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Exploiting Skewness To Build An Optimal Hedge Fund With A Currency Overlay.

by

C J Adcock
The University of Sheffield, UK

Abstract

This paper documents an investigation into the use of portfolio selection methods to construct a hedge fund with a currency overlay. The fund, which is based on number of international stock and bond market indices and is constructed from the perspective of a Sterling investor, allows the individual exposures in the currency overlay to be optimally determined. As well as using traditional mean variance, the paper constructs the hedge funds using portfolio selection methods that incorporate skewness in the optimisation process. These methods are based on the multivariate skewnormal distribution, which motivates the use of a linear skewness shock. An extension to Stein's lemma gives the ability to explore the mean-variance-skewness efficient surface without the necessity to be concerned with the precise form of an individual investor's utility function. The results suggest that it is possible to use mean variance optimisation methods to build a hedge fund based on the assets and return forecasts described. The results also suggest that the inclusion of a skewness component in the optimisation is beneficial. In many of the cases reported, the skewness term contributes to an improvement in performance over and above that given by mean variance methods.

Keywords: Currency hedging, multivariate skew normal distribution, portfolio selection,

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Address for correspondence

C J Adcock
School of Management
University of Sheffield
9, Mappin Street
Sheffield, S1 4DT
UK

Tel: +44 (0)114 222 3402
Fax: +44 (0)114 222 3458
Email: c.j.adcock@shef.ac.uk

1. Introduction

It is well known that it is possible in principle to use currency hedging techniques either to seek to increase the return of a portfolio of international securities, or to control its risk. For international investors, currency hedging using the futures market to reduce the effect of exposure to foreign exchange risk has obvious attractions. The use of hedging methods in conjunction with currency futures also offers the possibility of extra return. Accordingly, the implementation and management of international portfolios that use currency overlays is an activity of importance to many investors and their managers.

Numerous studies of currency hedging have been reported in the literature. Examples of some of the investigations into the benefits of hedging include papers by Eun and Resnick(1988) Glen and Jorion(1993) and De Roon et al(2001a, b). Eun and Resnick(1988) explain that a hedged portfolio should exhibit lower variance than its unhedged counterpart . This is not as strong a result as the "free lunch" argument suggested in Pérold and Schulmann(1988) in which hedging causes a reduction in volatility without a commensurate reduction in expected return However, both findings are consequences of the correlation structure of international assets, which is considered to be changing. In a recent study, Adcock(2003) suggests that a currency overlay on a portfolio of international stock and bond indices still has the potential to add expected return. There is an increase in volatility compared to an unhedged portfolio based on the same inputs and design parameters, but this is not large enough to neutralise the beneficial effect of the increases in expected return. Furthermore, the results of the same study suggest that strategies in which the currency exposures are optimally determined are to be preferred to those in which a fixed hedge ratio is set externally to the portfolio selection process. In common with most studies in this area that use formal methods for portfolio selection, the results reported in Adcock use mean variance optimisation methods. Non-normality in the multivariate probability distribution of asset returns is ignored.

The aim of this work reported in this paper is twofold. First, the paper documents an investigation into the use of mean variance portfolio selection methods to construct a hedge fund with a currency overlay. The fund is based on number of international stock and bond market indices and is constructed from the perspective of a Sterling Investor. It includes returns on several major currencies as an asset class and allows the individual exposures in the currency overlay to be optimally determined. Secondly, the paper constructs the hedge funds using portfolio selection methods that incorporate skewness in the optimisation process. A priori, there are good reasons to consider the inclusion of skewness. Since Samuelson(1970), skewness has been an issue in both asset pricing and portfolio selection – see for example Arditti and Levy(1975) and Harvey and Siddique(2000). Numerous authors report the presence of skewness in returns and discuss the implications. Examples are papers by Singleton and Wingender(1986), Chunhachinda et al(1997) and Sun and Yan(2003). From a theoretical point of view, the process of transforming returns in local market terms to a different base currency induces skewness even in the situation when the underlying asset returns in local market terms are normally distributed.

There is an increasing number of papers in the literature that seek to estimate skewness by using non-standard models for the probability distribution of asset returns. Examples include several papers by McDonald and his co-workers, McDonald and Newey(1988), McDonald and Nelson(1989), McDonald and Xu(1995), and more recently by Theodossiou(1998) and Harris et al(2004). However, the models reported in these papers are univariate. The increasingly well known multivariate skew normal, henceforth MSN, distribution, which was introduced by Azzalini and Dalla Valle(1997) and first used in finance by Adcock and Shutes(2001) provides a coherent starting point for portfolio selection. The MSN model has stimulated development of other multivariate distributions which incorporate skewness. Sahu et al(2003) and Azzalini and Capitanio(2003) are recent examples of papers that use the multivariate skew student distribution. Harvey et al(2002) are the first to apply the MSN model to portfolio selection. There are numerous papers in the literature that add a skewness term to the MV optimisation objective function. Use of the MSN model means that the approach to the inclusion of skewness taken in this paper is different from that usually followed. In addition, as shown in section 3, use of the MSN distribution motivates a different approach to the measurement of skewness. The approach requires N estimators for a universe of N assets. This is far more parsimonious than approaches based on co-skewnesses, for which the number of estimators needed is of the order of N cubed. As is shown, this model motivates the use of estimators based on sample skewness, as well as those produced by the MSN distribution.

The structure of this paper is as follows. Section 2 contains a summary of the MSN model and the properties that are needed in the rest of the paper. Section 3 describes portfolio selection based on the MSN model. This section is based in part on the negative exponential utility function. Under multivariate skew normality, this utility function exhibits some shortcomings, which may limit its effectiveness for portfolio selection in some situations. However, it is possible to avoid these shortcomings by an appeal to an extension to Stein's lemma, which is appropriate for the MSN distribution. A proof of the lemma, which is believed to be new, is in the appendix. In keeping with increasingly common practice, this has been omitted to save space, but it available on request. In section 4, there is a presentation of the efficient set mathematics for currency hedged portfolios. Section 5 describes the data, models and forecasts for an empirical study. Its results are reported in section 6. Results are presented in summarised form, but further details are available on request. Section 7 of the paper contains conclusions and an indication of future research in this area. All the computations reported were carried out in S-Plus.

