A STUDY OF THE FATIGUE PROPERTIES OF LIGHTWEIGHT AGGREGATE CONCRETE

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Technical Paper

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OF LIGHTWEIGHT AGGREGATE CONCRETE

TO: K. B. Woods, Director
Joint Highway Research Project

November 2, 1960

FROM: H. L. Michael, Assistant Director
Joint Highway Research Project

File: 7-4-7
Project: C-36-56G

Attached is a technical paper entitled "A Study of Fatigue Properties of Lightweight Aggregate Concrete" by Messrs. W. H. Grey, J. F. McLaughlin, and J. D. Antrim. The paper is proposed for presentation to a regional meeting of the American Concrete Institute in Tucson, Arizona.

The material contained in the paper is from recent research previously reported to the Board by Messrs. Grey and Antrim. The paper is presented to the Board for the record and for approval of the proposed presentation.

Respectfully submitted,

Harold L. Michael

Harold L. Michael, Secretary

HLM:kmc

Attachment

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Technical Paper

A STUDY OF THE FATIGUE PROPERTIES OF LIGHTWEIGHT AGGREGATE CONCRETE

by

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File No.: 7-4-7
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SYNOPSIS

Fatigue tests were conducted on two different lightweight aggregate concretes, one proportioned for a high strength and the other for a low strength. Specimens of approximately the same age were tested at stress levels of 40, 50, 60, 70, and 80 per cent of the ultimate static compressive strength of the respective mixes. Within the limits of the investigation, the fatigue behavior of high-strength lightweight concrete was similar to that of low-strength lightweight concrete. In addition, the fatigue behavior of the lightweight aggregate concrete appears to be similar to that found for a normal weight concrete in a previous study.

A STUDY OF THE FATIGUE PROPERTIES OF LIGHTWEIGHT AGGREGATE CONCRETE

INTRODUCTION

Lightweight aggregate concrete has proven itself to be a useful structural material; its uses and applications are becoming more numerous as engineers learn more about it. The literature contains the results of numerous research projects carried out to determine the properties of the various lightweight aggregates available and some on strength, volume change, and thermal properties of the finished concrete. Published information on the fatigue resistance of lightweight concrete is scant.

The resistance of a material to repeated loading is obviously an important factor in design. A working stress must be chosen such that the structure will not fail from repeated applications of loads below the ultimate strength. Knowledge of the relationship between the number of cycles to failure and applied stress is essential. Many studies have been conducted to determine the fatigue resistance of normal weight concrete (1,2,3,4,5) but more information is needed on the behavior of lightweight concrete subjected to this type of loading. The work reported in this paper had as its major objective, the establishment and comparison of the S-N relationship of two lightweight aggregate concretes, and the comparison of these two relationships with two curves previously established for a normal weight concrete.

TESTING PROGRAM

The testing program was divided into two parts. In the first part tests were conducted on concrete proportioned to have a static compressive strength of 3,500 psi. This concrete was called the low-strength concrete and will be referred to as the LL series. The second part consisted of conducting tests on concrete proportioned to have a static compressive strength of 6,000 psi. This concrete was called the high-strength concrete and will be referred to as the HL series. Each series was made up of five batches of concrete and from each batch 30 cylinders 3 inches in diameter and 6 inches in length were cast.

Materials and Proportioning

The fine and coarse aggregate used in this study was an expanded shale produced in a rotary kiln. Type I portland cement from a single clinker batch was used in both mixes and Darex, added at the mixer, was used for an air-entraining agent. The mixes were designed in accordance with the ACI "Proposed Recommended Practice for Selecting Proportions for Structural Lightweight Concrete," (7) except that the specific gravity factor was not computed.

The average strength of the low-strength concrete was 3,700 psi with a slump of 2 1/2 in. and an air content 7 per cent. A cement factor of 5.8 bags per cubic yard was used. The average strength of the high strength concrete was 6,200 psi with a slump of 2 1/2 in. and an air content of 6 1/2 per cent. A cement factor of 9.6 bags per cubic yard was used. The low-strength concrete contained 60 per cent fine aggregate and the high-strength concrete contained 65 per cent fine aggregate.

Specimens were cured in water for 28 days in accordance with ASTM Designation: C 129-52 T, after which, they were placed in an oven to dry for 4 to 5 days at a temperature of approximately 105°C. The object of drying was to prevent further hydration during the period of fatigue testing. After the specimens were removed from the drying oven, they were allowed to cool for 24 hours before capping. Caps were then placed on each end of all cylinders with a sulfur capping compound and the cylinders were stored at room temperature until testing.

Testing Methods

Shortly before fatigue testing, static compression tests were conducted on five randomly chosen specimens from each batch to estimate the ultimate strength of the batch. The static tests served as an estimate of the average batch strengths and the stress levels used in fatigue testing were determined from these results.

