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Quantum Phase Slips in the Presence of Finite-Range Disorder

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To study the effect of disorder on quantum phase slips (QPSs) in superconducting wires, we consider the plasmon-only model where disorder can be incorporated into a first-principles instanton calculation. We consider weak but general finite-range disorder and compute the form factor in the QPS rate associated with momentum transfer. We find that the system maps onto dissipative quantum mechanics, with the dissipative coefficient controlled by the wave (plasmon) impedance Z of the wire and with a superconductor-insulator transition at $Z = 6.5 \text{ k}\Omega$. We speculate that the system will remain in this universality class after resistive effects at the QPS core are taken into account.

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The possibility that quantum fluctuations destroy superconductivity in thin wires has attracted the attention of both experimentalists and theorists for a long time. Similarly to Little's analysis of thermal fluctuations [1], one concludes that the requisite quantum fluctuation should be sufficiently large, so as to allow the Ginzburg-Landau (GL) order parameter ψ_{GL} to vanish at the core, and the phase of ψ_{GL} to unwind. Such fluctuations are known as quantum phase slips (QPSs) [2]. On the experimental side, there has been a bit of controversy over precisely how superconductivity disappears in thin wires at low temperatures. Some experiments see a sharp superconducting-insulator transition (SIT) [3], while others do not [4].

Superconductivity takes place when it is difficult to transfer momentum from the moving condensate. In one dimension, a QPS unwinds a large momentum $P \sim \pi n_s$, where n_s is the linear superconducting electron density, and this momentum has to go somewhere. In uniform superfluids (such as a cold Bose gas in a ring trap), the requirement of momentum transfer constitutes a major bottleneck for QPSs [5]. In superconducting wires, on the other hand, there are some easily identifiable sinks of momentum. The most obvious, and as far as we know the only one that has been considered in the literature, is the normal electrons at the QPS core, which in turn transfer momentum to the disorder potential. It appears that in the existing theory of QPSs in wires [6] this process is assumed to be 100% effective, so that no trace of momentum conservation is left in the QPS rate. One should keep in mind, though, that this result is obtained using instantons of a disorder-averaged theory.

In this work, we analyze effects of suppressed momentum transfer explicitly, within a simple model where the QPS rate can be found from a first-principles instanton calculation—by first obtaining the rate for a given disorder configuration and then averaging over disorder. We consider the general case of weak but finite-range disorder. Our essential simplifying assumption is that electrons in the core are not effective in transferring momentum to the lattice, so that the transfer takes place via the gapless

plasmon mode [7]. In this limit (applicability of which is further discussed below), the rate can be computed within a plasmon-only effective theory [Eq. (1)].

Under these assumptions we find that the transition point is determined by the wave (plasmon) impedance of the wire Z . We find that the system is in the universality class of the dissipative quantum mechanics [8] (as opposed to the XY universality class found in Ref. [6]) and identify a SIT at $Z = \pi/2e^2 = 6.5 \text{ k}\Omega$. The Ohmic resistivity of the wire at the SIT, for weak disorder (the only case considered here), is small (much smaller than the normal-state resistivity) and has a nonuniversal value that depends on both the strength and the correlation length of disorder.

These results apply in the limit when the normal resistance R_{core} of the QPS core is effectively infinite. The effect of a finite R_{core} can be understood as follows. Plasmons produced by a QPS can be viewed as charge fluctuations in equivalent transmission lines, one such line on each side of the QPS core. A finite R_{core} will shunt the charge separation at the QPS core, thereby reducing the plasmon emission. This picture of two transmission lines shunted by R_{core} suggests that the universality class will remain the same even when dissipation is caused mostly by a finite R_{core} (which may very well be the case for existing experimental samples). The SIT will now be controlled by the total impedance formed by Z and R_{core} connected in parallel and thus occur across a straight line in the $(1/Z, 1/R_{\text{core}})$ plane.

Our results rely on a certain amount of impedance matching at the wire's ends. We assume that plasmons can leave the wire and go into the leads. This inhibits quantization of plasmon modes and translates, technically, into the possibility to consider the temperature T and the wire length L_w as independent infrared parameters. In the limit of short-range disorder, our results can be compared to those obtained by the Luttinger-liquid methods in Ref. [9] and reproduced in the instanton approach in Ref. [10]. Because we use the scaling $T \rightarrow 0$ with L_w fixed (as opposed to $T \propto L_w^{-1}$ used by those authors), we obtain a different value of the critical coupling. On the other hand, our value of the critical coupling coincides with that ob-

tained in the model where an effective resistor is connected to the wire's ends [11], provided we substitute Z for the resistance.

