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INTRODUCTION

A great deal of work has already been done in investigating analytically the behaviour of Roots Blower machines in order to obtain a better understanding of their mode of operation and assess means of improving performance in operating conditions. The work described here is part of a continuing programme of such investigations. Firstly described is an extension to the "characteristic equation" approach given in ref.1. This paper also describes the mode of operation of the Roots Blower, and its performance both as a single stage machine and as a multistaged unit. The advantages of multstaging arise particularly from the fall of efficiency of a single stage Roots Machine as the pressure ratio across it is increased, and also the inherent restriction of the design to relatively modest delivery pressures. Using more than one machine thus enables one to generate with improved efficiency pressures of which a single stage machine is just capable, or to enable pressures to be obtained which would otherwise be beyond the range of a single stage machine; an additional practical bonus lies in the reduction of loading on the timing gears which can arise if the pressure rise across a single machine is reduced. By way of example of what might be expected, it is considered feasible that a competitive 3 stage Roots Machine could be devised for a pressure ratio of around 6:1 to provide a convenient workshop supply at about 90 psi. This arrangement would have the additional advantage of being clean and eliminate the need for filters to remove traces of oil contamination from the compressor. Such a system is to be analysed in due course by considering 2 pairs of 2 stage machines with the second stage of the first pair identical to the first stage of the second pair, and will be reported at some later date. Here the analyses are particularly applied to 2 stage arrangements. The equations previously derived by this method have assumed each stage to be running at the same speed, and that the real leakage areas of the machine are directly related to the machine size. Extensions to the analysis described here allow for these conditions to be relaxed, to give greater freedom in definition of the machine conditions. The equations are also presented for a machine operating under motoring conditions. Practical use if made of a Roots machine to motor for the purpose of metering gas, and whilst it is not usual to extract power from this situation, the behaviour of such a machine may now be investigated. These equations are also used in the 2 stage analysis for the situation where the second stage motors.

Further benefits may be achieved from multistaging beyond those described above. Firstly intercooling may be allowed between the stages, and such a possibility is included in the characteristic equation analysis. An alternative is to close-couple the stages to reduce to a minimum the size of the intervolume trapped between the stages. This is of benefit to the machine because of the changes in size of the trapped volume during parts of the machine cycle giving rise to some isentropic compressions within the cycle and resulting improvement in efficiency. In this part of the paper, use is made of the equations developed previously for the geometry of the rotors (ref.2) and as a first approximation only an ideal arrangement with leakless machines is considered.

NOTATION

A	Leakage area
a	Acoustic velocity
B	Ratio of swept vols as of first to second stage
S	Ratio of shaft speeds as of second to first stage
C	Swept vol/rev (= 4 x vol of a cell between rotor and casing)
F	Leakage area factor
M	Mass flow rate
m	Mass
N	Non dimensional speed $= (nc/Aa)$
n	Shaft speed
R	Gas constant
T	Static temperature
v	Volume
W	Work input per unit mass of gas
γ	Ratio of specific heats (= 7/5 for air)
η_c	Intercooler effectiveness
η_{is}	Isentropic efficiency
η_v	Volumetric efficiency
Θ	Overall pressure ratio referred to intake pressure
θ	Stage pressure ratio
ϕ	Flow function of θ

Suffixes

1,2	Stage quantities
L	Leakage
B	Blowing condition
M	Motoring condition

THEORETICAL MODELS FOR CHARACTERISTIC EQUATION

The theoretical model used here was proposed by Cole, Groves and Imrie (ref.1) and is shown in fig.1. In the model the actual leakage paths around the rotor tips, end faces, and between the rotors themselves are replaced by a nozzle external to the machine. Perfect sealing is now assumed within the machine. The cross sectional area of the nozzle is initially taken to be equal to the total leakage area within the real machine, and flow through the nozzle assumed isentropic. It has been found that these assumptions overestimate the flow to some degree, but a suitable correction can be made by an empirical factor to give close agreement of the predicted performance with measured over a wide range of conditions. In this paper such a coefficient will be assumed to have been allowed for within the determination of the value of nozzle area. The principal assumptions of the analysis are as follows:-

1) The working fluid is a perfect gas
2) All processes are adiabatic
3) Mixing processes proceed instantaneously to homogeneous equilibrium
4) The leakage flow is considered isentropic to the throat of an equivalent area convergent nozzle. The subsequent throttling and mixing processes produce a rise of intake temperature. (Shaft leakage to outside the machine is ignored for present purposes).
5) The delivery space acts as an infinite receiver
6) Negligible wave motion effects

Using these assumptions the equations describing the machine performance may be derived and are given in ref.1.

