

COMPUTERIZED SLOPE STABILITY ANALYSIS; THE SLIDING BLOCK

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BY

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JHRP

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INDIANA STATE HIGHWAY COMMISSION

Technical Paper

COMPUTERIZED SLOPE STABILITY ANALYSIS; THE SLIDING BLOCK

TO: J. F. McLaughlin, Director December 28, 1972
Joint Highway Research Project

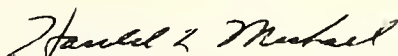
FROM: H. L. Michael, Associate Director Project: C-36-36J
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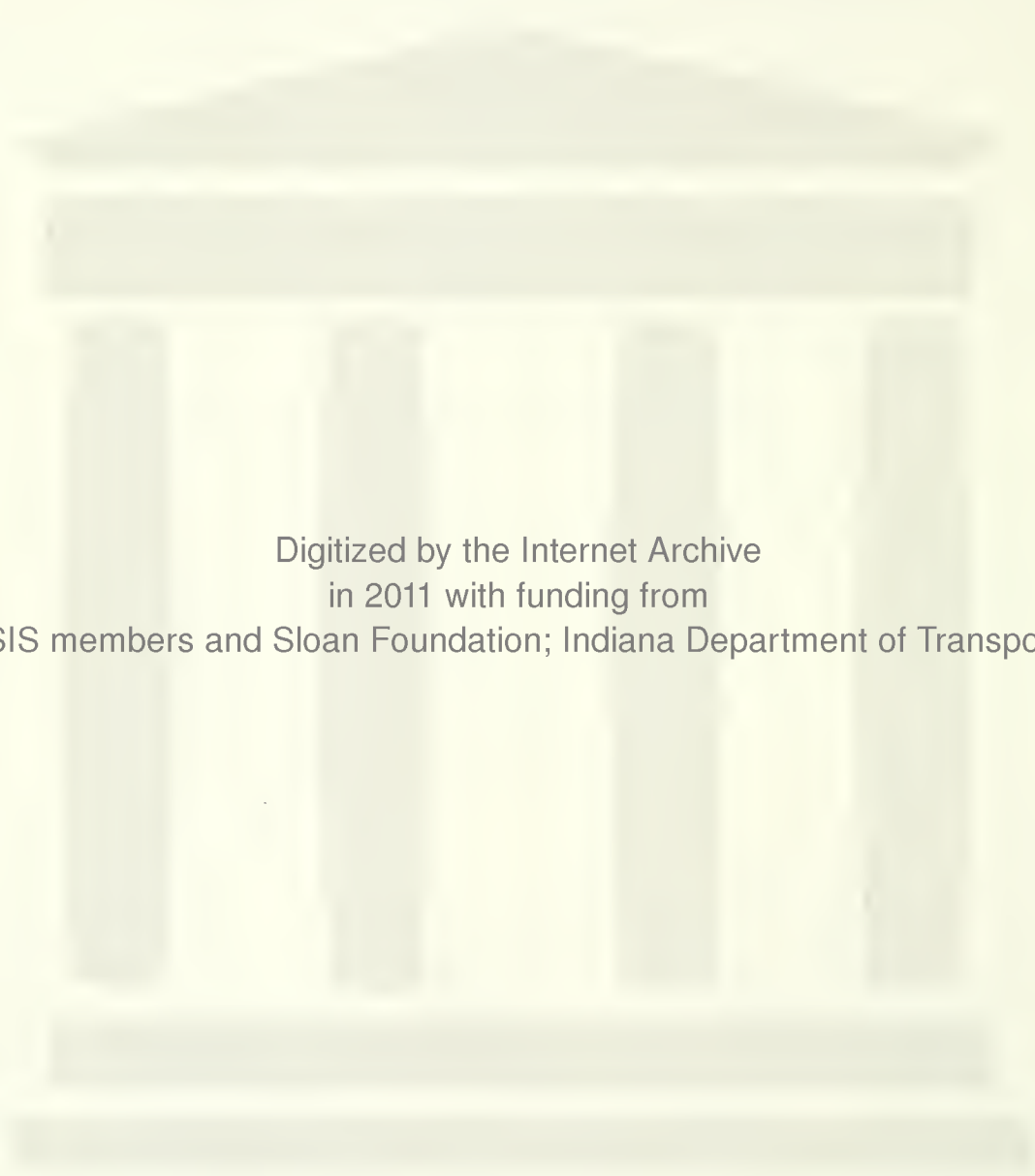
Respectfully submitted,



Harold L. Michael
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by

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ABSTRACT

This paper presents a computer program for a common type of analysis of the slope stability problem, viz., the possibility of slope failure by translation of a massive block along a weak layer of soil. The problem, which can occur in either natural or man-made slopes, is most generally referred to as the "sliding block problem".

Variation in the water surface position requires three subroutines or cases. The program automatically sequences selected potential sliding surfaces one by one, then selects the desired water surface case, and finally computes the factor of safety against sliding along the base of the central block.

The analysis is based on total unit weights and boundary forces. It is possible to consider ten different soil types having very different soil parameters, viz., unit weight, Mohr-Coulomb cohesion intercept and Mohr-Coulomb angle of friction. A maximum of twelve continuous soil layers at any inclination can be considered in the present program. A total of ten vertical strip loads of different intensities can be placed on the ground surface anywhere below the toe and above the crest. Finally, with all the above information, ten sliding surfaces can be concurrently analyzed for the factor of safety. Said factor is applied to the strength of the soil at the base of the central block, assuming limiting equilibrium for the active and passive earth pressure forces at the ends of the central block.

The paper is complete with a flow chart of the computer program and two illustration problems.