Since a period of at least two weeks was required to complete the fatigue testing on any one batch, static tests were conducted after fatigue testing on the remaining specimens in those batches that had enough specimens left to insure a reasonable estimate of the batch strength. These tests were to detect any gain in strength which took place during the time of testing.

Eight specimens were chosen from each batch for testing under a pulsating load at stress levels of 40, 50, 60, 70, and 80 per cent of the estimated static compressive strength of the batch. Specimens were tested in direct compression only and a minimum stress of between 70 and 170 psi was maintained on all specimens to eliminate any possibility of impact. In some cases, where power failures or testing machine break-downs occurred, it was not possible to test specimens from each batch at all stress levels. An unused specimen was used in every test and if the testing was interrupted, the specimen was discarded and the test was rerun on a new specimen.

Two testing machines were employed in the fatigue testing program, the Krouse-Purdue machine, and an Amsler machine. The Krouse-Purdue machine is of the constant deflection type which derives its force from hydraulic pressures acting on a piston (Figure 1). Two components of load are applied by this machine. The first is an average preload which is proportional to the difference in average pressures existing at opposite ends of a hydraulic cylinder. This load is automatically controlled by a hydraulic make-up pump. The second is a pulsating load controlled by varying the throw of an eccentric crank. This causes a loading which is alternately larger and smaller than the preload. This machine has a capacity of ± 60,000 lbs. and operates at 1,000 cycles per minute.

The Amsler fatigue testing machine is based on the same principle as the Krouse-Purdue machine except that the loading jack is separated from the pulsator. The preload of the Amsler machine is the minimum load to be applied and the pulsator increases the load from this minimum to the desired maximum. Loads are transmitted by hydraulic pressure through the tubing to the load jack which is mounted on a specially built frame. The jack develops its force by pushing against the frame and the concrete specimen which is placed on a bearing plate on the floor (Figure 2). This machine has a capacity of 110,000 lbs. and can operate at speeds of either 250 cycles per minute or 500 cycles per minute.