In the case of a machine operating under motoring conditions, providing the definitions of the variables are strictly adhered to the performance equations remain unchanged. The volumetric efficiency of the machine will, however, become greater than unity signifying that a greater volume of gas is passing through the machine than would be the case if it were completely leakless. The characteristic equation, however, will appear in simplified form because the mixing now occurs on the delivery side of the machine (Appendix 1).

The characteristic equations of the machine are thus as follows:

1) For a blowing condition:

$$N^2 = \left[\frac{nc}{a_o A_E}\right]^2 = \frac{2}{\gamma}\left[\frac{\phi_B \theta}{1-\eta_v}\right]^2 \left[1 + \frac{1}{\eta_v}\cdot\frac{\gamma-1}{\gamma}\cdot(\theta-1)\right] \quad \ldots (1)$$

$$\text{where } \phi_B = \sqrt{\frac{\gamma}{\gamma-1}}\left[\left(\frac{1}{\theta}\right)^{\frac{2}{\gamma}} - \left(\frac{1}{\theta}\right)^{\frac{\gamma+1}{\gamma}}\right]^{\frac{1}{2}} \text{ for } \theta \leqslant \left[\frac{\gamma+1}{2}\right]^{\frac{\gamma}{\gamma-1}}$$

$$\text{or } \phi_B = \left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}}\sqrt{\frac{\gamma}{\gamma+1}} = 0.484 \text{ for } \theta \geqslant \left[\frac{\gamma+1}{2}\right]^{\frac{\gamma}{\gamma-1}}$$

2) For a motoring condition:

$$N^2 = \left[\frac{nC}{a_o A_E}\right]^2 = \frac{2}{\gamma}\left[\frac{\phi_M}{1-\eta_v}\right]^2 \quad \ldots (2)$$

$$\text{where } \phi_M = \sqrt{\frac{\gamma}{\gamma-1}}\left[\theta^{\frac{2}{\gamma}} - \theta^{\frac{\gamma+1}{\gamma}}\right]^{\frac{1}{2}} \text{ for } \theta \geqslant \left[\frac{2}{\gamma+1}\right]^{\frac{\gamma}{\gamma+1}}$$

$$\text{or } \phi_M = \left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}}\sqrt{\frac{\gamma}{\gamma+1}} = 0.484 \text{ for } \theta \leqslant \left[\frac{2}{\gamma+1}\right]^{\frac{\gamma}{\gamma-1}}$$

Also necessary for a complete solution are the following equations:

$$\eta_v^3 - 2\eta_v^2 + \eta_v\left(1 - \frac{1}{E}\right) - \frac{1}{E}\cdot\frac{\gamma-1}{\gamma}\cdot(\theta-1) = 0 \quad \ldots (3)$$

$$\text{where } E = \frac{N^2}{\frac{2}{\gamma}\cdot\phi_B^2.\theta^2}$$

$$\frac{W}{RT_o} = \frac{1}{\eta_v}\cdot(\theta - 1) \quad \ldots (4)$$

APPLICATION OF CHARACTERISTIC EQUATIONS TO TWO-STAGE SYSTEM

For this part of the analysis, in addition to the previous assumptions, it is assumed that the interstage volume, whether cooled or not, is 'large' compared with the stage volumes, and creates no pressure loss. Previous analyses of the 2 stage system have depended upon equal stage speeds and complete geometric similarity between the stages. These conditions are here relaxed by the introduction of factors S and F, where S is defined as the ratio of second stage shaft speed to first, and F is defined by the condition

$$\frac{A_1}{A_2} = \frac{B^{\frac{2}{3}}}{F} \quad \ldots (5)$$

such that putting F = 1 reverts to the complete geometric similarity condition. A further extension arises from keeping the equations in general form such that allowance may be made in the computing for the leakage to become choked. Manipulation of the equations applied to each stage yield the following results:

$$B = \frac{\eta_{v2}}{\eta_{v1}}\cdot\theta_1 S \frac{1}{\left[1 + \frac{\gamma-1}{\gamma}\cdot\frac{1}{\eta_{v1}}(\theta_1 - 1)\right](1-\eta_c) + \eta_c} \quad \ldots (6)$$

$$\left[\frac{N_1}{N_2}\right]^2 = \left[\frac{n_1 C_1}{a_o A_1}\right]^2 \left[\frac{a_3 A_2}{n_2 C_2}\right]^2 = \frac{T_3}{T_o}.\frac{F^2 B^{\frac{2}{3}}}{S^2} \quad \ldots (7)$$

For both stages blowing

$$\left[\frac{N_1}{N_2}\right]^2 = \left[\frac{\phi_1\theta_1}{\phi_2\theta_2}\right]^2 \left[\frac{1-\eta_{v2}}{1-\eta_{v1}}\right]^2 \frac{\left[1 + \frac{1}{\eta_{v1}}\cdot\frac{\gamma-1}{\gamma}\cdot(\theta_1 - 1)\right]}{\left[1 + \frac{1}{\eta_{v2}}\cdot\frac{\gamma-1}{\gamma}\cdot(\theta_2 - 1)\right]} \quad \ldots (8)$$

For second stage motoring

$$\left[\frac{N_1}{N_2}\right]^2 = \left[\frac{\phi_1}{\phi_2}.\theta_1\right]^2 \left[\frac{1-\eta_{v2}}{1-\eta_{v1}}\right]^2 \left[1 + \frac{1}{\eta_{v1}}\cdot\frac{\gamma-1}{\gamma}\cdot(\theta_1 - 1)\right] \quad \ldots (9)$$

For the limiting case where $\theta_1 = 1$ and the second stage is blowing:

$$\left[\frac{N_1}{N_2}\right]^2 = \left[\frac{1-\eta_{v2}}{\phi_2\theta_2}\right]^2 \frac{\frac{\gamma}{2} N_1^2}{\left[1 + \frac{1}{\eta_{v2}}\cdot\frac{\gamma-1}{\gamma}\cdot(\theta_2 - 1)\right]} \quad \ldots (10)$$

$$\left[\frac{N_1}{N_2}\right]^2 = \left[\frac{\eta_{v2}}{\eta_{v1}}\right]^{\frac{2}{3}} \frac{F^2\theta_1}{S^{\frac{4}{3}}}^{\frac{2}{3}}\left\{\left[1 + \frac{1}{\eta_{v1}}\cdot\frac{\gamma-1}{\gamma}\cdot(\theta_1 - 1)\right](1 - \eta_c) + \eta_c\right\}^{\frac{1}{3}} \qquad \ldots (11)$$

$$\frac{W_1}{RT_o} = \frac{1}{\eta_{v1}}(\theta_1 - 1) \qquad \ldots (12)$$

$$\frac{W_2}{RT_o} = \frac{1}{\eta_{v1}}\cdot\frac{S}{B}(\Theta - \theta_1) \qquad \ldots (13)$$

$$\frac{W}{RT_o} = \frac{W_1}{RT_o} + \frac{W_2}{RT_o} \qquad \ldots (14)$$

The solution of these equations on a digital computer may be approached from two viewpoints. Firstly given operating conditions may be taken by specifying the parameters Θ and η_c and machine conditions by N_1 S and F. For a given first stage pressure ratio the solution is obtained thus. The first stage volumetric efficiency is obtained by numerical means from equation (3), choosing the appropriate root if more than one should lie within the range searched. Simultaneous solution of equations (11) and (8) (or (11) and (9)) will yield η_{v2} and N_2. B is obtained from equation (6) and the specific work for each stage and the total from equations (12)-(14). Overall efficiency may thus now be derived. Hence this method of solution gives the sizes of a pair of machines that would be required for a given set of operating conditions and the performance which could be expected from them. A variation could be used to prefix the size ratio B and hence determine the relevant speed ratio S. A sample set of computed results is shown in fig.2. The initial results of this type of analysis showed that with S and F equal to unity, the optimum efficiency was given closely by the condition $\theta_1 = \sqrt{\Theta}$. However by varying S and F it is seen that this condition does not always hold.