2. The Multivariate Skew Normal Distribution

The multivariate skew normal or MSN distribution was introduced by Azzalini and Dalla Valle(1996). It is an extension of the univariate skew normal distribution which was originally due to Roberts(1966) and, separately, O'Hagan and Leonard(1976) and which was developed in articles by Azzalini(1985, 1986). The standard form is obtained by considering the distribution of a random vector, **R** say, which is defined as:

$$\mathbf{R} = \mathbf{Y} + l U$$

The vector **Y** has a full rank multivariate normal distribution with mean vector m and variance-covariance matrix S. The scalar variable U, which is independent of **Y**, has a standard normal distribution that is truncated below at zero. The vector l is a vector of skewness parameters, which may take any real values. For applications in finance, a modification of this distribution is employed, as reported in Adcock and Shutes(2001), henceforth A&S. The vectors **R**, **Y** and l are defined as above. The scalar variable U has a normal distribution with mean τ and variance 1 truncated below at zero. This modification generates a richer family of probability distributions. In particular, it gives more flexibility in modelling skewness and kurtosis. The idea of adding a skewness shock to a multivariate normally distributed vector of asset returns is not new. It is suggested in Simaan(1993), which predates A&S.

The probability distribution of **R** is MSN with parameters m, Σ, l and τ, denoted as:

$$\mathbf{R} \sim \mathrm{MSN}(m, \Sigma, l, \tau)$$

The probability density function of this distribution is:

$$\mathrm{f}_{\mathrm{R}}(\mathbf{r}) = \mathrm{n}(\mathbf{r}; \boldsymbol{\mu} + \boldsymbol{\lambda} t, \boldsymbol{\Sigma} + \boldsymbol{\lambda}\boldsymbol{\lambda}^{\mathrm{T}}) \frac{\Phi\left(\dfrac{t + \boldsymbol{\lambda}^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}(\mathbf{r} - \boldsymbol{\mu})}{\sqrt{1 + \boldsymbol{\lambda}^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}\boldsymbol{\lambda}}}\right)}{\Phi(t)};$$

where Φ(x) is the standard normal distribution function evaluated at x. The notation n(**x;** w, **W**) denotes the probability density function, evaluated at **x**, of a multivariate normal distribution with mean vector w and variance covariance matrix **W**. This density function is essentially Azzalini and Dalla Valle's(1996) result with a change of notation and generalization to accommodate a non-zero value of τ. The distribution of any sub-vector of **r**, including the scalar variable r_i, is of the same form, based upon the corresponding sub-vectors of m and l and sub-matrix of S.

As reported in A&S, the moment generating function of this distribution is:

$$\mathrm{M}_{\mathrm{R}}(\mathbf{t}) = \mathrm{E}[\mathrm{e}^{\mathbf{t}^{\mathrm{T}}\mathrm{R}}] = \exp\left[\mathbf{t}^{\mathrm{T}}(\boldsymbol{\mu} + \boldsymbol{\lambda} t) + \frac{1}{2}\mathbf{t}^{\mathrm{T}}(\boldsymbol{\Sigma} + \boldsymbol{\lambda}\boldsymbol{\lambda}^{\mathrm{T}})\mathbf{t}\right] \frac{\Phi(\boldsymbol{\lambda}^{\mathrm{T}}\mathbf{t} + t)}{\Phi(t)};$$

The first two (multivariate) moments are given by:

$$E[\mathbf{R}] = \boldsymbol{\mu} + \boldsymbol{\lambda}\{t + x_1(t)\} = \boldsymbol{\gamma}, say$$

$$V[\mathbf{R}] = \Sigma + \boldsymbol{\lambda}\boldsymbol{\lambda}^{T}\{1 + x_2(t)\} = \boldsymbol{\Omega}, say \tag{1.}$$

The function ξ_k() is defined as:

$$x_k(x) = \frac{\partial^k ln\Phi(x)}{\partial x^k}, k = 1,2,...$$

Two properties of this function are required below. These are:

$$1+x_2(t)\geq 0, x_2(t)\leq 0 \qquad (2.)$$

The co-skewness of three variables, i, j and k say, and their co-kurtosis[1] with variable l are, respectively:

$$S[r_i,r_j,r_k] = l_i\, l_j\, l_k x_3(t),\; K[r_i,r_j,r_k,r_l] = l_i\, l_j\, l_k\, l_l x_4(t)$$

Skewness and kurtosis of a single variable are therefore, respectively:

$$\mathrm{SK}[\mathrm{r_i}] = l_\mathrm{i}^3 x_3(t), \mathrm{KU}[\mathrm{r_i}] = l_\mathrm{i}^4 x_4(t)$$

The parameters of the distribution may be estimated by the method of maximum likelihood.

3. Portfolio Selection

The negative exponential function is a natural choice of utility function to use in conjunction with the MSN distribution. Ignoring constants, the utility of R_p, the return on a portfolio, is:

$$\mathrm{U}(\mathrm{R_p}) = -\exp\left(-\frac{\mathrm{R_p}}{\theta}\right), \theta > 0$$

Since portfolio return is given by:

$$R_p = \sum_{i=1}^{n} w_i R_i = w^T R,$$

the expected value of the utility function follows from the moment generating function:

$$E[U(R_p)] = -M\left(-\frac{\mathbf{w}}{q}\right) = -\exp\left(-\frac{\mathbf{w}^T(\boldsymbol{\mu}+\boldsymbol{\lambda})}{q} + \frac{\mathbf{w}^T(\boldsymbol{\Sigma}+\boldsymbol{\lambda}\boldsymbol{\lambda}^T)\mathbf{w}}{2q^2}\right)\frac{\Phi\left(t-\frac{\mathbf{w}^T\boldsymbol{\lambda}}{q}\right)}{\Phi(t)}$$

After re-arrangement, the first order conditions, henceforth FOCs, for maximisation of the expected utility subject to the budget constraint are

$$\left(\boldsymbol{\Sigma}+\boldsymbol{\lambda}\boldsymbol{\lambda}^\mathrm{T}\right)\mathbf{w} = \theta\left\{\boldsymbol{\mu}+\boldsymbol{\lambda}\tau-\boldsymbol{\lambda}\zeta_1\left(\tau-\frac{\mathbf{w}^\mathrm{T}\boldsymbol{\lambda}}{\theta}\right)\right\}-\eta\mathbf{1} \qquad (3.)$$

where **1** denotes a vector of ones and η is the Lagrange multiplier of the budget constraint. The expected value of the utility function may be written as:

[1] This is defined as the 4th cumulant of the distribution.