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INTRODUCTION

The stability of man-made and natural slopes has always been an important topic of discussion in the field of civil engineering. Yet, failure of man-made fills and cuts probably occurs more frequently than all other failures of civil engineering structures combined. Although an understanding of the major factors which contribute to failure of slopes has improved considerably, our predictive ability remains less than satisfactory.

This paper addresses the problem of the "sliding block", i.e., an essentially rigid mass sliding in a weak layer. At first glance, this seems to be a rather simple problem. But when practical variations in soil profile are considered, as well as water levels, boundary geometries and loadings, and uncertainties of position and shape of the most critical sliding surface, the solutions require reasonably large computer systems.

When a slope is underlain by one or more strata of very soft or loose materials, the most critical sliding surface may not be even approximately circular, as shown in Figure 1. Rather there is a 3-plane surface of potential sliding in which a maximum amount of the surface lies within the weak material.

An initial programmed solution by one of the junior authors (Mendez (1972)) was quite general with respect to the shape of the 3-plane surface, but to accommodate this feature the profile was simplified to two soil layers, viz., a strong soil over a weak one. A second program, reported in this paper, makes simplifying assumptions with respect to the shape of the sliding surface, but is quite versatile with respect to the profile and boundaries. This second program seems to better meet the analytical requirements of the Indiana State Highway Commission.

Location of failure surface depends on relative strength and orientation of layers.

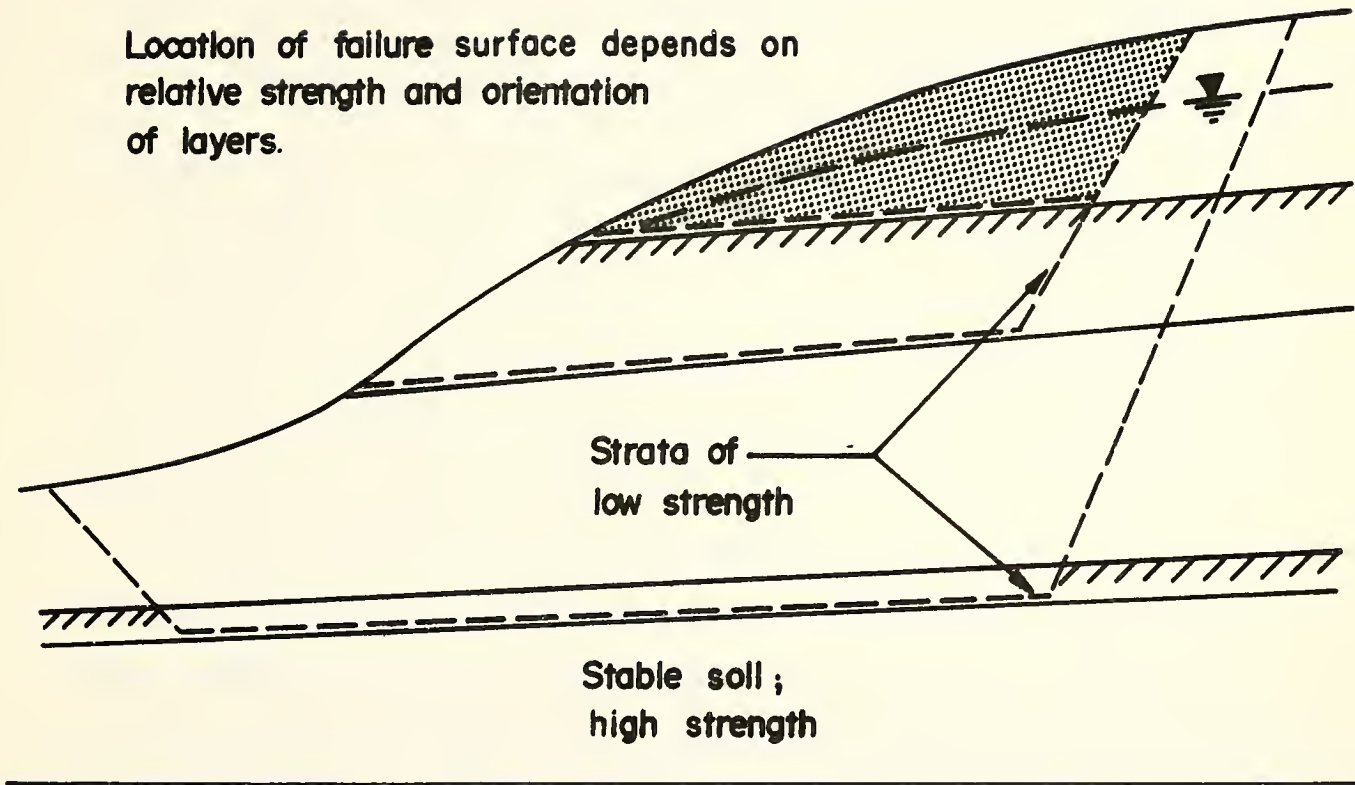


FIGURE I

SLOPE FAILURE BY SLIDING

The type of failure usually assumed in slope stability analysis is the one piece slide (HRB (1958)). The failure is one in which the moving body is essentially rigid and the failing mass is separated from the unmoved one by a surface of assumed shape. Where the soil is grossly homogeneous it seems logical that the failure surface would be roughly circular, and in the interest of simplicity it is usually made exactly so. A recent overview of the circular type analysis, by the well known methods of slices, is contained in Carter, Lovell and Harr (1971).

Where there is evidence of definite differences in shearing resistance in the soil profile, it is well to consider potential failure surfaces which follow the surfaces of weakness. Several methods of handling irregular surfaces are reported by Morganstern and Price (1965), Carter, Lovell and Harr (1971), and Mohan (1971).

A special case of the irregular sliding surface has been shown in Figure 1, where the potential failure planes have a maximum length in the weaker materials. The potential failing block is actually a combination of active and passive wedges, with a central trapezoidal block based in a weak layer. Examples of simplified solutions to this problem are given in Department of the Navy (1971) and United States Steel (1972), as well as Mendez (1972).

