A STUDY OF THE FATIQUE PROPERTIES OF AIR-ENTRAINED CONCRETE

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by

J. de ANTRIM

Joint Highway Research Project

PURDUE UNIVERSITY
LAFAYETTE INDIANA

FINAL REPORT

A STUDY OF THE FATIGUE PROPERTIES OF

AIR-ENTRAINED CONCRETE

TO: K. B. Woods, Director
Joint Highway Research Project

July 9, 1958

FROM: H. L. Michael, Assistant Director

File: 5-13-2
Project: C36-58-B

Attached is a final report entitled, "A Study of the Fatigue Properties of Air-Entrained Concrete," by John deCourcy Antrim. Mr. Antrim utilized this report as his thesis in obtaining the degree of Master of Science in Civil Engineering. The project was performed under the supervision of Prof. J. F. McLaughlin.

This report is presented for the record.

Respectfully submitted,

H. L. Michael

H. L. Michael, Secretary

HLM:acc

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A STUDY OF THE FATIGUE PROPERTIES OF AIR-ENTRAINED CONCRETE

by

John deCourcy Antrim
Graduate Assistant

Joint Highway Research Project
File No: 5-13-2
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Purdue University
Lafayette, Indiana

July 9, 1958

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ABSTRACT

Antrim, John deCourcy. M.S.C.E., Purdue University, August, 1958. A Study of the Fatigue Properties of Air-Entrained Concrete. Major Professor: Martin J. Gutzwiller.

This investigation was concerned with establishing whether or not there is a definite distinction between the resistance to fatigue of air-entrained plain concrete and non-air-entrained plain concrete. Two mix designs were used, one for the non-air-entrained concrete and one for the air-entrained concrete, each being designed to produce a compressive strength of 4000 psi. Cylindrical specimens, 3 inches in diameter by 6 inches in height, were used. Numerous cylinders were tested in static compression to arrive at an estimate of the ultimate static compressive strength of each type of concrete. Other cylinders were tested in compression at specific stress levels at a speed of 1000 cycles per minute in the Krouse-Purdue Axial-load Fatigue Testing Machine. A statistically sound analysis was insured by using over 50 specimens for the fatigue testing and over 200 specimens for the static testing.

It was found that within the limits of the investigation, the fatigue behavior of air-entrained plain concrete is similar to that of non-air-entrained plain concrete. The results also show, however, that the air-entrained concrete is more uniform than the non-air-entrained concrete with regard to both fatigue and static strength properties.

INTRODUCTION

As technology has progressed, there has been an increased need for information concerning the behavior of concrete under repeated applications of loads that are less than the ultimate. This type of loading, known as a fatigue loading, does not occur as frequently as the steady or static-type, but it is just as important a factor in design for those cases where it is present. Bridges which are subjected to loads varying from dead to dead plus live, and highway slabs which are subjected to repeated loads varying from practically zero to some maximum are two important examples of concrete in service being subjected to fatigue loadings.

A type of fatigue loading sometimes encountered is that in which the stress reverses during a cycle. In general, however, the particular fatigue limit of concrete which is of greatest concern is that limit for cycles of stress varying from zero to a maximum in one direction only.

Experience has shown that it is insufficient for the designer to know the fatigue strength of the concrete at a specified fatigue life, or in the case of certain metals, the value of the fatigue limit; he needs to know the shape of the S-N curve. From the S-N curve he can relate his particular working stresses to a fatigue life or, knowing the number of cycles of stress a structure will receive in its lifetime, he can select the maximum stress which will not cause failure.

The highway engineer, in addition to lacking adequate information on the fatigue of concrete, now needs information on the fatigue of air-entrained concrete because of the increased use of air-entrained concrete in pavements and reinforced concrete bridges. The available literature contains no information concerning the fatigue characteristics of this type of concrete. Over the past 15 years, the use of air-entrained concrete has increased considerably, and there is every indication that it will be used to an even greater extent in the future. At the present time, all but one state highway organization specifies the use of air-entrained concrete.

Although there has been no information reported on the fatigue behavior of air-entrained concrete, it is known from many studies (7)[a] that air-entrained concrete has certain properties which differ from those of non-air-entrained concrete. When concrete contains entrained air, the following are noticed:

1. The workability is improved.
2. The fine aggregate content can be reduced.
3. The water requirements can be reduced.
4. The modulus of elasticity decreases.

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[a] Numbers in parentheses refer to reference listed in REFERENCES CITED.

5. The compressive and flexural strengths may be reduced.
6. The durability in freezing and thawing may be increased.
7. Greater uniformity is obtained.

Since many properties of concrete are altered by the addition of entrained air, the question naturally arises as to what effect entrained air has on the fatigue properties of concrete. The work reported in this thesis had as its objective the establishment and comparison of S-N curves for non-air-entrained concrete and for air-entrained concrete.

SURVEY OF THE LITERATURE

A rather comprehensive literature search was conducted for articles in the English language related to or on fatigue of concrete. There was also conducted a less comprehensive survey of the literature on fatigue of metals that was limited to articles concerning the theory of fatigue failure and the more common fatigue properties of metals.

The information relating to metals is presented ahead of the information relating to concrete because it was felt that it provides a good background on fatigue behavior. A section on nomenclature has also been included so that the reader will be familiar with the terminology that appears throughout this thesis.

Nature of Fatigue

Fatigue is defined as the weakening and subsequent failure of a material resulting from a frequently-repeated stress. This stress is further defined as that stress which is less than the ultimate strength of the material as determined from a static test. When a material is subjected to repeated stresses of sufficient magnitude, a microscopic crack is produced, which, under the action of the repeated stresses, spread until failure occurs (1).

There are three basic forms of stresses that are encountered in fatigue action (11):

1. Stresses that reverse from tension to compression.
2. Stresses that may only be partly reversed.
3. Stresses that fluctuate from a maximum to a minimum yet remain entirely tensile or compressive.

Fatigue failure is shown by most metals and metallic alloys, by some plastics, by woods and plywoods, by concrete, and probably by other materials that have yet to be investigated with respect to fatigue. The majority of fatigue investigations have been conducted in the field of metals. From tests on these materials there has been collected a wealth of experimental data that permits explanations of some of the aspects of fatigue action.

In the case of metals, fracture always occurs through the crystals and not along the boundaries. The cracks start in regions of high local stress where a considerable amount of slip has taken place and they spread along slip, twin, or cleavage planes; but never along grain boundaries. For both the ranges of stress which cause failure and the ranges of stress which do not cause failure, slip lines appear and increase in number as the test goes on but the rate of appearance of new slip lines decreases because of strain hardening. If a safe range of stress is applied, a stage is finally reached where no additional slip lines develop and no serious damage to the lattice structure takes place.

With an unsafe range of stress, the production of slip lines does not come to a standstill, and lattice distortion is observable in many crystals after only a few cycles. As the number of cycles increases, the extent of distortion and the number of damaged crystals also increases (1).

The fatigue failure as applicable to metals can be summarized as follows:

1. Slip occurs, the result being strain hardening and lattice distortions.
2. The fatigue crack starts, its nucleus generally being positioned at some surface notch, nick, scratch, flaw, or sharp change of section or contour.
3. The crack spreads along the path of least resistance, due to stress concentration. The spreading continues until the cross-section of the metal is reduced so much that the remaining portion tears or breaks away suddenly; thus, fatigue failure usually exhibits two zones: the brittle zone due to the real fatigue action, and the fibrous zone having the same appearance as a failure under static stress.

Nomenclature

The nomenclature used by the early investigators when reporting their fatigue studies was not consistent. In 1949, the American Society for Testing Material's

Manual on Fatigue Testing (11) presented a standardized set of symbols and definitions. The nomenclature from this ASTM Manual which is applicable to this thesis is as follows:

Stress Cycle. - A stress cycle is the smallest section of the stress-time function which is repeated periodically and identically.

Nominal Stress, S. - The stress calculated on the net section by P/A without taking into account the variation in stress conditions caused by geometrical discontinuities such as holes, groves, fillets, etc. (For the purposes of this investigation "S" will be the percentage of the ultimate strength of the net section).

Maximum Stress, $S_{max.}$ - The highest algebraic value of the stress in the stress cycle, tensile stress being considered positive and compressive stress negative.

Minimum Stress, $S_{min.}$ - The lowest algebraic value of the stress in the stress cycle, tensile stress being considered positive and compressive stress negative.

Range of Stress, $S_{r.}$ - The algebraic difference between the maximum and minimum stress in one cycle, that is, $S_r = S_{max} - S_{min}$.

Mean Stress, S_m. - The algebraic mean of the maximum and minimum stress in one cycle, that is, $S_m = 1/2(S_{max} + S_{min})$.

Stress Ratio, R. - The algebraic ratio of the minimum stress to the maximum stress in one cycle, that is, $R = S_{min}/S_{max}$.

Fatigue Life, N. - The number of stress cycles which can be sustained for a given test condition.

S-N Diagram. - A plot of stress against number of cycles to failure.

Fatigue Limit (or Endurance Limit), S_e. - The limiting value of the stress below which a material can presumably endure an infinite number of stress cycles, that is, the stress at which the S-N diagram becomes horizontal and appears to remain so.

Fatigue Strength, S_n. - The greatest stress which can be sustained for a given number of stress cycles without failure.

Figure 1 illustrates a fluctuating compressive stress and further explains, diagrammatically, some of the above terms.

The S-N Diagram

A popular way of representing fatigue test results is through the use of the S-N diagram. The stress, S, is commonly plotted on the ordinary cartesian scale and the number of cycles, N, may be similarly plotted. More commonly N is

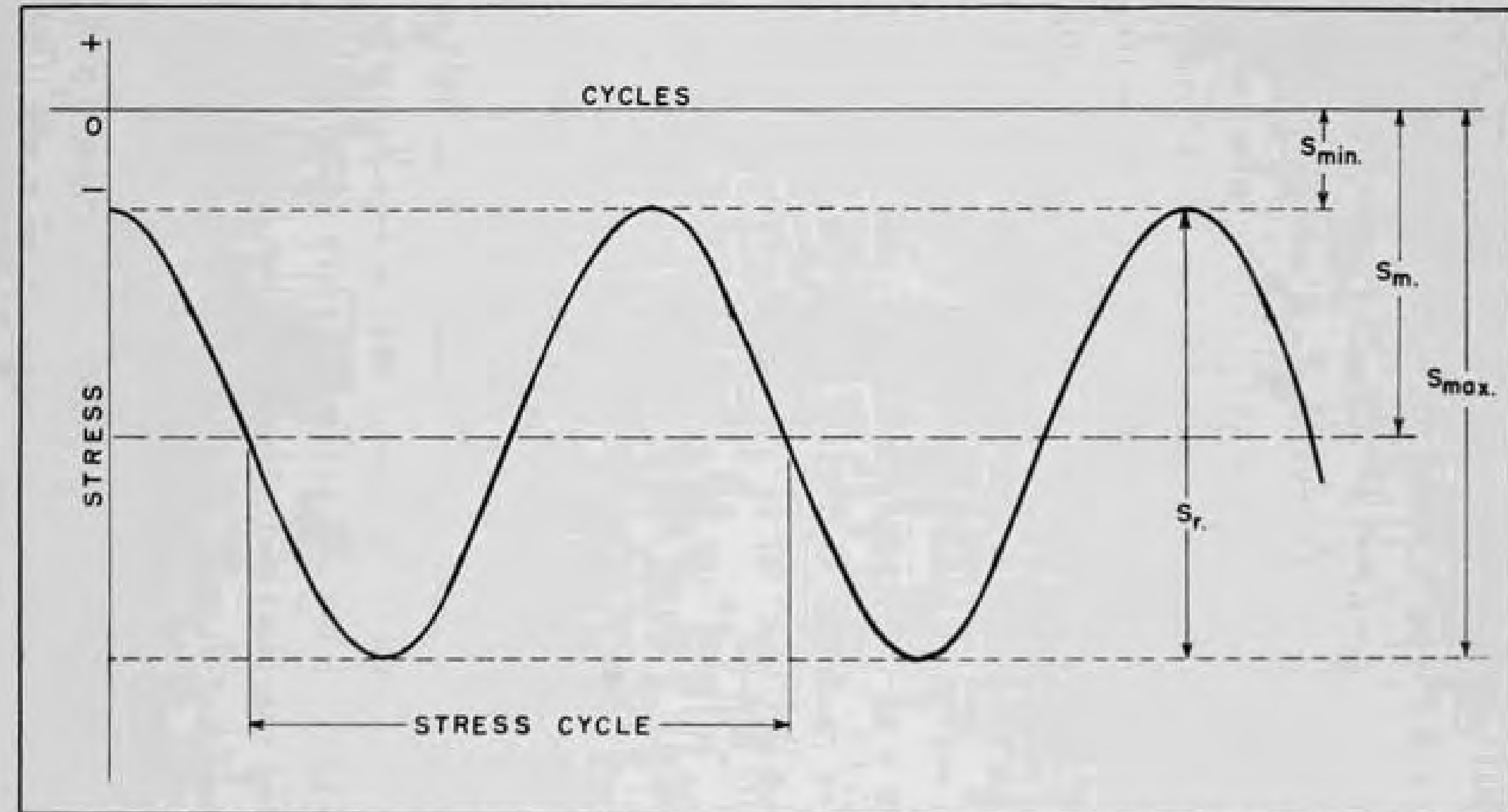

FIGURE I. TYPICAL FLUCTUATING COMPRESSIVE STRESS ENCOUNTERED IN FATIGUE TESTING plotted on a logarithmic scale ("semi-log" plot). In addition, it is sometimes convenient to plot the stress on a logarithmic scale ("log-log" plot). When the arithmetic method of plotting is used, the fatigue limit chosen is different from the limit chosen when the semi-log method of plotting is used. This difference is a result of the left hand portion of the S-N curve having considerably greater curvature than that encountered in the curves resulting from the other two methods of plotting. Figure 2 illustrates this difference in the S-N curves. The fatigue limit selected is usually at a lower number of cycles than is actually the case. Another advantage of the semi-log and log-log methods of plotting is that by using the log scale for N the percentage of error is the same for both small and large values of N (13).

Fatigue Properties of Metals

Studies of the fatigue behavior of ferrous metals and ferrous alloys have been sufficient in number to permit the establishment of a number of well defined patterns which are characteristic of metals undergoing fatiguing action. On the other hand, fatigue studies related to concrete are small in number and since certain similarities have been observed between the fatigue behavior of concrete and metals, a knowledge of the behavior of the latter is helpful in understanding the behavior of concrete.

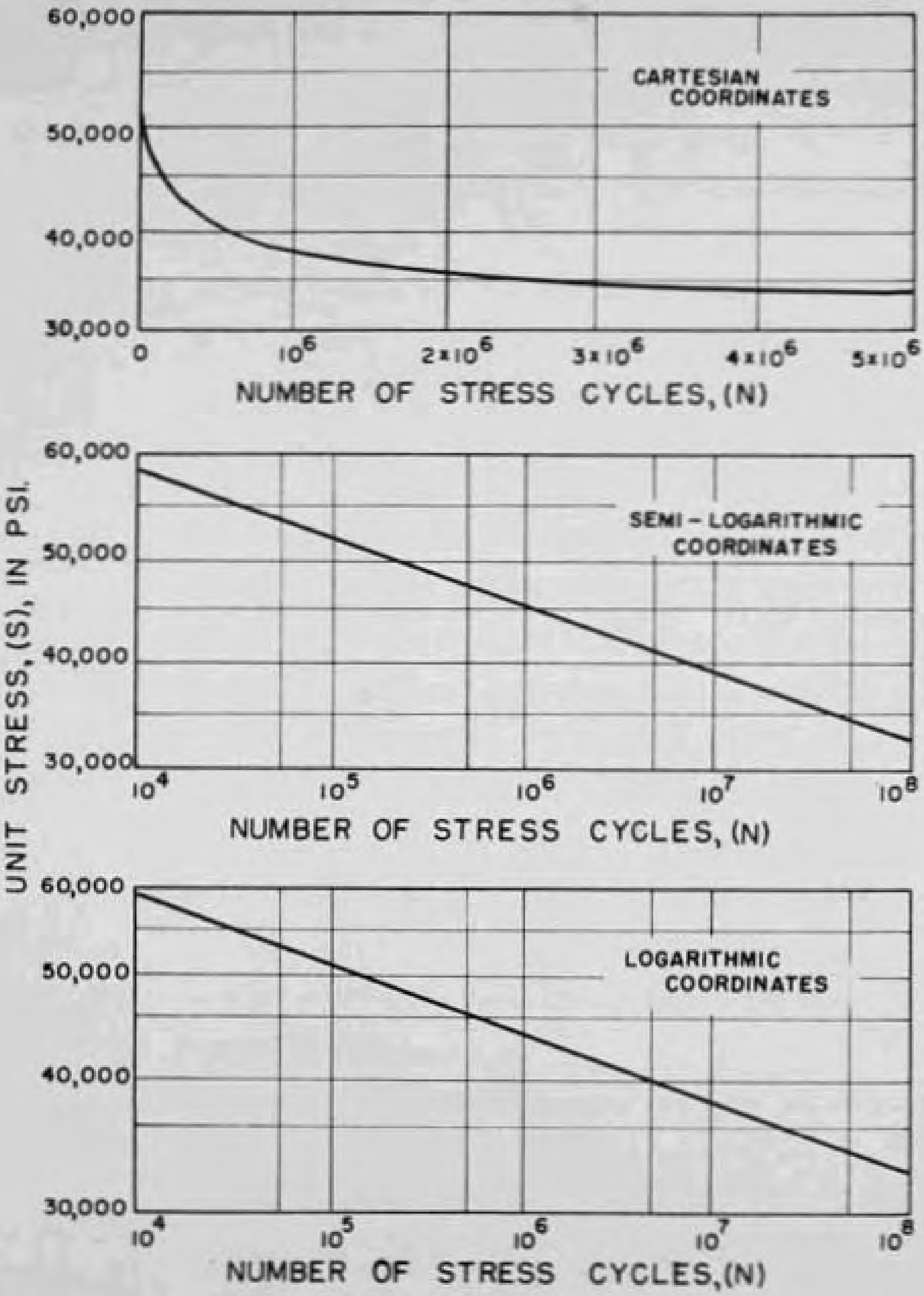

FIGURE 2. THE THREE TYPES OF S-N DIAGRAMS

In many cases, the behavior of one metal is not the same as another. However, there are a number of features which are common to all metals. This basic nature of fatigue can be summarized from the many references reviewed as follows:

1. Ferrous metals and ferrous alloys have a definite fatigue limit, that is, the S-N curve for these materials becomes practically horizontal.
2. Most nonferrous materials have no definite fatigue limit and the fatigue strength must be reported in terms of both a stress and the number of cycles endured at that stress.
3. Flaws or discontinuities either on the surface or in the interior of a piece have a detrimental effect on the range of stress which may be withstood for a given number of cycles.
4. Speed of testing and temperature under normal conditions have little, if any, effect on the fatigue strength of metals except under special conditions. High temperature is one of these special conditions.
5. If a metal is repeatedly stressed just below its fatigue limit (understressing), it may gain strength. When tested at a higher stress, its fatigue life is generally longer than the

fatigue life of a virgin piece initially stressed at the higher level.