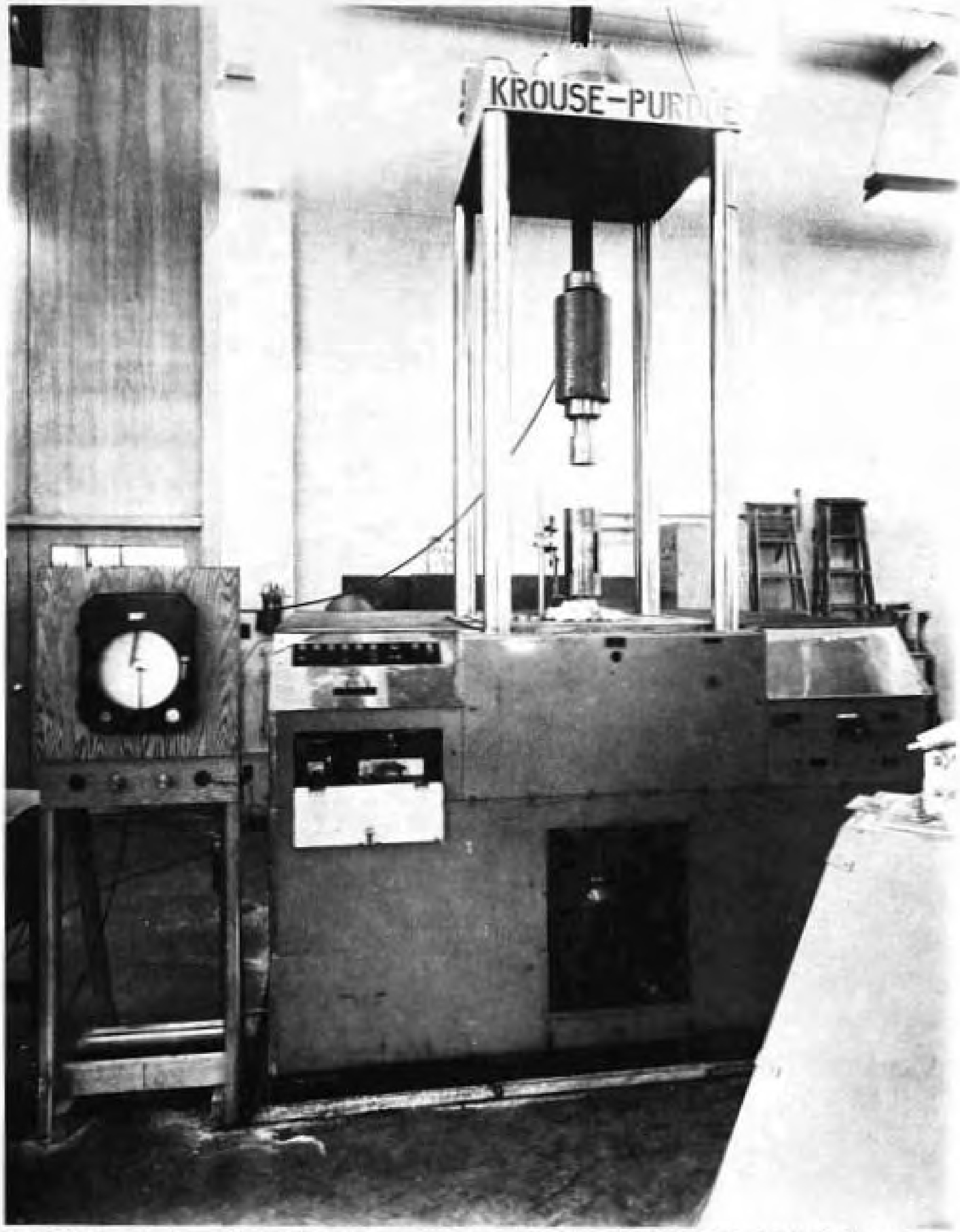


FIGURE I. KROUSE-PURDUE FATIGUE MACHINE

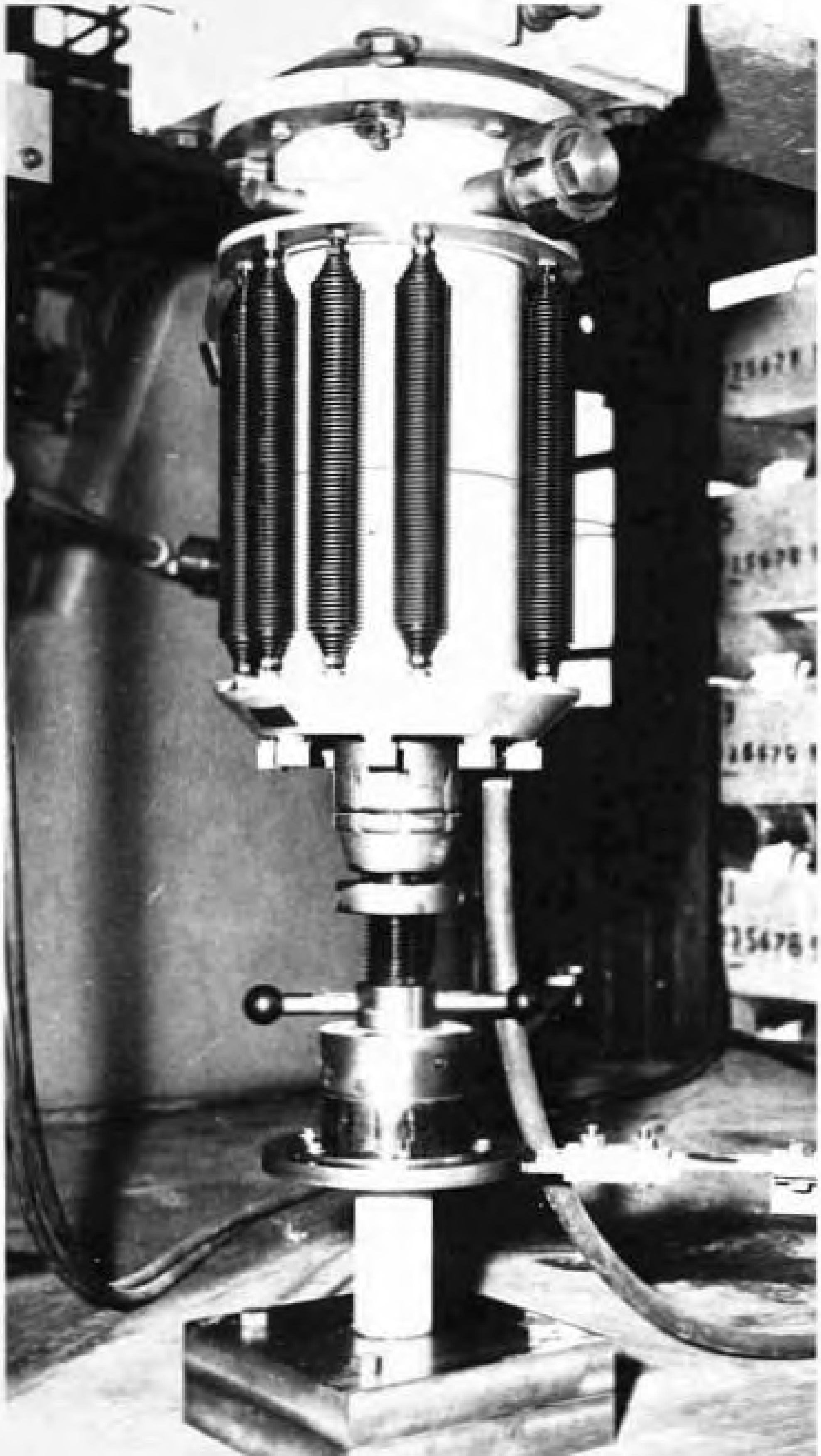


FIGURE 2. LOADING JACK

Since the Krouse-Purdue machine tested at a rate of 1,000 cycles per minute and the Amsler machine was used at the maximum speed of 500 cycles per minute, tests were conducted to see if this variation would affect the fatigue test results. Batch HL 1 was selected for this purpose with one-half of the specimens being tested in each machine. All of these specimens were tested at the 80 per cent stress level.

