Simplex method of direct search with penalty function.

4. Adrans - Random search strategy with accelerated pattern moves with penalty function.

5. Approx - Successive linear approximation search method.

6. Random - Random search strategy with shrinkage.

7. Seekl - Hooke and Jeeves direct search method with penalty function.

A further discussion of these methods is given in reference [7]. The starting position for all cases was chosen as

$$\bar{X} = (2, 1, 2)^T.$$

RESULTS

A comparative listing of the results of all seven methods is given for each of the three desired output functions in tables 1, 2 and 3. All seven methods handled the problem satisfactorally, but the Hooke and Jeeves direct search method arrived at the minimum in the least amount of computer time.

Graphs of the harmonics of the output volume velocity for all three desired functions are given in figures 3, 4, and 5. In all cases the desired function is not reached but the objective function is reduced by at least 60%.

CONCLUSION

The method developed for the optimal design of mufflers was demonstrated on a simple muffler element which had only three design parameters. The method, however, is applicable to any muffler configuration which can be modeled. The resulting design cannot be guaranteed to fit the desired function exactly but the resulting design will be as close as possible within the muffler class considered.

NOMENCLATURE

C = speed of sound in acoustic medium

D = diameter

F(X) = objective function to be minimized

j = imaginary number

k = wave number

L = length

p = pressure

S = crossectional area

u = volume velocity

X = vector containing design variables

$\bar{\rho}$ = mean density of acoustic medium

ϕ_i = ith inequality constraint

ψ_j = jth equality constraint

REFERENCES

1. Alfredson, R.J., "The Design and Optimization of Exhaust Silencers," Ph.D. Thesis, University of Southampton, 1970.

2. Kinsler, L.E., and Frey, A.R., Fundamentals of Acoustics, John Wiley and Sons, 1966.

3. Harris, C.M., Handbook of Noise Control, McGraw-Hill, 1957.

4. Mason, W.P., Electromechanical Transducer and Wave Filters, Van Nostrand, 1948.

5. Elson, J. P., and Soedel, W., "The Calculation of Discharge Pressure Oscillations in a Reciprocating Compressor," Submitted to "Sound and Vibrations" for publication.

6. Siddall, J.N., "Opti-Sep: Designer's Optimization Subroutines," McMaster University, Canada, 1971.

7. Siddall, J.N., Analytical Decision Making in Engineering Design, Prentice Hall, 1972.

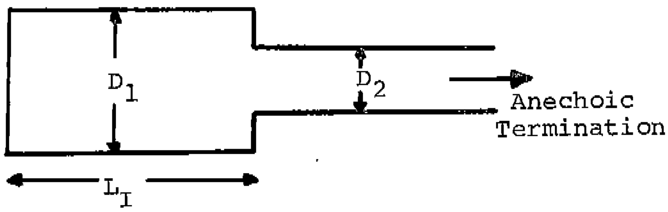

Figure 1 Simplified Muffler to be Optimized

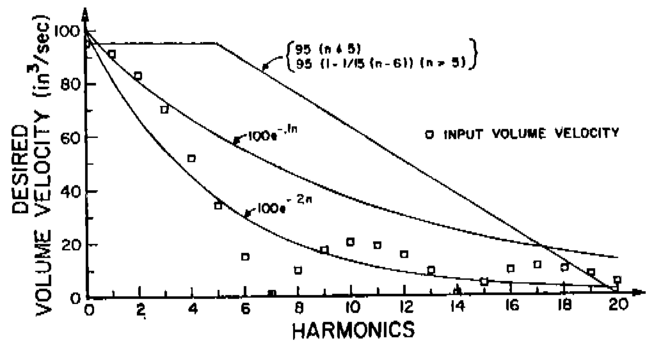

Figure 2 Desired Volume Velocities

METHOD	DAVID	MIELE	SIMPLX	ADRANS	APPROX	RANDOM	SEEK1
X(1)	1.2647	1.3113	1.0837	1.9529	1.5078	1.6628	1.0105
X(2)	3.0641	3.2129	2.6427	4.7551	2.4453	2.9443	2.4644
X(3)	3.9873	3.9937	3.9893	3.9889	2.2578	3.5259	3.9894
$F(\bar{X}_0)$	17785.	17785.	17785.	17785.	17785.	17785.	17785.
$F(\bar{X}_m)$	4128.9	4128.9	4128.8	4128.8	5831.8	4793.2	4128.8

Table 1 Comparative Results for Qdes = 100 $e^{-.1n}$

METHOD	DAVID	MIELE	SIMPLX	ADRANS	APPROX	RANDOM	SEEK1
X(1)	2.8538	2.1094	2.3596	4.5481	2.0078	4.4342	2.4377
X(2)	1.7986	1.3004	1.4919	2.5498	.9922	2.6605	1.1820
X(3)	.9368	.8822	.9324	.7020	.5234	.8228	.5000
$F(\bar{X}_0)$	2691.7	2691.7	2691.7	2691.7	2691.7	2691.7	2691.7
$F(\bar{X}_m)$	1579.3	1579.1	1579.4	1578.5	1578.1	1578.9	1578.1

Table 2 Comparative Results for Qdes = 100 $e^{-.2n}$

METHOD	DAVID	MIELE	SIMPLX	ADRANS	APPROX	RANDOM	SEEK1
X(1)	1.0000	1.0067	1.0419	1.0540	1.0703	1.6472	1.0000
X(2)	4.9999	3.7886	4.9510	4.7548	2.8047	4.2766	5.0000
X(3)	4.1108	4.1423	4.0173	4.0509	3.7578	3.5265	4.1109
$F(\bar{X}_0)$	50684.	50684.	50684.	50684.	50684.	50684.	50684.
$F(\bar{X}_m)$	13531.	14954.	13885.	13944.	15566.	16328.	13531.