6. If a metal is repeatedly stressed above the fatigue limit short of failure (overstressing), it will ultimately fail when tested at a stress below the fatigue limit.
7. At a given fatigue life the range of stress necessary to produce failure usually decreases as the mean stress is increased.

Numerous efforts have been made to correlate fatigue strengths with other properties of the materials (19). The fatigue strength of ferrous metals subjected to completely reversed stresses is best correlated with the tensile strength. This fatigue limit is generally about 50 percent of the ultimate tensile strength. However, the fatigue limit may vary between 40 and 60 percent of the ultimate tensile strength. Hardness is another property which shows a fair correlation with the fatigue limit when the Brinell and Rockwell hardness numbers are used in the comparison. Proportional limit, yield point, ductility, and impact values of wrought ferrous metals show very poor correlation with the fatigue limit. In addition, very poor correlation is shown in the case of nonferrous metals, where the fatigue limit varies from 18 to 50 percent of the ultimate tensile strength.

Fatigue Properties of Concrete

Information pertaining to the fatigue properties of concrete can be divided into a number of categories but for the purposes of this thesis the information is presented in two parts. The first part pertains to the fatigue of plain concrete and it is for this part that detailed information has been obtained, since the study was confined to plain concrete. The second part is concerned with the fatigue properties of reinforced concrete and prestressed concrete.

Plain Concrete

Most of the investigations concerned with the fatigue behavior of plain concrete were conducted during the period from 1900 to 1928. Although test procedures differed, some of the results from each investigation are similar. Probably the most important discovery was that the fatigue limit of plain concrete appeared to be approximately 55 percent of the ultimate static strength of the concrete.

Van Ornum (20) was the first to investigate the fatigue of concrete in compression. Tests were performed on 2-inch cubes of neat portland cement at the age of four weeks. The ultimate strength of the cubes was determined by static tests and then similar cubes were subjected to specific percentages of the ultimate strength, the range being from 55 to 95 percent. A minimum load of 4 percent of the ultimate strength was maintained and the testing was conducted at a rate of four repetitions per minute. Van Ornum's interpretation of an arithmetic plot of the data was that the fatigue limit for neat portland cement cubes lies in the vicinity of 55 percent of the ultimate strength. Later on, however, Moore and Kommers (13) plotted the same data on logarithmic coordinates, and they concluded that although the tests were not carried out to a sufficient number of cycles, there was an indication that the fatigue limit is about 50 percent of the ultimate static strength.

In a later series of tests, Van Ornum (21) investigated concrete beams in compression. The 5- by 5- by 12-inch prisms were cured in water for two weeks and then stored in air until testing. Testing was done at the ages of one month and one year, using a minimum load of 500 pounds and 4 to 8 repetitions of stress per minute. Van Ornum found that the curve for modulus of elasticity of the beams undergoing repeated stress, showed an increase for the second loading, a decrease during the next few loadings to its original value, and then a gradual straight-line decrease during the greatest portion of the test, terminating in a downward curve as the failure point was approached. Those specimens observed when the maximum stress was below the fatigue limit had a modulus curve through the first two stages similar to that described. Thereafter, it kept a constant value. Although he describes the shape of the modulus curve, Van Ornum does not mention how the modulus of elasticity was measured.

Another phenomenon observed by Van Ornum was a permanent set which occurred in all specimens during the first few loadings. For those specimens which finally failed, the permanent set became comparatively small during the third and fourth stages. Moore and Kommers (13) plotted the results of Van Ornum's tests on logarithmic coordinates; and concluded that there is some evidence of a break in the curve for the one year old beams which indicates a fatigue limit of 55 percent of the ultimate static stress. They also concluded that there is no indication of a fatigue limit for the one month old beams.

Clemmer (3) conducted flexure tests on 6- by 6- by 36-inch beams using a 1:2:3 1/2 mix and a 1:3:5 mix. A cantilever action, which repeated 40 times per minute on each beam, was induced by loading the free ends of the beams with rubber-tired wheels actuated by an axle. A definite loading period was used with the load being increased after each loading period until failure occurred. Clemmer concluded; a) that the fatigue limit of the concrete was 51 to 54 percent of the modulus of rupture as determined from one application of load, b) that repetitions of loads which produced less than the critical stress apparently increased the strength of the concrete, c) that a stress below the fatigue limit did not cause permanent deformation, and d) that for the same percentage of ultimate strength, a considerable reduction was found in the number of repetitions of loads which were required to cause failure in the 1:3:5 mix than was the case for the 1:2:3 1/2 mix.

Crepps (4) reported in 1923 on a continuation of the tests started by W. K. Hatt at Purdue University. Mortar beams, 4 by 4 by 30 inches, were tested by placing the beams upright and loading them by a bracket fixed to one end. A full reversal of bending stress was applied at a rate of 10 cycles per minute. It was found that the fatigue limit for the mortar beams tested after 28 days of curing appeared to be between 40 and 60 percent of the ultimate strength. For the four month tests, the limit was approximately 50 to 55 percent of the ultimate strength, and for the six month tests the limit was 54 to 55 percent of the ultimate strength. Another observation reported was that mortar undergoing a fatiguing action had an appreciable recovery of strength during a period of rest. A five week rest period occurred when the beams were five months old, and it was though that the strength did not increase through normal aging during the rest period.

Probst, as reported by Moore and Kommers (13), performed tests on 7- by 7- by 28-centimeter concrete compression specimens. The tests were concerned with the elastic properties of the concrete. The results Probst obtained reinforced the conclusions drawn by Van Ornum. Specimens undergoing fatigue action were subjected to a maximum stress of 38 percent of the ultimate static strength. The modulus of elasticity of these specimens reached a constant value of about 79 percent of the value of unfatigued specimens at the same stress. According to Moore and Kommers (13), Van Ornum's tests showed that a modulus of elasticity value of 67 percent of the value of unfatigued specimens existed at a maximum stress of 50 percent of the ultimate static strength.

Impact fatigue tests were part of an investigation on plain concrete conducted by Thompson (18). Four beams were tested using a testing program of 60 blows per minute for 3 hours and a rest period of 21 hours, the cycle being repeated continuously for 30 days. The beams were 277 days old at the start of the tests, and the fatigue limit appeared to be 55 percent of the ultimate strength.

Mehmel's work is also reported by Moore and Kommers (13), and like Probst, Mehmel was concerned with the elastic properties of the concrete. The 7- by 7- by 28-centimeter specimens that he used were subjected to stresses from a lower limit near zero to an upper limit which caused failure, and to an upper limit which did not cause failure. Mehmel concluded from his tests that the determination of the fatigue limit of concrete is measured much better by the change in the slope of the curve of elastic deformations. Another conclusion was that the fatigue limit of concrete subjected to stresses from zero to a maximum value lay between 47 and 60 percent of the ultimate static compressive strength.

Additional tests at Purdue University were reported by Mills and Dawson (12) in 1927. Mortar beams were subjected to two types of curing: a) aged 19 months and saturated by immersion in water for 200 hours prior to test and b) aged 4 months and kept continuously saturated. These beams were then tested in flexure. Under progressive loading, the mortar which was cured by method "a" showed a strength of 89 percent of a dry mortar. In the case of the mortar cured by method "b", the strength was 83 percent of the standard dry mortar. The fatigue limit of the mortar cured by method "a" was found to be 37 percent of its static strength and 33 percent of the strength of the standard dry mortar. The fatigue limit for the mortar cured by method "b" approached the values of 45 percent of its static strength and 37 percent of the strength of the standard dry mortar.

Williams (23) performed reversed and repeated loading tests on 4- by 5 1/8- by 32 1/2-inch concrete beams made with Haydite. The beams were tested in an upright position with the bottom end fixed and the upper end secured in a loading beam. The beams were made in pairs from a single batch and were kept in the moist room from 24 hours after casting until testing. It was found that below one million cycles of load repetition, the Haydite beams did not fail if the stress was below 40 and 50 percent of the static modulus of rupture for reversed and repeated loading, respectively.

The effect of speed of testing on the fatigue properties of plain concrete was investigated by Kesler (10) at the University of Illinois. Ninety-six 6- by 6- by 64-inch beams of two different design strengths (3600 psi and 4600 psi) were tested in flexure by loading at the third points on a 60-inch span. The minimum load was kept at less than seven percent of the maximum load. Testing speeds were 70, 230, and 440 cycles per minute. Curing was seven days moist and then in normal room atmosphere for three months or more until testing. Testing was continued to a maximum of ten million cycles, and no specimen failed at a stress less than 55 percent of the static stress. It was concluded that the speed of testing has practically no effect on the results.

The results of recent tests at the University of Illinois were reported by Murdock and Kesler (14). A total of 175 plain concrete beams, 6 by 6 by 64 inches, of two consistencies (wet and dry) and a design strength of 4500 psi were tested in fatigue. The testing speed varied from 400 to 440 cycles per minute and the loading arrangement was the same as that used in the previously mentioned tests. At ten million repetitions of stress, the fatigue strength was found to be 85, 73, 63, and 61 percent of the ultimate strength for minimum to maximum stress ratios of 0.75, 0.50, 0.25, and 0.13 to 0.18 respectively. They concluded: a) plain concrete subjected to repeated flexural loading exhibits no fatigue limit, at least through ten million cycles, b) the repeated loads which plain concrete may sustain for a finite number of repetitions without failure is a critical percentage of the static ultimate flexural strength and this percentage is a function of the range of stress to which the concrete is subjected, and c) the difference in consistency had no effect on the fatigue behavior.

Not many general conclusions can be drawn regarding the fatigue of concrete since an analysis of the results of all the previously described investigations shows that the methods of investigating the problem varied considerably. Loading arrangement, rate of loading, type and size of specimen, and age of specimen are only a few of the differences existing between investigations. The following general statements concerning properties of concrete summarize much of what appears in the literature:

1. The earlier investigations indicate that there is a definite stress that is approximately 50 to 55 percent of the ultimate static stress below which concrete can undergo fatiguing action indefinitely, and above which the number of cycles to failure decreases as the stress increases.

2. There appears to be little variation in this limit that is due to type of testing (compression, flexure, and tension) and speed of testing.
3. Permanent set occurs during the earlier stages of the fatigue action. If the maximum stress is below the fatigue limit, the permanent set reaches and maintains a constant value. Stresses above the fatigue limit cause progressive deformation.
4. When the applied stress is below the fatigue limit, the modulus of elasticity reaches and maintains a constant value.
5. The repetitions of a stress which is below the fatigue limit appears to increase the strength of concrete.

Nordby (15) goes further in summarizing the results of these same investigations and his conclusions are quoted below:

1. Under repetitive load the modulus of elasticity changes in various ways depending upon the intensity of load. The secant modulus decreases with repeated load; the slope of the stress-strain curve may decrease in the lower part of the curve and increase slightly in the upper portion to become concave upward.
2. Age and curing has a decisive effect on the fatigue strength. Inadequately aged and cured concrete is less resistant to fatigue than well-aged and cured concrete.

3. Rest periods seem to increase the endurance of concrete although test results are very scant.

4. Fatigue strength decreases slightly with leaner mixes and higher water cement ratios (data not extensive).

5. As the range of stress is decreased the upper limit of the stress (fatigue strength) is increased substantially. This phenomenon can be represented by the Modified Goodman Diagram.

One very important result obtained by all the earlier investigators is no longer believed to be true. These earlier investigators were of the belief that there existed a fatigue limit for concrete. The present day concept of fatigue of concrete is that there probably is not any fatigue limit, but rather that the S-N curve resulting from the plotted data will continue to slope downward, never quite becoming horizontal.

Reinforced and Prestressed Concrete

In addition to the investigations on fatigue of plain concrete which have been mentioned in detail, there have been investigations into two other areas of equal importance: fatigue of reinforced concrete and the fatigue of prestressed concrete. The fatigue of reinforced concrete and the fatigue of prestressed concrete are covered in Nordby's (15) recent and extensive review of literature pertaining to the fatigue of concrete.

Regarding the fatigue of reinforced concrete, Nordby states that there is a considerable need for more information before the subject will be completely understood.

Failure of reinforced concrete beams can result from the failure of the steel or the failure of the concrete; the concrete in turn may fail in flexure, diagonal tension, or bond. Nordby lists certain features which he found reappearing in many of the investigations, and they are quoted below:

1. Most failures of reinforced beams were due to failure of the reinforcing steel. The failures seemed to be connected with severe cracking and the possible stress concentration and/or abrasion connected these cracks. Beams critical in longitudinal reinforcement seemed to have an endurance limit of 60 - 70% of the static ultimate strengths for 1 million cycles.

2. Oftentimes it was pointed out that the concrete in the compression zone behaves in much the same way as axially loaded compression specimens. There is certainly no assurance of this, since it is not true for static tests and of course there is a strain gradient in the beam which does not exist in compression specimens. No fatigue compression failures were noted in any of the beams reported except those of Le Camus.

3. On occasion beams failed in diagonal tension fatigue but the real cause of failure was obscured by bond and shear combination failures. Tests have been reported in which beams have failed in shear in repeated loads as low as 40% of the ultimate strength. Data is very scarce on this phase.

4. Beams accumulate residual deflections under extensive fatigue loading much the same as plain concrete specimens; but recover somewhat during rest periods.

Turning to prestressed concrete, Nordby states that from the fatigue standpoint, the methods of failure are essentially the same as for conventionally reinforced beams with the addition that for post-tensioned beams, the fatigue failure can be in the anchorages and splices. An important factor in the fatigue of prestressed concrete is that, in the working load range, the variations in stress are small but they vary about some high mean stress in both the steel and the concrete. In summarizing the prestressed concrete results, Nordby cautions against regarding any conclusions as being definite, but lists the following:

1. In none of the tests did concrete fail by fatigue. The current working stresses seem to give adequate protection in this regard.

2. Fatigue failure of stressing wires or strands was the cause of all failures reported. These failures seemed to be related to the extent and severity of the cracks.

3. Bond failures were rare and were found only under unusual circumstances, i.e., short beams, short shear span.

4. The ultimate strengths of prestressed beams for static loads was unaffected by repetitive loading if they did not fail by fatigue.

5. Safety factors seemed to be approximately "two" against fatigue failure for most of the beams tested.

6. Prestressed beams seemed superior to conventional beams for resisting fatigue loading. In fact, in a recent paper, Eckberg and Walther analytically verified this by relating the modified Goodman diagram of both the concrete and prestressing steel to the theoretical stresses in both types of beam.

PURPOSE AND SCOPE

The purpose of this study was to establish whether or not there is a definite distinction between the resistance to fatigue of air-entrained plain concrete and non-air-entrained plain concrete.

Fatigue tests were performed on specimens of two types of concrete, each type being designed for the same 28-day compressive strength. The one concrete contained only "accidental" air, while the other contained intentionally entrained air which was maintained at a constant level. Mixes were prepared periodically from each mix design so that there was little variation in the ages of the specimens being tested in fatigue. The fatigue test specimens, which were selected from each mix, were tested in fatigue at several different stress levels. These stress levels were 50, 60, 70, 80, and 90 percent of the ultimate static compressive strength of the respective mixes.

TESTING PROGRAM

The tests required for this study were done in two parts. The first part consisted of performing tests on specimens made from a non-air-entrained concrete mix. This mix will be referred to as the FN series. The second part consisted of performing tests on specimens made from an air-entrained concrete mix. This mix will be referred to as the FA series. Each series consisted of eight batches made periodically during the course of the investigation. In general, a batch yielded about twenty-five concrete cylinders, 3 inches in diameter by 6 inches in height.

After 28 days of curing and 3 to 4 days of oven drying to dry the concrete and prevent further hydration, eight specimens from each batch were tested for their ultimate static compressive strength. The average ultimate strength of these eight specimens served as an estimate of the ultimate strength of the batch, and the stress levels at which the fatigue tests were performed were based on this estimate of the batch strength. Following the completion of the fatigue tests on each batch, the remaining specimens in the batch were tested for their ultimate static compressive strength. The object of these later compression tests was to determine if there was any further strength gain in the concrete during the time taken by the fatigue tests.

Additional tests performed in this study were: 7-day compression tests, linear traverse determinations of the air contents of two of the batches, and tests to measure deformation characteristics of the two types of concrete. These tests are described in detail in this section.