$$\mathrm{E[U(R_p)]} = -\exp\left(-\frac{m_p}{q} + \frac{s_p^2}{2q^2}\right)\mathrm{P}(d_p) \tag{4.}$$

where μ_p and σ_p^2 are, respectively, the portfolio expected return and variance. The parameter δ_p is a linear measure of portfolio skewness which is defined as:

$$\delta_p = \mathbf{w}^T l \tag{5.}$$

The function P() is:

$$\mathrm{P}(d_p) = \exp\left(d_p x_1(t) - \frac{1}{2} d_p^2 x_2(t).\right)\frac{\Phi\left(t - \frac{d_p}{q}\right)}{\Phi(t)} \tag{6.}$$

Using the expression for expected utility at equation (4.), the FOCs may be re-written in terms of the overall VC matrix of returns, W, and the vector of expected returns, g, which are defined at equation (1.). The FOCs become:

$$\mathbf{\Omega w} = q\gamma + \boldsymbol{\lambda}\left\{\mathbf{w}^T \boldsymbol{\lambda} x_2(t) + q x_1\left(t - \frac{w^T \boldsymbol{\lambda}}{q}\right) - q x_1(t)\right\} - h\mathbf{1} = g(w), say \tag{7.}$$

This equation may be solved iteratively by computing the sequence of vectors $\{\mathbf{w}_k\}$ given by $\mathbf{w}_k = g(\mathbf{w}_{k-1})$. The procedure may be followed in the presence of inequality constraints and implemented using a quadratic programming algorithm.

Equation (7.) indicates that there are some limitations of the negative exponential utility function. First, when the risk appetite θ increases without limit:

$$lt_{q\to\infty}\left(q\gamma + \lambda\left\{\mathbf{w}^T \boldsymbol{\lambda} x_2(t) + q x_1\left(t - \frac{w^T \boldsymbol{\lambda}}{q}\right) - q x_1(t)\right\}\right) = \gamma q$$

The right hand side of the FOCs becomes θg - η**1** and the portfolio is a point on the mean variance efficient frontier. Thus, the MSN model adds nothing for aggressive investors who use the negative exponential utility function. Secondly, when risk appetite θ tends to zero, the behaviour of the terms inside the parentheses above depends on the sign of portfolio skewness. When portfolio skewness is positive, as the risk appetite θ tends to zero:

$$lt_{q\to 0}\left(q\gamma + \lambda\left\{\mathbf{w}^T \boldsymbol{\lambda} x_2(t) + q x_1\left(t - \frac{w^T \boldsymbol{\lambda}}{q}\right) - q x_1(t)\right\}\right) = \mathbf{w}^T \boldsymbol{\lambda}\{1 + x_2(t)\}$$

Therefore, the portfolio is located on a mean variance efficient frontier for which the vector of expected values is l. Using equation (2.), the degree of risk appetite is the positive quantity $\mathbf{w}^T\boldsymbol{\lambda}\{1+x_2(t)\}$. Using (2.) again, when $\mathbf{w}^T 1$ is negative, the portfolio is located on the same efficient frontier at a point with risk appetite $\mathbf{w}^T\boldsymbol{\lambda}x_2(t)$. A feature of the negative exponential utility function when used in conjunction with the MSN distribution is that the minimum risk portfolio has a expected return and variance which are both greater than the corresponding values that arise from the MV minimum variance portfolio.

The properties summarised above may be interpreted as limitations of the negative exponential utility function. However, it is possible to overcome them by appealing to an extension of Stein's lemma, Stein(1981), for the MSN distribution. The implication of this lemma, which is believed to be a new result, is that for all well behaved utility functions which are differentiable twice and for which various expected values exist, the first order conditions are always expressible in the form:

$$\boldsymbol{\Omega}\mathbf{w} = q_1\boldsymbol{\gamma} + q_2\boldsymbol{\lambda} - h\mathbf{1} \tag{8.}$$

where the parameters θ_1 and θ_2 denote risk appetite or preference for, respectively, expected return and skewness. This portfolio selection problem may be solved by quadratic programming. The resultant portfolios are located on a frontier for which the VC matrix is W and the vector of expected returns is of the form:

$$\boldsymbol{\mu} + \frac{q_2}{q_1}\boldsymbol{\lambda}$$

As already noted in the introduction, a proof of this result is outlined in the separately available appendix. The key implication of this result is that the preferences for expected return and skewness may be set by the investor. Another feature of the MSN model is that portfolio skewness, δ_p defined at equation (5.), is a linear measure.

4. The MSN Efficient Surface and Efficient Set Mathematics

When the negative exponential utility function is used, the risk parameter θ_2 associated with skewness is a non-linear function of **w**, the vector of weights. In the form which the extension of Stein's lemma facilitates, the preference for skewness may be fixed. In either case, the expected value, variance and linear skewness measure, δ_p, of the portfolio may be computed. As shown in Adcock and Shutes(2001), the resulting equations may be manipulated to give an efficient surface.

If the mean, variance and linear skewness are denoted by E, V and S, the equation of the efficient surface is:

$$\mathrm{E} = \alpha_0 + \frac{\beta_0}{\beta_1}(\mathrm{S}-\beta_2) + \sqrt{\left\{\alpha_1 - \frac{\beta_0^2}{\beta_1}\right\}\left(\mathrm{V} - \alpha_2 - \frac{1}{\beta_1}(\mathrm{S}-\beta_2)^2\right)}$$

where the scalars $\alpha_{0,1,2}$ and $\beta_{0,1,2}$ are defined in terms of g, 1 and W as:

$$\alpha_0 = \frac{\boldsymbol{\mu}^{\mathrm{T}}\boldsymbol{\Omega}^{-1}\mathbf{1}}{\mathbf{1}^{\mathrm{T}}\boldsymbol{\Omega}^{-1}\mathbf{1}}$$

$$\alpha_1 = \boldsymbol{\mu}^{\mathrm{T}}\left\{\boldsymbol{\Omega}^{-1} - \frac{1}{\mathbf{1}^{\mathrm{T}}\boldsymbol{\Omega}^{-1}\mathbf{1}}\boldsymbol{\Omega}^{-1}\mathbf{1}\mathbf{1}^{\mathrm{T}}\boldsymbol{\Omega}^{-1}\right\}\boldsymbol{\mu}$$

$$\alpha_2 = \frac{1}{\mathbf{1}^{\mathrm{T}}\boldsymbol{\Omega}^{-1}\mathbf{1}}$$

$$\beta_0 = \boldsymbol{\lambda}^{\mathrm{T}}\left\{\boldsymbol{\Omega}^{-1} - \frac{1}{\mathbf{1}^{\mathrm{T}}\boldsymbol{\Omega}^{-1}\mathbf{1}}\boldsymbol{\Omega}^{-1}\mathbf{1}\mathbf{1}^{\mathrm{T}}\boldsymbol{\Omega}^{-1}\right\}\boldsymbol{\mu}$$

$$\beta_1 = \boldsymbol{\lambda}^{\mathrm{T}}\left\{\boldsymbol{\Omega}^{-1} - \frac{1}{\mathbf{1}^{\mathrm{T}}\boldsymbol{\Omega}^{-1}\mathbf{1}}\boldsymbol{\Omega}^{-1}\mathbf{1}\mathbf{1}^{\mathrm{T}}\boldsymbol{\Omega}^{-1}\right\}\boldsymbol{\lambda}$$

$$\beta_2 = \frac{\boldsymbol{\lambda}^{\mathrm{T}}\boldsymbol{\Omega}^{-1}\mathbf{1}}{\mathbf{1}^{\mathrm{T}}\boldsymbol{\Omega}^{-1}\mathbf{1}}$$

