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Quantum phase slips (QPS) in narrow superfluid channels generate momentum by unwinding the supercurrent. In a uniform Bose gas, this momentum needs to be absorbed by quasiparticles (phonons). We show that this requirement results in an additional exponential suppression of the QPS rate (compared to the rate of QPS induced by a sharply localized perturbation). In BCS-paired fluids, momentum can be transferred to fermionic quasiparticles, and we find an interesting interplay between quasiparticle scattering on QPS and on disorder.

Introduction.—Particle production and scattering during tunneling are of interest in many applications. They can significantly modify the transition amplitude and, in addition, destroy coherence between the states connected by tunneling. Of particular interest are situations when some scattering *must* occur, as a consequence of a non-zero momentum generated by the tunneling event.

One of the simplest such cases, and the one on which we concentrate in this Letter, is a quantum phase slip (QPS) in a narrow superfluid channel. Advances in containment and cooling of atomic Bose gases make it possible to contemplate a ring (toroidal) geometry—a channel closed in the longitudinal (x) direction. This is the case considered here, although many of our results are applicable also to a cigar-shaped geometry. In addition, since these gases are dilute, the correlation ("healing") length ξ can be made larger than the transverse size, making the system effectively one-dimensional (1D).

In a 1D system, the order parameter ψ can momentarily vanish at some point, allowing the phase of ψ to unwind. Such events are referred to as phase slips. Here we consider the case of very low temperatures when the main mechanism for phase slips is quantum, rather than thermal fluctuations. These are QPS. Their high-temperature counterpart, thermally activated phase slips, was considered, in the framework of the nucleation theory of Refs. [1–3], in Ref. [4].

The purpose of this Letter is to present a calculation of how the QPS rate is affected by interactions of QPS with quasiparticles (phonons). First, we show that unless there is a highly localized (or very strong) external perturbation, QPS without momentum transfer to phonons are strongly suppressed by destructive interference. This makes such QPS highly unlikely and makes interaction with phonons a principal effect. Next, we compute the rate of QPS assisted by phonon scattering and find that at low temperature T it is given by $\mathcal{R} \sim \exp(-v_s P/2T)$, where $P = 2\pi n$ is the momentum generated by unwinding the phase; n is the linear number density of the gas, v_s is the speed of Bogoliubov's phonons. We also estimate the

preexponent. The physical interpretation of this result is that the destructive interference is removed when momentum P is absorbed by the phonon subsystem, but that requires energy of at least $v_s P/2$ (to allow for tunneling between phonon states with total momenta $-P/2$ and $P/2$).

The above estimate for the rate is the main result of the present Letter. It applies to a uniform weakly-coupled Bose gas at temperatures $T \ll gn$ and represents a strong suppression, compared to the case when QPS is induced by a highly localized perturbation, and momentum conservation plays no role. We briefly discuss means to experimentally detect phonon scattering on QPS in atomic superfluids. Finally, we compare this case to the case of BCS-paired superconductors.

QPS without momentum transfer to phonons.—By unwinding the supercurrent, each QPS releases momentum $\pm P$. (In the ring-shaped geometry, a better conserved quantity would be the angular momentum, but this distinction will not be important in what follows.) This corresponds to an imaginary contribution to the Euclidean action of a QPS:

$$\Delta S_{\text{QPS}} = -2\pi i \int_0^{x_0} n(x)dx \approx -2\pi i \bar{n} x_0, \qquad (1)$$

where x_0 is the QPS location, and $\bar{n}$ is an average density. Then, the tunneling amplitude is

$$\mathcal{A} = \int dx_0 \mathcal{B}(x_0) e^{2\pi i \bar{n} x_0}. \qquad (2)$$

If the scale of variation of $\mathcal{B}(x_0)$ (due to possible variations of density, etc.) is some large ℓ, the amplitude (2) is suppressed by $\exp(-\text{const}\bar{n}\ell)$. For $\ell \gg \xi$, this suppression is much stronger than the semiclassical $\exp(-\text{const}v_s/g)$. It can be seen as a consequence of the destructive interference among QPS with different values of x_0.