Materials

The coarse aggregate (material retained on the No. 4 sieve) used in this investigation was a crushed limestone from central Indiana. The source (laboratory designation 67-25) is located in the St. Genevieve formation and is of the Mississippian Age. This material is fine grained and has a low porosity. The aggregate as obtained from the quarry met the specifications of the State Highway Department of Indiana for size number 5. It was then further crushed in the laboratory to a maximum size of 1/2-inch. The resulting gradation and the physical properties of the coarse aggregate are shown in Table 1.

The fine aggregate (material passing the No. 4 sieve) was a local sand obtained from a river terrace deposit (laboratory designation 79-1). This material met the specifications of the State Highway Department of Indiana for gradation 14 No. 2. The physical properties of this concrete sand and the gradation used in this study are shown in Table 2.

Type I portland cement manufactured in central Indiana and from a single clinker batch (laboratory designation 315) was used in both mixes. Because the cement was from one lot, it was assumed that its characteristics did not vary

TABLE 1

CHARACTERISTICS OF COARSE AGGREGATE 67-2S

Dry rodded unit weight .... 94.3 lbs./cu. ft.

Bulk specific gravity 2.68*

Absorption 0.7%*

* Previously established laboratory value

Gradation

Sieve size	Percent finer
1/2 inch	100
3/8 inch	50
1/4 inch	25
No. 4	0

TABLE 2

CHARACTERISTICS OF FINE AGGREGATE 79-1

Bulk specific gravity 2.61*

Absorption 1.65%*

* Previously established laboratory value

Gradation

Sieve size	Percent finer
No. 4	100
No. 8	87
No. 16	63
No. 30	20
No. 50	3
No. 100	1

Fineness modulus 3.25

significantly and therefore a chemical analysis has not been included.

Darex, added at the mixer, was used as the air entraining agent for air-entrained concrete.

Mix Design

The concrete mixes were designed by the b/b_o method (9), which is essentially the same as the "Recommended Practice for Selecting Proportions for Concrete" published by the American Concrete Institute (17). Both mixes were designed for similar strength and slump.

Trial mixes were used to establish the required water-cement ratios and proportions of coarse and fine aggregate. The trial mixes also served to establish the required amount of air entraining agent in the case of the air-entrained mix.

The non-air-entrained mix was designed for a strength of 4000 psi and a slump of three inches. The mix had a cement factor of 5.4 sacks per cubic yard of concrete and a water-cement ratio of 0.69 by weight.

The air-entrained mix was designed for a strength of 4000 psi, a slump of three inches, and an air content of seven percent. The mix had a cement factor of 5.0 sacks per cubic yard of concrete and a water-cement ratio of 0.61 by weight.

The differences in the cement factor and the water-cement ratio were the outcome of designing the two mixes for similar strength and consistency.

Mixing Procedure

The coarse aggregate, in a room-dry condition, was immersed in water for 24 hours before mixing to satisfy its absorption requirements. The fine aggregate which was stored in the laboratory in an oven-dried condition, was also saturated by adding water to it 24 hours prior to mixing. The excess water in the aggregates was determined immediately before mixing, and this amount was deducted from the calculated quantity of mixing water. When the air entraining agent was used, it was considered as part of the mixing water. No adjustment was made in the amount of mixing water since the purpose was to maintain a constant water-cement ratio for each batch within the given mix. If the slump was zero or exceeded six inches, the batch was discarded and a new batch was prepared. The mixing was done in a 1-1/2 cubic foot capacity Lancaster tub type counter current mixer. The total mixing time was four minutes for every batch; the first minute for the aggregates alone, the second minute with the cement added, and the last two minutes for the aggregates, the cement, and the mixing water.

Two measurements were made on the plastic concrete from each batch: slump and air content. The slump was determined in accordance with ASTM Designation:C143-52. Air content was measured gravimetrically according to ASTM Designation:C138-44, except that a 1/10 cubic foot measure was used because of the small volume of concrete made per batch. To eliminate any possible contamination of the fresh concrete remaining in the mixer, the fresh concrete used for the two measurements was discarded.

Molding and Curing of Specimens

Twenty-five cylindrical specimens, 3 inches in diameter and 6 inches in height, were cast from each batch in accordance with a procedure outlined in ASTM Designation:C192-52T. One modification of the standard procedure was necessitated by the use of the 3-inch diameter molds, and that was the substitution of a 3/8-inch round rod for the standard 5/8-inch rod.

Curing followed the procedure specified in ASTM Designation:C192-52T. Following casting, the specimens were covered to prevent evaporation and were stored at room temperature in the molds for 24 hours. They were then removed from the molds and stored for 27 days in a saturated lime solution having a temperature of 67 F.

Preparation of Specimens for Testing

The specimens from both the FN series and the FA series were prepared in essentially the same manner. Some minor differences that did occur resulted from scheduling difficulties and the availability of laboratory help. These differences are made clear in the following text and were assumed to have no effect on the results.

Drying

When the specimens were 28 days old, they were removed from the saturated lime solution and dried in a gas-fired oven at a temperature of approximately 220 F until constant weight was obtained. Upon removal from the oven, the specimens were stored under room conditions until testing. The FA specimens were kept in the oven one day longer than the FN specimens only because it was more convenient to remove them on the fourth day.

Capping

Each specimen was capped on both ends with a sulfur compound soon after removal from the oven; 48 hours after removal for the FN specimens and 24 hours after removal for the FA specimens. The capping compound (trade name: Vitrobond) is a sulfur and carbon mixture which is easily poured when its temperature is approximately 275 F. The manufacturer's description of the compound includes the following: a) tensile strength - 550 psi, b) modulus of rupture - 1600 psi, and c) compressive strength - 6000 psi.

A capping device which is shown in Figure 3 was especially designed for the 3-inch diameter cylinders. Cap thicknesses ranged from approximately 1/16 to 3/16 of an inch, depending on the end conditions of each specimen.

Figure 3. Capping Device

Static Compression Tests for Estimating Batch Strength

Twenty-four hours after capping, eight specimens were chosen at random from the batch and tested in compression for their ultimate strength. These static compression tests were performed in a Riehle screw type testing machine modified by the addition of a Graham variable speed drive. The machine has a capacity of 50,000 lbs. The no-load head speed of the testing machine was set at 0.05 inches per minute as specified in ASTM Designation:C39-49.

Fatigue Tests

The specimens for the fatigue tests were chosen at random from the remaining specimens in each batch, the eight specimens used for determining an estimate of the batch strength having been previously selected.

The fatigue tests were conducted, when conditions permitted, at five different stress levels. These stress levels were 50, 60, 70, 80, and 90 percent of the estimated ultimate strength of the batch. A minimum stress of approximately 70 psi was maintained regardless of the magnitude of the maximum stress. No particular order was observed when testing the specimens, the 50 percent specimen from one batch may have been tested first, while for another batch, the 80 percent specimen may have been tested first.

The process of establishing the desired maximum and minimum loads on the fatigue testing machine required a considerable number of cycles; therefore the loading was set by first inserting a dummy specimen in the machine. When the loads were established, the machine was stopped and the dummy specimen was replaced by a test specimen. Upon restarting the machine, usually little or no adjustment was necessary to obtain the desired loads.

No specific data could be obtained at the 90 percent stress level for the PA series because the test specimens failed before the maximum and minimum loads could be verified on the load-measuring apparatus. Test data at the 50 percent stress level are incomplete (fewer than eight specimens tested) for both series because it was deemed unnecessary to complete the tests at this level when the first few specimens tested at the 50 percent level withstood ten million cycles and similar specimens tested at the 60 percent level withstood ten million or nearly ten million cycles. Ten million cycles was selected at the beginning of the study as the maximum number of cycles any specimen would be permitted to endure. Other missing data are the result of machine stoppage while a test was in progress, and subsequent lack of time to re-run the particular test on a new specimen.

In every case, a virgin specimen was used to obtain the information that is reported in this thesis. If a test was interrupted for any reason, it was started over again with a new specimen.

The Krouse-Purdue Fatigue Machine

The Krouse-Purdue axial-load fatigue machine that was used in this study is shown in Figure 4. It is of the constant deflection type and derives its force from hydraulic pressure acting on a large piston directly connected to the test piece through a piston rod.

The load applied by the machine consists of two components: a preload and a pulsating load. The preload is directly proportional to the average differential pressure existing between the two ends of the hydraulic cylinder, and it is controlled by automatically controlling make-up oil pressures. The amount by which the pulsating load varies above and below the preload depends on the throw of a variable-throw crank that can be adjusted before or while the machine is operating. The deflection characteristics of the specimen, the specimen holder, the load screw, and the piston rod also influence the magnitude of the pulsating load. Figure 5 is a simplified line diagram of the hydraulic system of the machine.

Holding the specimen in place is a load screw which extends through the top head of the machine. This load screw may be adjusted for the required testing space by means of a reversible motor driving a rotating nut. When the desired screw setting is reached, a clamping nut is closed on the screw by means of a hydraulic cylinder. This action prevents an end motion of the screw when acted upon

Figure 4. General View of Fatigue Machine

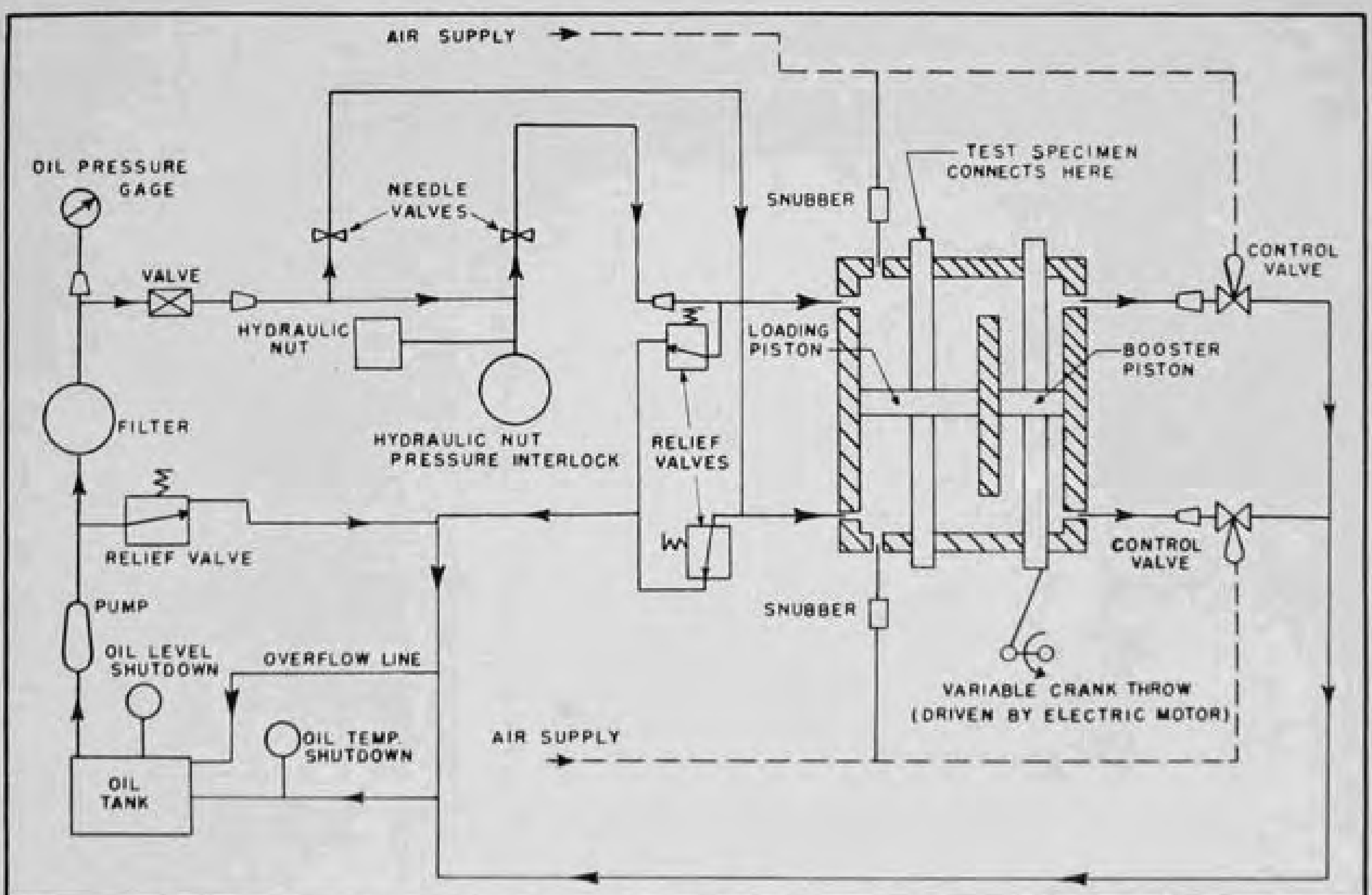

FIGURE 5. HYDRAULIC SYSTEM OF FATIGUE MACHINE

by the testing force.

The operating speed of the fatigue machine is 1000 cycles per minute and the machine has a maximum loading capacity of ± 60,000 lbs. The magnitude of the applied loads is measured by means of an electronic system which is actuated by a load cell that is a part of the lower end of the load screw. This portion of the apparatus is shown in Figure 6.

The load cell consists of a 3-inch diameter steel cylinder to which are attached four Type A-5 resistance strain gages (two vertical and two horizontal) connected in a full bridge circuit.

Two load reading systems were in operation during the testing program, one of which was in use during the testing of the FN series and the other of which was in use during the testing of the FA series. The first system consisted of a Baldwin strain indicator whose output was observed on a cathode-ray oscilloscope. The oscilloscope served to determine the existence of an instantaneous balance of the bridge corresponding to the maximum or to the minimum load. Loads with this system could be measured to within ± 500 lbs. The second system, which is an improvement of the first system, measures the loads in the same manner as does the first. It consists of a bridge bounce unit, an oscillator, an amplifier, and an oscilloscope. One of the advantages of the second system is that the loads can be measured to within ± 100 lbs.

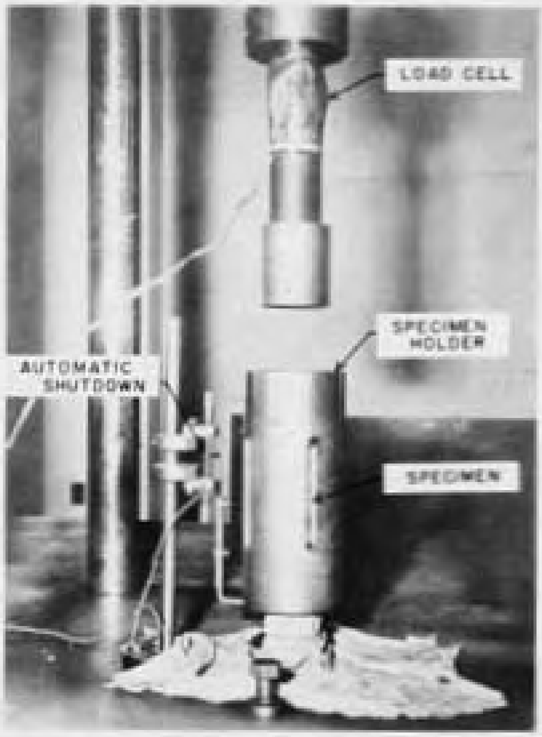

Figure 6. Load Cell and Specimen Holder

Static Compression Tests for Estimating Age Effect

Upon completion of the fatigue testing of a given batch, the remaining specimens in the batch were tested in compression for their ultimate strength. The testing procedure was the same as used for estimating the batch strength. The age effect tests were generally performed within a few days after the fatigue testing had terminated. One exception was the second batch of the FN series which, in addition to having eight specimens tested after the completion of the fatigue tests, had five other specimens tested at the age of 198 days. The other exception was the third batch of the FN series. In this case four specimens were tested at the age of 171 days but no specimens were tested immediately after the fatigue tests.

Seven-Day Compression Tests

Whenever possible, 7-day static compression tests were performed on three specimens from each batch. The purpose of these preliminary tests was to predict the 28-day strengths of the batches. The results of these tests were used as the basis for deciding if additional batches should be made.

All the specimens for the preliminary tests were removed from the saturated lime solution 48 hours prior to testing. They were then capped and immediately returned to the lime solution. The capping followed the same procedure as was previously described. The static compression tests were performed in the same manner as used in estimating the batch strength except in this case the specimens were kept moist until tested.

Air Content by the Linear Traverse Technique

The air contents of two batches, FN7 and FA1, were estimated by using the linear traverse technique on specimens of hardened concrete. A section through the long dimension of the specimen and approximately one-half inch thick was sawed from each test specimen. The two main faces of each section were then polished in accordance with a procedure used by F. K. Fears in his "A Study of the Air-Void Characteristics of Hardened Concrete"(8).

A total of 200 inches of traverse, 100 inches on each face, was obtained from each section. This total length of traverse of 200 inches was suggested by Fears for the determination of air content to within ± 0.5 percent of the true value at 90 percent confidence level.

Determination of Deformation Characteristics

Concern over whether or not the modulus of elasticity of the fatigue test specimens changed while a test was in progress, prompted an investigation of the deformation characteristics of both types of concrete. A single FN batch and a single FA batch were prepared, each batch producing enough concrete for nine 3-inch diameter specimens and for six specimens 6 inches in diameter by 12 inches in height.

The nine 3-inch diameter specimens and three of the 6-inch diameter specimens from each batch were tested statically in compression. The remaining three 6-inch diameter specimens from the batch were loaded to a predetermined level and then unloaded for five cycles. An estimate of the batch strength was obtained from the ultimate strengths of the three 6-inch cylinders tested in compression.