A GENERAL SOLUTION TO THE SLIDING BLOCK PROBLEM

Figure 2 shows the free body diagram with a full quota of complexities in boundary geometries and forces, i.e., these could be more simple in a given instance. Incorporation of a water surface and associated water

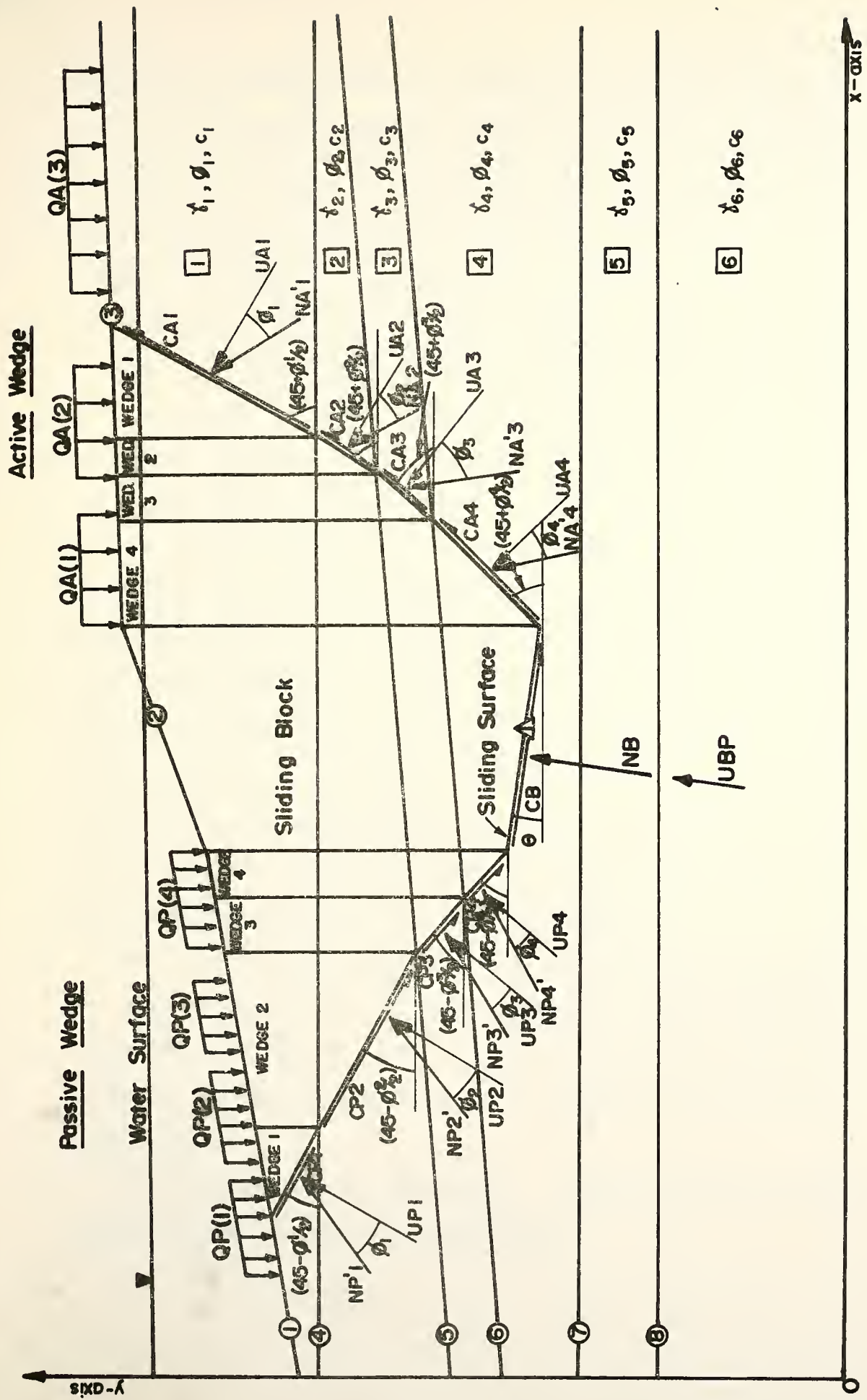


FIGURE 2

forces into the problem makes it convenient to consider three cases, each with its appropriate subroutine in the computer solution. The upper boundary slopes reading left to right in Figure 2 are referred to as the "down slope", the "middle slope" or simply the "slope" and the "upper slope". The cases are:

- Case 1. When the water surface is below the trial sliding surface.
- Case 2. When the water surface is partly above and partly below the ground surface, but above the trial sliding surface.
- Case 3. When the water surface is anywhere below the ground surface, but above the trial sliding surface.

It is assumed that the right hand wedges are in a state of limiting active earth pressure and the left hand wedges are in a state of limiting passive earth pressures. Simplifying assumptions are employed with respect to the inclinations of the wedges surfaces and the directions of the earth pressure forces. Although the right-hand and left-hand wedges are assumed to be on the verge of sliding, there is in general, an incomplete mobilization of the shearing resistance along the base of the block, i.e., the factor of safety is defined with respect to the shearing resistance-shearing force ratio along this surface.

The wedge inclination and earth pressure force direction assumptions are those which apply for a simple Rankine case. They are employed by others (Dept. of the Navy (1971)), and have been shown to be good approximations of the most critical values, for a number of cases tested by Mendez (1972).

To be certain that all assumptions inherent to the solution are understood, they are listed in detail below.