One way to avoid this suppression is to consider a localized perturbation with scale $\ell \ll \xi$. Calculations for this case have been done in Ref. [5]. Another possibility, which we consider here, is that the momentum

released by a QPS is absorbed by phonons. We expect that if phonons change their total momentum by $2\pi\bar{n}$, the recoil on the QPS will be zero, and the suppression by $\exp(-\text{const}\bar{n}\ell)$ will be lifted. For sufficiently uniform gases, such phonon-assisted tunneling may be the dominant QPS mechanism. Furthermore, the two mechanisms can be distinguished experimentally by measuring the momentum transfer (see below).

Phonon production at $T = 0$. —In this case, the phonon subsystem is initially in the ground state, so to transfer momentum to phonons we need to produce them in the final state. To set up come notation, we briefly discuss this case and show why, if the only source of energy available for phonon production is the energy due to unwinding of the supercurrent, such processes are kinematically forbidden. The formalism sketched here may be useful when there are other sources of energy, as for instance in various "cosmology in the lab" proposals [6], where the properties of the superfluid are changed in time to mimic certain cosmological spacetimes.

We consider a weakly-coupled 1D Bose gas. In 1D, the effective coupling constant g has dimension of velocity, and the dimensionless measure of the coupling strength is

$$\frac{g}{v_s} = 0.008\left(\frac{a}{10 \text{ nm}}\right)^{1/2}\left(\frac{10^{14} \text{ cm}^{-3}}{n^{(3)}}\right)^{1/2}\left(\frac{1 \text{ }\mu\text{m}}{R}\right)^2, \quad (3)$$

where v_s in the phonon speed, a is the scattering length, $n^{(3)}$ is the 3D number density, and R is the transverse radius of the channel. As we will see, the real part of the instanton action is of order $\pi v_s/g$ (times a logarithm).

The crossover between the weakly and strongly coupled regimes can be located using the results of Haldane [7]: it occurs at $g/v_s \approx \pi$. In what follows we assume a somewhat weaker coupling $g/v_s \sim 1$, which according to (3) can be achieved for $R \sim 0.1$ μm. Note that such small transverse radii may be necessary in any case, just to drive the system into the 1D regime. Indeed, the 1D regime requires $R \ll \xi$, while the typical values of ξ are a few tenths of micron.

So, if it were not for the destructive interference, an instanton action of order $S_{\text{QPS}} \sim \pi v_s/g$ would allow for a relatively large QPS rate in narrow channels.

Since the system is in the weakly-coupled regime, we can obtain a more precise estimate for S_{QPS} by using the Gross-Pitaevsky description. This is given by the Euclidean Lagrangian

$$L_E = \psi^\dagger \partial_\tau \psi + \frac{1}{2M}|\partial_x\psi|^2 + \frac{g}{2}|\psi|^4 - \mu|\psi|^2, \quad (4)$$

where $\tau = it$ is the Euclidean time. Outside the QPS core, fluctuations of density are small, and we can linearize the system by writing $\psi = \sqrt{n + \delta n}\exp(i\theta)$ and expanding in small δn. Integrating out δn and assuming the long-wavelength limit, we obtain

$$L_E \approx in\partial_\tau\theta + \frac{1}{2g}(\partial_\tau\theta)^2 + \frac{n}{2M}(\partial_x\theta)^2. \quad (5)$$

From this, we read off the sound velocity $v_s = \sqrt{gn/M}$. In what follows, we assume that n is x-independent and choose our unit of length so that $v_s = 1$.

Since (5) is a long-wavelength limit, it implies a certain choice of the reference frame: momenta of produced phonons should be small in comparison with $1/\xi$. Thus, the Galilean invariance of the original theory (4) now holds only approximately and only for boost speeds much smaller than $(M\xi)^{-1} \sim v_s$. This approximate symmetry, however, would be completely sufficient to restore the topological (first) term in Eq. (5).

