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THE MATCHING PROBLEM ON THE MODELING OF VAPOR COMPRESSION SYSTEMS. A TOOL TO ANALYZE THE SYSTEM BEHAVIOR.

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ABSTRACT

In this paper, a study of the well-posedness of the set of equations which form the model of a refrigeration system is presented. The conditions that the existence of some key components, such as the thermostatic expansion valve, orifice expansion device and the liquid receiver, are taken into account and fully discussed. For the purpose of the study, simplified models are considered for every component, relating the flow parameters and refrigerant properties. In any case a special care has been taken in order to assure a realistic physical behavior.

First, the chosen models are presented, and then a full discussion about the closure of the equations depending on the chosen elements is given. Finally, a sample of a parametric study performed with the described method is presented.

INTRODUCTION

A refrigeration unit is a thermal system composed of only a few components but, due to the fact that they are fully interconnected and that their characteristics are significantly different, the analysis of its operation and performance is a matter of major difficulty.

The future optimization of their operation and design will require the extensive use of computer based analysis. In this paper we present a simple way to develop a mathematical tool which allows the analysis of the global performance of a vapor compression system.

We must emphasize that the main objective of this work has been the analysis of how to define a mathematically well-posed problem depending on the characteristics of the elements which composed the system, and also the analysis of what kind of influence they have on the behavior of the global system. Therefore, highly simplified models are used for the different components, with the only required condition being that they behave similarly to the actual component in existence.

NOMENCLATURE

A	Area	$\dot{m}$	Mass flow rate
A_d	Diaphragm area	NTU	Number of transfer units
A_v	Valve area	T	Temperature
C_d	Discharge coefficient	U	Overall heat transfer coeff.
F	Force	x	Spring elongation
h	Specific enthalpy	X	Vapor mass fraction
K	Spring constant	ε	Efficiency
P	Pressure	ρ	Density
$\dot{Q}$	Heat rate		

COMPONENTS MODELING

Every component (compressor, evaporator, condenser,...) is treated as a black box, i.e. relation between the input variables and output variables, but in such a way that the formulae used to model it retain the physical behavior of the component. Figure 1 shows the scheme of the basic system. The main characteristics of the employed models are described in the following sections.

Compressor

For the compressor, we have chosen as independent variables the compression ratio and the inlet thermodynamic conditions, which can be represented by the inlet pressure and temperature, assuming that the flow at the inlet is always superheated. Two equations are required to characterize the compressor behavior, one for the mass flow rate and one for the outlet temperature

$$\dot{m} = f(\frac{P_2}{P_1}, P_1, T_1) \qquad (1)$$

$$T_2 = f(\frac{P_2}{P_1}, T_1) \tag{2}$$

The first equation is easily written as a function of the volumetric efficiency and then, the volumetric efficiency can be considered to be dependent on the compression ratio and on the death volume to displacement ratio. A constant can be easily fitted to the resulting equation in order to adjust the absolute value of the volumetric efficiency to actual values. In that way, equation (1) is able to provide reasonable results of the mass flow rate and also to show a trend to decrease the mass flow rate when either pressure ratio, or displacement to volume ratio, increase, and to be proportional to the inlet density. In regard to expression (2) a simple politropic law can be adopted for it. The exponent of the politropic law is easily adjusted in order to predict reasonable outlet temperatures.

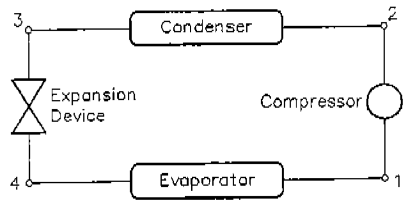

Figure 1. Scheme of a vapor compression refrigeration system

Evaporator and Condenser

Both elements are regarded as heat exchangers in which the mass flow rate and the inlet conditions of both fluids are known and the model must provide the fluid conditions at the outlet. The equation for the outlet temperature would be a complex function of the main parameters of the problem :

$$T_o = f(\dot{m}, T_i, P_i, T_{sat}(P_i), T_{air}, \dot{m}_{air}, U, A) \tag{3}$$

where U is the overall heat transfer coefficient and A the heat transfer area.