The alternative approach is to consider the performance that might be expected from a pair of machines over their whole working range. In this case the machines are defined (non-dimensionally) by setting values to the parameters N_1, B, S, F and η_c and the solution obtained for a given range of values of θ_1 by evaluating η_{v1} as previously, evaluating η_{v2} from equation (6), subsequently solving for N_2 and θ_2 from equations (11) and (8) (or (11) and (9)) and again obtaining the values of specific work and hence isentropic efficiency through equations (12)-(14). A sample set of results of these calculations is given in fig.3.

Elementary checks on these programs may be carried out by allowing N_1 to become large, simulating the ideal conditions of zero leakage area in the first stage machine. Under these conditions the first stage volumetric efficiency approaches unity and the resulting values derived approach the value obtainable from simplified expressions. In the second approach, at certain values of θ_1 the second stage will motor, and as θ_1 is reduced eventually the overall pressure ratio will fall to below unity.

Further reductions in θ_1 will eventually cause the computer program to give an error message when the work done by the first stage falls below unity as no allowance has been made for the first stage to motor. In the program so far developed, the limits imposed on θ_1 and θ_2 have been normally set at 2 as this is about the practical limit of most machines. However, trial runs have been made with θ_2 allowed to reach 20, and there does not appear from this to be an upper limit to the mathematical solution of the equations.

Further possible investigations arise from these solutions. By adjustment of the value of N_1, allowance may be made for the variation of the leakage area in the first stage machines. The second stage leakage area is then controlled by F in equation (5) and may either be restricted to its standard value or simultaneously (or independently) allowed to vary. Hence it may be predicted what possible variations are likely to arise due to the variation in "standard" machines due to the effect of normal machining tolerances on the leakage areas of the machines. Also consideration may be given to a pair of machines used in applications demanding varying operating conditions. In such a case it may be that benefits in efficiency can be obtained by suitable variation of the speed ratio between the stages. In such a case a suitable control system characteristic could be devised such that whatever the delivery pressure the relative speeds of the machines are chosen to give optimum economy at all times.

CLOSE-COUPLED TWO-STAGE ARRANGEMENT

Under conditions where it is not practicable to provide suitable intercooling between the stages in a 2 stage arrangement, some benefits are obtainable by mounting the machines in such a way that the inter-volume between them is kept to the smallest possible size. There comes a time between the transfer of an amount of gas in the first and second stages when the volume trapped between the stages undergoes changes in size and hence the gas undergoes an amount of isentropic compression. However, the amount of this compression achieved depends upon the relative phasing of the rotors, and also on the geometry of the machines, particularly the size of the first stage delivery port and the second stage inlet port. In this analysis, ideal machines have been used as a first approximation, having zero clearances between rotor and casing and unit volumetric efficiencies. The derivation of the equations governing the system is given in Appendix 2 and leads to the following governing equations, with notation as in fig.4.

For $0 \leqslant \beta \leqslant 45^\circ$ define

$$X = \frac{1}{v_o}\left[Dv_t + v_i + v_1\right] \qquad \ldots (15)$$

$$Y = \frac{1}{v_o}\left|Dv_1 + v_i + v_t\right| \qquad \ldots (16)$$

For $-45 < \beta < 0$ define

$$X = \frac{1}{v_o}\left[Dv_t + v_i + v_o + v_1\right] \qquad \ldots (17)$$

$$Y = \frac{1}{v_o}\left[D(v_o + v_1) + v_i + v_t\right] \qquad \ldots (18)$$

whence

$$\frac{P_3}{P_o} = \xi = \frac{1}{(1 + Y)\left[\frac{D + X}{1 + Y}\right]^{\gamma} - Y\left[\frac{X}{Y}\right]^{\gamma}} \quad . . . (19)$$

$$\frac{W}{RT_o} = \frac{\gamma}{\gamma - 1}\left\{\left[D\ \xi(1 + \frac{\gamma - 1}{\gamma}(\frac{\Theta}{\xi} - 1))\right] - 1\right\} . . (20)$$

$$\eta_{is} = \frac{\left[\Theta^{\frac{\gamma - 1}{\gamma}} - 1\right]}{D\ \xi\left[1 + \frac{\gamma - 1}{\gamma}(\Theta/\xi - 1)\right] - 1} \quad . . . (21)$$

To obtain optimum conditions equation (20) may be differentiated against D to give conditions governing minimum power expenditure, viz

$$\Theta_{opt} = \frac{\xi}{\gamma - 1}\left\{D\gamma\ \xi\left[\frac{D + X}{1 + Y}\right]^{\gamma - 1} - 1\right\} \quad . . . (22)$$

This condition has been used throughout the calculations.