Table 3 Comparative Results for Qdes = 95(n≤5), 95(1-1/15(n-6)) (n>t)

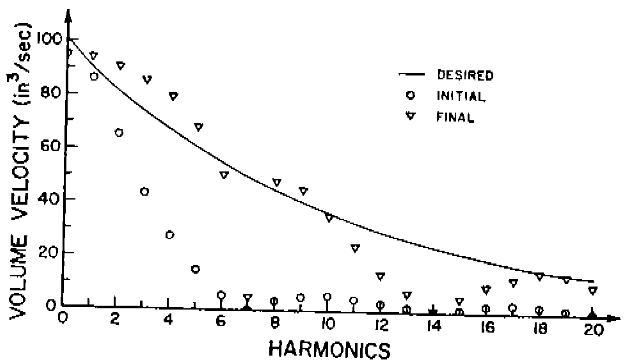

Figure 3 Output Volume Velocity for Qdes = 100 $e^{-.1n}$

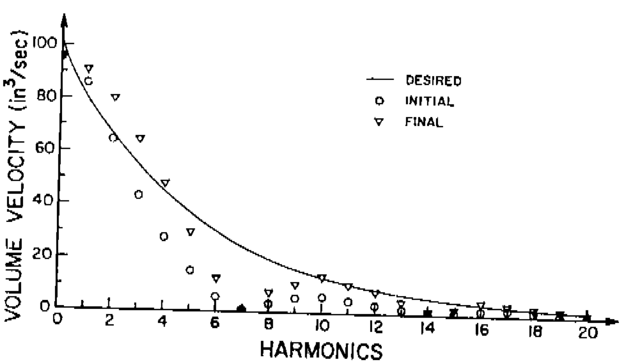

Figure 4 Output Volume Velocity For Qdes = 100 $e^{-.2n}$

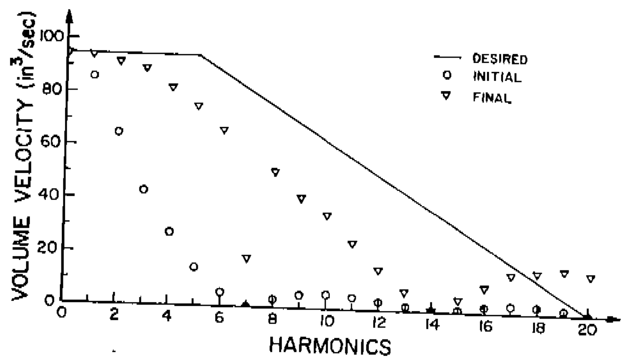

Figure 5 Output Volume Velocity for Qdes = 95(n≤5), 95(-1/15(n-6)) (n>5)

Addendum to "Optimal Muffler Design" by Eric Sandgren and Kenneth M. Ragsdell as presented at the 1974 Compressor Tecnology Conference

by

Eric Sandgren

It was noted that the desired output volume velocity functions chosen for the sample problem resulted in acoustical systems which are not in the ordinary sense mufflers. To demonstrate the case of muffler design in the conventional sense the results for a desired output volume velocity of the form $Q_{2_{des}} = 100e^{-.5n}$ are given below.

METHOD	DAVID	MIELE	SIMPLX	ADRANS	APPROX	RANDOM	SEEK1
X(1)	4.03	2.91	2.33	3.72	2.35	4.26	4.26
X(2)	.754	.795	1.00	.655	.805	1.49	.644
X(3)	.689	1.48	2.33	.609	2.35	2.44	.500
$F(\tilde{X}_o)$	2208	2208	2208	2208	2208	2208	2208
$F(\bar{X}_m)$	58.74	59.83	636.94	58.76	70.17	73.17	58.79

Comparitive Results for Qdes=100 $e^{-.5n}$

HARMONIC	Q_{2des}	Q_{2in}	Q_{2opt}
1	60.65	85.18	63.34
2	36.79	64.11	35.56
3	22.31	43.78	20.87
4	13.53	27.53	12.06
5	8.31	15.27	6.33
6	4.98	6.38	2.53
7	3.02	.28	.11
8	1.83	3.51	1.29
9	1.11	5.41	1.91
10	.67	5.83	1.97
11	.41	5.15	1.66
12	.24	3.75	1.15
13	.15	1.99	.58
14	.09	.20	.06
15	.06	1.37	.35
16	.03	2.53	.60
17	.02	3.14	.69
18	.01	3.16	.64
19	.007	2.61	.48
20	.005	1.59	.27