Line Drawing Interpretation in a Multi-View Context (Supplementary Material)

Jean-Dominique FAVREAU Florent LAFARGE Adrien BOUSSEAU
INRIA Sophia-Antipolis, France
firstname.lastname@inria.fr

Details on the computation of $\mathcal{M}(l)$

We detail here the resolution of the quadratic minimization problem under linear constraints formulated in Section 4.3. We want to minimize the function ϵ

$$\epsilon = \sum_{i \in \mathcal{V}} \|RP_i - SP_i p_i\|^2 \tag{1}$$

under the following linear constraints

$$(\forall i \in \mathcal{F}) \sum_{j \in \mathcal{E}} c_{ij} \lambda_j v_j = 0 \tag{2}$$

Each vertex P_i is defined in homogeneous coordinates using

$$P_i = P_0 + \sum_{j \in \mathcal{E}} \delta_{ij} \lambda_j v_j \tag{3}$$

where P_0 is the reference vertex expressed in the camera frame as

$$P_0 = \begin{pmatrix} \alpha\widehat{n} + \beta\widehat{u} + \gamma\widehat{v} \\ 0 \end{pmatrix} + \begin{pmatrix} 0_3 \\ 1 \end{pmatrix} \tag{4}$$

where $\widehat{n}$ is the camera direction. P_0 is rewritten in a more compact form as

$$P_0 = \omega + \sum_{k=1}^{2} \eta_k \widehat{n}_k \quad \text{with} \begin{cases} \eta_1 \widehat{n}_1 = \begin{pmatrix} \beta\widehat{u} \\ 0 \end{pmatrix} \\ \eta_2 \widehat{n}_2 = \begin{pmatrix} \gamma\widehat{v} \\ 0 \end{pmatrix} \\ \omega = \begin{pmatrix} \alpha\widehat{n} \\ 1 \end{pmatrix} \end{cases} \tag{5}$$

By first replacing the expression of P_0 from Eq. 5 into Eq. 3, and then replacing the expression of P_i into Eq. 1, we can formulate ϵ as

$$\epsilon = \sum_{i \in \mathcal{V}} \|R\omega - S\omega p_i + \sum_{k=1}^{2} \eta_k (R\widehat{n}_k - S\widehat{n}_k p_i) + \sum_{j \in \mathcal{E}} \delta_{ij} \lambda_j (Rv_j - Sv_j p_i)\|^2 \tag{6}$$

This can also be formulated as

$$\epsilon = \lambda^T A_0 \lambda + \lambda^T B_2 \eta + \eta^T J \eta + K^T \eta + C + B_1^T \lambda \tag{7}$$

where A_0, B_1, B_2, J, K and C are matrices defined by

$$\left\{\begin{array}{l}(\forall\,(j,k)\in [\![1,|\mathcal{E}|]\!]^2)\;A_0^{jk}=\sum_{i\in\mathcal{V}}<\delta_{ij}(Rv_j-Sv_jp_i),\;\delta_{ik}(Rv_k-Sv_kp_i)>\\(\forall\,(j,k)\in [\![1,|\mathcal{E}|]\!]\times[\![1,2]\!])\;B_2^{jk}=\sum_{i\in\mathcal{V}}2\delta_{ij}<Rv_j-Sv_jp_i,R\widehat{n}_k-S\widehat{n}_kp_i>\\(\forall\,(j,k)\in [\![1,2]\!]^2)\;J_{jk}=\sum_{i\in\mathcal{V}}<R\widehat{n}_j-S\widehat{n}_jp_i,R\widehat{n}_k-S\widehat{n}_kp_i>\\(\forall\,j\in [\![1,2]\!])\;K_j=\sum_{i\in\mathcal{V}}2<R\widehat{n}_j-S\widehat{n}_jp_i,R\omega-S\omega p_i>\\C=\sum_{i\in\mathcal{V}}<R\omega-S\omega p_i,R\omega-S\omega p_i>\\(\forall\,j\in [\![1,|\mathcal{E}|]\!])\;B_1^{j}=\sum_{i\in\mathcal{V}}2\delta_{ij}<Rv_j-Sv_jp_i,R\omega-S\omega p_i>\end{array}\right. \tag{8}$$

Eq. 7 can be rewritten as

$$\epsilon=\begin{pmatrix}\lambda^T & \eta^T\end{pmatrix}\begin{pmatrix}A_0 & B_3\\ B_3^T & J\end{pmatrix}\begin{pmatrix}\lambda\\ \eta\end{pmatrix}+\begin{pmatrix}B_1^T & K^T\end{pmatrix}\begin{pmatrix}\lambda\\ \eta\end{pmatrix}+C \tag{9}$$

where $B_3=\frac{1}{2}B_2$. We have indeed $\lambda^TB_2\eta=\lambda^TB_3\eta+\eta^TB_3^T\lambda$.

This is also equivalent to

$$\epsilon=X^TAX+BX+C \tag{10}$$

where X, A and B are matrices give by

$$X=\begin{pmatrix}\lambda\\ \eta\end{pmatrix},A=\begin{pmatrix}A_0 & B_3\\ B_3^T & J\end{pmatrix},B=\begin{pmatrix}B_1^T & K^T\end{pmatrix} \tag{11}$$

The equation (12) is the constraints defined in (2) rewritting in matrix mode using (13) and (14).

$$DX=0 \tag{12}$$

$$D=\begin{pmatrix}D_1 & 0\\ D_2 & 0\\ D_3 & 0\end{pmatrix} \tag{13}$$

$$\left\{\begin{array}{l}(\forall\,(i,j)\in [\![1,|\mathcal{F}|]\!]\times[\![1,|\mathcal{E}|]\!])\;D_1^{ij}=c_{ij}v_j^x\\(\forall\,(i,j)\in [\![1,|\mathcal{F}|]\!]\times[\![1,|\mathcal{E}|]\!])\;D_2^{ij}=c_{ij}v_j^y\\(\forall\,(i,j)\in [\![1,|\mathcal{F}|]\!]\times[\![1,|\mathcal{E}|]\!])\;D_3^{ij}=c_{ij}v_j^z\end{array}\right. \tag{14}$$

The new formulation of the problem is to find $\widehat{X}$ such that the equation (15) is respected.

$$\widehat{X}=\underset{X\in Ker(D)}{\operatorname{argmin}}\,(X^TAX+BX+C) \tag{15}$$

We solve Eq. (15) by Lagrange multipliers.