DISCUSSION OF RESULTS

The discussion of the results of this study has been divided into three parts. The first part is concerned with physical properties of the plastic concrete and the static tests on the hardened concrete. The discussion of the fatigue testing data is presented in the second part and the third part is concerned with the comparison of the results of this study with the results of a previous study on normal weight concrete.

Analysis of Mix Data

Since each series consisted of a group of five separately mixed batches, estimates of the properties of the concrete for each series were made from the properties measured on individual batches. The batch properties of slump, air content, and static ultimate strength are summarized in Tables 1 and 2.

The analysis of variance was used to test for any significant difference between the average batch strengths (8). Data from the low-strength concrete and the high-strength concrete were analysed separately, and then the observed strengths for the two series were compared. All comparisons were made at the 5 per cent significance level and the following results were found:

Table 1

PHYSICAL PROPERTIES OF CONCRETE

LOW-STRENGTH (LL) MIX

Batch Designation	Air Content Percent	Slump Inches	Average Ultimate Strength: After Oven Drying * psi	Average Ultimate Strength: After Fatigue Testing ** psi
LL 1	6.5	2-5/8	3500	3280
LL 2	6.1	2-3/8	3820	None tested***
LL 3	7.6	3-1/4	4060	3900
LL 4	7.5	3	3880	None tested
LL 5	6.2	1-1/2	3530	None tested

* Specimen age when tested was 34 days

** Specimen age ranged from 51 to 58 days

*** All specimens were used in fatigue testing

Table 2

PHYSICAL PROPERTIES OF CONCRETE

HIGH-STRENGTH (HL) MIX

Batch Designation	Air Content Percent	Slump Inches	Average Ultimate Strength: After Oven Drying * psi	Average Ultimate Strength: After Fatigue Testing ** psi
HL 1	6.8	2-3/8	5130	None tested***
HL 2	6.6	2-1/8	6310	None tested
HL 3	6.5	2-3/8	6360	None tested
HL 4	6.3	1-7/8	6360	6180
HL 5	6.8	3	6260	None tested
HL 6	7.0	2-3/4	6010	5790

* Specimen age ranged from 44 to 69 days

** Specimen age ranged from 84 to 94 days

*** All specimens were used in fatigue testing

1. The average strength of the low-strength series was 3760 psi with a standard error of 294 psi, and the average strength of the high-strength series was 6,260 psi with a standard error of 329 psi.
2. The variances of the batch strengths were not significantly different in either the high-strength or low-strength concrete.
3. The batch means were significantly different in the low-strength series but were not significantly different in the high-strength series.
4. The total variance of the low-strength series was not significantly different from the total variance of the high-strength series.
5. The average air content of the low-strength series is 6.8 per cent with a coefficient of variation of 9 per cent, and the average air content of the high-strength series was 6.5 per cent with a coefficient of variation of 5 per cent.
6. There was no significant change in ultimate strength during the time of fatigue testing.

These results indicated that two distinct populations were being tested and further that the batch-to-batch variability was not significant except for the case of strength in the LL series. Examination of the means shows a range of 560 psi in this series which was considered low enough to ignore for the purposes of fatigue testing. Therefore, specimens were chosen for fatigue testing at random from the entire number prepared and no batch effect was measured.

FATIGUE TEST RESULTS

The results of the fatigue data are summarized in Tables 3 and 4. These data are shown in graphic form in Figures 3 and 4, the S-N diagrams, where stress level in per cent is plotted against cycles to failure on a logarithmic scale. As is usual with most fatigue data, there is considerable variation evident, the presence of which makes the interpretation somewhat difficult. However, the purposes of this investigation would be satisfied if curves were fitted to the data from the two mixes, compared to each other, and compared to a comparable curve developed from normal weight concrete.

Since 10 million cycles was set as the upper limit of number of stress repetitions and many specimens tested at various stress levels endured this number, it was not possible to fit a curve by statistical means over the entire range of the data. For example, in the case of the low-strength concrete, all of the specimens tested at the 40 per cent stress level and all except one specimen tested at the 50 per cent level endured ten million repetitions of loading without failure. Specimen number two of batch LL 3 had endured 9.2 million when a power failure stopped the machine and no doubt would have gone to ten million repetitions.

