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ABSTRACT

The calculation of Ground Heat Exchanger (GHE) performance is very important for predicting the performance and estimating the initial cost of geothermal heat pump systems. In North America it is estimated that on average GHE are oversized by about 10% ~ 30%. A vertical Ground Heat Exchanger (VGHE) is most common because of its smaller environmental influence and land field requirement. This paper reviews the current theoretical, analytical and numerical models to calculate heat transfer in a single borehole and borehole field in VGHE systems. The purpose of this paper is to increase the awareness of the different assumptions and methodologies between these calculation methods. Their primary strengths and weaknesses are also presented.

1. INTRODUCTION

The demand for high efficient energy sources has led to an increase in the application of Ground Source Heat Pump (GSHP) systems in residential and commercial buildings. The high initial cost of the GHE is one of the main reasons that hinder the widespread application of GSHP systems. Cane and Forgas (1991) estimated that the length of GHE was oversized by about 10% ~30% in the North American market. Because the fluid in a closed-loop GHE does not contact the ground directly, the impact of a closed-loop GHE on the environment is much less than that of an open-loop GHE. There are two kinds of closed-loop GHE - horizontal and vertical. The advantages of the vertical GHE are: 1) to efficiently utilize land for installing a GHE; 2) to reduce the influence of near-surface ground temperature fluctuations on the performance of the GHE. In this paper, literature that deals with the heat transfer calculations of VGHE are reviewed and classified based on the methods used.

2. CALCULATION METHODS

The methods used to calculate heat transfer of VGHE have been under development since the 1940's. There appear to have been three phases in this development. The first phase, which covered a period from the 1940's to 1960's, was focused on the development of theoretical models. The second phase focused on analytical solutions, which covered a timeframe from the 1970's to the 1980's. Finally, with the advent of computers came the development of numerical models since the late of 1980's.

2.1 Theoretical models

Ingersoll et al (1954) proposed the line source model for the application of VGHE in GSHP systems. In the line source model, heat transfer from a single borehole is treated as a line heat source with constant heat strength in an infinite medium. The medium is assumed to be at a uniform initial temperature and the line heat source starts at time zero. The temperature distribution in the medium is given by:

$$T - T_0 = \frac{Q'}{2\pi k_g} \int_\eta^\infty \frac{e^{-\beta^2}}{\beta} d\beta = \frac{Q'}{2\pi k_g} I(\eta) \qquad (1)$$

where, η is defined as a dimensionless variable as

$$\eta = \frac{r}{2\sqrt{a_g t}}$$

The line source model is assumed to be a one-dimensional (radial) heat transfer with a known constant heat flux. For the application of GSHP, the extracted or ejected load profile varies with time. Ingersoll suggested the integral part in Equation (1) be split into parts on a monthly basis. A monthly average heat transfer rate is used to replace the constant heat transfer rate in Equation (1).

Carslaw and Jaeger (1959) developed the cylindrical source model based on one-dimensional heat conduction in a cylindrical coordinate system. The expression is:

$$T = \frac{2F_0 kt}{Ka} + \frac{F_0 a}{K}\left\{\frac{r^2}{2a^2} - \frac{1}{4} - 2\sum_{n=1}^{\infty} e^{-\frac{k\alpha_n^2 t}{a^2}}\frac{J_0\left(\frac{r\alpha_n}{a}\right)}{\alpha_n^2 J_0(\alpha_n)}\right\} \tag{2}$$

Hart and Couvillion (1986) defined the far-field radius to determine the range of the ground that would be influenced by a single borehole. The change of ground temperature beyond the far-field radius is negligible. The line source equation is expressed as follows, which is as same as Ingersoll's model.

$$T - T_0 = \frac{Q'}{4\pi k_g}\int_{\eta^2}^{\infty}\frac{e^{-\lambda}}{\lambda}d\lambda = \frac{Q'}{4\pi k_g}E1(\eta^2) \tag{3}$$

The integral part in Equation (3) is simplified as the sum of a power series of N terms. That is:

$$T - T_0 = \frac{Q'}{2\pi k_g}[ln\frac{r_\infty}{r} - 0.9818 + \frac{4r^2}{2r_\infty^2} - \frac{1}{4\times 2!}\left(\frac{4r^2}{2r_\infty^2}\right)^2 + \cdots + \frac{(-1)^{N+1}}{2N\times N!}\left(\frac{4r^2}{2r_\infty^2}\right)^N] \tag{4}$$

Some terms in Equation (4) could be truncated off if the absolute error is within a desired tolerance.

The main shortcomings of these theoretical models are: 1) the borehole is regarded as an infinite line or cylinder source in an infinite, homogeneous medium, and heat transfer in the vertical direction is neglected; 2) the heat flow rate of the borehole is assumed as a known constant. In the actual application, the heat flow rate is coupled to the ground and the fluid temperatures. It should be noted that the theoretical models provide rough approximation to the actual heat transfer process in VGHE systems.

