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SOME FURTHER ANALYSIS OF RECIPROCATING COMPRESSOR SYSTEMS

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INTRODUCTION

In three recent papers Benson and Ucer (1,2,3) presented the results of analytical and experimental investigations of single and multi reciprocating compressor installations. In the analytical studies the effect of friction in the pipes was included but the flow was assumed to be homentropic. In order to allow for entropy variations in the system and heat transfer an empirical method was introduced to adjust the entropy level in the pipe to give agreement between experiment and analysis. This method was based on a mass balance in the system. An additional correction was made to the pipe length to allow for valve box volumes and heat transfer. The adjustments referred to were made by comparing experiment and analysis for one test. No further corrections were made for all the remaining tests analysed. The results gave reasonable agreement between experiment and analysis and the method was considered to be satisfactory for engineering studies.

The advantage of the modified homentropic theory is that it could be programmed and run fairly economically for the type of computer commercially available at the time of the investigation. The current generation of computers are at least an order of magnitude faster than the previous generation of computers. These are being used commercially for calculations using a generalised non-steady theory allowing for friction, heat transfer and entropy variations for studies of wave action in internal combustion engines. It was decided to apply the same methods to compressor systems. Since the ducts in compressor systems are larger than in engine systems it was considered that the representation of friction and heat transfer in the pipes might be more important so two models were investigated. The first model, used in internal combustion engine calculations, assumed a constant friction factor and Reynolds analogy for the heat transfer coefficient. The second model assumed a friction factor related to the pipe Reynolds number and a heat transfer model related to the Stanton number.

In order to apply the general non-homentropic theory, the boundary conditions across the valves and at the pipe ends must allow for the entropy changes the boundary equations used in the original analysis were modified accordingly.

In this paper the results are presented of an analysis of the test results reported by Benson and Ucer (1) using the generalized non-homentropic theory.

THEORETICAL BACKGROUND

The generalized non-homentropic theory has been presented in a number of papers (4,5). The reader is referred to these for the development of the conservation equations and their numerical solution. The final form of the expressions used in the present analysis are given in the Appendix. The problem is divided into two parts, the pipes and the boundary conditions at the pipe ends. Within the pipes the Riemann variables λ, β, A_a are calculated at fixed mesh points. At the pipe ends the boundary equations calculate λ_{in}, λ_{out} and A_a. The basic characteristic equations are given in equations (1) to (3) in the Appendix. The two models for the friction factor and heat transfer are given by equations (4) and (5) for one model and equations (6) to (8) for the other.

The valve boundary equations given in the earlier paper (1) are modified to include the entropy changes across the valve. The final form of these equations are given in equations (14) to (24). These equations are solved by iterative techniques. With the non-homentropic theory we can examine heat transfer effects in the compressor cylinder. The methods used in the present analysis are based on the techniques used in internal combustion engine calculations. The heat transfer coefficient is calculated using Annand's equation (9).

With the modifications indicated above a number of calculations have been carried out and the results compared with experimental results reported earlier (1) and the modified homentropic calculations. For convenience we will refer to the two non-homentropic calculations as

Non-Homentropic I	Constant Friction and Reynolds Analogy.
Non-Homentropic II	Variable Friction Factor and Stanton Relationship.

RESUMÉ OF CALCULATIONS

The calculations were carried out in two separate groups. The first group of calculations were at 400, 500 and 600 rpm with the compressor system shown in Figure 1. Non-homentropic calculation I used the corrected length and calculation II the actual length. Indicator diagrams were obtained in the cylinder, the intake and the delivery pipe for each speed. Typical results are shown in Figures 2 to 4 at 600 rpm. In these figures we have in addition to the non-homentropic calculations the modified homentropic calculation and the experimenal results. In Table 1 a comparison is made between

TABLE 1

Comparison of Experimental Results with Homentropic and Non-Homentropic Calculations