Instantons corresponding to QPS are vortices of the theory (4) in the (x, τ) plane. Using the limit (5), one finds that away from the core the phase θ of the instanton solution is

$$\theta_I(x - x_0, \tau - \tau_0) = \arg[x - x_0 + i(\tau - \tau_0)], \quad (6)$$

where (x_0, τ_0) is the location of the QPS.

Consider an instanton−anti-instanton (IA) pair in the presence of a persistent supercurrent. The corresponding classical configuration is

$$\theta_c(x, \tau) = \theta_I(x - x_0, \tau - \tau_0) - \theta_I(x, \tau) + \frac{2\pi}{L}Nx \quad (7)$$

(here we have placed the anti-instanton at the origin). The last term corresponds to a supercurrent with winding number N over the longitudinal length L of the system. Substituting (7) into (5) and integrating, we obtain the Euclidean action of the pair to logarithmic accuracy

$$S_{\text{pair}} = -2\pi i n x_0 - E\tau_0 + \frac{2\pi}{g}\ln\frac{d}{\xi} + O\left(\frac{1}{g}\right). \quad (8)$$

where $d = (x_0^2 + \tau_0^2)^{1/2}$, and $\xi = (4gMn)^{-1/2}$ is the healing length (the size of the vortex core). Equation (8) applies when the separation d is much smaller than the longitudinal size L of the channel. Contribution from the QPS core is subleading and included in the $O(1/g)$ term.

The first term in (8) is the imaginary part anticipated in (1). The second term, with

$$E = E_N \equiv \frac{(2\pi)^2 N}{gL}, \quad (9)$$

comes from the cross term between the instanton and the supercurrent contributions to $\partial_x\theta$. Note that E_N is precisely the energy released by a single phase slip.

It is easy to see, however, that this energy alone is not sufficient to satisfy the kinematic condition for production of phonons with total momentum $P = 2\pi n$. Indeed, the minimal (critical) energy required for that is

$$E_{\text{cr}} = 2\pi n = \frac{\pi}{g\xi}. \quad (10)$$

Landau's criterion for superfluidity is $N/L < 1/4\pi\xi$, so in the superfluid state $E_N < E_{cr}$ ($E_N = E_{cr}$ precisely at the critical current). We conclude that at zero temperature QPS in a uniform gas are effectively impossible, unless one introduces additional sources of energy. This conclusion is consistent with Galilean invariance of the $T = 0$ state. One way to break Galilean invariance is to include thermal fluctuations.

Phonon scattering at $T \neq 0$.—In this case, there are preexisting phonons in the system, and we consider a QPS assisted by scattering of phonons, so that the momentum released by the QPS is absorbed by the phonon subsystem. We take the limit when the energy E_N is negligible compared to the energy of the requisite phonon state. At $T \neq 0$, the system can be regarded as compactified on a cylinder of circumference $\beta = 1/T$. Equivalently, instead of a single IA pair we should consider a periodic chain of such pairs:

$$\theta_c = \arg[1 - e^{-i(\tau-\tau_0)-(x-x_0)}] - \arg(1 - e^{-i\tau-x}). \quad (11)$$

Here, instantons are shifted relative to anti-instantons by amount x_0 in the x-direction, and by τ_0 in the τ-direction. These are "soft" collective coordinates, which will need to be integrated over. All lengths in (11) are in units of $\beta/2\pi$, and we assume that the total longitudinal length L is effectively infinite.

The action per period, to logarithmic accuracy, equals

$$S_{chain} = -iPx_0 + \frac{2\pi}{g}\ln\frac{1}{\xi} + \frac{\pi}{g}\ln(\cosh x_0 - \cos\tau_0). \quad (12)$$

The last term is the instanton–anti-instanton interaction. If it were not for the momentum P, the dominant configuration would have $x_0 = 0$ and coincide with the periodic instanton [8], which describes tunneling between quasiparticle states with zero total momenta. At nonzero P, configurations with $x_0 \neq 0$ become important.

The integrals over x_0 and $t_0 = -i\tau_0$ can be done by steepest descent. In the limit $\beta \gg \xi$ (that is, $T \ll gn$), the saddle point is at $\tau_0 = \pi$ and $x_0 \approx i\pi$. The saddle-point action is dominated by the first term in (12): $S_{chain} \approx P/2T$ (where we have returned to the physical units of length). Corrections to this result are controlled by the small parameter $T\xi$.