It should be noted that this surface is appropriate for any utility function used with the MSN model. It also holds for the more general case where the expected value of the utility function is increasing in expected value, decreasing in variance and where skewness is measured by a quantity of the form $\mathbf{w}^{\mathrm{T}} l$. This corresponds to the situation where the skewness in asset returns is generated by a single unobserved shock and the elements of the vector l measure the sensitivity of each stock to the shock. Although this is a restricted model of skewness, it does have the advantage of parsimony. As noted in the introduction, there is one skewness parameter to estimate for each security. Under the MSN model, skewness itself is proportional to $(\mathbf{w}^{\mathrm{T}} l)^3$. This suggests that a linear measure of skewness should be estimated by the cube root of the usual sample skewness statistic. The use of such an estimator corresponds to model in which it is assumed that the unobserved skewness shock is positive with a mean equal to unity.

Figure 1 shows a sketch of the mean-volatility-skewness efficient surface. This uses the ML estimators for the parameters of the MSN distribution based on 50 months of data for the period ending February 1999.

Figure 1 about here

To develop the criteria required for portfolio selection with a currency overlay, consider a universe of N_R real assets and N_C currencies. Normally N_C is less than N_R, but for ease of exposition it is assumed in this section that they are the same and both equal to N. Using the above notation with the addition of an subscript i to denote asset i, the return on a hedged portfolio is:

$$R_P = \sum_{i=1}^{N} w_{R,i} R_{R,i} + \sum_{i=1}^{N} w_{C,i} R_{C,i}$$

where $\{w_{R,i}\}$ are the weights on the real assets and $\{w_{C,i}\}$ are the weights on the currencies. The $\{R_{R,i}\}$are unhedged returns and the $\{R_{C,i}\}$ are the returns from the currency positions. Strictly speaking, this expression is non-linear in the underlying assets through its functional dependence on the product terms $R_{L,i}R_{SC,I}$, where R_L denotes the return on a real asset in local market terms. However, the usual approximation, in which the non-linearity is ignored, is followed. The 2N by 2N variance-covariance matrix is W, which is written in the standard partitioned form:

$$\mathbf{\Omega} = \begin{bmatrix} \mathbf{\Omega}_{RR} & \mathbf{\Omega}_{RC} \\ \mathbf{\Omega}_{CR} & \mathbf{\Omega}_{CC} \end{bmatrix}$$

To derive the efficient set portfolio, expected utility maximisation is subject only to the budget constraint on the weights $\{w_{R,i}\}$. See De Roon et al(2000a) for example when there is currency hedging. The FOCs give the following normal equations:

$$\begin{bmatrix} \mathbf{\Omega}_{RR} & \mathbf{\Omega}_{RC} \\ \mathbf{\Omega}_{CR} & \mathbf{\Omega}_{CC} \end{bmatrix} \begin{bmatrix} \mathbf{w}_R \\ \mathbf{w}_c \end{bmatrix} = q_1 \begin{bmatrix} \boldsymbol{\gamma}_R \\ \boldsymbol{\gamma}_C \end{bmatrix} + 2q_2 \begin{bmatrix} \boldsymbol{\lambda}_1 \\ \boldsymbol{\lambda}_2 \end{bmatrix} - \begin{bmatrix} h\mathbf{1} \\ \mathbf{0} \end{bmatrix}$$

The solution for the vector of weights for the real assets is given by the equation:

$$\mathbf{\Omega}_{R|C}\mathbf{w}_R = q_1 \boldsymbol{\gamma}_{R|C} + q_2 \boldsymbol{\lambda}_{R|C} - h\mathbf{1}$$

where:

$$\mathbf{\Omega}_{R|C} = \mathbf{\Omega}_{RR} - \mathbf{\Omega}_{RC}\mathbf{\Omega}_{CC}^{-1}\mathbf{\Omega}_{CR}$$

$$\boldsymbol{\gamma}_{R|C} = \boldsymbol{\gamma}_R - \mathbf{\Omega}_{RC}\mathbf{\Omega}_{CC}^{-1}\boldsymbol{\gamma}_C$$

$$\boldsymbol{\lambda}_{R|C} = \boldsymbol{\lambda}_R - \mathbf{\Omega}_{RC}\mathbf{\Omega}_{CC}^{-1}\boldsymbol{\lambda}_{\mathbf{C}}$$

This shows that there are situations in which investor preferences for expected return or skewness will have no effect. For mean variance portfolio selection, the contribution of the currency returns to the weights of the real assets may be tested using the F test reported by Gibbons Ross and Shanken(1989), henceforth the GRS test. This gives a test for the contribution of real returns for the weights of the currency assets. De Roon et al(2001a) solve the system of equations slightly differently and use $W_{C|R}$ instead of $W_{R|C}$. For portfolio selection under the MSN model, the development of a test is a subject that is beyond the scope of this paper. However, in section 6 the GRS test is reported as a diagnostic aid.

5. Data and Models

The empirical study described in section 6 of this paper is based on stock and bond indices for 15 markets, as listed in table 1 below. The weights shown in the third column of table 1 are used to construct a benchmark portfolio for the 30 assets.

For each of these markets, monthly returns in local currency terms are provided for 294 months ending July 1999. Also available for the same period are monthly returns on the spot exchange rate between the Pound Sterling and the currency of each overseas market. For those countries which are now in the Euro, the Euro is used throughout and it itself proxied by the Deutschemark for time periods prior to its inception. The returns data is used to compute the return in Sterling terms on each of the overseas assets. This data is used to compute one month ahead forecasts of the VC matrix of returns.

One month ahead forecasts of returns on the 37 assets have been provided by an asset manager, starting in March 1978. The return forecasts, which have been based on econometric models, are presented in Sterling terms and the forecasts for returns on the foreign exchange rate are computed from the perspective of a Sterling investor.