For the purpose of the present analysis a ε-*NTU* characterization has been chosen. The heat transfer area A is divided in two portions A_1 and A_2 ; A_1 is the portion needed to reach the saturation temperature $T_{sat}(P_i)$ and there, the overall heat transfer coefficient is estimated as a combination of standard air and gas heat transfer coefficients, and considered constant during the calculations. The rest of the heat exchanger A_2 is the core of the evaporator or condenser, and there, an adjusted value of the overall heat transfer coefficient is used in order to provide results close to measurements. A simple linear expression for U as a function of the Reynolds of the liquid has been fitted to the experimental values. This allows to take into account that when the velocity of the flow is increased the overall heat transfer coefficient tends to increase. A third portion of the heat exchanger should have been considered in order to allow a separate calculation of the subcooling at the condenser, or the superheating at the evaporator. However, it was decided not to include them in order to avoid a further complication of the system of equations. Anyway, the amount of heat transferred in those parts is small and therefore this process can be considered as included in the core of the heat exchanger.

For the outlet pressure, the equation should take into account the classical three components: friction, acceleration and gravity. However, that expression would be coupled with equation (3) and would significantly complicate the solution. Therefore, for the present analysis we will use the equation of negligible pressure drop.

With the described model, the conditions at the outlet of the condenser can be subcooled, saturated or two-phase. This fact allows a continuous behavior of the system of equations and therefore makes the solution finding process easier. In the case of two-phase flow, the outlet temperature is equal to the saturation temperature and then the vapor quality appears as a new dependent variable.

On the evaporator side, superheated conditions are required in order to feed the compressor only with vapor. At any rate this limitation is introduced in the algorithm in such a way that the evaporator model is only able to calculate when the outlet is on the superheated region.

Expansion device

A quite different behavior of the system can be obtained depending upon the employed type of expansion device. For the purpose of this study they could be classified as orifices with cross section constant or variable. In general, both systems could be modeled as an orifice, and therefore, two equations would represent its behavior : the iso-enthalpic process, and the orifice equation for the mass flow rate.

$$h(P_4,T_4,X_4)=h(P_3,T_3,X_3) \tag{4}$$

$$\dot{m}=f(\rho_3,\frac{P_4}{P_3},A_v) \tag{5}$$

The Bernouilli's equation is used to represent expression (5). This model, in practice, gives a possibility to obtain reasonable results, although at first view it could seem far away from the actual flow pattern. The value of the effective area of the orifice A_V is adjusted in order to reproduce the experimental results.

In some expansion devices, the area of the orifice is variable and varies depending on some parameter of the flow : typically, the evaporator pressure or the superheating. For instance, in a thermostatic valve the area of the orifice varies in order to approximately keep a certain degree of superheating.

In order to model the influence of a thermostatic valve on the behavior of the global system two different strategies can be followed. First, the simplest, is to eliminate equation (5) from the set of equations and substitute it by the condition of a fixed amount of superheating, following the way the actual valve tries to operate. Second, equation (5) is retained in the system of equations, but then a relationship between the orifice area and superheating must be included in order to close the problem. This can be made by a simple model of the regulation mechanism.

The needle position depends on the forces balance over the diaphragm of the valve; the acting forces are bulb pressure, evaporator pressure and the equivalent spring force. Let us assume that the bulb is charged with the same refrigerant as the system. Then, the spring force divided by the area of the diaphragm represents an equivalent pressure which has to balance the pressure in the bulb and the pressure in the evaporator, i.e.: $\frac{F_s}{A_d}=P_b-P_e$

The value of the pressure at the bulb is that which corresponds to the saturation pressure at the evaporator outlet temperature. The spring force depends on the displacement of the needle of the valve in a linear way:

$$F_s=F_0+Kx$$

where F_0 is the initial charge of the spring required to open the valve, which is fixed at the setting of the valve, and K is the spring equivalent constant.