Compressor Speed rpm		Mass Flow lb/sec	Volumetric Efficiency %	Maximum Cylinder Pressure psig
400	Experiment	0.00320	83.02	73.0
	Homentropic	0.00341	88.41	73.0
	Non-Homentropic I	0.00302	78.21	74.0
	Non-Homentropic II	0.00306	73.99	69.2
500	Experiment	0.00367	74.79	88.0
	Homentropic	0.00376	76.57	90.0
	Non-Homentropic I	0.00367	74.79	92.0
	Non-Homentropic II	0.00364	75.48	92.0
600	Experiment	0.00419	73.00	97.5
	Homentropic	0.00444	77.35	97.5
	Non-Homentropic I	0.00418	72.91	104.8
	Non-Homentropic II	0.00429	74.88	95.4

TABLE 2

Comparison of Variations in Input Data on Overall Compressor Performance

				Percentage Effect on:			
Parameter Changed	Original Data	New Value	Model	Maximum Cylinder Pressure	Volumetric Efficiency	Cylinder Heat Transfer	Power
Cylinder Heat Transfer Annand Coefficient (No Heat Transfer)	a = 0.1	a = 0	I	+ 3.0	+ 0.1	No Heat Transfer	+ 1.2
			II	- 1.1	+ 0.3	No Heat Transfer	+ 1.6
Cylinder Heat Transfer Annand Coefficient	a = 0.1	a = 0.04	I	+ 3.2	Negligible	- 57.2	+ 1.7
			II	- 1.8	+ 0.1	- 56.6	+ 1.4
Cylinder Wall Temperature	104°F	208°F	I	+ 1.6	- 1.7	- 50.7	+ 1.0
			II	- 0.9	- 1.5	- 56.2	+ 0.6
Delivery Pipe Wall Temperature	135°F	270°F	I	+ 2.9	- 0.4	- 4.9	- 0.8
			II	+ 7.8	- 1.6	+ 10.6	+ 3.9
Suction Valve Weight	0.0068 lbm	0.01 lbm	I	+ 2.2	+ 1.3	+ 5.3	+ 2.4
			II	+ 0.6	+ 0.5	+ 3.9	+ 0.4
Delivery Valve Weight	0.0076 lbm	0.01 lbm	I	+ 3.5	- 2.8	- 1.7	- 4.1
			II	- 5.5	+ 0.6	- 3.7	+ 1.1
Suction and Delivery Valve Weights	0.0068 lbm	0.01 lbm	I	+ 2.9	+ 0.4	+ 3.0	+ 1.0
	0.0076 lbm	0.01 lbm	II	- 5.5	+ 0.7	- 0.9	+ 2.1
Valve Area Data	Pressure Dependent	Pressure Independent	I	+ 1.2	+ 1.2	+ 21.5	Negligible
			II	- 2.6	Negligible	+ 17.5	- 0.8

the predicted mass flow, volumetric efficiency and maximum cylinder pressure and the various analyses for all three speeds.

In the second group of calculations the compressor speed was set to 600 rpm and step by step changes were made, changing one or two at the most variables only from the original value, and the influence of the step changes on the maximum cylinder pressure, volumetric efficiency, cylinder heat transfer and power were noted. The parameters changed were the cylinder heat transfer coefficient, the cylinder wall temperature, the delivery pipe wall temperature, the suction valve weight, the delivery valve weight and the pressure dependence of the valve area data. The results are shown in Table 2.

DISCUSSION OF RESULTS

It has already been stated that the modified homentropic calculations were corrected to give agreement with experiments for one basic test. Thus the homentropic results follow the experimental results. There are no adjustments in the non-homentropic theory. The cylinder pressure diagram (Figure 2) shows that the variable friction model gives better agreement with experiment than the constant friction model. In the delivery pipe (Figure 2) the variable friction model gives lower pressures than the constant friction model. In the intake pipe the pressure amplitudes predicted by the variable friction model are closer to the experimental results than the constant friction model. It would appear from these results that a variable friction model will give better agreement with experiment, but there might have to be some adjustment in the constant in the friction factor expression. This adjustment could be obtained by experiment on pipes.