If the assumption that specimen two of batch LL 3 endured ten million cycles without failure is accepted, the data at the 50 per cent stress level is incomplete. Using only the four specimens which failed at the 50 per cent stress level, when actually five were tested, would introduce a bias into the statistical interpretation of the fatigue data. For this reason a better approximation can be made of

Table 3

FATIGUE TEST RESULTS

LOW-STRENGTH CONCRETE

Batch Designation	Specimen Number	Maximum Fatigue Load	Minimum Fatigue Load	Number of Stress Cycles Endured
LL 1	1	9,900 (40)*	900 (3.6)*	10,304,600→ **
	2	12,400 (50)	800 (3.2)	3,147,600
	3	14,800 (60)	700 (2.8)	688,700
	4	17,300 (70)	900 (3.6)	43,000
	5	19,800 (80)	800 (3.2)	19,100
LL 2	1	10,800 (40)	900 (3.3)	10,005,400→
	2	13,500 (50)	800 (3.0)	4,926,400
	3	16,200 (60)	500 (1.9)	396,600
	4	18,900 (70)	800 (3.0)	52,800
	5	21,600 (80)	800 (3.0)	3,500
LL 3	1	11,500 (40)	700 (2.4)	10,464,100→
	2	14,300 (50)	500 (1.7)	9,204,100→
	3	17,200 (60)	900 (3.1)	1,610,000
	4	20,100 (70)	800 (2.7)	51,400
	5	23,000 (80)	500 (1.7)	1,600
LL 4	1	11,000 (40)	1,000 (3.6)	10,418,100→
	2	13,700 (50)	1,100 (4.0)	5,673,300
	3	16,500 (60)	900 (3.3)	1,217,000
	4	19,200 (70)	800 (2.9)	105,100
	5	22,000 (80)	900 (3.3)	8,600
LL 5	1	10,000 (40)	1,000 (4.0)	11,723,300→
	2	12,500 (50)	1,000 (4.0)	4,628,100
	3	15,000 (60)	1,000 (4.0)	2,262,500
	4	17,500 (70)	900 (3.6)	26,300
	5	19,900 (80)	1,100 (4.3)	1,900

*Figure in parentheses is the dynamic load expressed as a percentage of the average ultimate strength of the batch.

**→indicates that the specimen had not failed when the test was stopped.

Table 4

FATIGUE TEST RESULTS

HIGH-STRENGTH CONCRETE

Batch Designation	Specimen Number	Maximum Fatigue Load	Minimum Fatigue Load	Number of Stress Cycles Endured
HL 2	1	17,800 (40)*	1,000 (2.2)*	10,458,700 → **
	2	22,200 (50)	1,200 (2.7)	10,620,300 →
	3	26,700 (60)	1,100 (2.5)	720,500
	4	Specimen not tested		
	5	35,500 (80)	1,200 (2.7)	6,100
HL 3	1	18,000 (40)	1,100 (2.4)	10,216,500 →
	2	22,500 (50)	900 (2.0)	10,474,400 →
	3	27,000 (60)	1,000 (2.2)	9,673,500
	4	31,500 (70)	1,000 (2.2)	150,900
	5	36,000 (80)	1,000 (2.2)	6,600
HL 4	1	Specimen not tested		
	2	22,500 (50)	1,000 (2.2)	4,751,300
	3	22,500 (50)	1,200 (2.7)	5,957,200
	4	27,000 (60)	1,000 (2.2)	736,400
	5	31,000 (70)	1,000 (2.2)	166,100
	6	36,000 (80)	1,000 (2.2)	5,400
HL 5	1	Specimen not tested		
	2	22,200 (50)	1,300 (2.9)	10,499,400 →
	3	26,600 (60)	1,100 (2.5)	6,737,500
	4	31,000 (70)	1,100 (2.5)	611,900
	5	35,400 (80)	1,100 (2.5)	110,000
HL 6	1	Specimen not tested		
	2	21,200 (50)	1,200 (2.8)	10,196,100 →
	3	25,400 (60)	1,100 (2.6)	4,116,700
	4	29,700 (70)	1,100 (2.6)	753,600
	5	33,900 (80)	1,100 (2.6)	86,300

* Figure in parentheses is the dynamic load expressed as a percentage of the average ultimate strength of the batch.