Now, the displacement x must be related to the orifice cross section. This can be made by assuming that the effective area can be estimated by the product of a discharge coefficient and a minimum geometrical area left between the needle and the seat.

Therefore, at least three constants are required to model a thermostatic valve in the described way : $C_1=F_0/A_d$, $C_2=K/A_d$, and C_d

Other components

There are a few other components that are frequently found in refrigeration systems, for instance the connecting pipes. These elements can very easily be taken into consideration in the present analysis by expressing the outlet temperature and pressure as a function of the mass flow rate and the inlet conditions, in a similar way as for the heat exchangers.

Another element which is quite common is the liquid receiver. Due to the fact that during operation this element is only partially filled with liquid, the fluid properties at their inlet must be on saturation, and the outlet could only retain a small amount of subcooling, mainly due to the heat transferred from the bottle to the atmosphere. In fact, the presence of this element at the inlet of the expansion device drives the condenser pressure to force a saturated outlet. A very simple way to model this element is therefore to fix the condenser outlet point at the saturation line.

CLOSURE OF THE SYSTEM OF EQUATIONS

In the vapor compression system shown in figure 1 the list of unknowns, once the indoor and outdoor ambient conditions are given and the compressor speed and the geometry of every element are defined, is the following one : Circulating mass flow rate: $\dot{m}$, Compressor inlet and evaporator ρ_1,T_1,P_1, Compressor outlet and condenser inlet: ρ_2,T_2,P_2, Condenser outlet and expansion device inlet: ρ_3, T_3, P_3 Expansion device outlet and evaporator inlet: ρ_4,T_4,P_4, X_4.

It should be pointed out that the described problem is in fact to find the operating point of a system for a given set of operational conditions, in the same way as the system finds the operating point in reality. The problem therefore requires the determination of 14 unknowns, so that the same number of equations is required to obtained a well posed problem.

Two equations have been proposed and discussed above to model the behavior of the individual components, therefore, for the system of figure 1, 2 x 4 = 8 equations are provided from the models. Two equations of state can be written to relate density, pressure and temperature at points 1 and 2, and one additional one to express the density of point 3 normally as a function of the liquid temperature. Another two equations can be written for point 4, stating the relationship between saturation pressure and temperature and between the density, the temperature, and the vapor quality. A total of 13 equations is obtained from this analysis.

One equation is missing therefore to close the problem under the specified conditions. This equation is, as has been pointed out by other researchers [1] the equation which express the mass inventory for the refrigerant. This equation allows the problem to be closed.

In the case that a liquid receiver or a shell and tube condenser (which in practice acts as liquid receiver also) are included in the system, then a new equation comes into the balance expressing the obligation for the condenser to supply saturated liquid. In this case the system is closed without the necessity for the Mass inventory equation because then, the absolute amount of refrigerant charge would only influence the level of liquid in the reservoir. In fact that equation would only provide a way to estimate the level of refrigerant in the bottle and would not have any influence on the other considered variables.

This conclusion, which has been drawn from the analysis of the closure of the system of equations, has its proof also in the real behavior of a refrigeration system. In those systems which do not incorporate any place to accumulate the liquid, the charge of refrigerant has an influence on its behavior [2], meanwhile in those which do incorporate such a device the operation is independent of the charge, provided that a minimum charge has been provided for. In fact, that minimum charge should be the one required to have accumulated liquid. In the first system mentioned, for instance in small refrigeration units with air heat exchangers, variations in the rates of condensation or evaporation would affect the amount of liquid refrigerant in the system, thus affecting the subcooling, and therefore the performance of the whole system [3].