One-month ahead estimates of skewness are estimated using a rolling window of 50 prior months. Estimation is done in two ways. First, the vector of skewness parameters is estimated using the multivariate skew normal distribution. Secondly, the usual sample estimate is computed and the cube root taken. For each month from March 1979 to July 1999, the latest estimates of skewness from both procedures together with the one month ahead forecasts of expected returns and VC matrices form the input to a portfolio selection exercise based on the FOCs described in section 3. For the 12 month period from March 1978 to February 1979, for which forecasts of expected returns and the VC matrix only are available, skewness is set to zero.

Four portfolio selection strategies are considered, as follows:

Strategy 1: MV optimisation is performed using only forecasts of expected return and the one period ahead VC matrices. This is done for comparison purposes.

Strategy 2. This uses a fixed preference for skewness in conjunction with the sample estimates of skewness, equation (8.)

Strategy 3. This is based on the MSN model, but appeals to the extension to Stein's lemma and uses a fixed preference for skewness, equation (8.)

Strategy 4. This is based on the MSN model with the negative exponential utility function and results in a preference for skewness that is non-linear and time varying, equation (7.)

The optimisation approach used is the standard back-testing method. Each month, the forecast returns, VC matrix and skewness estimates are used as input to the portfolio selection process. The weights are used with asset returns for the following month to compute ex-post portfolio returns. A time series of portfolio returns and related performance statistics, such as turnover, are accumulated for analysis. Portfolio simulations are carried out using a combination of parameter values.

As the aim is to build a hedge fund with a currency overlay , two overall portfolio designs are investigated. The first is to construct portfolios at discrete points on the mean-variance-skewness efficient surface. There is no benchmark, but the weights for the 30 real assets are constrained to lie in the range [-0.2, 0.2]. This is ensure a modicum of diversification in the real assets. The weights for the currency overlay are not-constrained per se. However, no naked currency positions are allowed and the absolute value of a currency exposure is required not to exceed the absolute value of the sum of the weights of real assets denominated in the currency in question. Thus, if the exposure to US$ denominated stocks and bonds is 10%, the exposure to the US$ in the currency overlay is required to be within ±10%.

The second portfolio design is to optimise the holdings of the real assets relative to a benchmark portfolio. For this study, the benchmark has fixed weights, based on a conventional 60:40 split between stocks and bonds with weights for countries in four size groups. The weightings are shown in the third column of table 1. It should be noted that a fixed benchmark automatically generates a certain amount or turnover each month. There is no benchmark for currencies, equivalent in terms of methodology to benchmark weights being set to zero. Over the whole period covered by this study, the return on the benchmark portfolio was 12.5% per annum in Sterling terms.

Table 1 about here

For the currencies, the design parameters are as above. The real assets are required to lie within ±200% of the corresponding benchmark value. Thus for an asset with a benchmark weight of 0.05, the permitted range is [-0.05, 0.15]. Transactions costs are set to 20 basis points for all assets.

After some experimentation, the levels of risk appetite for expected return used in the study ranged from 0.001 to 0.1. When fixed skewness preference was used, this was set to be 10% of the corresponding preference for expected return. The results that are described in the next section focus on strategies 1 through 3. The strategy based on the negative exponential utility function and time varying preference for skewness was found to produce returns that were generally inferior to those generated by the two methods that use fixed skewness preference and often also generated higher turnover.

6. Results

An initial analysis of the statistical properties of the returns on the 37 assets indicates that there are substantial departures from normality. A brief summary of the analysis is shown in tables 2 and 3. As noted in the introduction, further details are available on request.

Table 2 shows a summary of the Bera-Jarque test for normality. This was applied as a diagnostic test to the 294 monthly returns on all 37 assets. The skewness and kurtosis components of the test were also computed for each asset. The table shows an analysis of skewness and kurtosis at the 5% level of significance. As the table shows, there are 18 stocks which are skewed. Of this number, 16 are both skewed and kurtotic and 2 are skewed only.

Tables 2 and 3 about here

Table 3 shows the Bera-Jarque test and its components for the benchmark portfolio. The table shows that the benchmark possesses both skewness and kurtosis.

To demonstrate that the currencies have the potential to add value, the Gibbons-Ross-Shanken test is shown in table 4. Both versions of the test are shown and indicate that this set of forecasts will generate portfolios in which the weights for real assets and currencies will vary with risk.

Table 4 about here

The MSN model was estimated each month from February 1979 using a rolling window of 50 months observations. The model was assessed using a standard likelihood ratio test. The null hypothesis was that returns have a multivariate normal distribution. To summarise the values of the likelihood ratio test: there are 74 months out of 245 where a likelihood ratio test does not achieve significance at the 5% level. Of the other 171 months, the level of significance achieved is 1% or smaller in 128 cases.

Table 5 about here

Table5 shows the results of the absolute optimisations. The table has three panels: (a) shows the results of the mean variance optimisations, (b) shows the results for optimisation using sample estimates of linear skewness and (c) the results based on the estimates from the MSN model. The results in the table show that all three approaches generate portfolios for which the ex-post returns increase with risk, this demonstrating that the forecasts used do indeed contain some signal concerning future expected returns. All three sets possess some common characteristics. The return-to-risk ratio declines as risk appetite increases. The return-to-risk ratios of the three strategies are comparable at each level of risk. The average hedge ratios are comparable at low and medium levels of risk. Furthermore, all three sets exhibit the characteristic that the hedge ratio increases with risk appetite. At low risk, depending on strategy, it is between –84% and –93%. For strategies (a) and (b) the hedge ratio increases with risk and is between 5 and 10% at the highest level of risk appetite considered. For strategy (c), however, the hedge ratio increases to a maximum of 65% at the highest level of risk.

In terms of performance, the two strategies that use skewness outperform mean variance optimisation at all levels of risk. This is because of a combination of higher achieved gross returns and lower portfolio turnover. It is interesting to note that strategy (b), based on sample estimates, does better than strategy (c), based on the MSN model, at low levels of risk, but that this position is reversed at higher levels of risk appetite.

Table 6 about here

Table 6 shows the corresponding results for the three strategies when the optimisation is performed relative to the benchmark portfolio, but uses the same levels of risk appetite. The table shows excess returns. For these strategies, the performance differences are far smaller. At low levels of risk appetite, all three strategies fail to beat the benchmark after the deduction of costs, or beat it by small amounts. At medium to high risk appetite, annualised excess returns of between 1.5 and 3.8% are achievable as shown in the table. In contrasts with the results of the absolute optimisation shown in table 4, the strategy that uses the MSN estimates of skewness does not do significantly better than mean variance optimisation. However, use of the sample skewness estimates does appear to give a significant advantage at medium to high risk.