1. Problem is two dimensional.
2. The ground surface is defined by three slopes, and a well defined toe and crest.
3. Soil strata are laterally continuous.
4. Soil properties in layers are defined by γ , c , and ϕ (where c or ϕ can be equal to zero).
5. Sliding surface at the base of the block and between the slide wedges is a plane.
6. All lateral forces on vertical wedge boundaries are normal to these boundaries, i.e., there are no shear forces on these boundaries.
7. The factor of safety is figured for the base of the sliding block only. The movement required to mobilize limiting active and passive pressures is smaller than the movement required to mobilize the shearing strength of the weak soil strata.
8. The wedge slip surfaces are at $(45 + \phi/2)$ and $(45 - \phi/2)$ with the horizontal for active and passive wedges, respectively.
9. The active and passive forces are computed by satisfying static equilibrium and after assumptions 6 and 8.
10. Seepage, if any, is in a steady state. However, water pressures are calculated at any point as if they were hydrostatic.

The analysis of forces is demonstrated in Figures 2, 3, and 4. The analysis is divided into three parts:

- (i) calculation of forces on central block due to active wedge;
- (ii) calculation of forces on central block due to passive wedge; and
- (iii) calculation of base forces on central block and of the factor of safety against sliding along this base.

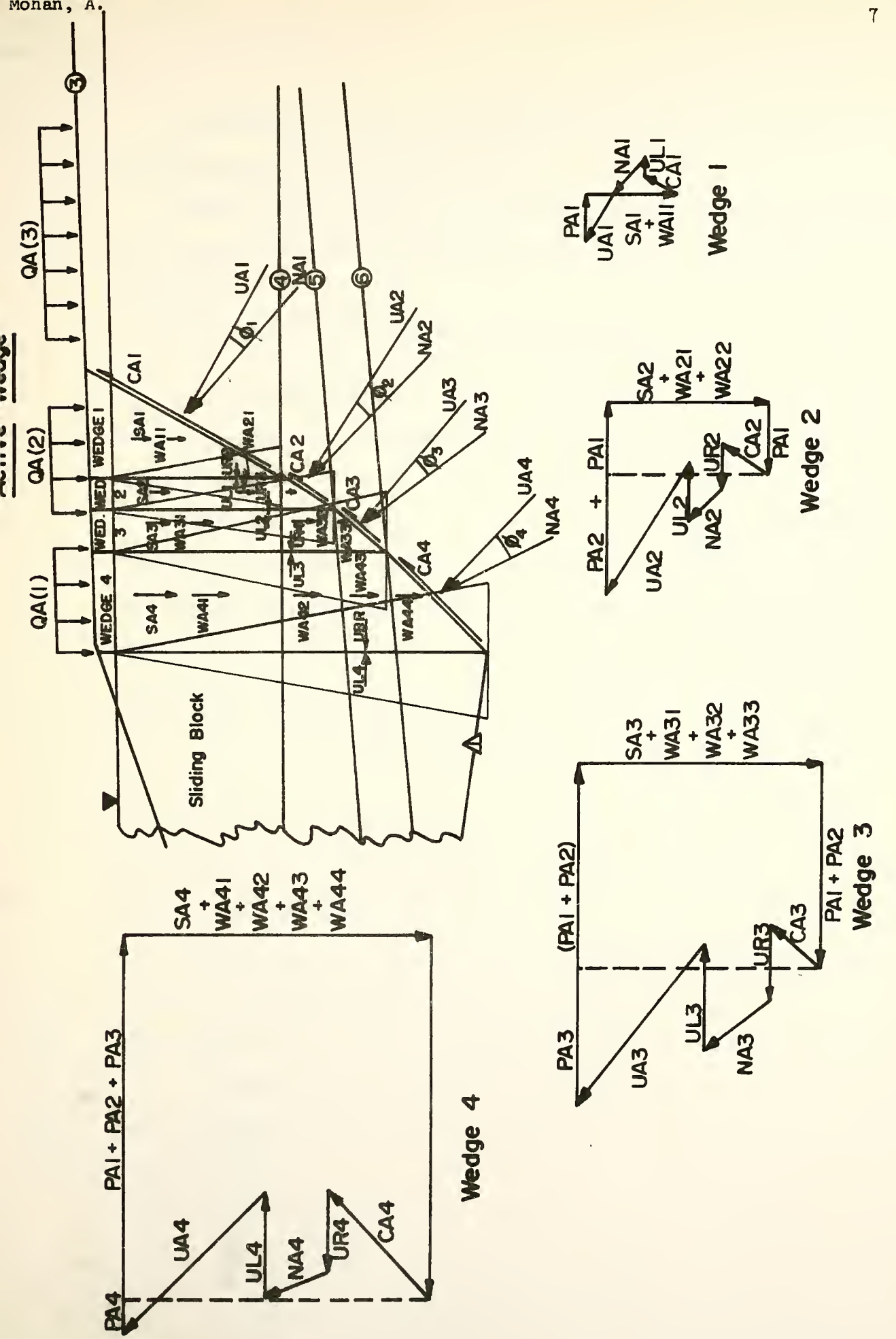


FIGURE 3

The analysis of forces is illustrated for water surface case 2, but the other cases follow directly.

Figure 2 shows a rather complex problem space section, with multiple soil layers at variable inclinations and with very different soil properties.

(i) Analysis of Active Forces on Central Block

Figure 3 shows the active wedge from Figure 2, divided into small wedges governed by the intersection of the assumed slip surface and soil boundaries.