The above action determines the exponential factor in the QPS rate. The preexponent is given by the determinant of field fluctuations near the saddle-point. As usual, the main effect is due to the collective coordinates. Two zero modes, corresponding to translations in space and time, each contribute a factor of order $1/\xi\sqrt{g}$, up to a power of $\ln(\beta/\xi)$. The saddle-point integrals over x_0 and τ_0 result each in a factor of order $(k_*\xi)^{-1/2}$, where k_* is the typical momentum of phonons scattered by the QPS. Other modes result in a factor of order unity. Assembling all the factors together, we obtain, for the QPS rate per length L, in the limit $\beta \gg \xi$,

$$\mathcal{R} \sim \frac{v_s^2 L}{g\xi^3 k_*} e^{-v_s P/2T} \quad (13)$$

(where we have restored v_s). A detailed study of the initial and final phonon states (to be presented elsewhere) gives $k_*^3 \sim (\xi^2\beta)^{-1}\ln(\beta/\xi)$, in the same limit. So, $k_* \ll \xi^{-1}$, which confirms that the longwave limit (5) is applicable to the present problem, with corrections controlled by $k_*\xi$.

The exponential in (13) can be interpreted by noting that at a finite temperature the quasiparticle system can tunnel from a state with momentum $-P/2$ to a state with momentum $P/2$, thus absorbing the full momentum P but requiring energy of only $P/2$. The main physical effect reflected in (13) is that transfer of a large momentum requires tunneling from a fairly high-energy state, resulting in an additional suppression. If no momentum transfer were necessary, the leading term in the exponent would be $(2\pi/g)\ln(T\xi)$ [8] (see also Ref. [5]), so the suppression would be much weaker.

Phonon scattering by QPS in atomic superfluids can in principle be detected experimentally by measuring the momentum distribution of atoms, using, say, the standard momentum imaging. In the ring-shaped geometry, this can be done by opening the trap in one place and looking at the atomic cloud emitted from there. For example, if we start with the state without supercurrent, a single QPS will create a unit of supercurrent and a compensating normal flow in the opposite direction. Since the "normal" atoms have comparatively large momenta, the cloud will be highly asymmetric.

Quasiparticle production in superconductors.—The case of BCS superconductors is especially interesting because nonzero resistance has been observed in ultrathin wires at low temperatures [9,10], and it has been interpreted in terms of QPS [11].

Curiously, the "hydrodynamic" part of the effective Lagrangian for superconductors is given by the same expression (4) as for atomic superfluids, except that ψ is now the field of Cooper pairs, which describes charge fluctuations in the wire (cf. Ref. [12]). Accordingly, the coupling constant is $g = 4e^2/C$, where C is the capacitance per unit length. The reason for this similarity is that in 1D screening is weak, and the plasmon mode is gapless (the Mooij-Schön mode [13]). From (4), we obtain the speed of plasmons as $c_0 = (e^2 n_s/m_* C)^{1/2}$, where $n_s \equiv 4\langle\psi^\dagger\psi\rangle m_*/M$, and m_* is the electron mass. This is indeed the speed of the Mooij-Schön mode.

For BCS superconductors, there is an additional Lagrangian describing the interaction of ψ with fermionic quasiparticles. Indeed, production of these quasiparticles is the least energy-consuming way of momentum production, since each quasiparticle carries a large momentum k_F but costs only a relatively small energy equal to the superconducting gap Δ. This mechanism is unavailable to atomic superfluids.

On general grounds, one expects that impurities, which break both the Galilean invariance and momentum conservation, may significantly affect the physics of QPS [14]. Here we present an explicit example of such interplay, using the simple, if somewhat artificial, single-channel limit, in which electrons are confined to motion in the x direction only, and their mean-free path λ is taken to be much larger than the QPS core size.