We start with the purely bosonic Euclidean Lagrangian density

$$L_E = \psi^\dagger \partial_\tau \psi + \frac{1}{2M} |\partial_x \psi|^2 + \frac{g}{2} |\psi|^4 - (\mu + V) |\psi|^2, \quad (1)$$

essentially the 1D Gross-Pitaevskii model in the presence of the disorder potential $V \equiv V(x)$. The field ψ is the “field of Cooper pairs” describing fluctuations of superconducting density and phase. Thus, $4\langle \psi^\dagger \psi \rangle m_*/M = n_s$ (m_* is the effective electron mass) is the density of superconducting electrons, while the coupling constant in Eq. (1) is $g = 4e^2/C$, where C is the wire capacitance per unit length. The effective theory (1) holds only at spatial scales larger than the size of a Cooper pair, i.e., the GL coherence length ξ . Thus, the potential $V(x)$ is coarse grained at the scale ξ in the x direction and at the scale of the wire thickness in the transverse directions.

We assume the disorder to be Gaussian with the correlators $\langle V \rangle = 0$, $\langle V(x)V(x') \rangle = V_0^2 f(x - x')$. The correlation function $f(x)$ is normalized so that $f(0) \equiv 1$, and V_0 is the rms disorder amplitude. The disorder correlation length is l , meaning that for $x \gtrsim l$, $f(x) \rightarrow 0$.

A major role in determining the QPS rate is played by interactions between QPSs at different locations. These interactions are determined by regions outside the QPS cores and can be accounted for in a phase-only model. We define $\psi = (\rho + \delta\rho)^{1/2} e^{i\theta}$ and, assuming weak disorder, expand Eq. (1) in powers of the small density fluctuation $\delta\rho$ around the stationary point with a given phase gradient $\theta' = \bar{\theta}'$. This phase gradient takes into account the biasing current $I = 2e\bar{\theta}'\rho/M$. At the classical level the stationary point is characterized by the local minimum $\rho = \rho_*$ of the effective potential

$$U(\rho) \equiv \frac{g}{2} \rho^2 - \mu \rho - \frac{MI^2}{8e^2 \rho}. \quad (2)$$

The minimum exists below the critical current, $I < I_c$, where $3[MI_c^2 g^2 / (4e^2)]^{1/3} = 2\mu$.

Integrating out the density fluctuations $\delta\rho$, we obtain the Euclidean Lagrangian density for the phase fluctuations $\theta_1 \equiv \theta - \bar{\theta}$, describing gapless plasmons [7] propagating with speed $c_0 = (\rho_* g / M)^{1/2}$ on the background moving with superfluid velocity $u \equiv I / (2e\rho_*)$:

$$L_E = i\rho_* \dot{\theta}_1 + \frac{1}{2} D_\tau \theta_1 \hat{K}^{-1} D_\tau \theta_1 + \frac{\rho_*}{2M} (\theta'_1 + \bar{\theta}')^2, \quad (3)$$

where $D_\tau \theta_1 \equiv \dot{\theta}_1 - iu\theta'_1 - iV$ is the covariant time derivative in the moving reference frame and $\hat{K} \equiv g(1 - r_s^2 \nabla^2)$ is the differential operator with the screening length $r_s = (4M\rho_* g)^{-1/2}$. For realistic values of the parameters, this screening length is much smaller than the GL coherence length ξ . Our starting point (1) is already coarse grained at scale ξ ; in what follows, we set $\hat{K} = g$.

We can now find the exponential factor in the QPS rate by computing the action of a suitable classical configuration. We begin with the case of strictly zero temperature, $T = 0$. The leading effect is due to a single phase-slip-antislip pair, or equivalently a vortex-antivortex pair in the (x, τ) plane. Away from the vortex cores,

$$e^{2i\theta_1} = \frac{x - x_0 + iv_+(\tau - \tau_0)}{x - x_0 - iv_-(\tau - \tau_0)} \frac{x - x'_0 - iv_-(\tau - \tau'_0)}{x - x'_0 + iv_+(\tau - \tau'_0)}, \quad (4)$$

where $v_\mp \equiv c_0 \mp u$ are the up- (down-)stream velocities. For $u = 0$ (i.e., $v_+ = v_-$), this is the configuration familiar from the studies of the planar XY model [12].