The 6-inch cylinders were tested in a Riehle 200,000 lb. screw type testing machine located in the Materials Testing Laboratory. The slowest no-load head speed of this machine was 0.07 inches per minute which is not in accordance with the ASTM Designation:C39-49 specification of 0.05 inches per minute. The 3-inch diameter specimens were tested in the same testing machine and in the same manner as all the previous 3-inch specimens had been tested. To determine if the higher testing speed used on the 6-inch cylinders had an effect on the ultimate strength, a special mix was prepared. Thirty specimens were made, fifteen of which were tested in the 50,000 lb. machine at the correct speed and fifteen of which were tested in the 200,000 lb. machine at its high speed.

The deformation characteristics of the two mixes were observed through the use of a deflection yoke with a 10-inch gage length and two diametrically opposed Ames dials. The maximum load a cylinder was subjected to during a cycle was set at a predetermined level. This level was 50, 60, or 70 percent of the estimated batch strength which was described above. The testing machine used for these repeated load tests was a Southwark-Emery hydraulic machine having a capacity of 100,000 pounds which is also located in the Materials Testing Laboratory. The rate of loading at which the tests were conducted was approximately 10 psi per second. After being subjected to the five cycles, each cylinder was then tested in compression in the 200,000 lb. testing machine.

DISCUSSION OF RESULTS

The results of this study and the discussions of these results have been divided into three parts. The first part is concerned with the analyses of the physical properties of the plastic concrete and the hardened concrete used in this research work. The results of the fatigue testing program are covered in the second part, and the third part is concerned with additional tests which became necessary as the study progressed.

Analysis of Mix Data

The purpose of the study was to compare the fatigue behavior of non-air-entrained concrete and air-entrained concrete that were similar except for air content. Since the two concretes were prepared in batches, the properties of the two mix designs had to be estimated from the corresponding batch properties. These batch properties, slump and air content of the plastic concrete, and strength of the hardened concrete at various ages expressed as means, are tabulated in Tables 3 and 4.

A statistical procedure known as the "analysis of variance" (16) was used for testing for significant differences among the means. When using the analysis of variance method, the F-test is used to decide if apparent differences in the means are significant or are the result of chance. The analysis is based on the fact that when the means of subgroups are greatly different, the variance of the group means

TABLE 3

PHYSICAL CHARACTERISTICS OF MIX DESIGN FN

Batch Designation	Air Content	Slump	Average Ultimate Compressive Strength at Age of 7 Days (3 specimens tested)	Average Ultimate Compressive Strength after Oven Drying[1] (8 specimens tested)	Average Ultimate Compressive Strength after Completion of Fatigue Tests[2] (7 to 8 specimens tested)
	percent	inches	psi.	psi.	psi.
FN1	0.6	3	None tested[3]	4070	3970
FN2	1.2	3½	None tested[3]	3560	3520
FN3	1.8	6	2690[4]	3440	None tested[5]
FN4	1.5	2½	2510	4010	4150
FN5	0.5	3	2470	4150	4120
FN6	0.1	2½	2980	4270	4080
FN7	0.8	6	2760	3820	3980
FN8	1.0	6	2780	4230	4180

1. Specimen age ranged from 44 to 36 days.
2. Specimen age ranged from 47 to 65 days.
3. Specimens used in the fatigue tests.
4. Specimens tested at the age of 11 days.
5. Specimens tested at a later date.

TABLE 4

PHYSICAL PROPERTIES OF MIX DESIGN FA

Batch Designation	Air Content	Slump	Average Ultimate Compressive Strength at Age of 7 Days (3 specimens tested)	Average Ultimate Compressive Strength after Oven Drying[1] (8 specimens tested)	Average Ultimate Compressive Strength after Completion of Fatigue Tests[2] (7 to 8 specimens tested)
	percent	inches	psi.	psi.	psi.
FA1	7.6	1 3/4	2560	4020	None tested[3]
FA2	10.5	2 3/4	2740	4380	4260
FA3	7.7	2	2860	4490	4410
FA4	7.5	1 3/4	2900	4540	4430
FA5	7.7	2	2710	4300	4250
FA6	7.6	1 3/4	None tested[4]	4620	4560
FA7	9.0	2	2670	4500	4160
FA8	8.2	2 1/4	2490[5]	4170	3910
FA9	8.5	2 1/2	2640	4620	4510

1. Specimen age ranged from 34 to 36 days.
2. Specimen age ranged from 43 to 63 days.
3. Fatigue tests not run, machine out of operation for a month.
4. Scheduling difficulties.
5. Specimens tested at age of 6 days.

is much larger than variances within separate groups. By comparing the F-value calculated from the observations with the theoretical F-value (see, for example, Appendix 6 in Reference 16), it is possible to determine if there is any statistically significant difference among the means. If an F-ratio calculated from the data is less than the F-ratio for the population variance (for the given degrees of freedom and the selected significance level), there is insufficient evidence to reject the hypothesis of equal means. On the other hand, if the calculated F-ratio is larger than the F-ratio for the population variance, there is reliable evidence of significant difference among means.

The test for significance of differences in the analysis of variance is valid only if the observations are from normally distributed populations and the variance of each group is the same (homogeneous variance). In testing for homogeneity of variances, two methods were employed: one based on the F distribution* and the other based on the chi-square distribution (Bartlett's Test).**

Non-Air-Entrained Concrete

The data for the non-air-entrained mix which are summarized in Table 3 and given in their entirety in Tables 15 through 22, Appendix A, were considered first with regard

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* Reference 5, pg. 90
** Reference 16, pg. 242

to homogeneity of variance and then in terms of strength, age effect, air content, and slump.

Mix Strength. The strength data for each batch of non-air-entrained concrete were tested by Bartlett's Test and the F-test for homogeneity of variance. Both tests indicated that the variances of the cell means were non-homogeneous. The F-test gave an F-ratio of 2.75 and, therefore, the hypothesis that the variance of the fifteen normally distributed populations were equal had to be rejected since the F-ratio for the given degrees of freedom and a significance level of one-half percent is 2.31. The calculations for this F-test are contained in Table 32, Appendix B.

Heterogeneous variances, as have occurred here, sometimes can be transformed into a different form which tends toward a normal distribution. However, in this case, neither a logarithmic nor a square root transformation produced homogeneous variances.

Theoretically, the rejection of the hypothesis that the variances were equal, makes an analysis of variance of the strength data somewhat involved. However, it was felt that such an analysis could be of value since it would show any distinct properties of the data if any existed.

A two-way classification analysis of variance was performed on the strength data by the usual techniques. The results of this analysis are shown in Table 5. The type of analysis of variance used relies upon an equal number of

TABLE 5

ANOVA TABLE
STRENGTH DATA, NON-AIR-ENTRAINED CONCRETE

Source of Variation	Degrees of Freedom	Sum of Squares	Mean of Squares	$F_{obs.}$	$F_{f,f,\alpha}$	Decision
Age	1	45	45	0.08	$F_{.90}$ (1,96) = 2.76	Accept H_{o_1}
Batch	6	50,876	8,479	15.68	$F_{.95}$ (6,96) = 2.19	Reject H_{o_2}
Age vs Batch	6	3,647	608	1.12	$F_{.95}$ (6,96) = 2.19	Accept H_{o_3}
Error	96	51,887	541			
Total	109	106,455				

Hypotheses: H_{o_1} = no difference between age

H_{o_2} = no difference between batches

H_{o_3} = no interaction

observations per cell and since such was not the case, adjustments had to be made. Two of the cell blocks were each lacking one measurement. The missing measurement was approximated by using the mean of the original observations in the incomplete cell. This action necessitated an adjustment in the degrees of freedom which was accomplished by decreasing by two, the degrees of freedom of the error term.

In interpreting the results shown in Table 5, consideration has been given to the fact that one of the assumptions underlying the analysis of variance is homogeneous variances. Some of the consequences of this are discussed briefly in the section summarizing these analyses. Even though this assumption has been violated, it is reasonable to assume that there was no significant aging of the concrete during the period following the oven drying and lasting until the fatigue tests had been completed for the given batch. It also seems reasonable to state that there is no evidence of interaction. That is, the age effect and the batch to batch effect do not combine to produce an added effect not due to the sum of the separate effects. Thirdly, the analysis indicates that the batch strength means are not from the same population.

A large observed F-value, such as the one which appears in Table 5 for the batch variation, is sometimes caused by a single mean which is not part of the population. A test known as Duncan's Procedure (6), which is a significance test for linear contrasts among any number of means, was utilized next. This test can detect whether any two means are significantly different at the selected significance level by means of a calculated difference. The purpose of using this test was to obtain the best estimate of the strength of the mix. The age effect was assumed to be nil on the basis of the previously performed analysis of variance and accordingly the batch strengths were taken as the averages of all the specimen strengths. The test indicates that at the five percent significance level, there is reliable evidence that the mean of batch FN2 is significantly different from any of the other means as shown below.

	Batch Designation						
	FN1	FN2	FN4	FN5	FN6	FN7	FN8
Mean Strength (coded)	102	54	107	114	118	90	120

Allowable difference at 5 percent level (see Table 33, Appendix B) = 19

Actual difference between lowest and next lowest mean = 36

The mix strength was calculated by taking the average of all the individual strengths tabulated in Table 32, Appendix B, except for those listed for batches FN2 and FN3. Batch FN3 was not included because the data were not complete and also because the mean of the available data is smaller than the mean of batch FN2 which had previously been shown as not being part of the mix population. The best estimate of the mix strength, based on 94 specimens, was 4050 psi.

Air Content and Slump. In view of the fact that the slump measurements and the air content measurements are limited in their magnitude and, therefore, are not likely to be normally distributed, calculations were limited to finding the means, the standard deviations, and the coefficients of variation. A mean, designated by $\bar{Y}$, is the arithmetic average of the respective measurements. A coefficient of variation, designated by C, is the ratio, expressed as a percentage, of the standard deviation, designated by s, to the mean. The table below summarizes these calculations.

Property	Number of Batches	$\bar{Y}$	s	c
Slump	8	4 in.	1.6 in.	40%
Air Content	8	0.9%	0.55%	58%

Summary. In some of the preceding analyses an assumption was made that the within-cell variances of the strength data were homogeneous when such was not the case. Summary points 2 and 3 are based on the analyses which used this assumption and therefore there is the question of how reliable are these two statements. According to Cochran (2) there generally is a loss of efficiency in the estimates of treatment effects and a loss of sensitivity in tests of significance when ordinary analysis of variance methods are used and there is heterogeneity of errors present. Cochran also states that the validity of the F-test for all treatments is probably the least affected, but since some treatment comparisons may have much smaller errors than others, t-tests from a pooled error may give a serious distortion of the significance levels. It was felt that becasuse of the nature of the tests performed, the statements made in summary points 2 and 3 would not have been altered if the data had been homogeneous.

1. The within-cell variances of the strength data for the non-air-entrained concrete were not homogeneous.
2. There was no significant aging of the concrete during the periods of fatigue testing.
3. The best estimate of the mix strength is 4090 psi.
4. The average slump of the mix was 4 inches and the coefficient of variation was 40 percent. This coefficient of variation is very high when compared to an expected coefficient of variation of about 5 percent for laboratory work on portland cement concrete.
5. The average air content of the plastic concrete was 0.9 percent and the coefficient of variation was 58 percent. Again the coefficient of variation is considerably above that which is desirable.

Air-Entrained Concrete

The data for the air-entrained mix which are summarized in Table 4 and given in their entirety in Tables 23 through 31, Appendix A, were considered with regard to strength, age effect, air content, and slump. As was done in the previous section on non-air-entrained concrete, the strength test data were first tested for homogeneity of variance.

Mix Strength. The strength data for each batch of air-entrained concrete were tested by Bartlett's Test and the F-test for homogeneity of variance. Both tests indicated that the variances of the cell means were homogeneous. The F-test gave an F-ratio of 0.97 which is less than the corresponding theoretical F-ratio of 1.68 for a five percent significance level, and therefore, the hypothesis that variance of the sixteen normally distributed populations were equal was accepted. The calculations for this F-test are contained in Table 34, Appendix B.

Since the variances were found to be homogeneous, as contrasted to the heterogeneous variances of the non-air-entrained mix, an analysis of variance of the air-entrained mix strength data can be relied upon for a true picture of any relationships that might exist in the data.

A two-way classification analysis of variance was performed on the strength data by the usual techniques. The results of this analysis are shown in Table 6. The type of analysis of variance used relies upon an equal number of

TABLE 6

ANOVA TABLE
STRENGTH DATA, AIR-ENTRAINED CONCRETE

Source of Variation	Degrees of Freedom	Sum of Squares	Mean of Squares	$F_{obs.}$	$F_{f_1,f_2,\alpha}$	Decision
Age	1	6,685	6,685	28.43	$F_{.90}(1,109) = 2.75$	Reject H_{o_1}
Batch	7	36,017	5,145	21.88	$F_{.95}(7,109) = 2.30$	Reject H_{o_2}
Age vs Batch	7	3,073	439	1.87	$F_{.95}(7,109) = 2.30$	Accept H_{o_3}
Error	109	25,630	235			
Total	124	71,405				

Hypotheses: H_{o_1} = no difference between age

H_{o_2} = no difference between batches

H_{o_3} = no interaction

observations per cell and since such was not the case, adjustments had to be made. Three of the cell blocks were each lacking one measurement. The missing measurement was approximated by using the mean of the original observations in the incomplete cell. This action necessitated an adjustment in the degrees of freedom which was accomplished by decreasing by three, the degrees of freedom of the error term.

Referring to Table 6, it is seen that there is evidence of an age effect during the duration of the fatigue tests. This age effect was not in the form of a gain in strength, but rather a loss in strength. On a batch to batch basis, the loss of strength indicated by the batch means ranged from 50 psi to 340 psi, the average loss for the mix being approximately 140 psi. There is also evidence that the batch strength means are not from the same population. There is, however, no evidence of any interaction between age and individual batches.

As in the case of the PN mix, the observed F-value for the batch variation in the FA mix is fairly large. Duncan's Procedure was applied to the mean strengths of the batches to determine if one of the means did not belong to the population. Although there is reliable evidence of a loss of strength, the batch strengths were taken as the averages of all the specimens strengths. The reason for disregarding the age effect is that the strength of the mix at the time of the fatigue tests is of primary interest. The test indicates that at the five percent significance level, there is reliable evidence that the mean of batch FA8 is significantly different from any of the other means as shown in the table below:

	Batch Designation							
	FA2	FA3	FA4	FA5	FA6	FA7	FA8	FA9
Mean Strength (coded)	32	45	49	27	39	33	04	56

Allowable difference at 5 percent level (see Table 35, Appendix B) = 14

Actual difference between lowest and next lowest mean = 23

The mix strength was calculated by taking the average of all the specimen strengths tabulated in Table 34, Appendix B, except for those specimen strengths for batch FA8. The best estimate of the mix strength, based on 109 specimens, was 4430 psi.

The strength data for batch FA1 were not included in any of the previously described analyses because fatigue tests were not conducted on any specimens from this batch. A breakdown in the fatigue machine made it inoperative for approximately a month.

Air Content and Slump. The same limitations that existed for the air content and slump of the non-air-entrained concrete exist for the air entrained concrete, therefore calculations were limited, as before, to finding the means, the standard deviations, and the coefficients of variation. The table below summarizes these calculations:

Property	Number of Batches	$\bar{Y}$	s	C
Slump	9	2 in.	0.3 in.	15%
Air Content	9	8.3%	0.98%	12%

Summary. The following summarize the results of the previously mentioned analyses which were performed on the data for the air-entrained concrete.

1. The within-cell variances of the strength data for the air-entrained concrete were homogeneous.
2. There was a significant effect of age on the strength of the concrete, but it was a loss which averaged 140 psi rather than a gain.
3. The best estimate of the mix strength is 4430 psi.
4. The average slump of the mix was 2 inches and the coefficient of variation was 15 percent.
5. The average air content of the plastic concrete was 8.3 percent and the coefficient of variation was 12 percent.

Comparison of the Two Mix Designs

The two mix designs were compared to determine if the estimate of the strength of the non-air-entrained concrete was significantly different from the estimate of the strength of the air-entrained concrete. The hypothesis tested was that the two mixed strengths were equal, the alternate hypothesis being that the strength of the air-entrained concrete was greater than the strength of the non-air-entrained concrete. This significance test which is outlined in Table 36, Appendix B, states that at a 5 percent significance level there is no reliable evidence of any difference between the strengths of the two mix designs.

Many comparisons of the non-air-entrained mix design with the air-entrained mix design emphasizes the greater variation in properties that one encounters with non-air-entrained concrete or conversely the greater uniformity of air-entrained concrete. These characteristics have been reported many times in the literature and present a strong case for the use of air-entrained concrete wherever possible at least for experimental work.

The following is an enumeration of some comparisons that were made. The last three were not recorded as data during the course of this study but were observed purposely each time a batch was prepared.

1. The within-cell variances of the strength data for the air-entrained concrete were homogeneous while the variances for the non-air-entrained concrete were not homogeneous.
2. The air-entrained concrete lost a significant amount of its strength during the periods of fatigue testing while the non-air-entrained

concrete showed no significant gain. The loss in strength of the air-entrained, although statistically significant, averaged only 140 psi and accordingly was disregarded in the determination of the mix strength.

3. There was no significant difference between the strength of the non-air-entrained concrete and the strength of the air-entrained concrete at the time of the fatigue tests.
4. Better control of slump was possible with the air-entrained concrete, its coefficient of variation being 15 percent in contrast to a value of 40 percent for the non-air-entrained concrete.
5. The workability of the air-entrained concrete was considerably better than the workability of the non-air-entrained concrete.
6. Bleeding was practically nonexistent in the air-entrained concrete while it was considerable for the non-air-entrained concrete.
7. Finishing was easier and faster with the air-entrained concrete than with the non-air-entrained concrete.