Because of the accuracy required in solving these equations where small differences of large numbers are involved, a computer was used. Equations giving the sizes of the volumes involved for a typical pair of commercially available machines were taken from ref.2. The solutions have been obtained for a varying size of intervolume although it is considered that a size equal to one first stage cell volume would be about the smallest practical limit that could be achieved (fig.5). The results for varying rotor phasing are given in fig.6. The discontinuity which arises at $\beta = 0$ is caused by the additional interaction of a second stage cell volume. A further investigation was that of altering the port sizes to note the effect of a delay in the communication of the first stage cell volume with the interstage volume (fig.7). In this case considerations of a flow restriction through a port of reduced size do not arise since it is found that maximum efficiency is obtained with maximum port size that can be accommodated within a particular geometry.

It has been found (ref.1) that with this arrangement certain undefined gas dynamic effects within the intervolume cause deviations from the expected figures in an actual machine. It would be prudent therefore if such an arrangement were to be considered that some tests be carried out during the commissioning to ensure that the optimum arrangement had been achieved.

Actual power savings with this system would appear to be small. However, if a situation arose where a 2 stage machine was required without intercooling being available between the stages, then the benefits that might be obtained suggest this system should be considered. Further, in situations where the gas is not particularly required to be in a dry state, some cooling might be effected by means of water spray injection either at the inlet or between the stages, and again the benefits obtainable from this geometry might be advantageous.

CONCLUSIONS

This paper extends the means available for predicting and examining the operation of a pair of Roots Blowers. In the case of the characteristic equation approach, new methods are available for assessing the effects of different stage speeds and leakage areas, and will make it possible for easier optimisation of new designs and improvement of existing arrangements. Where intercooling is not possible, enhanced efficiency by means of close coupling gives a way of making some small saving in power consumption.

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APPENDIX 1

Derivation of characteristic equation for Roots Motor

The theoretical model is as shown in fig.1, but with the leakage taking place forward from the high pressure (inlet) to low pressure (delivery) sides. The states of the gas are denoted by suffix 0 at inlet, 1 immediately downstream of the blower at plane B, and 2 at delivery after mixing with leakage flow.

In this model the leakage mass flow rate is denoted by M_L, or m_ℓ per unit mass entering the system. M represents actual inlet mass flow to the system as a whole. As for the Roots blower, it may be verified that the leakage mass flow is given by, under the assumptions given in the text,

$$M_L = m_\ell . M = A_E\ P_o\sqrt{\frac{2g}{RT_o}}\phi_M \quad . . . (23)$$

$$\text{where } \phi_M = \sqrt{\frac{\gamma}{\gamma - 1}\left[\theta^{\frac{2}{\gamma}} - \theta^{\frac{\gamma+1}{\gamma}}\right]^{\frac{1}{2}}} \text{ for } \theta \geq \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\text{or } \phi_M = \left(\frac{2}{\gamma + 1}\right)^{\frac{1}{\gamma-1}}\sqrt{\frac{\gamma}{\gamma + 1}} \text{ for } \theta \leq \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\text{and } \theta = \frac{P_2}{P_o} \leq 1$$

The volume flow rate through the machine is given by

$$V_o = \eta_v\ V_1 = \frac{RT_o}{P_o} \quad . . . (24)$$

where η_v is defined by (Actual volume throughput)/(Ideal volume throughput in leakless machine) and is in this case ≥ 1.
If unit mass be supposed induced in r revolutions, then the volume of air passing through the blower proper is given by

$$V_1 = Cr = (1 - m_\ell)V_o \quad . . . (25)$$

Hence from (24) and (25)

$$m_\ell = 1 - \frac{1}{\eta_v} \quad . . . (26)$$

Since unit mass is delivered in r revolutions, and actual mass flow rate is M at speed n

$$M = \frac{n}{r} \quad \ldots (27)$$

Using equations (24), (25) and (27)

$$M = Cn\,\frac{\eta_v P_o}{RT_o} = \frac{\gamma g . Cn\, P_o\, \eta_v}{a_o^2} \quad \ldots (28)$$

Combining equation (28) with (23) and using (26) leads to

$$(1 - \frac{1}{\eta_v})\frac{\gamma g\, Cn\, P_o\, \eta_v}{a_o^2} = A_E\, P_o\sqrt{\frac{2g}{RT_o}}\,\phi_M \quad \ldots (29)$$

which may be manipulated to yield the equation (2).