Table 7 about here

The performances of all three models for absolute and relative optimisation are summarised in table 7. The net returns from table 5 are recast as excess returns over the benchmark. In general, it is not intended to compare the merits of absolute versus relative optimisation. This is because it is clear from the turnover percentages that the two sets of strategies are very different. However, one comment is appropriate. The hedge ratios for all strategies optimised relative to the benchmark are positive at all levels of risk. The hedge ratios are most aggressive for the portfolios based on the MSN skewness estimates.

Although the details are not reported, the effect of the design parameters given in the previous section has been to ensure that all portfolios remain well diversified. In general, very few zero holdings in the real assets were experienced at any time period. The number of currency holdings tended to be more volatile: an average of 5 in most simulations, but with as few as two holdings in some months. All sets of simulations exploited the ability to take short positions in the stock and bond indices as well as the currencies.

Figures 2 and 3 about here

Figures 2 and 3 give a summary of the temporal performance characteristics of the simulated portfolios. The graph in figure 2 is based on a medium risk portfolio, risk appetite equal to 0.01, constructed using absolute optimisation with skewness estimates from the MSN model. The excess returns over the benchmark have been indexed. As the graph shows, the excess performance was consistent for long periods of time. However, it should be noted that performance in the late 1990s was not so encouraging and that there was a period of poor performance in the late 1980s. Figure 3 shows rolling twelve-month average excess returns for a portfolio with the same risk and skewness estimates, but constructed using relative optimisation. This also exhibits under-performance in the late 1980s but excess returns in the late 1990s were positive.

Finally, figure 4 shows the computed skewness for the benchmark and for a low risk portfolio constructed using relative optimisation. Skewness was computed each month using the portfolio weights obtained at the start of the month in conjunction with the estimates of skewness computed at the end of the month. As the chart shows, skewness clearly changes. The evidence from the figure also suggests that there are trends in skewness, which are not permanent and which ultimately reverse. It is not the purpose of this paper to investigate the dynamics of skewness, but this evidence may be interpreted as being supportive to some extent of the ideas of Singleton and Wingender(1986) who suggest that skewness is not persistent, ie it does change.

Figure 4 about here

Conclusions

The results reported in this paper suggest that it is possible to use mean variance optimisation methods to build a hedge fund based on the assets and return forecasts described. The results also suggest that the inclusion of a skewness component in the optimisation is beneficial. In most of the cases reported, the skewness term contributes to an improvement in performance over and above that given by mean variance methods. This applies when skewness is modelled by the multivariate skew normal distribution. It also applies when it is based on suitable sample estimates of the linear skewness measure defined in section 3 of the paper.

The empirical study has been aided by two aspects of the methods reported in sections 2 and 3. First, the MSN model motivates the use of the linear skewness shock. This offers an alternative to co-skewness which is both parsimonious and easy to compute. Secondly, the extension to Stein's lemma gives the ability to explore the mean-variance-skewness efficient surface without the necessity to be concerned with the precise form of an individual investor's utility function.

However, as all users of optimisation are well aware, these results are inevitably conditional on the various forecasts and estimates used. The insights offered by the results reported in section 6, should not detract from the requirements for good forecasts. As well as being dependent on the forecasts, they also reflect the correlation structure of international asset returns, which is believed to be dynamic.

The results of the simulations also offer some insights into the dynamics of skewness. This is an interesting topic for future study in its own right. As indicated in section 4, there is a need for an extension of the Gibbons Ross Shanken test. Other potential extensions of this work include incorporation of kurtosis by using the multivariate skew student distribution and by extending the model to handle more than one skewness shock. The motivation for the former is well understood. The motivation for the latter is the hypothesis that different asset classes and markets are likely in reality to be susceptible to different shocks.

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Table 1 – Markets and Currencies

Country	Currency	Weighting
Australia	A$	3.5
Austria	Euro	3.5
Belgium	Euro	3.5
Canada	C$	3.5
Denmark	Euro	3.5
France	Euro	10
Germany	Euro	10
Italy	Euro	3.5
Japan	Yen	10
Netherlands	Euro	3.5
Norway	Norwegian Kroner	3.5
Sweden	Euro	3.5
Switzerland	Swiss Franc	3.5
United Kingdom	Pound Sterling	20
United States	US$	15

Table 2 - Summary of the Components of the Bera Jarque Test

Based on 294 monthly returns from February 1975 to July 1999. Table shows number of assets in each category which are significant at the 5% level.

	Not-kurtotic	Kurtotic	Total
Not-skewed	9	10	19
Skewed	2	16	18
Total	11	26	37

Note
Each component of the Bera Jarque test is computed separately and compared with critical values of Chi-squared distribution with one degree of freedom.

Table 3 – Summary of Skewness and Kurtosis of the Benchmark

Based on 294 monthly returns from February 1975 to July 1999.

Skewness component of BJ test	4.5307
P-value	0.0333
Kurtosis component of BJ test	34.5716
P-value	0.0000
Bera-Jarque test	39.1023
P-value	0.0000

Note
Each component of the Bera Jarque test is computed separately and compared with critical values of Chi-squared distribution with one degree of freedom.

Table 4 – Gibbons-Ross-Shanken Test

Based on 294 monthly returns from February 1975 to July 1999.

	F.stat	P.val	Dof.n	Dof.d
Cur|real	24.8137	0	7	257
Real|cur	10.8696	0	30	257

Note

Gibbons Ross Shanken test computed to test effect of currency returns on real asset weights and vice versa.

Legend

Cur|real	Tests for effect of real assets on currencies.
Real/cur	Tests for effect of currencies on real assets.
P.val	P value computed to 4 decimal places.
Dof.n/d	Degrees of freedom for numerator/denominator.

Table 5 – Summary of Returns and Related Statistics for Hedge Funds Based On Absolute Optimisation

Based on 257 forecasts from March 1978 to July 1999 and ex post computation of returns and related statistics. Transactions costs computed at 20bp.