When the overall results are examined in Table 2, at all speeds the non-homentropic model gives better agreement than the homentropic results, with the variable friction model giving slightly better agreement than the constant friction model.

The results given in Table 2 give a broad picture of the influence of design parameters on the overall compressor results. Variations in output ± 1% could be considered to be within the accuracy of the numerical procedures. As pointed out earlier, the two friction models give different results. This is due mainly to the difference in the numerical value of f and q. The surprising feature in these calculations is the small effect of all the individual changes on the volumetric efficiency. Except for one result with the variation in delivery valve weight, the only significant variable affecting the volumetric efficiency is the cylinder wall temperature even though the heat transfer rate is affected almost the same for the heat transfer coefficient change as for the cylinder wall temperature change. This implies that the cylinder wall temperature has a greater effect during the suction stroke than during the delivery stroke. Another surprising feature is the relatively small effect the heat transfer rate has on the compressor power absorption. The calculations show that in general the valve weights might have a significant but small effect on the power, but it should be noted that the step change in weight was of the order of 35 to 45%. It is quite clear that the effect of valve weight is to reduce valve flutter and this in turn influences the maximum cylinder pressure and power. A useful result is the apparent lack of influence of the pressure dependence of the valve areas. The delivery pipe wall temperature appears to influence the results with the variable friction models. Since the pressure diagrams show that this model gives better agreement in the pressure diagrams it is reasonable to assume that delivery pipe temperature is an important variable.

CONCLUSIONS

The non-homentropic models give good correlation with the experimental results. However, the modified homentropic theory also gives reasonable results. This theory is however not so flexible as the non-homentropic theories and does not allow such variables as wall temperatures and heat transfer coefficients to be examined. Thus it would not be possible to examine a water cooled compressor with this theory. The question as to which theory to use will depend on the type of computer available, the economics of the run time and the problem under investigation. Of the two models for friction the variable friction factor model appears to be marginally better than the constant friction model.

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APPENDIX

BASIC EQUATIONS

In this appendix we present the final form of the equations used in the analysis. They are presented under two headings: the characteristics equation for one-dimensional non-steady non-homentropic flow and the boundary conditions at the pipe ends.

NOTATION

a	speed of sound, Annand constant
A	non-dimensional $a = \frac{a}{a_{ref}}$
a_{ref}	reference speed of sound
a_A	speed of sound after isentropic change of state to reference pressure p_{ref}
A_a	non-dimensional $a_A = \frac{a_A}{a_{ref}}$
b	Annand constant
C_p	specific heat at constant pressure
D	pipe diameter

D_c	cylinder diameter
f	friction factor
k	ratio of specific heats
L	length
L_{ref}	reference length
m	mass
Nu	Nusselt number
p	pressure
q	rate of heat transfer per unit mass per unit time
R	characteristic gas constant
Re	Reynolds number
St	Stanton number
t	time
T	temperature
u	particle velocity
U	non-dimensional velocity $= \frac{u}{a_{ref}}$
V_c	cylinder volume
V_p	piston velocity
Z	non-dimensional time $= \frac{a_{ref}t}{L_{ref}}$
λ	Riemann variable $A + \frac{k-1}{2} U$
β	Riemann variable $A - \frac{k-1}{2} U$
ψ	instantaneous valve throat area/pipe area
ρ	density
τ_w	wall shear stress.