** → indicates that the specimen had not failed when the test was stopped.

FIGURE 3. S-N DIAGRAM FOR LOW STRENGTH CONCRETE

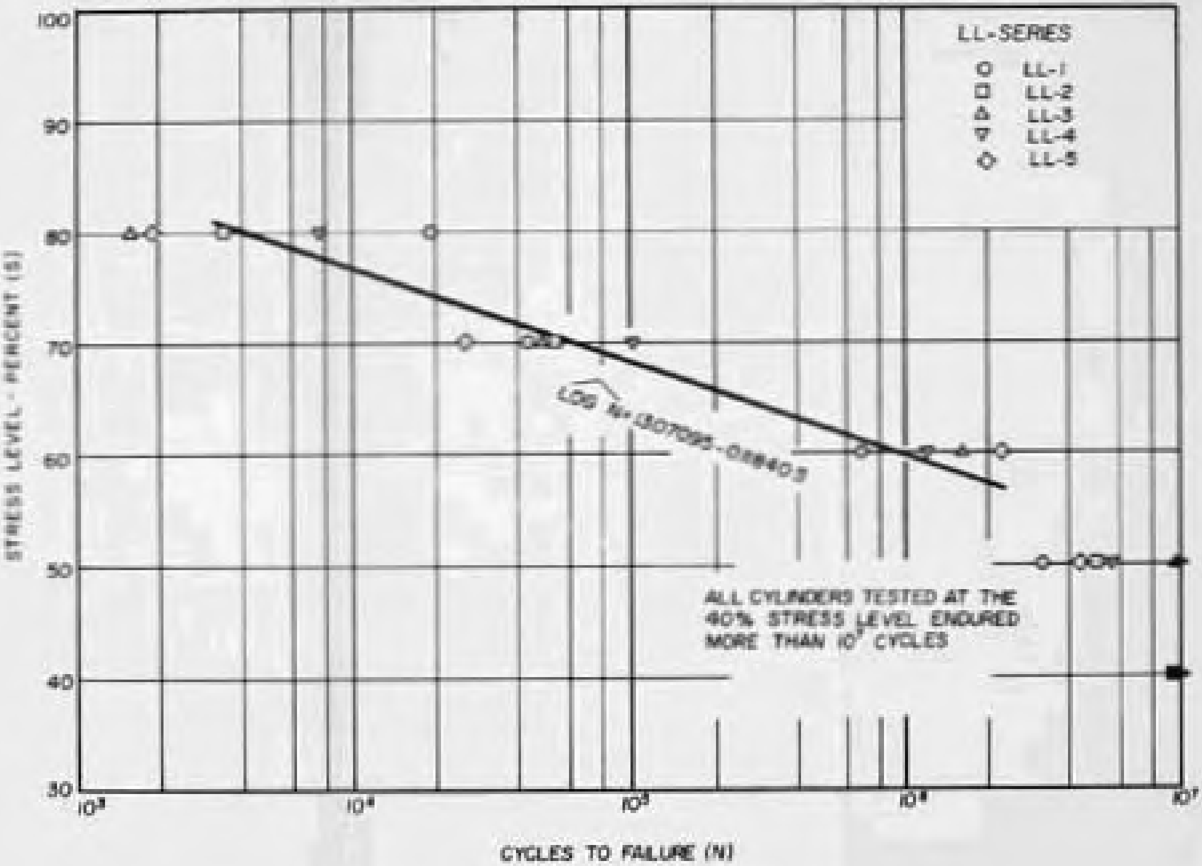
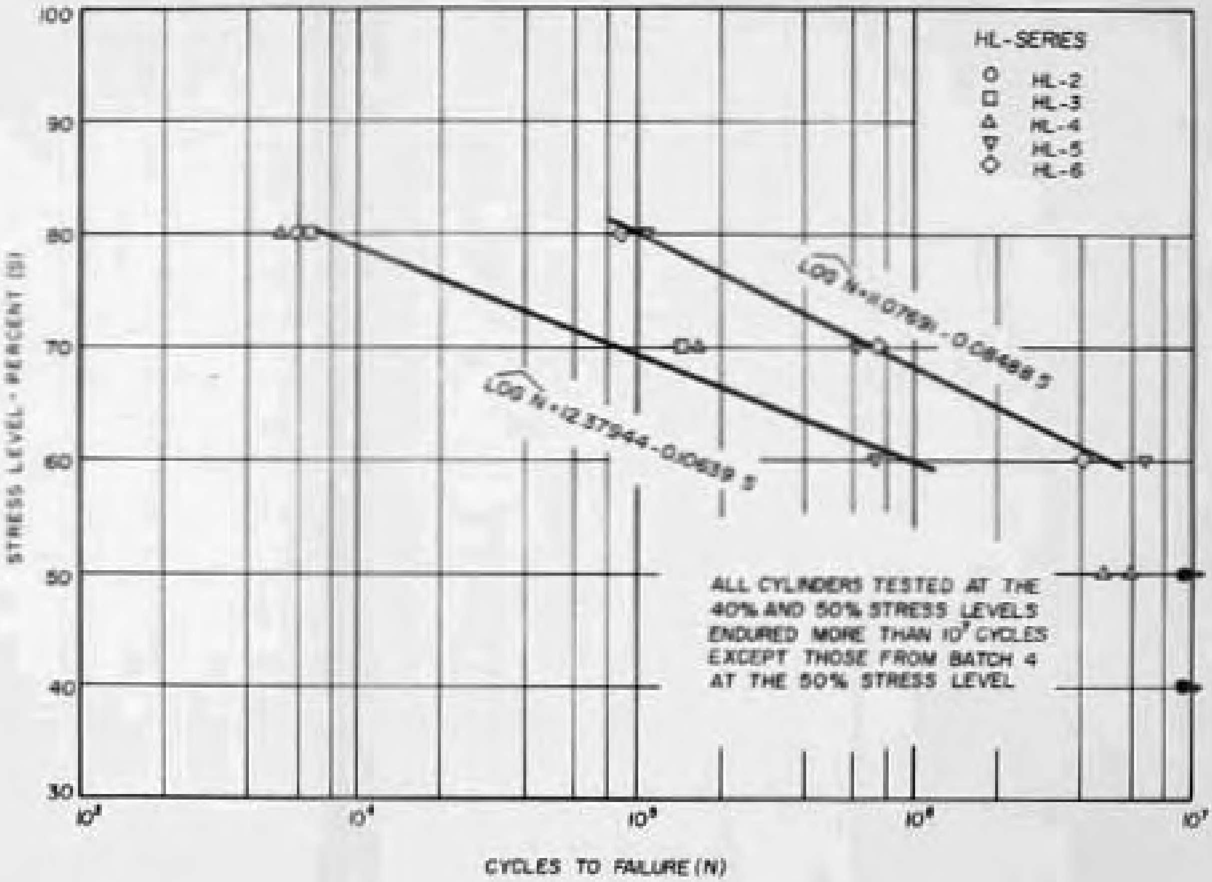