Consider a typical polygon of forces for any (nth) wedge in Figure 3. Summation of all the forces in the x and y directions and equating to zero yields the following equations,

$$\Sigma F_x = 0$$

$$P_{An} = UAR_n - UAL_n - UAn \cos (45 - \phi_n/2) + NA'n \cos (45 + \phi_n/2) - CAn \cos (45 + \phi_n/2) \quad (1)$$

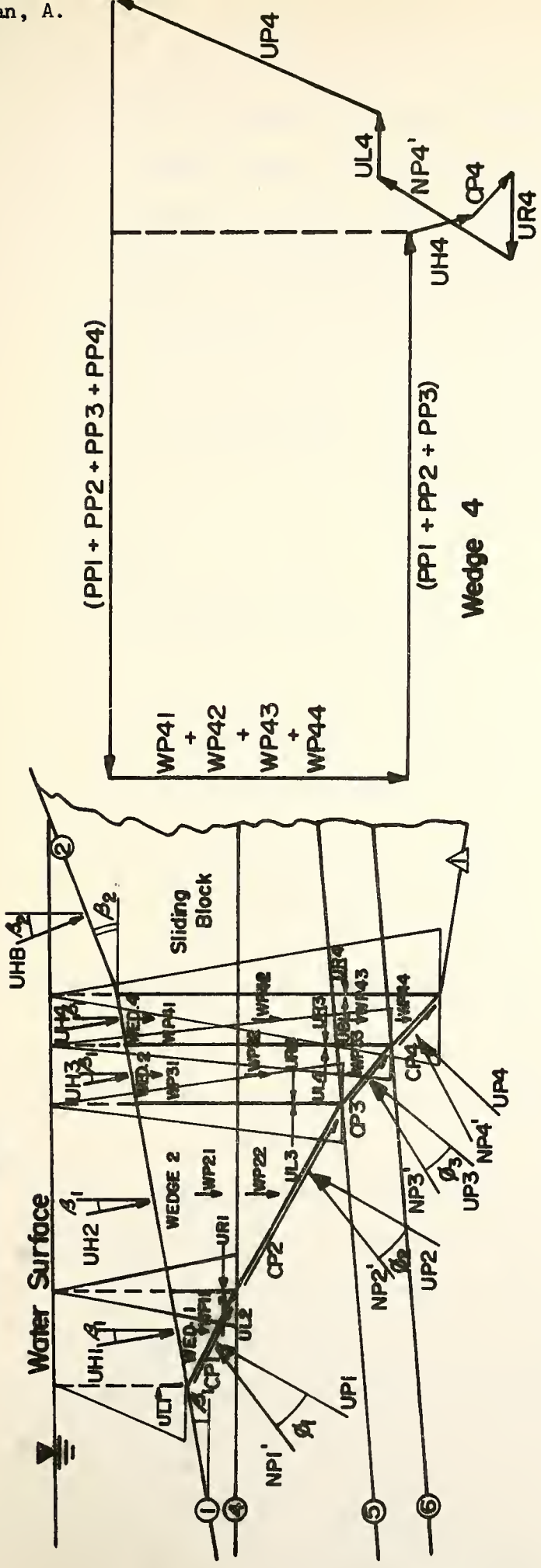
$$\Sigma F_y = 0$$

$$WAN = CAn \sin (45 + \phi_n/2) + UAn \sin (45 - \phi_n/2) + NA'n \sin (45 + \phi_n/2) \quad (2)$$

Elimination of $NA'n$ from equations (1) and (2) yields an expression for the incremental active force for the nth wedge,

$$P_{An} = WAN \tan (45 - \phi_n/2) - 2 CAn \cos (45 + \phi_n/2) + (UAR_n - UAL_n) + UAn \left\{ \cos (45 - \phi_n/2) - \tan (45 - \phi_n/2) \sin (45 - \phi_n/2) \right\} \quad (3)$$

Passive Wedge



$(PP1 + PP2 + PP3 + PP4)$

WP41
+
WP42
+
WP43
+
WP44

$(PP1 + PP2 + PP3)$

Wedge 4

$(PP1 + PP2)$

WP21
+
WP22

Wedge 1

Wedge 2

Wedge 3

Wedge 4

FIGURE 4

(ii) Analysis of Passive Forces on Central Block

Figure 4 shows the forces acting on the passive wedge from Figure 2. Consider a typical polygon of forces acting for an nth passive wedge in Figure 4. Sum forces in the x and y directions, and equate to zero for equilibrium.

$$\Sigma F_x = 0$$

$$PP_n = U\beta_n \sin \beta_1 + CP_n \cos (45 - \phi_n/2) + UL_n + UP_n \cos (45 + \phi_n/2) - UR_n + NP'_n \cos (45 - \phi_n/2) \quad (4)$$

$$\Sigma F_y = 0$$

$$WP_n = NP'_n \sin (45 - \phi_n/2) - U\beta_n \cos \beta_1 - CP_n \sin (45 - \phi_n/2) + UP_n \sin (45 + \phi_n/2) \quad (5)$$

Elimination of NP'_n from equations (4) and (5) yields an expression for the incremental passive force for the nth wedge.

$$PP_n = WP_n \tan (45 + \phi_n/2) + 2 CP_n \cos (45 - \phi_n/2) + U\beta_n \left\{ \sin \beta_1 + \cos \beta_1 \tan (45 + \phi_n/2) \right\} + (UL_n - UR_n) + UP_n \left\{ \cos (45 + \phi_n/2) - \sin (45 + \phi_n/2) \tan (45 + \phi_n/2) \right\} \quad (6)$$

(iii) Analysis of Forces on Central Block and Calculation of Factor of Safety

Figure 5 shows the appropriate free body from Figure 2. The factor (FS) is commonly called the factor of safety, although it is better interpreted as a strength reduction factor, i.e., if the real strength were divided by this factor, a reduced strength would obtain at which failure would impend. Note that the base sliding surface can be inclined up

(θ^+) or down (θ^-) with respect to the horizontal, or may be horizontal ($\theta = 0$).