Integrating the Euclidean Lagrangian density (3) with the configuration (4) over x and τ , we obtain the corresponding classical action

$$S_E = S_1 + S_2 + S_{\text{dis}}, \quad (5)$$

where the combination of uniform linear in θ_1 terms,

$$S_1 = iP\Delta x_0 - E\Delta\tau_0, \quad (6)$$

with $\Delta x_0 = x_0 - x'_0$, $\Delta\tau_0 = \tau_0 - \tau'_0$, accounts for the Berry phase of each QPS [$P = 2\pi\rho_*$ is the momentum released by unwinding the supercurrent] and the released energy $E = 2\pi\rho_* u = 2\pi I / (2e)$,

$$S_2 = \frac{\pi c_0}{g} \ln \left[\frac{(\Delta x_0 + iv_+ \Delta\tau_0)(\Delta x_0 - iv_- \Delta\tau_0)}{r_C^2} \right] \quad (7)$$

is the plasmon-mediated interaction between the phase slips coming from the terms quadratic in θ_1 , and

$$S_{\text{dis}} = -\frac{2\pi i}{g(1 - u^2/c_0^2)} \int_{x'_0}^{x_0} dx V(x) \quad (8)$$

is the effect of the disorder. In Eq. (7), we have used the QPS core size r_C (which is not determined by the present theory) as the short-distance cutoff.

These expressions illustrate the effect of the superfluid velocity u on the QPS action and can be useful, for instance, in weakly nonideal Bose gases, where u may in principle approach c_0 . Thus, for example, factor $(1 - u^2/c_0^2)^{-1} = (v_+^{-1} + v_-^{-1})c_0/2$ in Eq. (8) arises because the up- (down-)stream plasmons spend longer (shorter) time at the place with the given density. In thin superconductors, however, we typically have $u \ll v_F \lesssim c_0$. In the following, we neglect $u/c_0 \ll 1$, to get

$$S_E = S_1 + \frac{2\pi c_0}{g} \ln \frac{\Delta r_0}{r_C} - \frac{2\pi i}{g} \int_{x'_0}^{x_0} dx V(x), \quad (9)$$

where $\Delta r_0 = (\Delta x_0^2 + c_0^2 \Delta\tau_0^2)^{1/2}$. We observe that the last term here can be interpreted as the modification of S_1 due to the local correction to the density $\delta\rho_*(x) = -V(x)/g$ caused by disorder. We will assume that $V(x)$ incorporates all mechanisms leading to linear density inhomogeneity: nonuniform wire cross section, magnetic impurities, etc. The weak-disorder approximation is applicable for

$|\delta\rho_*| \ll \rho_*$, which gives the dimensionless measure of the disorder strength,

$$\alpha \equiv V_0/Mc_0^2 = V_0/g\rho_*. \quad (10)$$

For a given disorder configuration, the QPS rate (per unit length) can be found as the imaginary part of the partition sum of the pair,

$$\mathcal{R} = \frac{C_1 c_0^4}{g^2 r_C^4} R, \quad R \equiv \text{Im} \int \frac{dx_0 dx'_0}{L_w} \int d\Delta \tau_0 e^{-S_E}, \quad (11)$$

where the dimensionful prefactor with the coefficient $C_1 \sim 1$ incorporates the fluctuation determinant and the Jacobian of transformation to the collective coordinates. The integration over x_0, x'_0 is extended over the length L_w of the wire. For large enough L_w the rate (11) is self-averaging with respect to disorder. In this case, the effective action is obtained by disorder averaging,

$$S_{\text{eff}} = S_1 + \frac{2\pi c_0}{g} \ln \frac{\Delta r_0}{r_C} + \frac{1}{2} P^2 \alpha^2 \Delta(x_0 - x'_0), \quad (12)$$

where the function

$$\Delta(x) = 2 \int_0^x dx' (x - x') f(x') \quad (13)$$

is proportional to the average square of the disorder-induced phase between the slip and antislip. [The disorder correlator $f(x)$ is defined after Eq. (1).] For finite-range disorder, $\Delta(x)$ has a diffusive form at large distances, $|x| \gg l$,

$$\Delta(x) = 2l_1 |x|, \quad l_1 \equiv \int_0^\infty dx f(x). \quad (14)$$

For single-scale disorder, $l_1 \sim l$.