To write a realistic Mass inventory equation is a very difficult task because, on one hand it is necessary to perfectly characterize the capacity of every element and measure very accurately the charge, but on the other hand, it is also necessary to perform a detailed modeling of every element, including piping and auxiliary elements, and especially the heat exchangers, in such a way that it could allow the estimation of the amount of the refrigerant mass which they contain.

The authors have used a simplified method to express that equation in this study. First of all it is considered that in those kind of systems almost all the mass is concentrated as liquid in the liquid line and in a portion of the length of the condenser. Hence, in a certain way the charge would mainly affect the portion of the condenser which is dedicated to cool the liquid and thus would fix the amount of subcooling required. Therefore, one can substitute the Mass inventory equation by a linear equation for the subcooling fitted to reproduced measured values of this magnitude at different amounts of refrigerant charge and once the other operating conditions have been introduced in the described model.

SOLUTION STRATEGY

The above described system of equations consists of a series of non-linear equations. The solution for this kind of systems is not a simple matter and it is essential to define very strictly the solution domain for every equation. In the present study, the attention was mainly focused on the analysis of the matching of the equations and on the way of defining them in a simple, but physically consistent, way. Therefore, a standard mathematical library was used to solve the system of equations. In some cases, the solution could not be obtained, then we switched on a Minimizing standard procedure instead of using the Solving procedure. This allowed us to find the point that the solution tried to reach and check the reason why the solution could not be obtained. This turned out to always happen when some point of the solution tried to be placed in a region where the considered equations were not valid. The following phase of this study will be to carefully establish the physical domain of each equation, and provide for alternative equations or boundaries in order to continuously cover all the regions in which the solution could be looked for.

RESULTS

Figures 2, 3 and 4 show a sample of the calculated results obtained with the described methodology. The calculations have been performed for a small refrigeration unit of 3 kW of capacity which was tested at the laboratory in order to adjust the models in the way it has been described above.

The pressure enthalpy diagram is shown on each figure for the following cases ; Figure 2 shows the effect of changes on the indoor ambient temperature, Figure 3 shows the effect of changes on the outdoor ambient temperature, and Figure 4 shows the effect of small modifications on the expansion device effective area for the case in which the effective area of the valve is fixed but could be modified by hand. In the case of figures 3 and 4, the system is supposed to operate with a thermostatic valve which forces a superheat of 7 degrees centigrade. In the case of figure 4, the superheat is not fixed and it depends on the rest of operating parameters and also on the fixed effective area at the valve.

As can be observed in Figure 2 an increase of the indoor ambient temperature leads to a greater increase in the condensing temperature and pressure, although in relative terms, that increase is lower than the corresponding evaporator pressure increase so that pressure ratio remains almost constant, showing a slight decrease. The mass flow rate increases, mainly due to an increase in the density at the compressor's inlet, leading to an increase in the refrigeration capacity although the enthalpy variation across the evaporator decreases. The COP decreases because the compressor work increases more than capacity.

In regard to Figure 3, it shows that an increase of the outdoor ambient temperature leads to a greater condensing pressure and temperature. The evaporator temperature and pressure are not significantly affected, thus pressure ratio increases as the outdoor temperature increases. Mass flow rate remains almost unchanged, although the pressure ratio has increased. This is due to the fact that the volumetric efficiency formula which has been used seems not to recognize adequately the influence of the pressure ratio on the breathing of the compressor. Finally, the COP decreases significantly due to both the decrease in capacity and the increase in compressor work.

Figure 4 shows first that very small variations of the effective area of the valve lead to important modifications of performance. An increase of the area produces an increase of mass flow rate but all the other parameters of the cycle are also changed. The evaporation and condensation temperatures both increase producing an important increase of the pressure ratio and a decrease of the superheat. The refrigeration capacity is reduced and the COP decreases.