2.2 Analytical models
IGSHPA (1991) defined the ground formation resistance of a single borehole based on the line source model, as expressed in Equation (1). An approximation for the integral part in Equation (1) is given when η is in the range of $(0, 1]$ and $(1, \infty)$. The superposition method is used to consider the influence of adjacent boreholes on the borehole that is studied. IGSHPA also provided a design method to estimate the length of cooling and heating as follows:
For heating,

$$L_H = \frac{Capacity_H(COP_H - 1)(R_P + R_g * RunFraction_H)}{COP_H(T_{g,min,a} - T_{f,min})} \tag{5}$$

For cooling,

$$L_C = \frac{Capacity_C(COP_C + 1)(R_P + R_g * RunFraction_C)}{COP_C(T_{f,max} - T_{g,max,a})} \tag{6}$$

The larger of these two lengths will be used as the design length of the borehole. Then, a monthly performance analysis can be performed based on the monthly bin method. The procedure is: first, the $T_{f,min}$ and $T_{f,max}$ in each month are assumed; second, the heat pump run fractions are calculated using the bin method; third, the design length

of the borehole is used to calculate the temperature differences $(T_{g,min,a} - T_{f,min})$ and $(T_{f,max} - T_{g,max,a})$ from Equation (5) and (6); fourth, the calculated and assumed $T_{f,min}$ and $T_{f,max}$ are compared, if necessary, a new set of $T_{f,min}$ and $T_{f,max}$ should be assumed until the convergent solution is obtained.

Kavanaugh (1985) further developed a model based Carslaw's cylinder source model. He used a correction term to consider the non-uniform heat flow rate at the zones close to the pipes and the number of U-tubes inside the borehole. The average water temperature is shown as below:

$$T_{ave,f} - T_0 = \frac{Q'}{k_g} G(z,p) + \frac{Q'}{CN2\pi r_0 h_{eq}} \tag{7}$$

When N=2, C=0.85; when N=4, C=0.6-0.7. Kavanaugh also included the interference of two legs into his model. The short circuit losses are added into Equation (7) to correct the average fluid temperature as below:

$$\Delta T_{sc} = \frac{Q'_{sc} L}{m_w c_w} \tag{8}$$

ASHRAE (1999) presented a steady-state model based on the method of Ingersoll and Zobel (1954). That is,

$$L = \frac{qR}{T_g - T_f} \tag{9}$$

To represent the variable heat flow rate of an actual application, the numerator in Equation (7) is broken into two terms. One is the long term factor, which shows the influence of the net annual energy input to the ground. The other is short term factor, which is influenced by pipe resistance, and ground resistance in monthly and daily pulses. The part load factor and short circuit losses are also considered. For the short term, a 4-hour block is recommended. The design lengths for cooling and heating are:
For heating,

$$L_H = \frac{Q_a R_{ga} + (q_{lh} - W_h)(R_p + PLF_m R_{gm} + R_{gd} F_{sc})}{T_g - \frac{T_{fi} + T_{fo}}{2} - T_p} \tag{10}$$

For cooling,

$$L_C = \frac{Q_a R_{ga} + (q_{lc} - W_c)(R_p + PLF_m R_{gm} + R_{gd} F_{sc})}{T_g - \frac{T_{fi} + T_{fo}}{2} - T_p} \tag{11}$$

A chart is provided in ASHRAE (1999) to calculate the resistances of the ground to different heat flow pluses. However, in-situ testing is recommended to determine the resistance of the ground by an inverse heat transfer method. The local deep ground temperature T_g can be obtained from local water well logs and geological surveys.

The analytical models mentioned above provide engineering design methods for VGHE systems. In these models, the important parameters can be found from the literature and other sources. These models appear to be the most popular methods for sizing a VGHE system with an acceptable engineering accuracy.

2.3 Numerical models
The theoretical and analytical models simplify borehole geometry and the ground formation (i.e. different layers of sand, soil, and/or rock that are part of the borehole field) to adequately size VGHE systems. With the introduction of computing technology, numerical models have been developed to delve into the performance of VGHE with high accuracy.

Eskilson (1987) proposed a dimensionless temperature response factor, called g-function model. It describes the response of a single borehole to a unit step heat pulse in a two-dimensional (axial-vertical) coordinate system. Then the energy flow rate profile is decomposed as the sum of a set of unit step heat pulses. The response of a single borehole to each unit step heat pulse is superimposed to determine the overall response. The purpose of this model is

to predict the performance of VGHE in long term. Exact information about the borehole geometry is not included in this model, so it cannot accurately predict the short term performance.