The Fermi surface now consists of two points, at $k = \pm k_F$, so there are right-movers, denoted by subscript $(+)$, and left-movers, with subscript $(-)$. Each variety can have up or down spin, corresponding to $\sigma = \uparrow, \downarrow$. We define new fermionic operators $a_{\sigma\pm}$ related to the original electron operators $c_{\sigma\pm}$ by

$$c_{\sigma\pm}(x, t) = e^{\pm ik_F x} a_{\sigma\pm}(x, t). \qquad (14)$$

The new operators can be conveniently assembled into two Dirac spinors: $\Psi_\uparrow = (a_{\uparrow+}, a_{\uparrow-})$, $\Psi_\downarrow = (a_{\downarrow+}^\dagger, a_{\downarrow-}^\dagger)$.

In the case without disorder, $\lambda \to \infty$, the Lagrangian of fermions can be written in the relativistic form:

$$L_F = i \sum_{\sigma=\uparrow,\downarrow} \overline{\Psi}_\sigma \gamma^\mu \partial_\mu \Psi_\sigma - g'\psi \overline{\Psi}_\uparrow \Psi_\downarrow - g'\psi^* \overline{\Psi}_\downarrow \Psi_\uparrow. \quad (15)$$

Here $\mu = 0, 1$, the $(1 + 1)$ γ-matrices are $\gamma^0 = \sigma_1$, $\gamma^1 = -i\sigma_2$, and $\overline{\Psi} = \Psi^\dagger \gamma_0$. In this section, we set $v_F = 1$.

For any ψ with a nontrivial winding at infinity, the Euclidean equation of motion for Ψ has a nontrivial normalizable solution (zero mode). The zero mode is known explicitly for ψ of the form $\psi(r, \vartheta) = f(r) \times \exp(i\vartheta)$ (in polar coordinates) [15], and its existence in the general case is guaranteed by an index theorem [16]. The zero mode causes the amplitude for tunneling without fermions to vanish identically. As has been explained by 't Hooft [17] in the context of gauge theories in four dimensions, this means that only certain anomalous Green functions, involving fermions, are nonzero. In our case, the relevant Green function is $\langle a_{\uparrow-}^\dagger(x) a_{\uparrow+}(y) \rangle$. It corresponds to creation of a quasiparticle-quasihole pair at the QPS core. The total momentum produced in this process is $2k_F$. The phenomenon is a 1D analog of "momentogenesis" [18] by vortex motion in a 2D array of Josephson junctions.

Note that $2k_F$ is precisely the momentum necessary to cancel the momentum $-P$ released by unwinding of the current. Indeed, $P = 2\pi \langle \psi^\dagger \psi \rangle = \pi n$, where n is the electron density. Since $n = 2k_F/\pi$, we have $P = 2k_F$. If the spatial dependence of the zero mode for $a_{\uparrow+}$ is $\chi(x - x_0)$, then for the original operator $c_{\uparrow+}$ it is $\exp[ik_F(x - x_0)]\chi(x - x_0)$. In this way, we find

$$\langle c_{\uparrow-}^\dagger(\mathbf{x}) c_{\uparrow+}(\mathbf{y}) \rangle \sim \int dx_0 d\tau_0 e^{-S'} e^{ik_F(x+y)}$$
$$\times \chi(\mathbf{x} - \mathbf{x}_0)\chi(\mathbf{y} - \mathbf{x}_0), \qquad (16)$$

where $\mathbf{x} = (x, \tau)$, and S' is the real part of the instanton action.

At $T = 0$ and in the absence of disorder, production of a quasiparticle-quasihole pair via the matrix element (16) is kinematically forbidden in the same way as phonon production was in the case of an atomic superfluid (Landau's criterion now being that the velocity of the superflow must be smaller than the depairing velocity Δ/k_F). However, scattering on an impurity potential $W(x)$ introduces an additional Lagrangian

$$\Delta L_F = W(x) \sum_\sigma c_{\sigma-}^\dagger c_{\sigma+} + \text{H.c.} \qquad (17)$$

Computing to the first order in W and absorbing the fermion operators into the anomalous correlator (16), we find that in the presence of disorder the amplitude of QPS without quasiparticle production becomes nonzero.

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