	Total return		Turnover	Volatility	Hedge ratio	Return-risk ratio	
Risk	Gross	Net				Gross	Net
(a) Mean Variance Portfolio							
0.0000	8.10	7.53	21.99	3.06	-0.93	2.65	2.46
0.0010	8.61	7.32	50.21	3.15	-0.86	2.74	2.32
0.0025	9.14	6.83	89.54	3.45	-0.79	2.65	1.98
0.0050	10.22	6.78	132.73	4.13	-0.71	2.47	1.64
0.0100	12.31	7.59	180.22	5.67	-0.59	2.17	1.34
0.0250	16.59	9.85	250.58	9.41	-0.34	1.76	1.05
0.0500	21.14	13.04	292.07	13.34	-0.14	1.58	0.98
0.1000	27.14	17.88	320.41	19.47	0.06	1.39	0.92
(b) Sample skewness with fixed risk preference							
0.0010	9.09	7.55	59.52	3.31	-0.86	2.75	2.28
0.0025	10.35	7.86	96.04	4.01	-0.78	2.58	1.96
0.0050	12.01	8.67	127.06	5.30	-0.71	2.27	1.64
0.0100	15.05	10.71	161.71	7.37	-0.60	2.04	1.45
0.0250	20.28	14.76	198.21	10.80	-0.39	1.88	1.37
0.0500	24.74	18.44	219.60	14.44	-0.15	1.71	1.28
0.1000	28.96	21.89	239.56	17.21	0.10	1.68	1.27
(c) MSN model skewness with fixed risk preference							
0.0010	8.92	7.51	54.60	3.30	-0.84	2.70	2.28
0.0025	10.06	7.47	99.84	4.09	-0.75	2.46	1.83
0.0050	11.88	8.09	144.50	5.64	-0.63	2.10	1.43
0.0100	14.83	9.76	189.82	8.95	-0.43	1.66	1.09
0.0250	22.30	15.71	234.12	15.69	0.00	1.42	1.00
0.0500	28.36	21.47	234.17	21.82	0.41	1.30	0.98
0.1000	32.22	25.05	237.41	25.51	0.65	1.26	0.98

Legend – Under table 7

Table 6 – Summary of Excess Returns and Related Statistics for Hedge Funds Based On Optimisation Relative To The Benchmark Portfolio

Based on 257 forecasts from March 1978 to July 1999 and ex post computation of returns and related statistics. Transactions costs computed at 20bp.

	Total return		Turnover	Volatility	Hedge ratio	Return-risk ratio	
Risk	Gross	Net				Gross	Net
(a) Mean variance portfolio							
0.0010	0.32	-0.72	43.15	0.91	0.08	0.35	-0.79
0.0025	0.78	-0.69	61.52	1.53	0.12	0.51	-0.45
0.0050	1.39	-0.43	75.37	2.23	0.16	0.62	-0.19
0.0100	2.29	0.11	89.95	3.27	0.19	0.70	0.03
0.0250	4.48	1.72	111.64	5.25	0.18	0.85	0.33
0.0500	5.62	2.57	122.50	7.31	0.19	0.77	0.35
0.1000	5.92	2.61	132.72	8.55	0.20	0.69	0.31
(b) Sample skewness with fixed risk preference							
0.0010	0.57	-0.49	44.15	1.20	0.07	0.47	-0.41
0.0025	1.18	-0.22	57.97	1.95	0.11	0.61	-0.11
0.0050	2.01	0.35	68.67	2.72	0.13	0.74	0.13
0.0100	3.08	1.10	81.03	3.75	0.13	0.82	0.29
0.0250	4.82	2.42	96.89	5.44	0.11	0.89	0.44
0.0500	5.83	3.21	104.87	6.41	0.12	0.91	0.50
0.1000	6.60	3.77	112.38	7.10	0.14	0.93	0.53
(c) MSN model skewness with fixed risk preference							
0.0010	0.50	-0.57	44.44	1.18	0.10	0.43	-0.48
0.0025	1.39	-0.10	61.65	2.26	0.17	0.61	-0.05
0.0050	2.44	0.62	74.59	3.53	0.23	0.69	0.18
0.0100	3.56	1.50	83.84	5.67	0.30	0.63	0.26
0.0250	4.28	2.04	90.78	9.30	0.42	0.46	0.22
0.0500	4.57	2.12	98.86	10.34	0.50	0.44	0.21
0.1000	5.33	2.67	106.86	10.91	0.50	0.49	0.24

Legend – under table 7

Table 7 – Comparative Analysis of Portfolio Performance

Based on 257 forecasts from March 1978 to July 1999 and ex post computation of returns and related statistics. Transactions costs computed at 20bp.

Risk	Net excess	Turnover	Net excess	Turnover	Net excess	Turnover
	Mean variance		Sample skewness		MSN skewness	
Relative to benchmark						
0.0010	-0.72	43.15	-0.49	44.15	-0.57	44.44
0.0025	-0.69	61.52	-0.22	57.97	-0.10	61.65
0.0050	-0.43	75.37	0.35	68.67	0.62	74.59
0.0100	0.11	89.95	1.10	81.03	1.50	83.84
0.0250	1.72	111.64	2.42	96.89	2.04	90.78
0.0500	2.57	122.50	3.21	104.87	2.12	98.86
0.1000	2.61	132.72	3.77	112.38	2.67	106.86
Absolute optimisation measured relative to benchmark						
0.0000	-4.47	21.99				
0.0010	-4.66	50.21	-4.46	59.52	-4.49	54.60
0.0025	-5.10	89.54	-4.19	96.04	-4.53	99.84
0.0050	-5.14	132.73	-3.46	127.06	-3.98	144.50
0.0100	-4.42	180.22	-1.65	161.71	-2.50	189.82
0.0250	-2.42	250.58	1.95	198.21	2.79	234.12
0.0500	0.42	292.07	5.21	219.60	7.91	234.17
0.1000	4.72	320.41	8.28	239.56	11.08	237.41

Legend

Gross	Annualised monthly returns, shown as a percentage.
Net	Annualised net monthly returns after deduction of transactions costs, shown as a percentage.
Turnover	Monthly turnover shown as a percentage, computed as sum of absolute weights changes.
Volatility	Volatility of returns or excess returns, shown as annual percentage.
Hedge ratio	Sum of weights in currencies. NB sum of weights in real assets is always one.
Return risk ratio	Units of gross and net return per unit of volatility, expressed as percentage.

Note: **Tables 6 and 7 show excess returns. Table 5 shows total returns**

Figure 1 – Sketch of the Mean –Variance-Skewness Efficient Surface

Based on the universe of 37 assets used in the paper and maximum likelihood estimation of the parameters of the MSN distribution using 50 months of data ending February 1999.