According to Eqs. (12) and (14), disorder binds together the x coordinates of the phase slip and antislip within a pair. For the case of short-range disorder, this effect was noted previously in Ref. [10] and was taken into account by using a sharp δ function, $\delta(x_0 - x'_0)$, in the partition sum. We see, however, that in general there is a finite characteristic separation Δx_0 within the pair. This separation depends on both the range and the strength of the disorder potential. As the strength of disorder increases and the pairs become more tightly bound in the x direction, the wire effectively becomes more and more like a dissipative quantum-mechanical system with a logarithmic interaction, $\ln|\tau_0 - \tau'_0|$, between the instantons—a spatially extended version of the familiar resistively shunted Josephson junction [8].

We also note that the coefficient in front of the logarithm in Eq. (12) is proportional to the inverse impedance of the wire viewed as two transmission lines attached to the phase-slip region. This is consistent with the mapping onto dissipative quantum mechanics, since plasmons are the only source of dissipation in our present model.

Turning to the time integration in Eq. (11), we observe that while the integral is formally divergent, the corresponding imaginary part is finite and can be found by an

analytic continuation. We obtain

$$R = \frac{\pi^{3/2} r_C}{c_0} \int dx e^{iPx - (1/2)\alpha^2 P^2 \Delta(x)} \left(\frac{Er_C}{2c_0 |x|} \right)^\nu \frac{J_\nu(E|x|/c_0)}{\Gamma(\nu + 1/2)}, \quad (15)$$

where $J_\nu(z)$ is the Bessel function and the dimensionless index ν is inversely proportional to the interaction constant, $\nu + 1/2 \equiv \pi c_0/g$ ($= K^{-1}$ in notations of Ref. [9]). This explicit expression for the QPS rate (at $T = 0$) in the plasmon-only theory in the presence of weak but finite-range disorder is the main result of this work. Although Eq. (15) was obtained assuming that disorder induces only weak fluctuations of the density (as expressed by the weak-disorder condition $\alpha \ll 1$), it is nonperturbative with respect to the phase fluctuations and accounts for multiple scattering to all orders.

In the absence of disorder [$\alpha = 0$] the integral in Eq. (15) is zero for any $E < c_0 P$, consistent with the Galilean invariance of the $T = 0$ state [5]. The opposite case is when the convergence of the integral (15) is dominated by disorder. In this case, the Bessel function can be replaced by the first term of its small-argument expansion, and the QPS rate becomes

$$\mathcal{R} = \frac{C_2 c_0^3}{g^2 r_C^3} \left(\frac{Er_C}{2c_0} \right)^{2\nu} l_1 A_d, \quad A_d \equiv \int \frac{dx}{l_1} e^{iPx - (1/2)P^2 \alpha^2 \Delta(x)} \quad (16)$$

with $C_2 = \pi^{3/2} C_1 / [\Gamma(\nu + 1)\Gamma(\nu + 1/2)]$. This limit corresponds to the entire momentum P being absorbed by the disorder, with no momentum carried away by plasmons. For weak disorder, where Eq. (16) is applicable, the dimensionless form factor A_d is small.

At nonzero temperature, the simplest case is $T \gg gE/\pi c_0$. Then, the energy released by unwinding the supercurrent is insignificant, and instead of a single pair (4) we can use a periodic chain of such pairs with period $\beta = 1/T$. Properties of this chain are described in Ref. [5]. An especially simple result applies in the disorder-dominated regime, when the spatial separation in each pair is small, $\Delta x_0 \ll c_0/\pi T$: for an estimate, it is sufficient to make the replacement $E \rightarrow \pi c_0 T/g$ in Eq. (16). Then, at $\nu \sim 1$, the rate of thermally assisted QPSs per unit length is

$$\mathcal{R}_T \sim \frac{c_0}{r_C^3} \left(\frac{Tr_C}{g} \right)^{2\nu} l_1 A_d. \quad (17)$$

The voltage across the wire is

$$V = \frac{2\pi}{e} \mathcal{R}_T L_w \sinh \frac{\pi I}{2eT}. \quad (18)$$

We see that resistance becomes T independent at $\nu = 1/2$, which defines the SIT point. The dimensionless measure of QPS pair density near $\nu = 1/2$ is

$$\mathcal{R}_T \Delta x_0 \Delta \tau_0 \sim \frac{l_1 \Delta x_0}{r_C^2} \left(\frac{Tr_C}{g} \right)^{2\nu-1} A_d, \quad (19)$$

where $\Delta \tau_0 = 1/2T$ is the characteristic size of a pair in the τ direction. Thus, the density is small in the infrared at any

$\nu > 1/2$, and for sufficiently small A_d even at $\nu = 1/2$. At $\nu < 1/2$, pairs proliferate, resulting in the insulating behavior. Writing the impedance of the wire as $Z = 2(L/C)^{1/2}$, where $L = m_*/e^2 n_s$ is the “kinetic” inductance per unit length, and the factor of 2 corresponds to two transmission lines (one on each side of the phase slip), we see that $\nu = 1/2$ is equivalent to $Z = \pi/2e^2 = 6.5 \text{ k}\Omega$.