For θ^- and where forces are summed normal (N) and tangential (θ) to the sliding surface,

$$\Sigma F_N = 0$$

$$\begin{aligned} NB' + UBP = PAA \sin \theta - PPP \sin \theta + WB \cos \theta + UBH \cos \beta_2 \cos \theta \\ - UBL \sin \theta + UBR \sin \theta - UBH \sin \beta_2 \sin \theta \end{aligned} \quad (7)$$

$$\Sigma F_\theta = 0$$

$$\begin{aligned} \frac{CB}{FS} + \frac{NB' \tan \phi}{FS} = PAA \cos \theta - PPP \cos \theta - WB \sin \theta \\ - UBH \cos \beta_2 \sin \theta - UBH \sin \beta_2 \cos \theta \\ - UBL \cos \theta + UBR \cos \theta \end{aligned} \quad (8)$$

Elimination of NB' from equations (7) and (8) yields an expression for the factor of safety for a particular trial sliding surface,

$$\begin{aligned} FS = \frac{CB + (PAA \sin \theta - PPP \sin \theta + WB \cos \theta + UBH \cos \beta_2 \cos \theta \\ - UBL \sin \theta + UBR \sin \theta - UBP - UBH \sin \beta_2 \sin \theta) \tan \phi}{(PAA - PPP) \cos \theta - WB \sin \theta - UBH \cos \beta_2 \sin \theta \\ - UBH \sin \beta_2 \cos \theta - UBL \cos \theta + UBR \cos \theta} \end{aligned} \quad (9)$$

For θ^+

$$\begin{aligned} FS = \frac{CB + (PPP \sin \theta - PAA \sin \theta + WB \cos \theta + UBH \cos \beta_2 \cos \theta \\ + UBL \sin \theta - UBR \sin \theta - UBP + UBH \sin \beta_2 \sin \theta) \tan \phi}{(PAA - PPP) \cos \theta + WB \sin \theta + UBH \cos \beta_2 \sin \theta \\ - UBH \sin \beta_2 \cos \theta - UBL \cos \theta + UBR \cos \theta} \end{aligned} \quad (10)$$

For $\theta = 0$ (Horizontal slope)

$$FS = \frac{CB + (WB - UBP + UBH \cos \beta_2) \tan \phi}{(PAA - PPP) - UBH \sin \beta_2 - UBL + UBR} \quad (11)$$

THE COMPUTER PROGRAM AND ITS CAPABILITIES

The flow chart for the program is shown in Figure 6. The program has been written in FORTRAN IV language, and at present it is workable on the CDC 6500 computer. It is made up of a main program and six supporting sub-routines. The program makes use of common storage to optimize usage of high speed core and minimum computation time.

The program is capable of handling the following variables.

- (i) Multiple (up to 11) continuous soil layers at any inclination; layer boundaries are straight.
- (ii) Top ground surface made up of three slopes and well defined toe and crest points.
- (iii) Soil properties defined by γ , c and ϕ (c or ϕ can equal zero).
- (iv) Multiple (up to 10) uniform strip loads on ground surface of the upper and/or lower slopes.
- (v) Water surface anywhere in the problem space. The water surface is defined by continuous straight lines and/or by a non-linear surface defined by 7 or less known coordinates.
- (vi) Multiple trial sliding surfaces at the bottom of the central block. These can be at any inclination and as many as 10 can be analyzed in a single run.

Specific trial surfaces are input for analysis. No searching technique (for identification of a minimum FS) is recommended, although some ideas on this are contained in Carter, Lovell and Harr (1971).

The active and passive force subroutines are potentially valuable of themselves in the solution of lateral earth pressure problems.