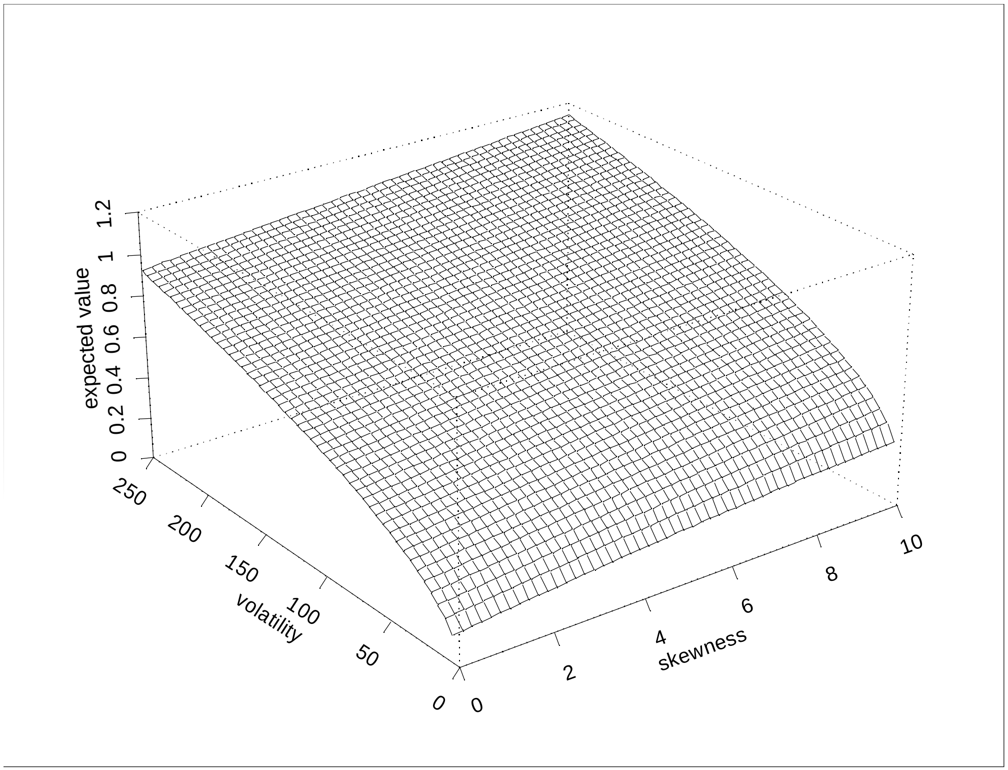

Figure 2 – Indexed Excess Returns

The figure shows indexed monthly excess returns of a medium risk portfolio from March 1978 to July 1999 based on absolute optimisation, constructed with a fixed preference for risk using skewness estimates from the MSN model.

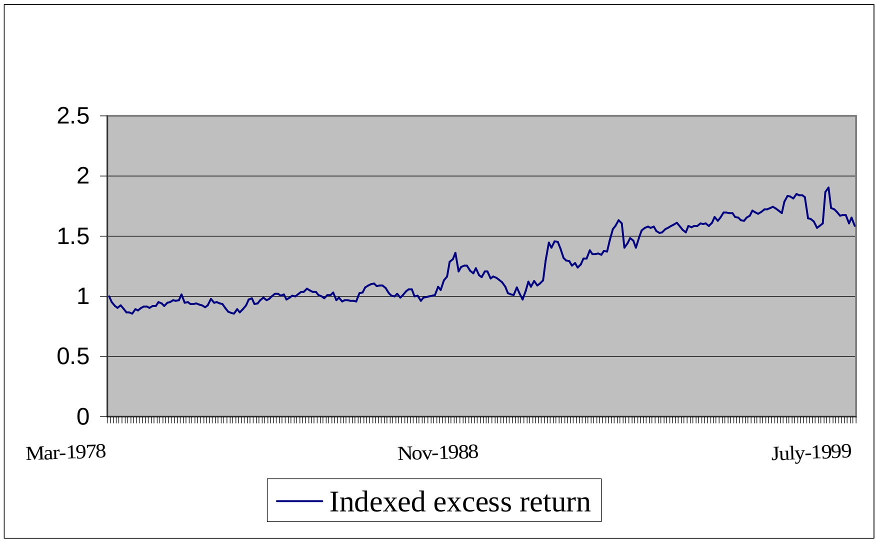

Figure 3 – 12 Months Rolling Average Returns of A Portfolio Based On Optimisation Relative to a Benchmark

The figure shows12 months rolling average excess returns of a medium risk portfolio from March 1978 to July 1999 based on relative optimisation, constructed with a fixed preference for risk using skewness estimates from the MSN model.

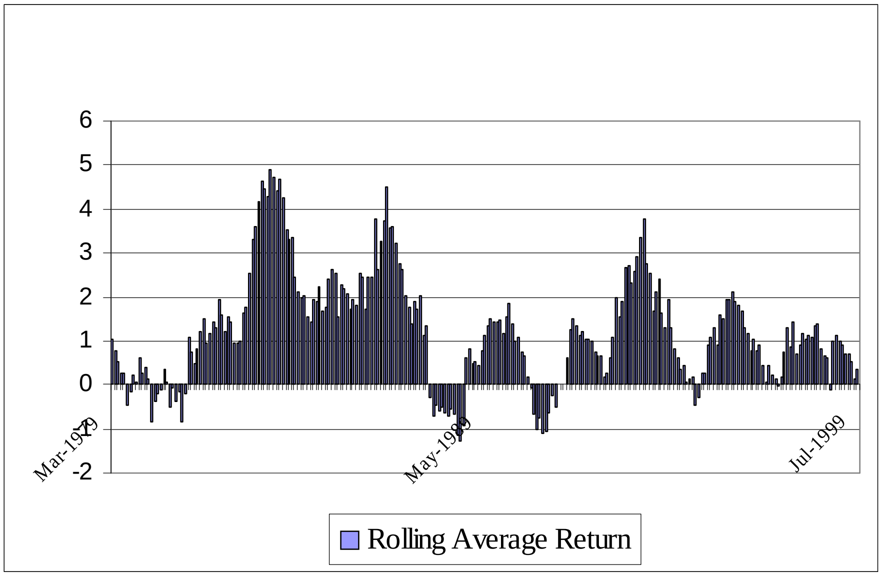

Figure 4 – Evidence of Skewness in the Benchmark and Portfolio Returns

The figure shows12 estimated skewness of the benchmark and a low risk portfolio from March 1978 to July 1999. The skewness is computed using the portfolio weights and the estimated skewness parameters from the MSN model. The portfolio is based on optimisation relative to the benchmark, constructed with a fixed preference for risk using skewness estimates from the MSN model.