FIGURE 4. S-N DIAGRAM FOR HIGH STRENGTH CONCRETE



the S-N relationship if only the data at the 60, 70, and 80 per cent stress levels are included in the calculations.

In the case of the high-strength concrete, the first two specimens tested at the 40 per cent stress level endured the maximum of ten million cycles of loading without failure. Since all of the specimens of the LL series tested at the 40 per cent stress level endured ten million repetitions of loading, it was felt that no further information could be gained from testing the remaining scheduled specimens of the HL series at the 40 per cent stress level. At the 50 per cent stress level only two specimens failed before they had endured ten million cycles of loading so for purposes of regression analysis, data collected at the 50 per cent level was not to be used to interpret the S-N relationship of the HL series. Again, the S-N relationship was computed from data collected at the 60, 70, and 80 per cent stress levels.

Comparison of the Two Mixes

Since the form of the distribution of fatigue data was unknown, it was felt that a test should be employed to determine whether or not the two sets of data (LL and HL) represent populations having the same frequency distribution. A method called "Runs" was chosen for this purpose (9). When this test was used to test the data that was applicable, it indicated that, at the 5 per cent significance level, there was no reason to believe that the data at each stress level did not represent the same population.

The S-N Diagrams

A semi-log coordinate system was used to plot the S-N relationship. The data collected from the LL series are plotted in Figure 3 and the data collected from the HL series are plotted in Figure 4.

When the data from the HL series were plotted there appeared to be two separate and distinct relationships. The concrete of batches HL 5 and HL 6 seemed to have different fatigue characteristics than the concrete of batches HL 2, HL 3, and HL 4. The reason for this separation of data is unknown. Three curves were fitted to the fatigue test data, one for the LL series and two for the HL series. For purposes of calculation, the LL series will be referred to as sample I. Batches HL 2, HL 3, and HL 4 will be referred to as sample II and batches HL 5 and HL 6 will be referred to as sample III. Curves were fitted to each set of data by the method of least squares.

The least squares linear regression equation obtained from the data of sample I (LL series) is:

$$\widehat{\log N} = 13.077 - 0.118\ S$$

where N is the number of cycles to failure and S is the stress expressed as a percentage of the static ultimate stress. This equation can be used to estimate N only when the value S lies between 60 and 80 per cent because this is the range of S used in the calculations.

The least squares linear regression equation obtained from the data of sample II is:

$$\widehat{\log N} = 12.379 - 0.106\ S$$

The use of this equation is also limited to values of S between 60 and 80 per cent.

The least square linear regression equation obtained from the data of sample III is:

$$\widehat{\log N} = 11.077 - 0.085\ S$$

The use of this equation is limited to values of S between 60 and 80 per cent.

Calculations for correlation coefficients were also included in the linear regression analyses. For all three samples there is obviously a very high degree of association between cycles to failure and stress level. The correlation coefficients were found to be -0.952, -0.982, and -0.956 for samples I, II, and III respectively. Comparison tests showed that there is no reason to believe that these correlation coefficients are different. Hence, the degree of association between the stress level and the number of cycles to failure can be assumed to be the same for all three samples.