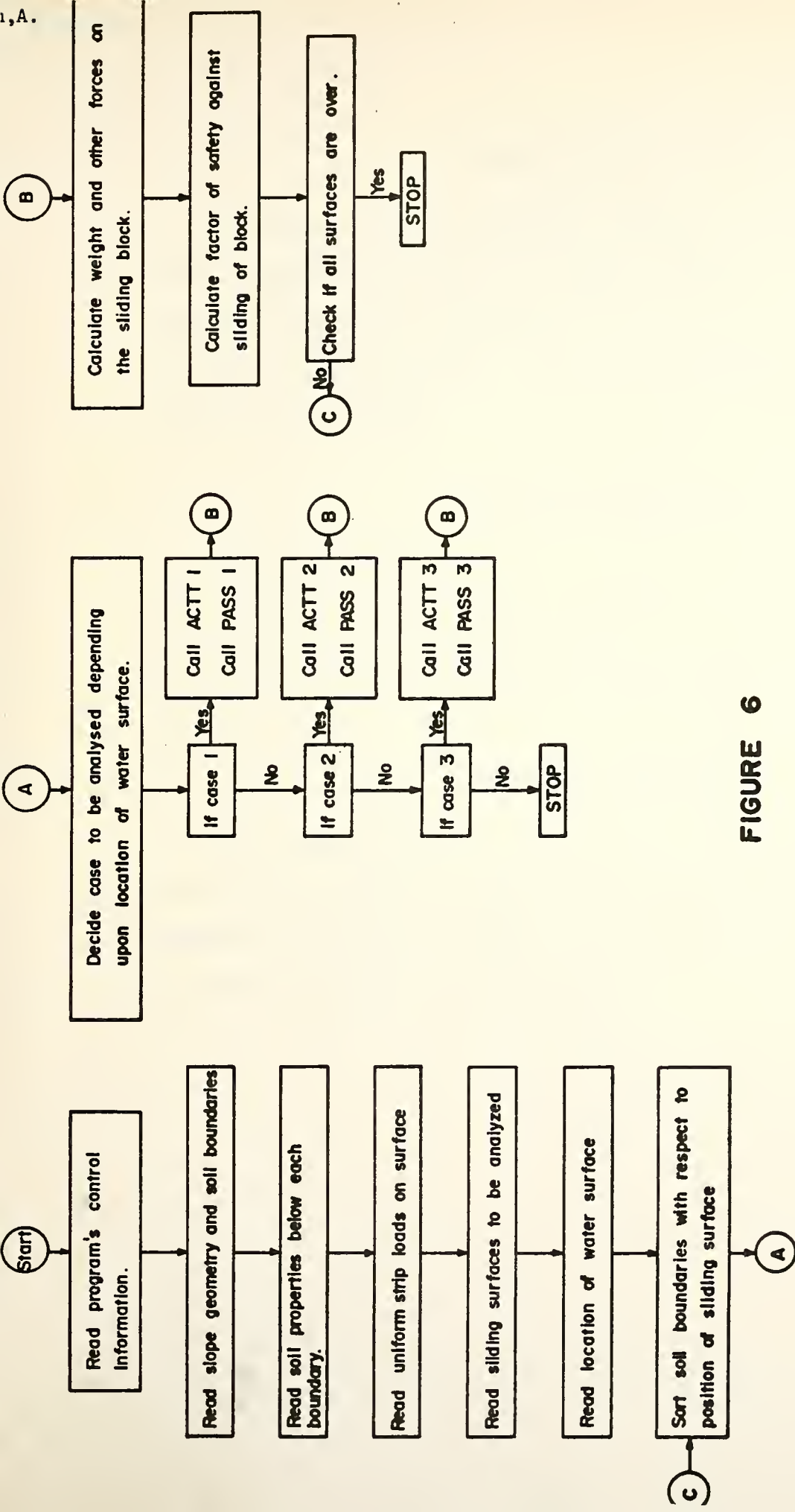


FIGURE 6

ILLUSTRATION PROBLEMS

The purpose of the illustration problems is three fold:

(1) to demonstrate the use of the computer program; (2) to show the versatility and several options of the program; and (3) to serve as a check for duplicated decks. Two separate hypothetical problems are chosen for this purpose.

Illustration Problem No. 1

This first illustration problem involves a simple soil profile shown in Figure 7. Solutions are obtained for three central block sliding surfaces and for three location of the water surface for each sliding surface. The results are given in Table 1.

Illustration Problem No. 2

This second problem is more complex and is shown in Figure 8. This problem is also solved for three slopes of sliding surfaces in combination with three locations of the water surface for each sliding surface. The results are again shown in Table 1.

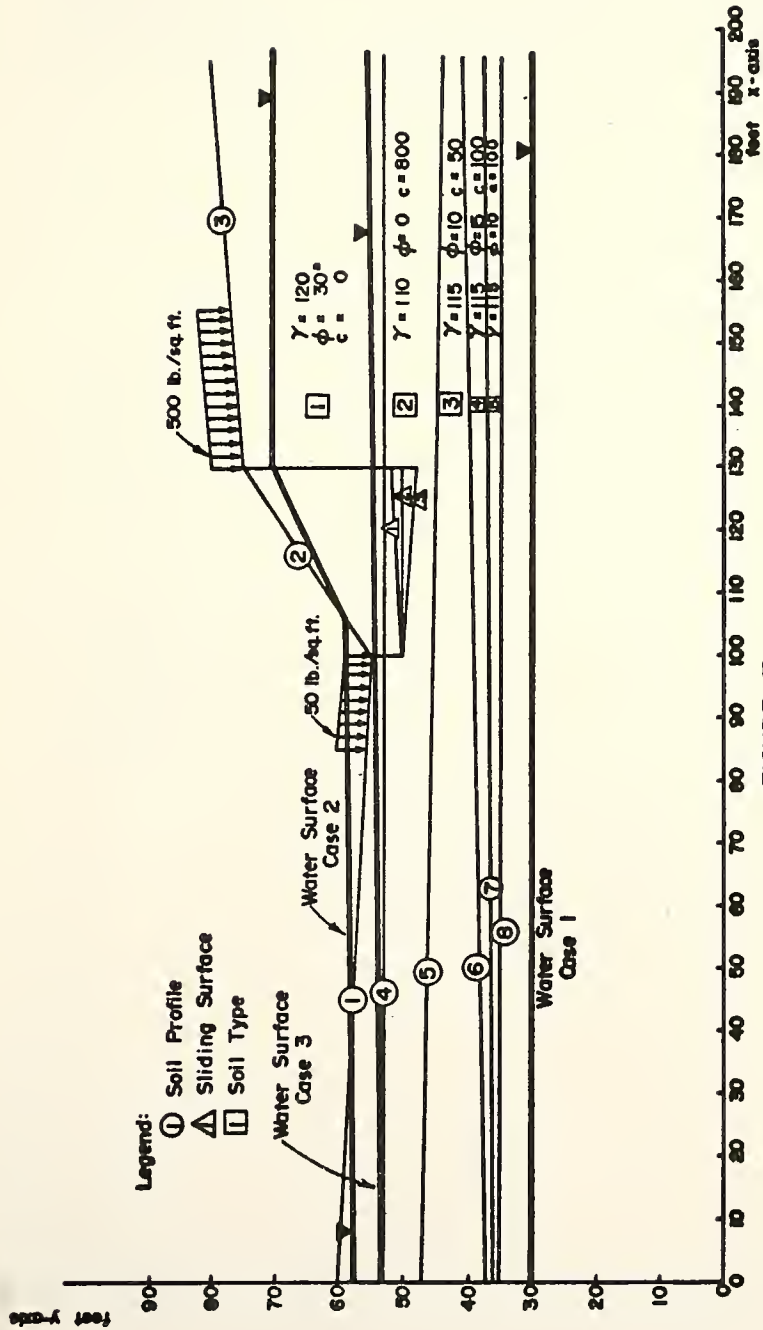


FIGURE 7

TABLE 1

Summary of Results for Two Illustration Problems

Case Analysed	Number and Slope of Sliding Surface	Factor of Safety	
		Problem No. 1	Problem No. 2
Case 1	1. θ^+	1.94	2.07
	2. $\theta = 0$	1.87	2.24
	3. θ^-	1.83	2.42
Case 2	1. θ^+	1.66	1.83
	2. $\theta = 0$	1.65	1.98
	3. θ^-	1.64	2.12
Case 3	1. θ^+	1.84	1.84
	2. $\theta = 0$	1.52	1.81
	3. θ^-	1.66	1.75