Fatigue Test Results

A plot of stress level versus cycles to failure was selected as the best device to detect differences between the fatigue behavior of the non-air-entrained concrete and the air-entrained concrete. When a plot of this type shows a definite pattern or trend, it is customary to show the trend by means of a curve known as an S-N curve. Selecting a truly representative S-N curve is perhaps the most difficult task in analyzing fatigue data because of the scatter that is characteristic of fatigue test results.

The existence of scatter in fatigue test results is not peculiar to fatigue tests alone but because fatigue tests are destructive tests, their results are expected to have more variation than the results of many other types of tests. Actually, variation can be found in repeated measurements on any object, quantity, or quality such that the more accurate the measuring device is, the more variation can be found from piece to piece and even along a single piece. Because fatigue tests are destructive tests, the observed variation will probably include effects of errors in loading and measuring, and also effects of specimen selection and preparation. From the design standpoint, fatigue test results, like the results of any destructive tests, can be used only when certain assumptions are made. One such assumption is that the test specimens are representative of the part or parts to be used in service.

Another feature of fatigue test data which makes an analysis difficult is that there is a question as to what type of distribution the measurements follow. At the higher stress levels the distribution becomes extremely skewed, to the extent of being essentially one-sided at the highest stress level. At the lowest stress level, it is unknown whether or not a particular level serves as a division point between a finite number of stress cycles being endured or an infinite number of cycles.

Any analysis of fatigue test data is also hindered by the limited amount of data collected in a normal testing program. To collect a desirable amount of data at the lower stress levels is not practical because of the prohibitive amount of time necessary to test a single specimen. The common practice is to terminate such tests when a predetermined number of cycles has been sustained by the specimen.

Analysis of the Fatigue Data

The analysis of the fatigue data tabulated in Tables 7 and 8 was limited by the characteristics of such data previously mentioned. The actual range of stress cycles endured is unknown for both the 50 and 60 percent stress levels of both the non-air-entrained concrete and the air-entrained concrete because the testing was stopped when the specimens had sustained ten million cycles without failing. There is, of course, a lower boundary for the data representing the 60 percent stress level. It was felt that any

TABLE 7

FATIGUE TEST RESULTS, NON-AIR-ENTRAINED CONCRETE

Batch Designation	Specimen Number	Maximum Fatigue Load (lbs.)	Minimum Fatigue Load (lbs.)	Number of Stress Cycles Endured
FN1	1	14,500 (52)[1]	500 (1.8)[1]	10,155,000 → [2]
	2	17,000 (60)	500 (1.8)	10,355,000 →
	3	19,500 (69)	700 (2.5)	12,000
	4	22,700 (81)	700 (2.5)	282,000
	5	25,400 (90)	500 (1.8)	100
FN2	1	12,300 (50)	500 (2.0)	10,568,000 →
	2	14,800 (60)	500 (2.0)	10,106,000 →
	3	17,800 (72)	500 (2.0)	31,000
	4	19,500 (79)	500 (2.0)	2,200
	5	22,200 (90)	1000 (4.0)	900
FN3	1	12,300 (51)	300 (1.2)	10,330,100 →
	2	14,400 (60)	500 (2.1)	3,630,400
	3	16,500 (69)	700 (2.9)	25,800
	4	18,700 (78)	700 (2.9)	19,800
	5	20,500 (85)	500 (2.1)	300
FN4	1	14,100 (49)	500 (1.8)	10,562,000 →
	2	16,500 (58)	500 (1.8)	10,472,000 →
	3	19,000 (67)	500 (1.8)	2,349,600
	4	22,600 (79)	800 (2.8)	8,900
	5	Specimen not tested		
FN5	1	Specimen not tested		
	2	17,400 (60)	500 (1.7)	10,740,000 →
	3	20,300 (70)	500 (1.7)	99,700
	4	22,500 (78)	500 (1.7)	1,500
	5	26,100 (90)	700 (2.4)	3,700
FN6	1	Specimen not tested		
	2	18,000 (62)	500 (1.7)	1,308,400
	3	21,000 (72)	500 (1.7)	278,700
	4	24,000 (82)	800 (2.7)	1,000
	5	26,500 (91)	500 (1.7)	300

(continued)

TABLE 7 (continued)

Batch Desig- nation	Specimen Number	Maximum Fatigue Load (lbs.)	Minimum Fatigue Load (lbs.)	Number of Stress Cycles Endured
FN7	1	Specimen not tested		
	2	Specimen not tested		
	3	18,800 (69)	500 (1.8)	54,500
	4	21,500 (79)	1000 (3.7)	4,600
	5	24,200 (89)	500 (1.8)	10,900
FN8	1	Specimen not tested		
	2	17,800 (61)	500 (1.7)	1,388,700
	3	20,500 (70)	800 (2.7)	52,200
	4	23,500 (80)	1000 (3.4)	1,700
	5	26,700 (91)	500 (1.7)	500

1. Figure in parentheses is the fatigue load expressed as a percentage of the average ultimate compressive strength of the batch. Average strength based on 15 to 16 specimens except for batch FN3 which had 8 specimens.

2. → indicates that specimen had not failed when test was stopped.

TABLE 8

FATIGUE TEST RESULTS, AIR-ENTRAINED CONCRETE

Batch Designation	Specimen Number	Maximum Fatigue Load (lbs.)	Minimum Fatigue Load (lbs.)	Number of Stress Cycles Endured
FA1	No tests performed, Fatigue Machine under repair			
FA2	1	Specimen not tested		
	2	18,350 (61)[1]	500 (1.7)	10,048,000 → [2]
	3	21,550 (71)	550 (1.8)	702,200
	4	23,900 (79)	500 (1.7)	2,800
	5	Loads not verified		~100
FA3	1	15,800 (51)	400 (1.3)	10,314,000 →
	2	19,200 (62)	500 (1.6)	6,294,500
	3	22,350 (72)	500 (1.6)	109,400
	4	25,300 (81)	500 (1.6)	600
	5	28,800 (93)	500 (1.6)	300
FA4	1	Specimen not tested		
	2	19,000 (60)	600 (1.9)	8,022,600
	3	22,000 (70)	450 (1.4)	204,800
	4	25,300 (80)	600 (1.9)	3,500
	5	Loads not verified		~100
FA5	1	Specimen not tested		
	2	18,000 (60)	600 (2.0)	9,092,900
	3	21,300 (71)	400 (1.3)	1,683,800
	4	24,200 (81)	500 (1.7)	1,500
	5	Loads not verified		~100
FA6	1	Specimen not tested		
	2	19,500 (61)	500 (1.6)	10,157,000 →
	3	22,300 (69)	600 (1.9)	226,900
	4	25,900 (81)	500 (1.6)	3,900
	5	Specimen not tested		
FA7	1	15,600 (51)	600 (2.0)	10,105,000 →
	2	18,800 (62)	500 (1.6)	10,256,000 →
	3	22,100 (73)	500 (1.6)	361,800
	4	24,900 (82)	500 (1.6)	600
	5	Specimen not tested		

(continued)

TABLE 8 (continued)

Batch Designation	Specimen Number	Maximum Fatigue Load (lbs.)	Minimum Fatigue Load (lbs.)	Number of Stress Cycles Endured
FA8	1	14,400 (51)	700 (2.5)	10,760,000 →
	2	17,400 (62)	650 (2.3)	10,109,000 →
	3	20,500 (72)	500 (1.8)	215,700
	4	23,600 (83)	300 (1.1)	900
	5	Specimen not tested		
FA9	1	Specimen not tested		
	2	19,400 (61)	500 (1.6)	10,034,000 →
	3	22,700 (71)	400 (1.3)	534,900
	4	26,000 (82)	500 (1.6)	400
	5	Specimen not tested		

1. Figure in parentheses is the fatigue load expressed as a percentage of the average ultimate compressive strength of the batch. Average strength based on 15 to 16 specimens.

2. → indicates that specimen had not failed when test was stopped.

curve fitted to all the data for each type of concrete would undoubtedly be a very poor approximation of the true curve; therefore, the curves that were drawn only represent the relationship that exists at the 70 and 80 percent levels.

The S-N Diagrams. The semi-log coordinate system used for the two S-N diagrams shown in Figures 7 and 8 was chosen because it presented the fatigue test results in a form which is easily viewed. The possibility of using either an arithmetic or log-log plot instead of the semi-log plot was investigated, but the results showed the semi-log plot to be superior for these data.

Referring to the two S-N diagrams, it can be seen by the position of the points on the right hand portion of both diagrams that it is unlikely that a definite fatigue limit exists in the vicinity of ten million cycles for either type of concrete. It is also apparent that the fatigue test results for the air-entrained concrete are characterized by less scatter than the test results for the non-air-entrained concrete.

Comparison of the FA Mix to the FN Mix. A nonparametric statistical technique was utilized in comparing the fatigue test results from the two mixes, because the type of distribution to which the data belonged was unknown. Nonparametric techniques compare distributions without specifying their form. In this analysis a method called "Runs" was

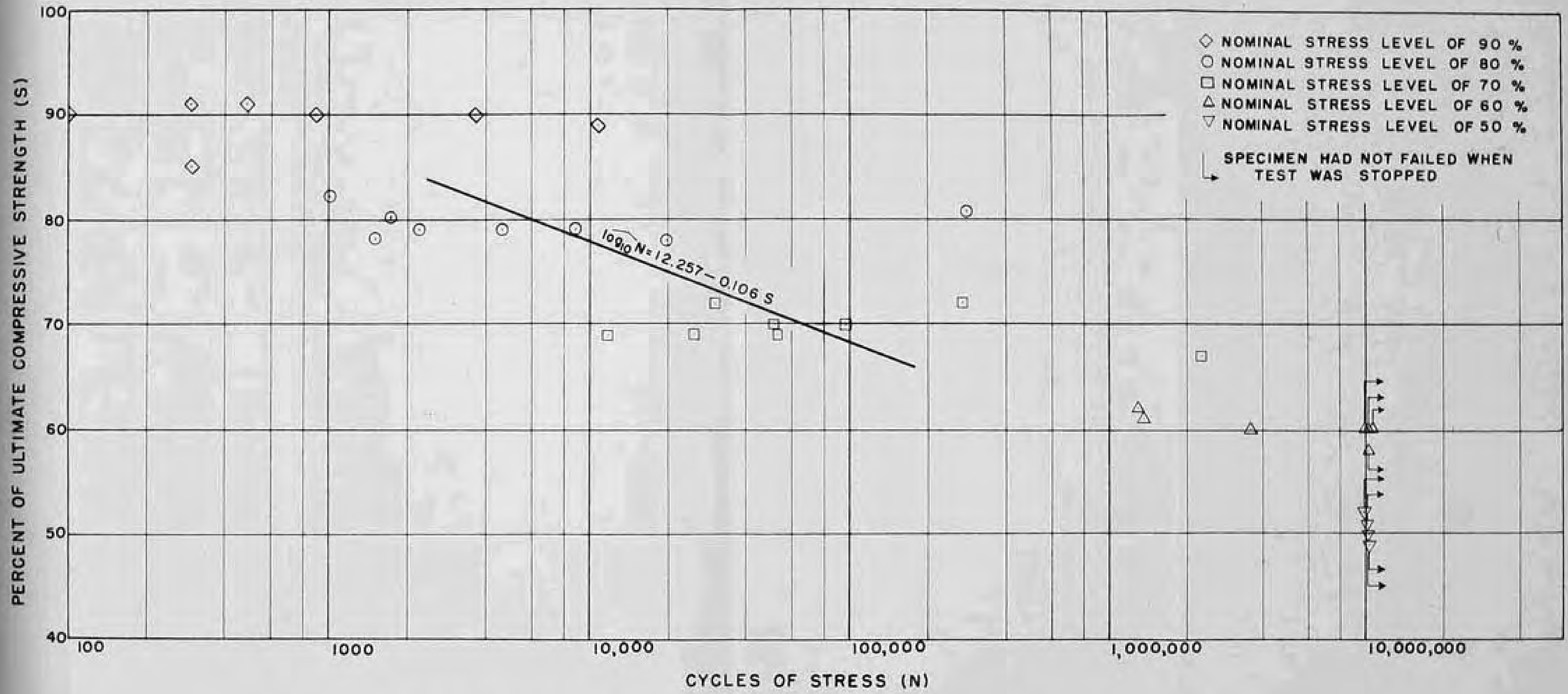

FIGURE 7. S-N DIAGRAM FOR NON-AIR-ENTRAINED CONCRETE

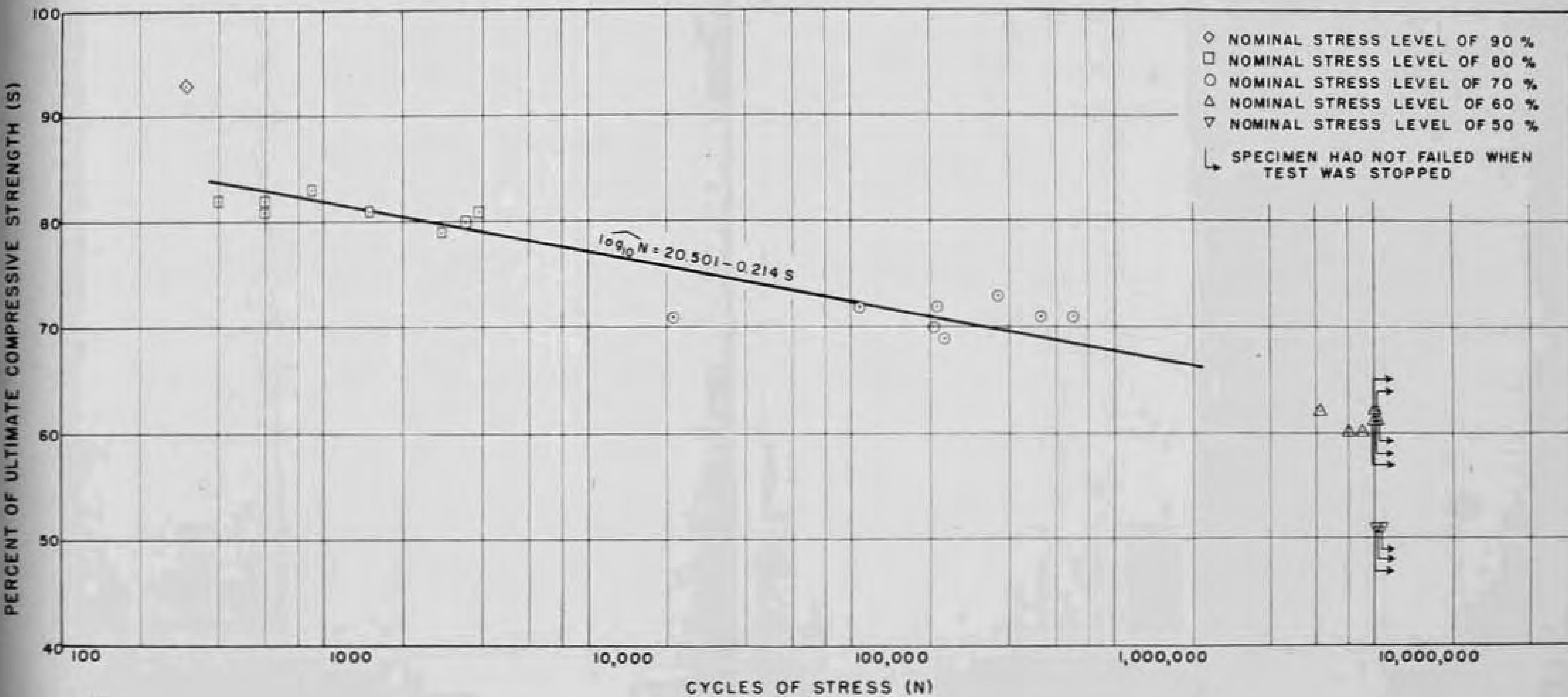

FIGURE 8. S–N DIAGRAM FOR AIR-ENTRAINED CONCRETE

used.[9] Runs can be used to test whether or not two random samples came from populations having the same frequency distribution. The hypothesis that the two populations have the same distribution is rejected when there are fewer or more runs than would be expected by chance if both populations are the same. This technique, when used to test the data to which it was applicable, indicates that there is reliable evidence at the 5 percent significance level that at the respective stress levels the data are from the same population.

The test could not be applied to the 50 percent stress level data because the number of cycles to failure was unknown. In this case, testing was terminated when a specimen had sustained at least ten million cycles without failing. A similar condition existed in the case of the 60 percent stress level data since a number of the specimens had endured the prescribed ten million cycles. However, it seemed reasonable to apply the run test to data from those specimens that actually failed at less than ten million cycles. Performing the test at the 80 and 90 percent stress levels presents a problem in the form of identical numerical values recorded for each of the two mixes. There occurs a situation in which the number of runs will depend on the order in which the identical values are considered. In both cases under consideration, the maximum and minimum number of

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[9] Reference 5, pg. 334

possible runs fell within the limits called for at the 5 percent significance level. Appendix C contains the details of the Run test.

Linear Regression Analysis. A consequence of plotting the cycles to failure to a logarithmic was a contraction or drawing together of the N values. This contraction was in the form of reducing the spread between the high N values. The data in this transformed condition appear to have a normal distribution with respect to cycles to failure. It is on this assumption that the following analyses are performed.

Only the data for the 70 and 80 percent stress levels were considered in the analyses because it is only at these levels that the data are complete. By complete it is meant that an equal number of specimens from each mix design had been tested, while in the case of the 90 percent stress level data was available for only one of the mixes and in the cases of the 50 and 60 percent levels many of the specimens tested did not fail. In addition, at the 70 and 80 percent stress levels there are a sufficient number of specimens tested to indicate any relationships that might exist at these two stress levels.

Fitting a curve to the data of each mix design was accomplished by the method of least squares. Only a linear relationship was considered and the calculations for the linear regression analysis are contained in Appendix C.