Subscripts

c	cylinder stagnation
b	pipe adjacent to cylinder
g	gas
o	stagnation condition
ref	reference condition
in	in to boundary
in.c	in correct
in.n	in estimated
out	out from boundary
out.c	out correct
out.n	out estimated
w	wall

One-Dimensional Non-Steady Non-Homentropic Flow

The basic characteristic equations for non-homentropic flow are for constant pipe area (4,5)

λ Characteristics

$$\frac{dX}{dZ} = U + A \tag{1(a)}$$

$$d\lambda = A\frac{dA_a}{A_a} - \frac{k-1}{2}\frac{2fL}{D}U^2\frac{U}{|U|}\left[1-(k-1)\frac{U}{A}\right]dZ + \frac{(k-1)^2}{2}\frac{qL}{a_{ref}^3}\frac{1}{A}dZ \tag{1(b)}$$

β Characteristics

$$\frac{dX}{dZ} = U - A \tag{2(a)}$$

$$d\beta = A\frac{dA_a}{A_a} + \frac{k-1}{2}\frac{2fL}{D}U^2\frac{U}{|U|}\left[1+(k-1)\frac{U}{A}\right]dZ + \frac{(k-1)^2}{2}\frac{qL}{a_{ref}^3}\frac{1}{A}dZ \tag{2(b)}$$

A_a Characteristics (Path Lines)

$$\frac{dX}{dZ} = U$$

$$dA_a = \frac{k-1}{2}\frac{A_a}{A^2}\left[\frac{qL}{a_{ref}^3} + \frac{2fL}{D}\frac{U}{|U|}U^3\right]dZ \tag{3(b)}$$

Friction and Heat Transfer

Model 1

This model assumes a constant friction factor given by

$$f = \frac{\tau_w}{\frac{1}{2}\rho u^2} \tag{4}$$

and Reynolds analogy for heat transfer which gives

$$q = \frac{2f\ u\ C_p(T_w - T_g)}{D} \tag{5}$$

Model 2

In this model we assume the friction factor is given by (6,7,8)

$$f = \frac{\text{Constant}}{Re^{0.214}} = \frac{0.07}{Re^{0.214}} \tag{6}$$

and the heat transfer is

$$q = \frac{4k}{k-1}R\ u(T_w - T_g)\frac{St}{D} \tag{7}$$

where the Stanton number St is given by

$$St = \frac{\text{Constant}}{Re^{0.2}Pr^{0.6}} = \frac{0.04}{Re^{0.2}} \tag{8}$$

The constants in (6) and (8) suggested by Issa (7) and Azim (8) are for constant Prandtl number (Pr) of 0.7.

In the Reynolds number (Re) the pipe velocity (u), density (ρ) and viscosity based on temperature T, are obtained from the Riemann variables λ, β, A_a.

$$U = \frac{u}{a_{ref}} = \frac{\lambda-\beta}{k-1} \tag{9}$$

$$A = \frac{a}{a_{ref}} = \sqrt{\frac{T}{T_{ref}}} = \frac{\lambda+\beta}{2} \tag{10}$$

$$\frac{\rho}{\rho_{ref}} = \left[\frac{\lambda+\beta}{2A_a}\right]^{\frac{2}{k-1}} \tag{11}$$

Boundary Conditions

The following boundary conditions are used in the analysis:

(1) Inflow to a pipe from constant stagnation conditions.
(2) Outflow from pipe to a constant pressure.
(3) Flow across a valve from a cylinder to a pipe.
(4) Flow across a valve from a pipe to a cylinder.
(5) Pressure condition in the cylinder.

The Riemann variables are λ_{in} (known), λ_{out} (unknown) and entropy A_a (sometimes known, sometimes not known). If there is an entropy change at the pipe entry the known value of $\lambda_{in} = \lambda_{in.n}$ is modified by the entropy change to $\lambda_{in} = \lambda_{in.c}$. The entropy level changes from A_{an} to A_{ac}.