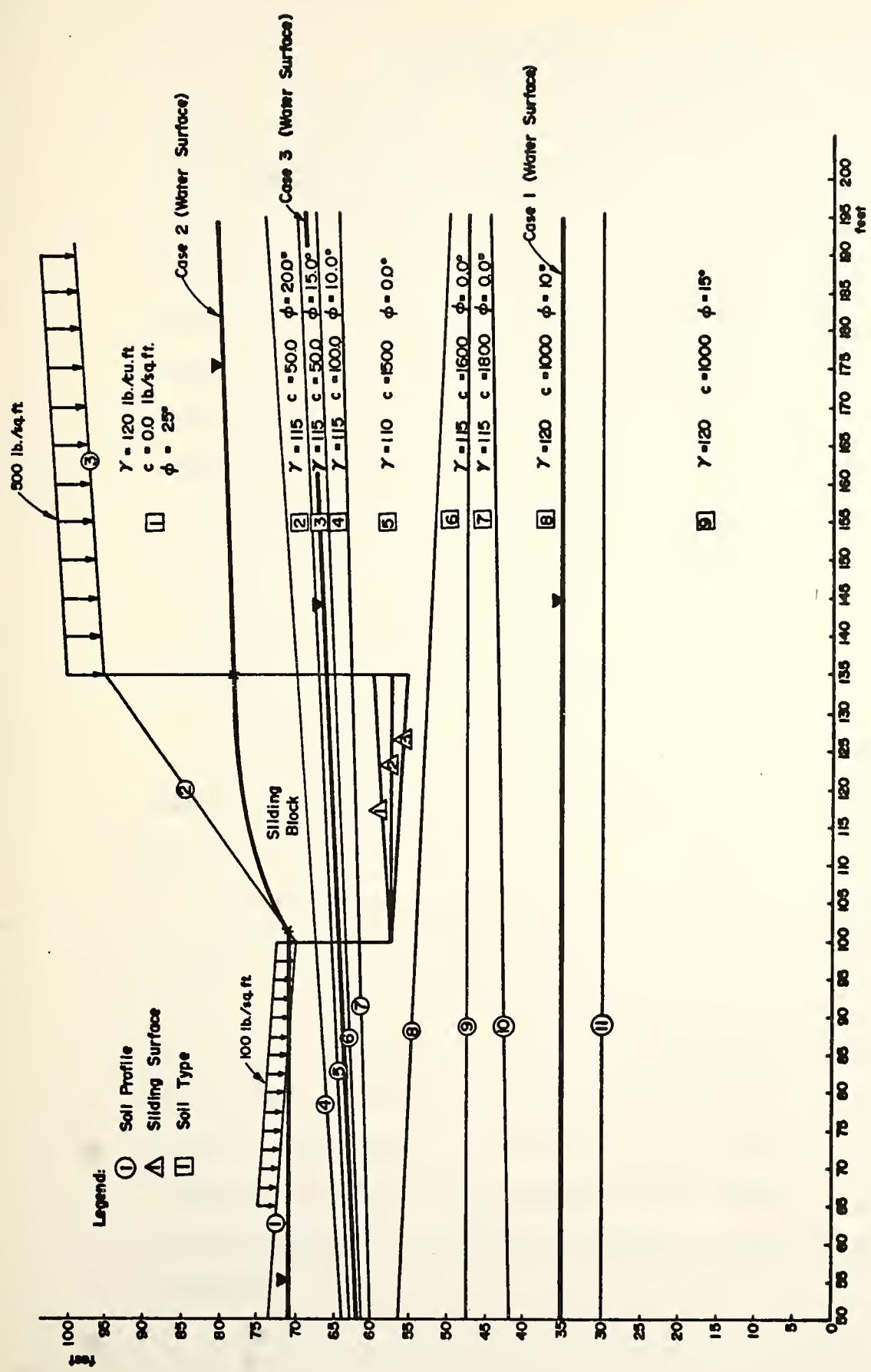


FIGURE 8

CONCLUSIONS AND RECOMMENDATIONS

The primary objective of this research was the development of a computer-assisted system for rapid prediction of the factor of safety of slopes where the mode of failure is a sliding block. The resulting program is sufficiently versatile to accommodate a 3-slope ground surface and a subsurface profile with spatial variations in material properties, a steady state flow domain, and uniform strip ground surface loadings. Up to ten trial sliding surfaces can be analyzed concurrently, with the base of the central wedge at any inclination in any selected soil layer. The program automatically sequences the trial sliding surfaces, computing a factor of safety for each. Since many sliding surfaces may have to be examined, i.e., there is no systematic search technique which assures identification of a minimum, this is a most important feature.

It was desired to develop a system which could be used on smaller computers. Consequently, the program uses a small storage and short computation time.

Hopefully, the program will enable a designer to check against this mode of instability for any slope where there is reason to suspect that it may occur. Such suspicion would ordinarily accrue from study of boring logs and profiles. Sliding blocks can be based in any soil stratum of below-average strength. When there is no evidence of weak layers, it is likely that some common form of the circular or rotational slump analysis will be employed. In questionable cases, both types of analysis may be undertaken and factors of safety compared.

Any computer program should be tested for reliability by generation of solutions to common problems through different programs or manual calculations. Unfortunately, this is usually possible for only simple examples, since the motivation for development of the new programs is an inability of old ones to accommodate the desired level of physical complexity.

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