APPENDIX 2

Analysis of Close-Coupled 2 stage system

The notation for this analysis is given in fig.4. It is assumed throughout that each stage has equal diameter ports, the blowers are assumed geometrically similar, and for the purposes of the calculations involute rotor profiles having minimum pressure angles have been considered.

For the advanced system with relative rotor phasing such that $0 \leq \beta \leq 45^o$, the cycle events for a simple first stage cell volume are with X and Y defined by equations (15) and (16).

1) Induction and carry round in first stage v_o at P_o, T_o

2) Communication and adiabatic mixing of cell volume with intervolume $(Dv_1 + v_i + v_t)$ at P_2, T_2 to give $(Dv_1 + v_i + v_o + v_t)$ at P_1, T_1 whence

$$P_1 = \frac{P_o v_o + P_2(Dv_1 + v_i + v_t)}{Dv_1 + v_i + v_t + v_o} = \frac{P_o + P_2 Y}{1 + Y} \quad \ldots (30)$$

3) Isentropic compression 1 $(Dv_1 + v_i + v_t + v_o)$ at P_1, T_1 to $[D(v_t + v_o) + v_i + v_1]$ at P_3, T_3 whence

$$P_3 = P_1\left[\frac{Dv_1 + v_i + v_t + v_o}{Dv_t + Dv_o + v_i + v_1}\right]^\gamma = P_1\left[\frac{1 + Y}{D + X}\right]^\gamma \quad (31)$$

4) Cut off by second stage Dv_o at P_3, T_3

5) Isentropic compression 2 in intervolume, $(Dv_t + v_i + v_1)$ at P_3, T_3 to $(Dv_1 + v_i + v_t)$ at P_2, T_2 (ready for next communication) whence

$$P_2 = P_3\left[\frac{Dv_t + v_i + v_1}{Dv_1 + v_i + v_t}\right]^\gamma = P_3\left[\frac{X}{Y}\right]^\gamma \quad \ldots (32)$$

6) Transfer in second stage and backflow compression Dv_o at P_3, T_3 to P_4, T_4

In the case of a retarded system, an additional second stage cell volume is involved in the initial communication. Provided the definitions of X and Y are modified as in equations (17) and (18), the analysis proceeds identically to the advanced case.

Eliminating P_2 and P_1 from equations (30)-(32) yields an expression for the interstage pressure ratio (although this is actually varying P_2, P_1, P_3 . . .)

$$\xi = \frac{P_3}{P_o} = \frac{1}{(1 + Y)(\frac{D + X}{1 + Y})^\gamma - Y(\frac{X}{Y})^\gamma} = \text{constant} \quad \ldots (33)$$

By applying the principle of conservation of mass in the stages, evaluating the isentropic temperature rises during the compressions and applying the expression for temperature rise in a Roots Blower to the second stage, it may be shown that:

$$\frac{T_4}{T_o} = D\xi\left[1 + \frac{\gamma - 1}{\gamma}(\frac{\Theta}{\xi} - 1)\right] \quad \ldots (34)$$

Finally equating the work input per unit mass to the change of enthalpy from states 0 - 4, and using expression (34) yields the specific work for the machine as given in equation (20).