1) Inflow to a pipe from constant stagnation conditions A_o

$$\lambda_{out} = \left(\frac{3-k}{k+1}\right)+\left(\frac{2}{k+1}\right)\left(2(1-k)\lambda_{in}^2+(k^2-1)A_o\right)^{\frac{1}{2}} \tag{12}$$

2) Outflow from a pipe to a constant pressure p_b

$$\lambda_{out} = 2A_a\left(\frac{p_b}{p_{ref}}\right)^{\frac{k-1}{2k}} - \lambda_{in} \tag{13}$$

3) Flow across a valve from a pipe to cylinder

The following group of equations represents the flow.

a) Effect of entropy change across the valve on the Riemann variable λ_{in}

$$\lambda_{in.c} = \lambda_{in.n} + \left(\frac{\lambda_{in.c}+\lambda_{out.c}}{2}\right)\left(\frac{A_{ac}-A_{an}}{A_{an}}\right) \tag{14}$$

b) Relationship between the pipe pressure p, the cylinder pressure p_c and the Riemann variables

$$\frac{\lambda_{in.c}+\lambda_{out.c}}{2} = \left(\left(\frac{p}{p_c}\right)\left(\frac{p_c}{p_{ref}}\right)\right)^{\frac{k-1}{2k}} \tag{15}$$

c) Relationships between pipe pressure, cylinder pressure, valve area ratio (ψ), pipe velocity (u) and speed of sound in cylinder (a_c).

i) Subsonic flow in valve throat

$$\left(\frac{p}{p_c}\right) = \left(\frac{1}{2C}\left(\psi\sqrt{\psi^2+4C} - \psi^2\right)\right)^{\frac{k}{k-1}} \tag{16}$$

$$C = \frac{\frac{k-1}{2}U_p^2}{\left(1-\frac{k-1}{2}U_p^2\right)^2} \tag{17}$$

$$\psi = \frac{\text{Valve area}}{\text{Pipe area}}, \quad U_p = \frac{u}{a_c} = \frac{U}{A_c}$$

ii) Sonic flow in valve throat

$$\left(\frac{p_p}{p_c}\right) = \psi\left(\frac{2}{k+1}\right)^{\frac{k+1}{2(k-1)}}\left(1-\frac{k-1}{2}U_p^2\right)\frac{1}{U_p} \tag{18}$$

d) Energy equation for flow from cylinder to valve for given stagnation speed of sound in cylinder is the same as (12) with $A_c=A_o$.

4) Flow across valve from pipe to cylinder

The following group of equations represent the flow

a) Subsonic flow in valve throat

$$\left(A^{*\frac{4}{k-1}} -\psi^2\right)\left(\lambda_{in}^* - A^*\right)^2 - \frac{k-1}{2}\psi^2\left(A^{*2}-1\right) = 0 \tag{19}$$

$$\text{where } A^* = \left(\frac{\lambda_{in}+\lambda_{out}}{2A_a}\right)\left(\frac{p_{ref}}{p_c}\right)^{\frac{k-1}{2k}} \tag{20}$$

$$\lambda_{in}^* = \frac{\lambda_{in}}{A_a}\left(\frac{p_{ref}}{p_c}\right)^{\frac{k-1}{2k}} \tag{21}$$

b) Sonic flow in valve throat

$$\lambda_{out} = \lambda_{in}\left(\frac{1-\frac{k-1}{2}\psi B}{1+\frac{k-1}{2}\psi B}\right) \tag{22}$$

where $B = \left(\frac{p_t}{p}\right)_{critical}^{\frac{k+1}{2k}}$

$\left(\frac{p_t}{p}\right)_{critical}$ = critical static pressure ratio across the valve dependent on ψ

5) Pressure conditions in the cylinder

From the general energy equation the pressure change in the cylinder is given by

$$\frac{dp_c}{dt} = \frac{k-1}{V_c}\frac{dq}{dt} + \frac{1}{V_c}\left(\left(\frac{dm}{dt}\right)_{in}a_{o.in}^2-\left(\frac{dm}{dt}\right)_{out}a_{o.out}^2\right) - \frac{kp_c}{V_c}\frac{dV_c}{dt} \tag{23}$$

where

$\frac{dq}{dt}$ = heat transfer rate

$\left(\frac{dm}{dt}\right)$ = mass flow rate into or out of cylinder

V_c = cylinder volume

The heat transfer coefficient is given by (9)

$$Nu = \frac{hD_c}{k} = a\,Re^b \tag{24}$$

where

$Re = \frac{\rho_c D_c V_p}{\mu_c}$

D_c = cylinder bore

V_p = mean piston speed

μ_c = dynamic viscosity in cylinder

a,b = Annand's coefficients

CORRECTED LENGTH = 1·6859'
ACTUAL LENGTH = 1·445'