The least squares linear regression equation resulting from an analysis of the data for the non-air-entrained concrete is:

$$\widehat{\log_{10} N} = 12.257 - 0.106\, S \text{ .... } 1$$

where N is cycles to failure and S is percent of the ultimate stress. This equation can only be used to estimate N when the value of S is between 66 and 84 percent since this is the range of the S values used in the analysis.

The least squares linear regression equating resulting from an analysis of the data for the air-entrained concrete was calculated to be:

$$\widehat{\log_{10} N} = 20.501 - 0.214\, S \text{ .... } 2$$

The same limits on S that apply to the equation for the non-air-entrained concrete, apply to this equation when estimating N.

Along with the linear regression equations, the correlation coefficients for the two sets of data were calculated. The degree of association among the data for the air-entrained concrete is very high, the correlation coefficient being -0.94. The degree of association for the non-air-entrained concrete is less than that for the air-entrained data, the correlation coefficient in this case being -0.62. A significance test for differences between correlation coefficients was performed as outlined in Appendix C, and at the 5 percent significance level there is reliable evidence that the two correlation coefficients are significantly different. This test only compares the linear association of the data and does not compare the curves determined by the regression analysis. However, it does show that the fatigue data for the air-entrained concrete has considerably greater uniformity than the fatigue data for the non-air-entrained concrete.

A test was also performed to find whether or not the slopes of the two equations were significantly different. Theoretically this test can be performed only if the two populations have a common variance. This was not the case for this set of data. An F-test on the variances indicates that there is a difference in the variances. There is no test available for the condition of unequal variances, therefore it seemed advisable to perform the test with the assumption that the variances were equal. The use of unequal variances in this test is critical only in those cases where the calculated t-value is not significantly different from the t-value corresponding to the selected significance level. In those cases where the two t-values are similar, the presence of unequal variances could result in the hypothesis being rejected when actually it should have been accepted or the reverse situation might apply in which the hypothesis is accepted when it should be rejected. The test, which is outlined in Appendix C, indicates that at the 5 percent significance level there is no reliable evidence that the slopes are significantly different. This indicated equality of the slopes suggests very strongly that the relationship existing between the stress level and the cycles to failure for the non-air-entrained concrete is the same as the relationship for the air-entrained concrete.

Prediction Intervals. Although the estimating equations for both mix designs have been calculated, it must be realized that if a number of tests were to be performed at a given S value, the observed N's would cluster about the calculated or estimated N. It is therefore advisable to determine the limits of the predicted value of an individual N for a given S value.

Ninety-five percent prediction intervals were calculated for each of the two S-N curves by the procedure outlined in Appendix C. As shown in Table 9, these intervals are extremely wide, especially in the case of the non-air-entrained concrete. It can be seen that for a given S value the predicted N for the air-entrained concrete lies within the limits for the predicted N for the non-air-entrained concrete. The fact that there are overlapping prediction intervals along with essentially equal slopes is good reason to assume that there is no difference between the fatigue behavior of the two types of concrete.

Type of Failure

Observation of the actual progress of a failure of a fatigued specimen was not possible because of the absence of any sizable openings in the specimen holder. Even if the

TABLE 9

NINETY-FIVE PERCENT PREDICTION INTERVALS FOR NON-AIR-ENTRAINED AND AIR-ENTRAINED CONCRETE

Stress Level (percent)	Prediction Intervals: Non-Air-Entrained Concrete		Air-Entrained Concrete	
	Lower Limit (cycles)	Upper Limit (cycles)	Lower Limit (cycles)	Upper Limit (cycles)
80	9.67×10^{-5}	3.26×10^{13}	8.94	5.17×10^{7}
75	1.41×10^{-5}	5.51×10^{13}	1.53	5.24×10^{8}
70	7.00×10^{-4}	3.14×10^{14}	12.18	9.08×10^{9}

specimens had been visible, it is doubtful that the progress of a failure could have been observed, since the final breaks were extremely rapid and without any warning. All the failures were of the type shown in Figure 9 and, as can be seen, the fatigue failure is similar in character to that exhibited by the specimens which were tested statically. Little or no crushing force was exerted on the specimens following their failure because of an automatic shutdown on the testing machine which was triggered the instant that the specimen failed.

Other Observations

It was noticed that if an air-entrained specimen was exchanged for a non-air-entrained specimen in the fatigue machine without any adjustment in the load control mechanisms, the loads applied to the air-entrained specimen were different from those that had been established for the non-air-entrained specimen. This change consisted of an increase in the minimum load and a corresponding decrease in the maximum load. The change is explained by the ability of air-entrained specimen to compress farther under a similar load. The desired loads were easily obtained by increasing the crank throw, which in turn increased the travel of the loading piston and thereby compensated for the decrease in height of the air-entrained specimen.

Another phenomenon noticed was an indicated change in the applied load while a test was in progress. This was

Figure 9. Typical Failures

Fatigue Failure on the Left

Static Failure on the Right

noticed only during the testing of air-entrained specimens and it was not detected for all specimens. In those cases where the load change was detected, it was only after the specimens had been subjected to a million or more cycles. A strange feature was that a number of the specimens endured ten million cycles without any load changes occurring. Although no load changes were noticed in the case of the non-air-entrained specimens, it is entirely possible that the same phenomenon was taking place during these tests since the load reading system in use during these tests was not sensitive enough to detect small load changes.

These changes in the loadings consisted of an increase in the minimum load and an increase or a decrease in the maximum load. The changes, however, were of different magnitudes ranging from a few hundred pounds to over a thousand pounds. In an effort to explain the phenomenon, the three possible causes of any changes in the loadings were considered separately and in combination. The three possibilities were: (a) a malfunctioning of some circuit in the load reading system, (b) a change in the physical properties of the specimen, and (c) an inconsistency in the functioning of some portion of the fatigue machine. The first and third possibilities were eliminated because the load reading system had sufficient sensitivity to detect changes in the loads of the magnitude that occurred, and either of the two changes possible in the machine would cause the loads to alter in equal increments. The second possibility, a change in the physical properties of the specimen, appears to be the best explanation. The type of load change that occurred suggests that the specimen deformations produced by the maximum load changed at a rate different from the rate by which the deformations produced by the minimum load changed.

Summary of Fatigue Test Results

The fatigue test results show considerable scatter even though efforts had been made to confine most of the variation to the fatigue tests alone. A non-varying program for preparing specimens attempted to minimize batch to batch differences of air content, slump, and strength. The same quantities of materials and the same mixing procedure were used for each batch. Curing and the preparation of specimens for testing were also the same for each batch. Some of the resulting variability can probably be attributed to temperature and humidity changes which were encountered in a complete cycle of seasons, spring through winter. But the main reason for the variability is that small variations in specimen composition and condition become more pronounced in fatigue tests than they would in static tests. The short duration of a static test tends to minimize many of the variations since they do not get a chance to become effective, while in the case of fatigue tests, the loading is repeated numerous times, and the variations have a chance to produce whatever effect is associated with them.

The analyses of the results of 65 specimens tested in fatigue indicate the following:

1. The linear regression equation for the non-air-entrained concrete for values of S between 66 and 84 percent is:

$$\widehat{\log_{10} N} = 12.257 - 0.106\ S$$

2. The linear regression equation for the air-entrained concrete for values of S between 66 and 84 percent is:

$$\widehat{\log_{10} N} = 20.501 - 0.214\ S$$

3. There is no evidence of any difference between the fatigue behavior of non-air-entrained plain concrete and air-entrained plain concrete when the stress condition is the fluctuating compressive type. The minimum stress varied from one to four percent of the ultimate static compressive stress and the maximum stress ranged from 50 to 90 percent of the ultimate stress.

4. The fatigue test results for the air-entrained concrete show less scatter than shown by the non-air-entrained concrete.

5. The S-N diagrams obtained by considering all of the fatigue test data seem to support the theory that there is no definite fatigue limit for concrete.

Results of Additional Tests

The tests covered in this section were prompted by the need for explanations of some of the peculiarities which were observed during the main testing program. Included in this section are the results of the linear-traverse air content determinations, the results of the deformation tests, and some additional information on the effect of age on the strength of the concrete.

Air Content by the Linear Traverse Technique

The air content of two samples of hardened concrete was determined by the linear traverse technique for comparison with air contents determined by the gravimetric method. Equipment for measuring air content by the linear traverse method had been used in a previous investigation and was still available in the laboratory.

The use of the linear traverse technique was prompted by the need for a check on the air content of batch FN7 as measured by gravimetric method. This air content, which was 6.2 percent according to the gravimetric determination, was questioned because the mix design used was for non-air-entrained concrete and because the air contents of the other seven batches of the same mix design did not exceed two percent. A check of the individual measurements disclosed a weight determination for batch FN7 which was slightly different from the corresponding weight determinations for the other seven batches. The weight had been recorded as 20.00

pounds, while a value of 20.90 pounds was more reasonable. A calculated air content of 0.8 percent resulted when the value of 20.90 pounds was used. The air content of the concrete from batch FN7 as estimated from two hundred inches of traverse on a section taken from one specimen was 1.6 percent as shown in Table 10a. This value was much closer to the revised gravimetric value of 0.8 percent than to the original gravimetric value of 6.2 percent, and therefore the 0.8 percent value was taken as the correct gravimetric method air content.

To check the linear traverse method with respect to a high and a low air content, the air content of a sample of concrete from the FA1 batch was determined. The air content as estimated from two hundred inches of traverse on a section taken from one specimen was 10.0 percent and the air content of the same batch by the gravimetric method was 7.6 percent.

A comparison of the air contents of the batches as determined by the gravimetric method and by the linear traverse method is shown in Table 10b. The comparison indicates that the two methods are more compatible at the higher air contents than at the lower air contents. It is also apparent that the linear traverse method is capable of distinguishing between high and low air contents.

TABLE 10a

AIR CONTENTS BY THE LINEAR TRAVERSE METHOD

Source of Section (batch)	Face	Total Distance Across Air Voids (inches)	Total Traverse Length (inches)	Total Number of Bubbles	Air Content* (percent)
FN7	1	2.12	100.01	87	1.6
	2	1.13	100.13	48	
FA1	1	9.41	100.01	1042	10.0
	2	10.69	100.89	838	

$$* \text{ Air Content} = \frac{\text{Total Distance Across Air Voids}}{\text{Total Traverse Length}} \times 100$$

TABLE 10b

COMPARISON OF AIR CONTENTS

Batch	Air Content (percent) Gravimetric Method (plastic concrete)	Air Content (percent) Linear Traverse Method (hardened concrete)
FN7	0.8	1.6
FA1	7.6	10.0

Results of the Deformation Tests

The behavior of some of the air-entrained concrete specimens when undergoing fatiguing action, prompted a limited number of slow speed repetitive load tests. It was hoped that some characteristic of the concrete could be detected which would offer an explanation of why the loads changed during the testing of some of the air-entrained fatigue specimens. Three 6-inch diameter by 12-inch high cylinders from both the non-air-entrained mix and the air-entrained mix, were loaded and unloaded in compression through four cycles to predetermined maximum loads. These maximum loads were intended to be 50, 60, and 70 percent of the estimated static compressive strength of the batch. This estimate was obtained by testing in compression three additional 6-inch diameter by 12-inch high cylinders from each batch.

These repeated-load tests did not produce any significant results. Reference to Table 11, which contains the deformations that were measured at the induced maximum and minimum loads, shows no trend in the data that can explain the observed behavior of the specimens. Any interpretation of the results of these tests is hindered by the fact that the Ames dials on the deflection yoke performed very erratically. The inconsistencies in the dial readings can be seen by referring to the actual dual readings listed in Tables 37 through 42, Appendix D.

TABLE 11

CONDENSATION OF DEFORMATION TEST DATA

Batch Designation	Specimen Number	Loading Min. (lbs.)	Loading Max. (lbs.)	Loading Sequence and Resulting Deformations in Inches Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.
FSN (non-air)	1	500	50,000	0	.0055	.0005	.0056	.0006	.0056	.0006	.0057	.0006
	2	500	60,000	0	.0070	.0008	.0072	.0009	.0073	.0010	.0074	.0010
	3	500	70,000	0	.0087	.0012	.0091	.0013	.0093	.0015	.0095	.0015
FSA (air)	1	500	50,000	0	.0060	.0005	.0061	.0006	.0061	.0006	.0061	.0006
	2	500	60,000	0	.0076	.0009	.0077	.0010	.0077	.0010	.0078	.0011
	3	500	70,000	0	.0082	.0004	.0083	.0005	.0083	.0005	.0083	.0005

Effect of Size of Specimen on Strength

Nine 3-inch specimens were cast from each of the two batches which provided the 6-inch specimens used in the deformation tests. By considering the test data obtained from the three 6-inch specimens which had been used for estimating the ultimate strength of each batch in the deformation tests, and that obtained from the nine 3-inch specimens from each batch, it was possible to arrive at an estimate of the effect of the size of the specimen on the compressive strength of the concrete. Testing facilities introduced one factor which had to be evaluated separately. This factor was the use of the 50,000-pound testing machine for testing the 3-inch specimens and the 200,000-pound testing machine for testing the 6-inch specimens. This testing machine effect is discussed in the next section. The results of the size effect tests are shown in Table 12.

Although the small number of 6-inch specimens tested does not give a very reliable estimate of the strength, there is a strong indication that the small cylinders gave a higher compressive strength than the large cylinders. The average compressive strengths of the small cylinders were 1640 psi and 1530 psi higher than the average strengths of the large cylinders for the non-air-entrained concrete and the air-entrained concrete respectively.

TABLE 12

AVERAGE COMPRESSIVE STRENGTHS, SIZE EFFECT STUDY

Batch Designation	Specimen Age When Tested (days)	Specimen Diameter (inches)	Average Ultimate Static Compressive Strength (psi.)	Number of Specimens Tested
FSN (non-air)	36	3	5110	9
	37	6	3470	3
FSA (air)	34	3	4860	9
	36	6	3330	3

Effect of Testing Machines on Strength

It was known that the 200,000-pound testing machine operated at a speed greater than the 0.05 inches per minute prescribed by ASTM. To determine if part of the difference in strength due to size was actually due to the faster testing speed, an additional concrete mix was prepared. Thirty 3-inch specimens were cast from the mix and cured in the saturated lime solution for 31 days before capping and the subsequent testing. Static compression tests were conducted on fifteen specimens by the 50,000-pound testing machine and on fifteen specimens by the 200,000-pound testing machine. The data from the tests were analyzed for significance of difference at a significance level of 5 percent.

There is reliable evidence of a difference in the compressive strengths produced by the two testing machines. The results of the compression tests are summarized in Table 13.

The lower average compressive strength (550 psi lower) obtained by use of the 200,000-pound testing machine indicates that the difference in strength is at least partially due to the nature of the testing machine, since it is an established fact (22) that an increase in testing speed will result in an increase in the ultimate compressive strength of concrete cylinders.

TABLE 13

AVERAGE COMPRESSIVE STRENGTHS - MACHINE EFFECT STUDY

Testing Machine	Testing Speed (in./min.)	Number of Specimens Tested	Average Ultimate Static Compressive Strength (psi.)	Coefficient of Variation (percent)
200,000-lb.	0.07	15	4340	9
50,000-lb.	0.05	15	4890	3

Note: All specimens from same batch and all were tested at the age of 35 days.

Effect of Additional Aging of the Concrete

Specimens taken from three of the batches made during the regular testing program, were tested in compression after having been stored under normal room conditions for as long as 5-1/2 months. The specimens were subjected to static compression tests using the same testing procedure that had been used in the earlier compression tests. The results of these tests are shown in Table 14.

Significance of difference tests were performed to determine if the increases in the average compressive strength of each of the three batches was significant. At a significance level of 5 percent, there is reliable evidence of an increase in the average strength of each batch.

In contrast to the loss of strength sustained during the fatigue testing of the air-entrained concrete, the compression tests on specimens from batch FA1 indicate that the air-entrained concrete increased its strength significantly over a period of almost 2-1/2 months. The loss of strength sustained by the air-entrained concrete during the fatigue testing averaged 140 psi over a period of time which averaged 16 days. Table 14 shows that the gain in the average strength of batch FA1 over a period of 82 days was 450 psi. In both cases the initial compression tests were performed when the age of the air-entrained concrete was from 34 to 36 days. The only explanation that can be given for the subsequent gain in strength of the air-entrained concrete is

TABLE 14

AVERAGE COMPRESSIVE STRENGTHS - EXTENDED AGE STUDY

Batch Desig-nation	Number of Specimens Tested	Specimen Age when Tested (days)	Average Ultimate Static Compressive Strength (psi.)	Number of Specimens Tested	Specimen Age when Tested (days)	Average Ultimate Static Compressive Strength (psi.)
FN2	8	36	3560	5	198	3780
FN3	8	33	3440	4	171	3750
FA1	8	34	4020	12	116	4470

that the specimens absorbed moisture from the surrounding air. An explanation of the preliminary loss of strength has not been attempted because no adequate reason has presented itself.

SUMMARY OF RESULTS

The following is a brief recapitulation of the major findings of this investigation in the order in which they were considered in the previous section:

1. The mean strengths of the non-air-entrained concrete mix and the air-entrained mix, 4090 and 4450 psi respectively, were not significantly different.

2. The average air content of the non-air-entrained mix was 0.9 percent and the average air content of the air-entrained mix was 8.3 percent. The coefficient of variation of the air content for the non-air-entrained mix was an abnormally high 50 percent as against a coefficient of 12 percent for the air-entrained mix.

3. The average slumps of the non-air-entrained and the air-entrained mixes were 4 inches and 2 inches respectively, with corresponding coefficients of variation of 40 and 15 percent.

4. There was no significant gain in strength of the non-air-entrained concrete during the periods of fatigue testing.

5. There was a significant decrease in the strength of the air-entrained concrete during the periods of fatigue testing. The decrease averaged 140 psi; however, it was assumed that this change did not materially affect the results of the fatigue tests.