APPENDIX 3

References

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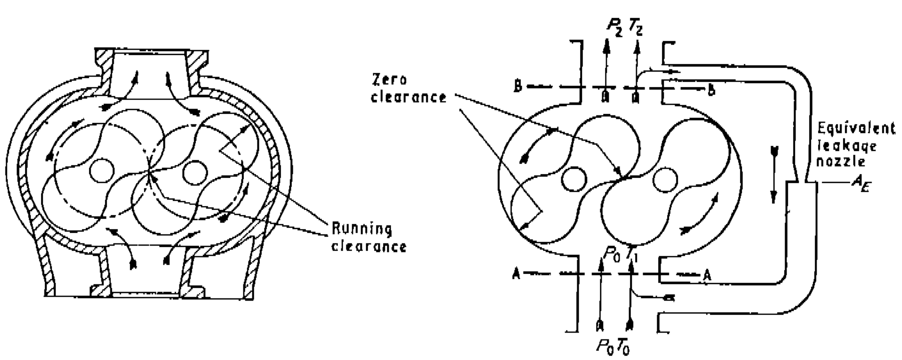

a Physical system

b Model system

FIG 1. Physical and Model Systems of Roots Blower

θ_1	η_{v1}	η_{v2}	N_2	B	Specific work	η_{is}
1.6000	0.7487	0.3985	2.301	0.6040	3.1229	0.4133
1.7000	0.7244	0.4251	2.179	0.6999	2.8894	0.4467
1.7321	0.7167	0.4333	2.142	0.7320	2.8340	0.4554
1.8000	0.7006	0.4504	2.069	0.8025	2.7428	0.4705
1.8929	0.6788	0.4731	1.977	0.9046	2.6678	0.4838

Fig 2: Sample evaluations of blower sizes and performance for F = 1.5, S = 0.75, η_c = 0.75, N_1 = 4 and H = 3.

θ_1	θ_2	η_{v1}	η_{v2}	N_2	H	Specific work	$\eta_{is.}$
1.1000	Failed - No rts of X2						
1.1500	0.6932	0.8821	1.2940	1.845	0.7971	-0.0700	
1.2000	0.8680	0.8630	1.2185	1.841	1.0416	0.1216	0.3373
1.4000	1.0004	0.8008	0.9874	1.824	1.4005	0.4999	0.7074
1.6000	1.0689	0.7487	0.8246	1.806	1.7103	0.8897	0.6519
1.8000	1.1871	0.7006	0.7016	1.785	2.1367	1.4303	0.5928
2.0000	1.3050	0.6531	0.6038	1.763	2.6100	2.0916	0.5277

Fig 3: Sample evaluations of performance obtainable with F= 1.5, S = 0.75, η_c = 0.75, N_1 = 4, B = 1.25.

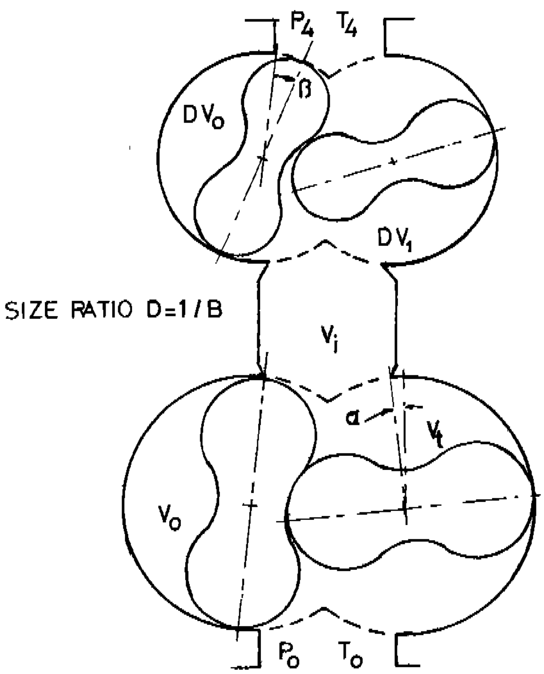

FIG 4. NOTATION FOR CLOSE-COUPLED SYSTEM.