CONCLUSIONS

The main conclusions that can be drawn from the performed study are the following ones:

- The physical modeling of the elements provide two equations per element. It has been considered that the compressor and the expansion device would force the satisfaction of a relationship among the mass flow rate, upstream conditions and pressure ratio, and would also require an equation to define the process across them : politropic for the compressor and iso-enthalpic for the expansion device. On the other hand, the condenser and the evaporator determine both outlet pressure and temperature depending on their characteristics and on the inlet conditions and mass flow rate.
- If a the thermostatic valve is used, then the modeling of the orifice and the regulation mechanism can be substituted by fixing a point for the approximated superheat that the valve attempts to maintain.
- If a refrigeration system does not include a liquid receiver or any other element which could accumulate liquid refrigerant, then the mass inventory equation is required to close the system of equations. Alternatively, a fixed amount of subcooling can be used to close it.
- When the expansion device is fed by liquid coming from a liquid receiver or from a condenser which accumulates saturated refrigerant, then the subcooling must be very low and can be modeled as zero. The mass inventory equation is not then required. In fact it would only be useful to calculate the level of liquid in the bottle, making this value irrelevant for the operation of the equipment.
- A program based on the described models and system of equations can be very useful for analyzing the behavior of a refrigeration system when any of the main operating conditions are changed.
- The presented model could also be used as a method to generate an initial guess of the solution for a more complex global model.