6. The fatigue test data for the air-entrained concrete had considerably less variation than the data for the non-air-entrained concrete.

7. The portion of the S-N curve for the non-air-entrained concrete between the stress levels of 66 and 84 percent has, as determined by a linear regression analysis, the relationship:

$$\widehat{\log_{10} N} = 12.257 - 0.106\ S$$

The portion of the S-N curve for the air-entrained concrete between the stress levels of 66 and 84 percent has, as determined by a linear regression analysis, the relationship:

$$\widehat{\log_{10} N} = 20.501 - 0.214\ S$$

It was found, however, that the slopes of these lines are not significantly different and in addition, that one prediction interval lies within the other. On this basis it may be said that the relationship of stress level to cycles to failure is not significantly different for the two types of concrete when a fluctuating compressive stress condition is maintained and the minimum stress is held constant at slightly in excess of zero stress.

8. The fatigue test data accumulated in this study suggest that the S-N curve is continually sloping downwards; that is, there is no indication of a definite fatigue limit for either the non-air-entrained or the air-entrained concrete.

CONCLUSIONS

The following conclusions are based on a laboratory investigation of fatigue of concrete in which 3- by 6-inch cylinders were tested by a fluctuating compressive stress in which the maximum was varied and the minimum stress was maintained at a constant value near zero. Subject to these qualifications, the following conclusions appear to be justified:

1. The fatigue behavior of non-air-entrained plain concrete and air-entrained plain concrete are not significantly different.

2. It appears that there is considerably less variation present among fatigue test data for the air-entrained plain concrete than there is for the non-air-entrained plain concrete.

SUGGESTIONS FOR FURTHER WORK

The research reported in this thesis has considered the behavior of air-entrained concrete in fatigue with respect to only two major variables, a single air content and a fluctuating compressive stress. There are numerous other variables that could be and should be investigated singularly or in combinations. Unfortunately investigations of certain variables can easily be extended over a period of years in order to obtain a sufficient amount of data.

The results of this study suggest that a better estimate of the fatigue life of concrete at a given stress condition could be obtained by taking the necessary number of fatigue specimens from the same batch. Such a procedure would eliminate the batch-to-batch variation that was present in this study. With proper storage of the test specimens, aging could be kept negligible except possibly when specimens are tested at low stress levels and can be expected to remain in the fatigue machine for a considerable period of time.

The following recommendations for future research are ones which are not only of major interest, but are such that the testing program involved could be completed in a reasonable period of time.

1. Additional tests should be run at other stress levels to supplement the data presented in this thesis and

thereby permit the establishment of an S-N curve extending over a greater number of stress levels.

2. The effect of stress gradients should be investigated. That is, the stress condition could be one that varies from tension to compression, or it could be one that has a small stress range which is either tensile or compressive. In the case of the small stress range the mean stress could be large or small.

3. The effect of the strength of the concrete should be investigated. It could be that the extremes of a high and a low cement content have a pronounced effect on the fatigue properties of the concrete.

4. Not much information is available on the fatigue properties of lightweight aggregate concrete. Investigations in this area would be valuable.

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APPENDIX A

TEST DATA FOR CONCRETE MIXES

TABLE 15

DATA SHEET FOR MIX DESIGN FN BATCH 1

Plastic characteristics
slump = 3 inches
air content = 0.6 percent

Curing
1 day in molds
27 days in saturated lime solution

Drying
age at start = 28 days
age at finish = 32 days

Capping
at age of 35 days

Static compression tests

age when tested (days)	36	63
breaking stress (psi.)	3640	4160
	4030	3840
	4460	4250
	4050	3910
	4340	3910
	4070	3980
	3870	3770
	4070	

Fatigue tests
age of specimens: at start = 39 days
at finish = 61 days

minimum load (lbs.)	maximum load (lbs.)	stress cycles endured
500	14,500	10,155,000
500	17,000	10,355,000
700	19,500	12,000 failed
700	22,700	282,000 failed
500	25,400	100 failed

TABLE 16

DATA SHEET FOR MIX DESIGN FN BATCH 2

Plastic characteristics
slump = 3-1/2 inches
air content = 1.2 percent

Curing
1 day in molds
27 days in saturated lime solution

Drying
age at start = 28 days
age at finish = 32 days

Capping
at age of 34 days

Static compression tests

age when tested (days)	36	58	198
breaking stress (psi.)	3350	3630	3880
	3540	3610	3670
	3630	3440	3780
	3600	3450	3800
	3610	3350	3760
	3420	3570	
	3770	3650	
	3540	3490	

Fatigue tests
age of specimens: at start = 37 days
at finish = 56 days

minimum load (lbs.)	maximum load (lbs.)	stress cycles endured
500	12,300	10,568,000
500	14,800	10,106,000
500	17,800	31,000 failed
500	19,500	2,000 failed
1000	22,200	900 failed

TABLE 17

DATA SHEET FOR MIX DESIGN FN BATCH 3

Plastic characteristics
slump = 6 inches
air content = 1.8 percent

Curing
1 day in molds
27 days in saturated lime solution
10 days in saturated lime solution for cylinders tested at 11 days

Drying
age at start = 28 days
age at finish = 31 days

Capping
at age of 32 days
at age of 10 days for cylinders tested at 11 days

Static compression tests

age when tested (days)	11	33	171
breaking stress (psi.)	2750	3200	3640
	2630	3830	3700
		3680	3900
		3550	3770
		3200	
		3380	
		3370	
		3270	

Fatigue tests
age of specimens: at start = 36 days
at finish = 51 days

minimum load (lbs.)	maximum load (lbs.)	stress cycles endured
300	12,300	10,330,100
500	14,400	3,630,400 failed
700	16,500	25,800 failed
700	18,700	19,800 failed
500	20,500	300 failed

TABLE 18

DATA SHEET FOR MIX DESIGN FN BATCH 4

Plastic characteristics
slump = 2-1/2 inches
air content = 1.5 percent

Curing
1 day in molds
27 days in saturated lime solution
6 days in saturated lime solution for cylinders tested at 7 days

Drying
age at start = 28 days
age at finish = 32 days

Capping
at age of 33 days
at age of 6 days for cylinders tested at 7 days

Static compression tests

age when tested (days)	7	35	69
breaking stress (psi.)	2530	4410	4230
	2440	4200	4260
	2550	3860	4090
		4190	4170
		3910	4060
		3600	4200
		3780	4030
		4130	

Fatigue tests
age of specimens: at start = 35 days
at finish = 55 days

minimum load (lbs.)	maximum load (lbs.)	stress cycles endured
500	14,100	10,562,000
500	16,500	10,472,000
500	19,000	2,349,600 failed
800	22,600	8,900 failed

TABLE 19

DATA SHEET FOR MIX DESIGN FN BATCH 5

Plastic characteristics

slump = 3 inches
air content = 0.5 percent

Curing

1 day in saturated lime solution
27 days in saturated lime solution
6 days in saturated lime solution for cylinders tested at 7 days

Drying

age at start = 28 days
age at finish = 32 days

Capping

at age of 33 days
at age of 5 days for cylinders tested at 7 days

Static compression tests

age when tested (days)	7	34	48
breaking stress (psi.)	2460	4240	3940
	2480	4300	4190
	2480	4250	4020
		4230	4250
		4160	4120
		4240	4130
		3970	4110
		3810	4220

Fatigue tests

age of specimens: at start = 34 days
at finish = 47 days

minimum load (lbs.)	maximum load (lbs.)	stress cycles endured
500	17,400	10,740,000
500	20,300	99,700 failed
500	22,500	1,500 failed
700	26,100	3,700 failed

TABLE 20

DATA SHEET FOR MIX DESIGN FN BATCH 6

Plastic characteristics
slump = 2-1/2 inches
air content = 0.1 percent

Curing
1 day in molds
27 days in saturated lime solution
6 days in saturated lime solution for cylinders tested at 7 days

Drying
age at start = 28 days
age at finish = 32 days

Capping
at age of 33 days
at age of 5 days for cylinders tested at 7 days

Static compression tests

age when tested (days)	7	34	47
breaking stress (psi.)	2940	4560	4240
	2960	3410	4300
	3040	4190	3920
		4590	4130
		3990	4070
		4350	4060
		4660	3540
		4380	4410

Fatigue tests
age of specimens: at start = 35 days
at finish = 39 days

minimum load (lbs.)	maximum load (lbs.)	stress cycles endured
500	18,000	1,308,400 failed
500	21,000	278,700 failed
800	24,000	1,000 failed
500	26,500	300 failed

TABLE 21

DATA SHEET FOR MIX DESIGN FN BATCH 7

<u>Plastic characteristics</u>

slump = 6 inches
air content = 0.8 percent

<u>Curing</u>

1 day in molds
27 days in saturated lime solution
6 days in saturated lime solution for cylinders tested at 7 days

<u>Drying</u>

age at start = 28 days
age at finish = 32 days

<u>Capping</u>

at age of 33 days
at age of 5 days for cylinders tested at 7 days

<u>Static compression tests</u>

age when tested (days)	7	34	54
breaking stress (psi.)	2770	3910	3840
	2690	4290	3570
	2830	3700	4150
		4360	4030
		3460	3610
		3510	4180
		3370	4300
		3960	4180

<u>Fatigue tests</u>

age of specimens: at start = 47 days
at finish = 47 days

minimum load (lbs.)	maximum load (lbs.)	stress cycles endured
500	18,800	54,500 failed
1000	21,500	4,600 failed
500	24,200	10,900 failed

TABLE 22

DATA SHEET FOR MIX DESIGN FN BATCH 8

Plastic characteristics
slump = 6 inches
air content = 1.0 percent

Curing
1 day in molds
27 days in saturated lime solution
6 days in saturated lime solution for cylinders tested at 7 days

Drying
age at start = 28 days
age at finish = 32 days

Capping
at age of 33 days
at age of 5 days for cylinders tested at 7 days

Static compression tests

age when tested (days)	7	34	53
breaking stress (psi.)	2880	4180	4210
	2690	4270	4280
	2770	4570	4140
		4340	4120
		4200	3970
		4060	4100
		4270	4200
		3960	4330

Fatigue tests
age of specimens: at start = 34 days
at finish = 35 days

minimum load (lbs.)	maximum load (lbs.)	stress cycles endured
500	17,800	1,388,700 failed
800	20,500	52,200 failed
1000	23,500	1,700 failed
500	26,700	500 failed

TABLE 23

DATA SHEET FOR MIX DESIGN FA BATCH 1

Plastic characteristics

slump = 1-3/4 inches
air content = 7.6 percent

Curing

1 day in molds
27 days in saturated lime solution
6 days in saturated lime solution for cylinders tested at 7 days

Drying

age at start = 28 days
age at finish = 31 days

Capping

at age of 32 days
at age of 5 days for cylinders tested at 7 days

Static compression tests

age when tested (days)	7	34	116
breaking stress (psi.)	2120	4000	4360
	2810	3710	4450
	2740	3970	4600
		4320	4670
		4200	4480
		3930	4360
		4050	4420
		3990	4510
			4620
			4590
			4220
			4380

TABLE 24

DATA SHEET FOR MIX DESIGN FA BATCH 2

Plastic characteristics
slump = 2 inches
air content = 10.5 percent

Curing
1 day in molds
27 days in saturated lime solution
6 days in saturated lime solution for cylinders tested at 7 days

Drying
age at start = 28 days
age at finish = 32 days

Capping
at age of 33 days
at age of 5 days for cylinders tested at 7 days

Static compression tests

age when tested (days)	7	34	63
breaking stress (psi.)	2690	4210	3960
	2810	4470	4210
	2720	4500	4590
		4600	4220
		4390	4280
		4300	4150
		4210	4120
		4380	4520

Fatigue tests
age of specimens: at start = 36 days
at finish = 54 days

minimum load (lbs.)	maximum load (lbs.)	stress cycles endured
500	18,350	10,048,000
550	21,550	702,200 failed
500	23,900	2,800 failed
~500	~27,500	~100 failed

TABLE 25

DATA SHEET FOR MIX DESIGN FA BATCH 3

Plastic characteristics

slump = 2 inches
air content = 7.7 percent

Curing

1 day in molds
27 days in saturated lime solution
6 days in saturated lime solution for cylinders tested at 7 days

Drying

age at start = 28 days
age at finish = 32 days

Capping

at age of 33 days
at age of 5 days for cylinders tested at 7 days

Static compression tests

age when tested (days)	7	34	57
breaking stress (psi.)	2990	4280	4590
	2770	4370	4410
	2830	4540	4630
		4710	4310
		4520	4460
		4520	4160
		4450	4300
		4550	

Fatigue tests

age of specimens: at start = 41 days
at finish = 56 days

minimum load (lbs.)	maximum load (lbs.)	stress cycles endured
400	15,800	10,314,000
500	19,200	6,294,500 failed
500	22,350	109,400 failed
500	25,300	600 failed
500	28,800	300 failed

TABLE 26

DATA SHEET FOR MIX DESIGN PA BATCH 4

Plastic characteristics
slump = 1-3/4 inches
air content = 7.5 percent

Curing
1 day in molds
27 days in saturated lime solution
6 days in saturated lime solution for cylinders tested at 7 days

Drying
age at start = 28 days
age at finish = 32 days

Capping
at age of 35 days
at age of 5 days for cylinders tested at 7 days

Static compression tests

age when tested (days)	7	36	49
breaking stress (psi.)	2830	4550	4250
	2900	4570	4160
	2970	4390	4670
		4640	4530
		4480	4570
		4530	4290
		4680	4550
		4480	

Fatigue tests
age of specimens: at start = 37 days
at finish = 45 days

minimum load (lbs.)	maximum load (lbs.)	stress cycles endured
600	19,000	8,022,600 failed
450	22,000	204,800 failed
600	25,300	3,500 failed
~500	~28,500	~100 failed

TABLE 27

DATA SHEET FOR MIX DESIGN FA BATCH 5

Plastic characteristics
slump = 2 inches
air content = 7.7 percent

Curing
1 day in molds
27 days in saturated lime solution
6 days in saturated lime solution for cylinders tested at 7 days

Drying

age at start = 28 days
age at finish = 32 days

Capping
at age of 33 days
at age of 5 days for cylinders tested at 7 days

Static compression tests

age when tested (days)	7	34	44
breaking stress (psi.)	2780	4330	4130
	2600	4230	4060
	2750	4530	4240
		4000	4180
		4290	4300
		4330	4280
		4340	4500
		4320	4290

Fatigue tests
age of specimens: at start = 34 days
at finish = 44 days

minimum load (lbs.)	maximum load (lbs.)	stress cycles endured
600	18,000	9,092,900 failed
400	21,300	1,683,800 failed
500	24,200	1,500 failed
~500	~27,100	~100 failed

TABLE 28

DATA SHEET FOR MIX DESIGN FA BATCH 6

Plastic characteristics
slump = 1-3/4 inches
air content = 7.6 percent

Curing
1 day in molds
27 days in saturated lime solution

Drying
age at start = 28 days
age at finish = 32 days

Capping
at age of 35 days

Static compression tests

age when tested (days)	36	58
breaking stress (psi.)	4420	4680
	4710	4380
	4750	4490
	4480	4710
	4620	4380
	4630	4510
	4820	4670
	4550	4640

Fatigue tests
age of specimens: at start = 46 days
at finish = 55 days

minimum load (lbs.)	maximum load (lbs.)	stress cycles endured
500	19,500	10,157,000
600	22,300	226,900 failed
500	25,900	3,900 failed

TABLE 29

DATA SHEET FOR MIX DESIGN FA BATCH 7

Plastic characteristics
slump = 2 inches
air content = 9.0 percent

Curing
1 day in molds
27 days in saturated lime solution
6 days in saturated lime solution for cylinders tested at 7 days

Drying
age at start = 28 days
age at finish = 32 days

Capping
at age of 34 days
at age of 5 days for cylinders tested at 7 days

Static compression tests

age when tested (days)	7	35	53
breaking stress (psi.)	2760	4490	4220
	2600	4650	4330
	2650	4670	4220
		4350	4020
		4510	4170
		4410	4110
		4420	4040
		4530	

Fatigue tests
age of specimens: at start = 37 days
at finish = 52 days

minimum load (lbs.)	maximum load (lbs.)	stress cycles endured
600	15,600	10,105,000
500	18,800	10,256,000
500	22,100	361,800 failed
500	24,900	600 failed

TABLE 30

DATA SHEET FOR MIX DESIGN FA BATCH 8

Plastic characteristics
slump = 2-1/4 inches
air content = 8.2 percent

Curing
1 day in molds
27 days in saturated lime solution
5 days in saturated lime solution for cylinders tested at 6 days

Drying
age at start = 28 days
age at finish = 32 days

Capping
at age of 33 days
at age of 5 days for cylinders tested at 6 days

Static compression tests

age when tested (days)	6	34	57
breaking stress (psi.)	2640	4080	4250
	2430	4110	3990
	2400	4010	3630
		4340	3890
		4170	3940
		4280	3980
		4300	3750
		4100	3830

Fatigue tests
age of specimens: at start = 35 days
at finish = 54 days

minimum load (lbs.)	maximum load (lbs.)	stress cycles endured
700	14,400	10,760,000
650	17,400	10,109,000
500	20,500	215,700 failed
300	23,600	900 failed

TABLE 31

DATA SHEET FOR MIX DESIGN PA BATCH 9

Plastic characteristics
slump = 2-1/2 inches
air content = 8.5 percent

Curing
1 day in molds
27 days in saturated lime solution
6 days in saturated lime solution for cylinders tested at 7 days

Drying
age at start = 28 days
age at finish = 32 days

Capping
at age of 33 days
at age of 5 days for cylinders tested at 7 days

Static compression tests

age when tested (days)	7	34	43
breaking stress (psi.)	2820	4430	4730
	2640	4760	4510
	2450	4600	4490
		4600	3980
		4410	4480
		4740	4490
		4770	4760
		4610	4580

Fatigue tests
age of specimens: at start = 34 days
at finish = 41 days

minimum load (lbs.)	maximum load (lbs.)	stress cycles endured
500	19,400	10,034,000
400	22,700	534,900 failed
500	26,000	400 failed

APPENDIX B

STATISTICAL ANALYSIS OF THE STATIC TEST DATA

TABLE 32

F-TEST FOR HOMOGENEITY OF VARIANCE FOR STRENGTH DATA
NON-AIR-ENTRAINED CONCRETE

Coding: $Y_i = \dfrac{(X_i - 3000)}{10}$ where X_i represents the individual strength measurements (psi.) which are recorded in Tables 15 through 22, Appendix A.