An analysis of covariance was used to determine whether the slopes of the three regression curves were significantly different. The computation form for this analysis is described in reference (8). The results of this test indicate that there is no reason to believe that the slopes of the three regression equations are different.

A two-way analysis of variance test was conducted on the combined data of all three samples to see if there was any difference in the intercepts of the regression equation. This analysis indicates that there is a significant difference between intercepts. A Newman-Kuels sequential range test showed that the intercept of sample III was different than the intercepts of the other two samples.

Prediction intervals were also calculated for the three S-N relationships. When the prediction intervals were compared it appeared that the data of sample III was different than the data of sample I and II.

The relationship between the stress level and the number of cycles to failure is represented most clearly by the slope of the linear regression equation. The results of the analysis of slopes suggest that the data of all three samples have the same slope and therefore suggest that all three samples have the same S-N relationship. The prediction intervals tend to weaken the conclusion that all three samples have the same S-N relationship but this test is not as strong as the analysis of slopes because individual errors rather than a total combined error is used. It is, therefore, felt that the strongest and most likely conclusion that can be drawn from this analysis is that the fatigue properties of all concrete tested in this study are the same regardless of differences in the static properties.

Effect of Rate of Load Application

Nine specimens were tested in each machine (Krouse-Purdue and Amsler) at the 80 per cent stress level and a t-test was made on the means obtained. Specimens tested in the Krouse-Purdue machine were tested at a rate of 1,000 cycles per minute and those tested in the Amsler machine were tested at a rate of 500 cycles per minute. The statistical analysis of these fatigue tests indicated very strongly that, between the limits of 500 and 1000 cycles per minute, the rate of load application has no effect on the fatigue properties of the lightweight concrete used in this study.

Summary of Fatigue Test Results

The results obtained from the analysis of fatigue test data may be summarized as follows:

1. The linear regression equation of the S-N relationships for the low-strength concrete for values of S between 60 and 80 per cent is:

$$\widehat{\log N} = 13.077 - 0.118\ S$$

2. The data of the high-strength series appeared to form two separate S-N relationships. The linear regression equations of these two S-N relationships for values of S between 60 and 80 per cent are:

$$\widehat{\log N} = 12.379 - 0.106\ S$$

and

$$\widehat{\log N} = 11.077 - 0.085\ S$$

3. There is no reason to believe that a difference exists between the fatigue properties of low-strength and the high-strength concrete. Hence there is no reason to believe that the gradations used in this study have any effect on the fatigue properties of lightweight aggregate concrete.
4. The S-N diagrams appear to show no fatigue limit for lightweight aggregate in the neighborhood of ten million repetitions of loading.
5. There is no reason to believe, from the results obtained on the 18 specimens tested, that the rate of load application has an effect on the fatigue properties of lightweight aggregate concrete when the rate of load application lies between the values of 500 and 1,000 cycles per minute.

<u>Comparison of Lightweight Concrete</u>

<u>With Normal Weight Concrete</u>

In May of 1959 the results of a study comparing the fatigue properties of air-entrained concrete with the fatigue properties of non-air-entrained concrete was reported (5). That study was made in the same laboratory as the present study. The testing programs were nearly identical except for the aggregates used. The coarse aggregate utilized in the 1959 study was a crushed limestone from central Indiana and the fine aggregate was from a local river terrace deposit.

A comparison was made to determine whether the fatigue properties of lightweight concrete are different from the fatigue properties of normal weight concrete observed in the earlier study. Statistical tests indicated that at the 5 per cent significance level there is no significant difference in the slopes of the S-N curves for lightweight and normal weight concrete. Since the slope is the property which best defines the S-N relationship, it seems reasonable to conclude that the fatigue properties of the concrete used in the present study do not differ from the fatigue properties of normal concrete.

A test for difference in correlation coefficients indicated that there is a significant difference between the correlation coefficients of the five S-N relationships tested. By comparing these coefficients visually it can be seen that the correlation coefficient for the non-air-entrained series reported in reference (5) is about 30 per cent lower than the other four values compared. This comparison indicates that the degree of association between stress level and number of cycles to failure is much higher for concrete containing entrained air.

CONCLUSIONS

The following conclusions can be drawn from the fatigue data collected in this study. These conclusions are based on the fatigue testing of forty-seven 3 inch by 6 inch lightweight aggregate concrete specimens tested in direct compression at a rate of loading which varied from 1,000 to 500 cycles per minute.

1. The fatigue properties of lightweight aggregate concrete are not significantly different over large variations in strength level of the concrete.
2. The fatigue properties of lightweight aggregate concrete are not significantly different from the fatigue properties of normal weight concrete.

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