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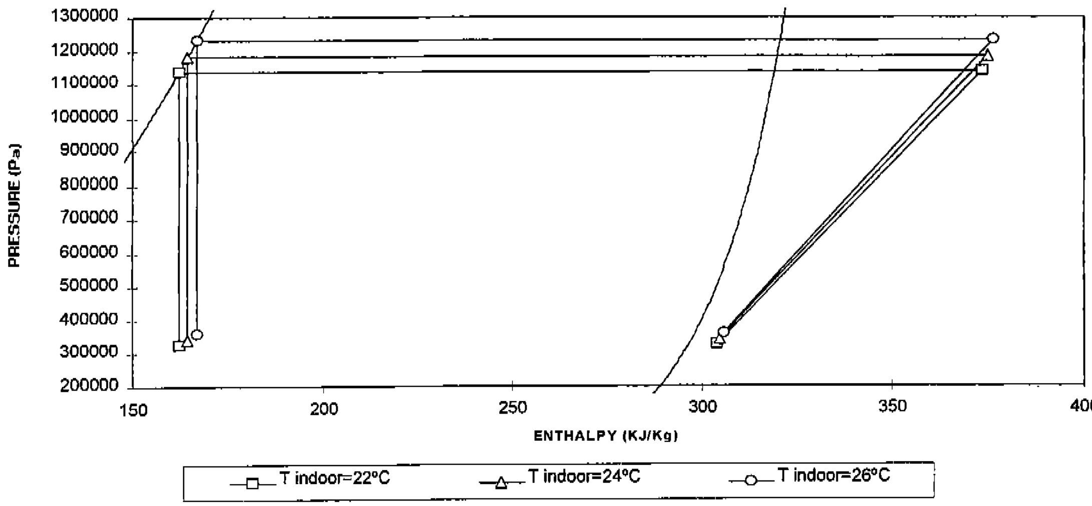
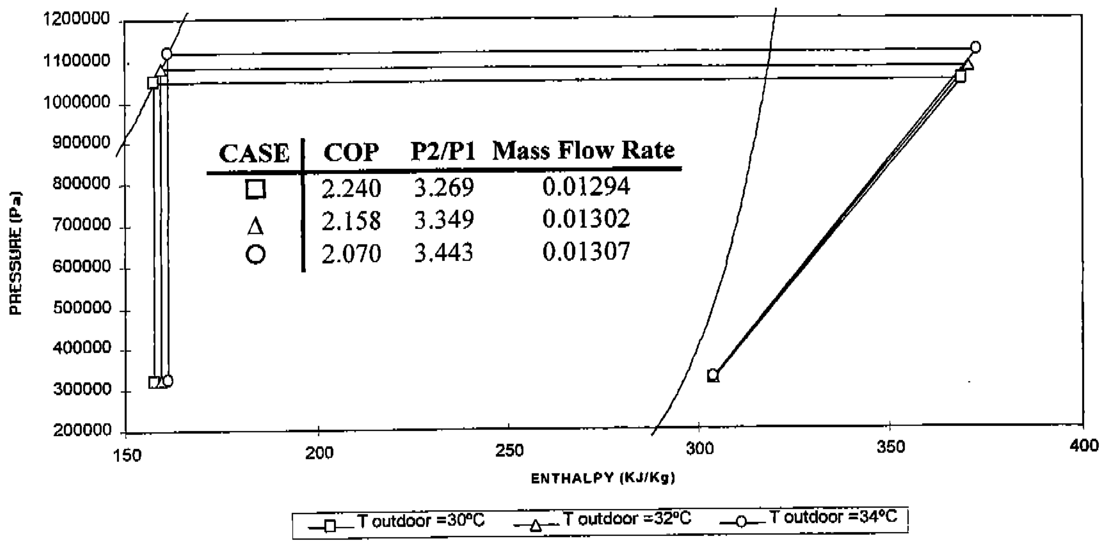
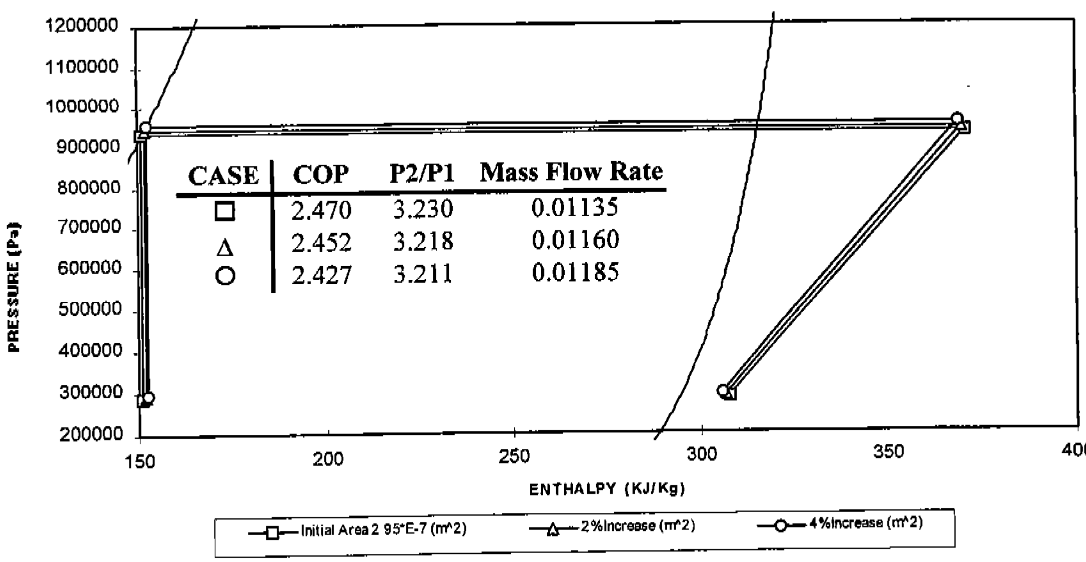


Figure 2. Effect of a modification of the indoor ambient temperature.



CASE	COP	P2/P1	Mass Flow Rate
□	2.240	3.269	0.01294
Δ	2.158	3.349	0.01302
O	2.070	3.443	0.01307

Figure 3. Effect of a modification of the outdoor ambient temperature



CASE	COP	P2/P1	Mass Flow Rate
□	2.470	3.230	0.01135
Δ	2.452	3.218	0.01160
O	2.427	3.211	0.01185

Figure 4. Effect of a modification of the effective area of the expansion valve.