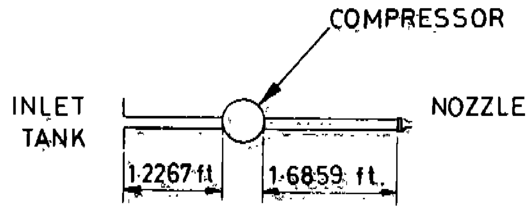

FIG.1. COMPRESSOR SYSTEM

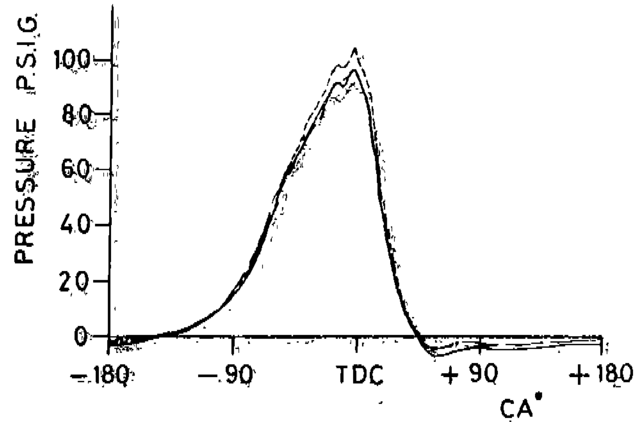

FIG. 2 CYLINDER – 600 R.P.M.

——— EXPERIMENTAL
– – – – – NON-HOMENTROPIC I
.... NON-HOMENTROPIC II
–·–·–·– HOMENTROPIC

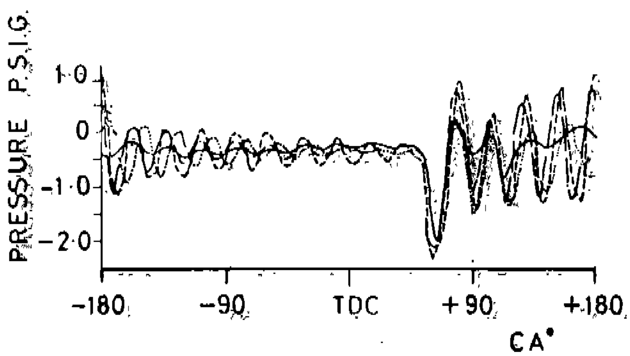

FIG. 3 INTAKE PIPE AT CYLINDER – 600 R.P.M.

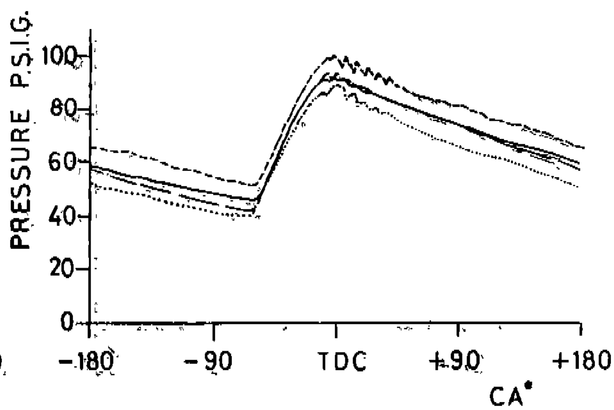

FIG 4 DELIVERY PIPE AT CYLINDER – 600 R.P.M.