Hellstrom (1991) developed a duct storage model for a densely packed GHE, which was used for seasonal thermal storage. The ground formation is subdivided into two different parts: a local region and a global region. The local region, which is composed of the ground in the immediate vicinity of a single borehole, is treated as quasi-steady state with constant heat flux in a time step. The difference between the local average temperature and the average fluid temperature inside the borehole is calculated by the constant heat extraction/rejection rate multiplying the thermal resistance between the fluid and the ground in the local region. The global region is the ground between the boundary of the local region and the far-field radius. Heat transfer in the global region is treated as three components: a steady-state heat loss component, a thermal build-up component and a periodic heat loss component. In order to calculate the total temperature change under a time-varying heat transfer profile, the spatial superposition is used first to add up the impacts of a unit step heat pulse from the local region and global region. Then, the heat profile is decomposed into a series of unit step heat pulses and the impacts in time series are superimposed.

Rottmayer et al (1997) developed a thermal resistance network for VGHE. In order to use a two-dimensional finite difference formulation, the circular legs of the U-tube are modified into a pie sector shape with the same perimeter, but the fluid convective heat transfer coefficient is consistent with that of the circular legs. Conduction in the vertical direction is neglected, but the ground temperature in each section is coupled with the fluid temperature. The shape change may alter the short circuit between the two legs, so a geometry factor of the order of 0.3-0.5 is proposed to adjust the influence of shape change on the heat transfer rate calculation.

Shonder and Beck (1999) developed an equivalent diameter model for a single borehole with a U-tube inside. The heat capacity of the U-tube and the fluid is represented by a thin film that immediately surrounds the equivalent diameter of the U-tube. So this model assumes one-dimensional transient heat conduction through the film, the grout, and the ground surrounding the borehole. The finite difference method and the Crank-Nicolson scheme are used to solve this model. Based on this numerical calculation, a parameter estimation procedure is proposed to predict the effective thermal conductivity of different soil formations.

Lee and Lam (2007) proposed a three-dimensional implicit finite difference method with a rectangular coordinate system. An equivalent thermal circuit is used to describe quasi-steady state heat transfer inside the borehole. The borehole heat transfer rate is numerically calculated under the prescribed borehole temperature profile. Each borehole is approximated by a square column using a loading factor of 1.047 to adjust for the shape change. The calculated borehole heat transfer rate is used as the boundary condition to calculate the ground temperature and borehole wall temperature. Iteration is used to solve these coupled equations. The numerical result shows that the temperature profile and load profile are not constant along the borehole.

Li and Zheng (2009) developed a three-dimensional unstructured finite volume model for a single borehole. This model uses Delaunay triangulation method to mesh the cross-section of borehole, which keeps the geometric structure in the borehole unchanged. The fluid inlet temperature is used as the boundary condition. The conjugate heat transfer solutions between the fluid temperature and ground (grout) temperature are achieved when the fluid temperature at the bottom of the borehole for each leg is convergent. This model has been verified by testing a ground sink direct cooling system with good agreement.

Numerical models use finite difference or finite element methods to describe heat transfer in every cell. They can accurately investigate heat transfer processes within a borehole in the short term. The finite element analysis can keep the structure of borehole unchanged to provide more accurate solutions. Although the numerical models take a relatively long time to simulate long term operation, accurate solutions will be helpful for the design and optimization of GHE systems that integrate with cooling towers and/or solar collectors.

3. CONCLUSIONS

The models to calculate heat transfer for boreholes in VGHE systems have been under development since the 1940's. The line source model and cylinder source model are the fundamental theoretical methods. They provide rough approximation to the actual heat transfer process in VGHE systems. Based on the theoretical methods, the analytical methods are developed using the superposition method. The various authors have used experimental or measured

data to adjust or modify the analytical solutions to size the VGHE system. Because the analytical models simplify the geometry of the borehole and the formation of the ground, they just provide a method to evaluate the performance of VGHE in a long term situation. The numerical models use the finite difference or finite element method to analyze every cell in the domain of the borehole and ground (bore field). They can delve into the performance of VGHE over the short term with accurate solutions, which provide the engineering solution to design or optimize the VGHE systems.

NOMENCLATURE

a	thermal diffusivity	(m/s^2)	a	annual
F	factor		ave	average
J_0	zero order term of Bessel function		c	cooling
k	thermal conductivity	$(W/m \cdot K)$	d	daily
L	length of borehole for cooling or heating	(m)	f	fluid
PLF	partial load factor		g	ground
p	distance from the borehole	(m)	h	heating
Q'	line heat flux	(W/m)	i	inlet
R	thermal resistance	$(m \cdot K/W)$	l	load
r	radial distance from borehole	(m)	m	monthly
T	temperature	$(°C)$	max	maximum
t	time	(s)	min	minimum
z	depth of position in vertical direction	(m)	o	outlet
η	dimensionless variable		p	pipe
			sc	short circuit

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