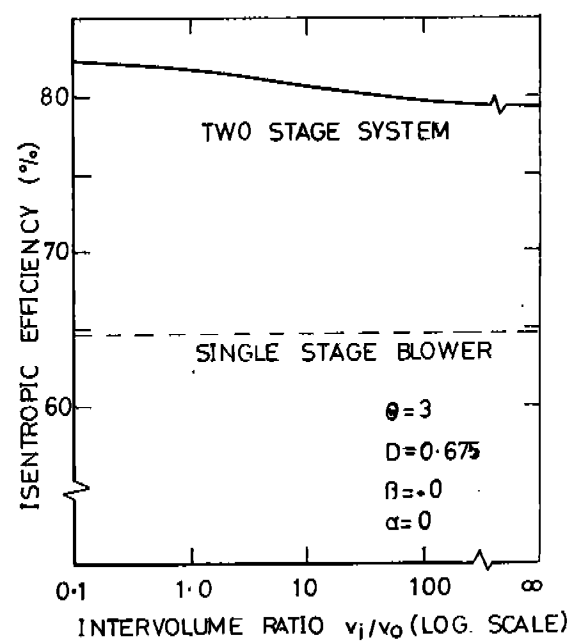

FIG 5. EFFECT OF INTERVOLUME SIZE

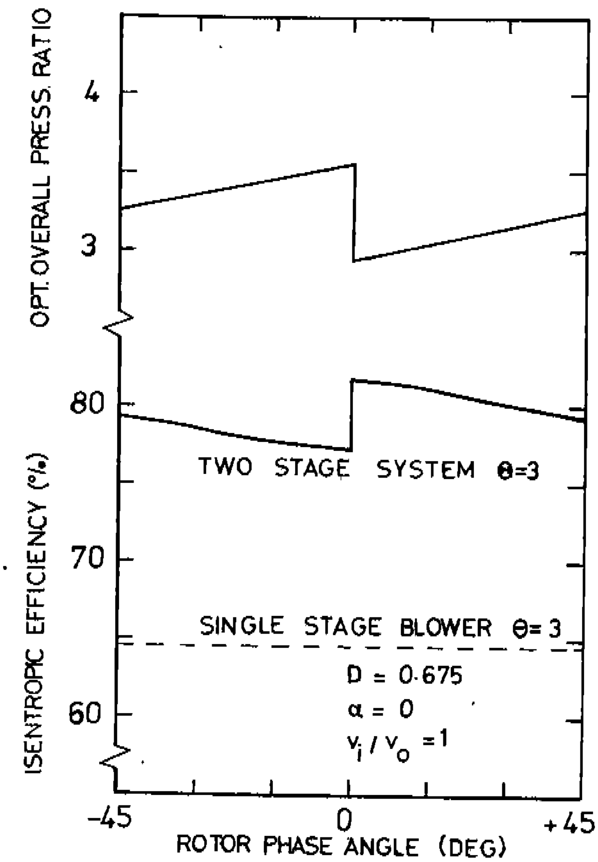

FIG 6. EFFECT OF ROTOR PHASING

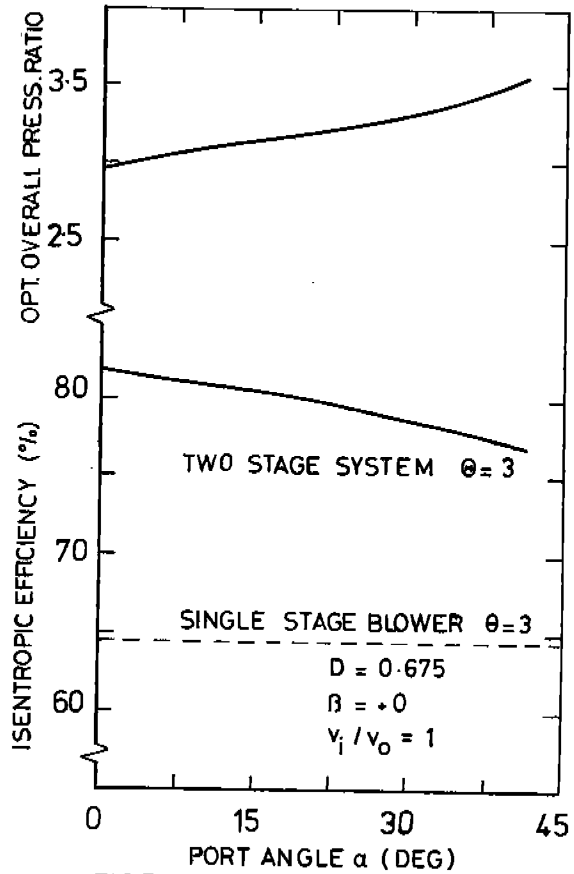

FIG 7. EFFECT OF PORT SIZE