We now turn to discussion of the form factor A_d . The important parameter here is $\alpha^2 Pl$. We begin with the case $\alpha^2 Pl \ll 1$. If the stronger condition $\alpha Pl \ll 1$ is also satisfied, we can (upon integrating by parts twice) expand the integrand in (16) to the linear order in α^2 . This corresponds to the entire momentum P being absorbed in a single scattering event. Then, $A_d = (\alpha^2/l_1) \int e^{iPx} f(x) dx$. Thus, for $Pl \ll 1$ the form factor is universal, $A_d = 2\alpha^2$, while for $l \gtrsim P^{-1}$ it depends on details of the correlations. For $Pl \gg 1$, it is determined by the ordinate l_0 of the singularity of $f(x)$ closest to the real axis and scales exponentially, $A_d \propto e^{-Pl_0}$. If $\alpha Pl \gtrsim 1$ (but still $\alpha^2 Pl \ll 1$), effects of multiple scattering result in a correction $\sim \alpha^2 P^2 l^2$ in the exponent.

In the opposite limit, $\alpha^2 Pl \gg 1$, it is the disorder that determines convergence of the integral (16). The integral converges at $x \sim (\alpha^2 P)^{-1} \ll l$, where we can replace $\Delta(x)$ by its short-distance form, $\Delta(x) = x^2$. Then, the integral becomes Gaussian and gives $A_d = (2\pi)^{1/2} (\alpha Pl_1)^{-1} e^{-1/(2\alpha^2)}$. In this case the process of momentum transfer to disorder is clearly a result of a large number of scattering events.

The nature of the crossovers between different scattering regimes as the disorder correlation length l is increased can be understood in a model with correlation function $f(x) = (1 + x^2/l^2)^{-3/2}$. Integration in Eq. (13) gives $\Delta(x) = 2l^2[(1 + x^2/l^2)^{1/2} - 1]$, and the coordinate integration in Eq. (15) can be performed explicitly,

$$A_d = \frac{2\alpha^2 Pl}{[1 + (\alpha^2 Pl)^2]^{1/2}} e^{\alpha^2 P^2 l^2} K_1(Pl[1 + (\alpha^2 Pl)^2]^{1/2}).$$

For $Pl \ll 1$, we can use $K_1(z) = z^{-1} + \mathcal{O}(z \ln z)$, which gives the short-range limit (with some logarithmic corrections due to the power-law tail of the correlation function), while for $Pl \gg 1$, the large-argument asymptote $K_1(z) \approx (\pi/2z)^{1/2} e^{-z}$ can be used to restore the other discussed scattering regimes.

Our discussion of A_d has so far assumed that the scattering is characterized by a single distance scale, l , which may not necessarily be the case. For example, a model correlation function $f(x) = e^{-|x|/l}$ has a singularity (derivative discontinuity) at the real axis, which gives distance $l_0 = 0$, and the exact integration in Eq. (16) shows that A_d is independent of P for the entire range $\alpha Pl \ll 1$. Conversely, for a correlation function with the power-law long-distance tail, $f(x) \propto x^{-m}$, $0 < m < 1$, the distance l_1 is infinite, and instead of Eq. (14) one obtains superdiffusive form

$$\Delta(x) = \frac{l_*^2}{2-m} \left(\frac{x}{l_*}\right)^{2-m}, \quad x \gg l_*,$$

where l_* is a convenient distance scale. Then, perturbation theory breaks down already for arbitrarily weak disorder; we get $\ln(A_d) \propto -[\alpha^2 (Pl_*)^m]^{-m/(1-m)}$.

In conclusion, the plasmon-only theory provides a useful laboratory for studying the effect of disorder on QPSs from first principles. We have computed the QPS rate for general finite-range weak disorder; as seen from Eq. (15), disorder drives the system into the universality class of dissipative quantum mechanics, with the dissipative coefficient determined by the wave impedance Z of the wire. From an extension of this result to finite temperatures [Eq. (17)] we have found a superconductor-insulator transition at $Z = \pi/2e^2$. We see no reason why the universality class should change when resistive effects at the QPS core are taken into account. Rather, we expect that it will remain the same, but the dissipative coefficient will now be determined by the total impedance, including both plasmon and resistive effects.

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