Column Headings: "A" represents specimens tested before fatigue testing of other specimens from same batch.
"B" represents specimens tested after fatigue testing of other specimens from same batch.

	Batch FN1		Batch FN2		Batch FN3		Batch FN4	
	A	B	A	B	A	B	A	B
	64	116	35	63	20	None tested	141	123
	103	84	54	61	83		120	126
	146	125	63	44	68		86	109
	105	91	60	45	55		119	117
	134	91	61	35	20		91	106
	107	98	42	57	38		60	120
	87	97	77	65	37		78	103
	107	-	54	49	27		113	-
n_i	8	7	8	8	8		8	7
$\sum Y_i$	853	682	446	419	348		808	804
$\sum Y_i^2$	95469	68232	26040	22751	18880		86572	92820
$(\sum Y_i)^2$	727609	465124	198916	175561	121104		652864	646416
$n_i \sum Y_i^2$	763752	477624	208320	182008	151040		692576	649740
L_{yy}	36143	12500	9404	6447	29936		39712	3324
$n_i - 1$	7	6	7	7	7		7	6
s_i^2	645.41	297.62	167.93	115.13	534.57		709.14	79.14
$\ln s_i^2$	6.4699	5.6958	5.1235	4.7460	6.2814		6.5640	4.3712

(continued)

TABLE 32 (continued)

	Batch FN5		Batch FN6		Batch FN7		Batch FN8	
	A	B	A	B	A	B	A	B
	124	94	156	124	91	84	118	121
	130	119	41	130	129	57	127	128
	125	102	119	92	70	115	157	114
	123	125	159	113	136	103	134	112
	116	112	99	107	46	61	120	97
	124	113	135	106	51	118	106	110
	97	111	166	54	37	130	127	127
	81	122	138	141	96	118	96	133
n_i	8	8	8	8	8	8	8	8
$\sum Y_i$	920	898	1013	867	656	786	985	942
$\sum Y_i^2$	107832	101544	140085	98991	63620	82608	123639	111892
$(\sum Y_i)^2$	846400	806404	1026169	751689	430336	617796	970225	887364
$n_i \sum Y_i^2$	862656	812352	1120680	791928	508960	660864	989112	895136
L_{yy}	16256	5948	94511	40239	78624	43068	18887	7772
$n_i - 1$	7	7	7	7	7	7	7	7
s_i^2	290.29	106.21	1687.70	718.55	1404.00	769.07	337.27	138.79
$\ln s_i^2$	5.6709	4.6654	7.4311	6.5772	7.2471	6.6452	5.8209	4.9329

(continued)

Note: $L_{yy} = n_i \sum Y_i^2 - (\sum Y_i)^2$

$$s_i^2 = \frac{L_{yy}}{(n_i)(n_i - 1)}$$

TABLE 32 (continued)

Hypothesis: variance of k normally distributed populations are equal.

Significance Level: 0.5%

$$\sum\left[(n_i-1)s_i^2\right] = 55628.98 \qquad \sum\left[(n_i-1)\ \ln s_i^2\right] = 607.6035$$

$$\sum\left[\frac{1}{n_i-1}\right] = 2.1896 \qquad N - k = n_i - k = 118 - 15 = 103$$

$$s_p^2 = \frac{\sum\left[(n_i-1)\ s_i^2\right]}{N-k} = 540.087 \qquad \ln s_p^2 = 6.29174$$

$$M = (N-k)\ \ln s_p^2 - \sum\left[(n_i-1)\ \ln s_i^2\right] = 40.4182$$

$$A = \frac{1}{3(k-1)}\left[\sum\left(\frac{1}{n_i-1}\right) - \frac{1}{N-k}\right] = 0.05188$$

$$f_1 = k-1 = 14 \qquad f_2 = \frac{k+1}{A^2} = 5944.64$$

$$b = \frac{f_2}{1 - A - \frac{2}{f_2}} = 6267.74$$

$$F_{obs} = \frac{f_2\ M}{f_1\ (b-M)} = 2.747$$

$F_{.995}(14, 5945) = 2.31 < F_{obs}$ $\therefore$ reject the hypothesis

TABLE 33

SIGNIFICANCE TESTS FOR DIFFERENCE BETWEEN MEANS
(DUNCAN PROCEDURE)
NON-AIR-ENTRAINED CONCRETE

Batch Designation	FN1	FN2	FN4	FN5	FN6	FN7	FN8
Average Strength of Batch (coded)	102	54	107	114	118	90	120

ANOVA Table - One-Way Classification

Source	d.f.	S.S.	M.S.
mix	6	50,876	8,479.3
error	103	55,579	539.6
total	109	106,455	

$K = 7 \quad n = 16 \quad s^2 = 539.6 \quad d.f. = 103$

$$s_{\bar{Y}} = \sqrt{\frac{539.6}{16}} = 5.81$$

$$\gamma_{(7,103,0.05)} = 3.22$$

$$D = (5.81)(3.22) = 18.7 \rightarrow 19$$

$54 + 19 = 73 < 90$ $\qquad$ $90 + 19 = 109 > 102$

∴ at a 5% significance level the mean strength of batch FN2 is significantly different from the other means.

TABLE 34

F-TEST FOR HOMOGENEITY OF VARIANCE FOR STRENGTH DATA
AIR-ENTRAINED CONCRETE

Coding: $Y_i = \frac{(X_i - 4000)}{10}$ where X_i represents the individual strength measurements (psi.) which are recorded in Tables 24 through 31, Appendix A.

Column Headings: "A" represents specimens tested before fatigue testing of other specimens from same batch. "B" represents specimens tested after fatigue testing of other specimens from same batch.

	Batch FA2		Batch FA3		Batch FA4		Batch FA5	
	A	B	A	B	A	B	A	B
	21	-04	28	59	55	25	33	13
	47	21	37	41	57	16	23	06
	50	59	54	63	39	67	53	24
	60	22	71	31	64	53	00	18
	39	28	52	46	48	57	29	30
	30	15	52	16	53	29	33	28
	21	12	45	30	68	55	34	50
	38	52	55	-	48	-	32	29
n_i	8	8	8	7	8	7	8	8
$\sum Y_i$	306	205	394	286	432	302	237	198
$\sum Y_i^2$	13056	8279	20568	13364	23932	15294	8537	6130
$(\sum Y_i)^2$	93636	42025	155236	81796	186624	91204	56169	39204
$n_i \sum Y_i^2$	104448	66232	164544	93548	191456	107058	68296	49040
L_{yy}	10812	24207	9308	11752	4832	15854	12127	9836
$n_i - 1$	7	7	7	6	7	6	7	7
s_i^2	193.07	432.27	166.21	279.81	86.29	377.48	216.55	175.64
$\ln s_i^2$	5.2631	6.0691	5.1133	5.6341	4.4577	5.9335	5.3778	5.1684

(continued)

TABLE 34 (continued)

	Batch FA6 A	Batch FA6 B	Batch FA7 A	Batch FA7 B	Batch FA8 A	Batch FA8 B	Batch FA9 A	Batch FA9 B
	42	68	49	22	08	25	43	73
	71	38	65	33	11	-01	76	51
	75	49	67	22	01	-37	60	49
	48	71	35	02	34	-11	60	-02
	62	38	51	17	17	-06	41	48
	63	51	41	11	28	-02	74	49
	82	67	42	04	30	-25	77	76
	55	64	53	-	10	-17	61	58
n_i	8	8	8	7	8	8	8	8
$\sum Y_i$	498	446	403	111	139	-74	492	402
$\sum Y_i^2$	32296	26140	21195	2487	3415	3070	31632	24180
$(\sum Y_i)^2$	248004	198916	162409	12321	19321	5476	242064	161604
$n_i \sum Y_i^2$	258368	209120	169560	17409	27320	24560	253056	193440
L_{yy}	10364	10204	7151	5088	7999	19084	10992	31834
$n_i - 1$	7	7	7	6	7	7	7	7
s_i^2	185.07	182.21	121.70	121.14	142.84	340.79	196.29	568.46
$\ln s_i^2$	5.2207	5.2052	4.8016	4.7970	4.9617	5.8313	5.2796	6.3429

(continued)

Note: $L_{yy} = n_i \sum Y_i^2 - (\sum Y_i)^2$

$$s_i^2 = \frac{L_{yy}}{(n_i)(n_i - 1)}$$

TABLE 34 (continued)

Hypothesis: variance of k normally distributed populations are equal.

Significance Level: 5%

$$\Sigma\left[(n_i-1)s_i^2\right] = 25722.31 \qquad \Sigma\left[(n_i-1)\ \ln\ s_i^2\right] = 581.8338$$

$$\Sigma\left[\frac{1}{n_i-1}\right] = 2.3571 \qquad N - k = n_i - k = 125 - 16 = 109$$

$$s_p^2 = \frac{\Sigma\left[(n_i-1)\ s_i^2\right]}{N - k} = 235.984 \qquad \ln\ s_p^2 = 5.46392$$

$$M = (N-k)\ \ln\ s_p^2 - \Sigma\left[(n_i-1)\ \ln\ s_i^2\right] = 13.7335$$

$$A = \frac{1}{3(k-1)}\left[\Sigma\left(\frac{1}{n_i-1}\right) - \frac{1}{N-k}\right] = 0.05218$$

$$f_1 = k-1 = 15 \qquad f_2 = \frac{k+1}{A^2} = 6244.39$$

$$b = \frac{f_2}{1 - A + \frac{2}{f_2}} = 6585.91$$

$$F_{obs} = \frac{f_2\ M}{f_1\ (b-M)} = 0.8699$$

$F_{.95}\ (15,6244) = 1.68 > 0.8699$ $\therefore$ accept the hypothesis

TABLE 35

SIGNIFICANCE TESTS FOR DIFFERENCE BETWEEN MEANS
(DUNCAN PROCEDURE)
AIR-ENTRAINED CONCRETE

Batch Designation	FA2	FA3	FA4	FA5	FA6	FA7	FA8	FA9
Average Strength of Batch (coded)	32	45	49	27	59	33	4	56

ANOVA Table - One-Way Classification

Source	d.f.	S.S.	M.S.
mix	7	36,017	5,145
error	117	35,388	302.5
total	124	71,405	

$K = 8$ $n = 16$ $s^2 = 302.5$ $d.f. = 117$

$$s_{\bar{Y}} = \sqrt{\frac{302.5}{16}} = 4.35$$

$$\gamma_{(8,117,0.05)} = 3.26$$

$$D = (4.35)(3.26) = 14.2 \rightarrow 14$$

$4 + 14 = 18 < 27$

$\therefore$ at a 5% significant level the mean strength of batch FA8 is significantly different from the other means.

TABLE 36

SIGNIFICANCE OF DIFFERENCE TEST FOR COMPARISON OF MIX STRENGTHS

FN Mix:	$n = 94$	$\bar{Y} = 4090$ psi.	$s^2 = 68{,}231$ psi.^{2}
FA Mix:	$n = 109$	$\bar{Y} = 4430$ psi.	$s^2 = 40{,}179$ psi.^{2}

Test on variability

H_0: $\sigma_1^2 = \sigma_2^2$

H_1: $\sigma_1^2 > \sigma_2^2$

$\alpha = 5\%$

$s_1^2 = 68{,}231$ d.f. = 93

$s_2^2 = 40{,}179$ d.f. = 108

$$F_{obs} = \frac{68{,}231}{40{,}179} = 1.70 \qquad F_{.95}(93,108) = 1.39$$

$1.39 < 1.70$ $\therefore$ reject H_0, accept H_1

Test on means

H : $\mu_1 = \mu_2$

H : $\mu_1 > \mu_2$

$\alpha = 5\%$

$\bar{Y}_1 = 4430$ $s_1^2 = 40{,}179$ d.f. = 108

$\bar{Y}_2 = 4090$ $s_2^2 = 68{,}231$ d.f. = 93

$$t_{obs} = \frac{4430 - 4090}{\sqrt{\frac{68231}{94} + \frac{40179}{109}}} = 1.05 \qquad t_{0.10} = 1.66$$

$1.66 > 1.05$ $\therefore$ accept H_0

APPENDIX C

STATISTICAL ANALYSIS OF THE FATIGUE TEST DATA

CALCULATIONS FOR RUN TEST

Test procedure described in Reference 5, p. 354.

H_0: two populations have the same distribution

α = 5 percent

Stress level: 50% ± 3%

FN 10,568,000→ 10,562,000→ 10,330,100→ 10,156,000→

FA 10,760,000→ 10,314,000→ 10,105,000→

Test not possible since there are not any specific N's, i.e., no failures occurred

Stress level: 60% ± 3%

FN 10,740,000→ 10,472,000→ 10,355,000→ 10,106,000→ 3,630,400 1,338,700 1,306,400

FA 10,256,600→ 10,157,000→ 10,109,000→ 10,048,000→ 10,034,000→ 9,092,900 8,022,600 6,294,500

Test possible only for those N's which are specific

Runs = FA FA FA FN FN FN $\quad n_{FN} = 3 \quad n_{FA} = 3$

$\mu = 2 \quad \mu_{.025} = \text{—} \quad \mu_{.975} = 6 \quad \therefore$ accept H_0

Stress level: 70% ± 3%

FN 2,349,600 278,700 99,700 54,500 52,200 31,000 25,800 12,000

FA 1,683,800 702,200 534,000 361,800 256,900 215,700 204,800 109,400

(continued)

CALCULATIONS FOR RUN TEST (continued)

Runs = FN FA FA FA FA FN FA FA FA FA FN FN FN FN FN FN

$n_{FN} = 8 \quad n_{FA} = 8$

$\mu = 5 \quad \mu_{.025} = 4 \quad \mu_{.975} = 13 \quad \therefore$ accept H_o

Stress level: 80% ± 3%

FN 282,000 19,900 8,900 4,600 2,200 1,700 1,500 1,000

FA 3,900 3,500 2,500 1,500 900 600 600 400

Runs = FN FN FN FN FA FA FA FN FN $\frac{FN^*}{FA}$ FN FA FA FA FA

* same numerical value therefore in figuring the runs every possible combination of these two identical values are used.

$n_{FN} = 8 \quad n_{FA} = 8 \quad \mu_1 = 6,\ \mu_2 = 6 \quad \mu_{.025} = 4 \quad \mu_{.975} = 13$

$\therefore$ accept H_o

Stress level: 90% ± 5%

FN 10,900 3,700 900 500 300 300 100

FA 300 100 100 100

Runs = FN FN FN FN FN $\frac{FN}{PA}$ FA FA FA FN

$n_{FN} = 7 \quad n_{PA} = 4 \quad \mu_1 = 3,\ \mu_2 = 5 \quad \mu_{.025} = 2 \quad \mu_{.975} = 9$

$\therefore$ accept H_o

CALCULATIONS FOR LINEAR REGRESSION

FN

S	$n(=\log_{10}N)$
69	4.07918
72	4.49136
69	4.41161
67	6.37099
70	4.99870
72	5.44514
69	4.73640
70	4.71767
81	5.45025
79	3.34242
78	4.29667
79	3.94939
78	3.17609
82	3.00000
79	3.66276
80	3.23045

For calculation purposes: S = X and n = Y

$$\bar{X} = \frac{\Sigma X_i}{N} = \frac{1194}{16} = 74.625$$

$$\bar{Y} = \frac{\Sigma Y_i}{N} = \frac{69.35908}{16} = 4.33494$$

$$x_i = X_i - \bar{X} \qquad y_i = Y_i - \bar{Y}$$

estimating equation: $\hat{y} = bx$

$$b = \frac{\Sigma x_i y_i}{\Sigma x_i^2} = \frac{-43.922445}{413.750} = -0.106156$$

$$\hat{y}_{FN} = -0.106156\ x_{FN}$$

$$r_{XY} = \frac{\Sigma x_i y_i}{\sqrt{\Sigma x_i^2\ \Sigma y_i^2}} = \frac{-43.922445}{\sqrt{(413.750)(12.18372)}}$$

$$= -0.618625$$

FA

S	$n(=\log_{10}N)$
71	5.84646
72	5.03902
70	5.31133
71	4.32838
69	5.35583
73	5.55847
72	5.33385
71	5.72827
79	3.44716
81	2.77815
80	3.54507
81	3.17609
81	3.59106
82	2.77815
83	2.95424
82	2.60206

$$\bar{X} = \frac{1218}{16} = 76.125$$

$$\bar{Y} = \frac{67.37539}{16} = 4.21085$$

$$b = \frac{-90.253729}{421.750} = -0.213998$$

$$\hat{y}_{FA} = -0.213998\ x_{FA}$$

$$r_{XY} = \frac{-90.253729}{\sqrt{(421.750)(22.03059)}} = -0.936319$$

(continued)

CALCULATIONS FOR LINEAR REGRESSION (continued)

estimating equation in terms of the original observation units is: $\hat{Y} = a + bX$ where $a = \bar{Y} - b\bar{X}$

$a_{FN} = 4.33494 + (0.106156)(74.625) = 12.25683$

$a_{FA} = 4.21085 + (0.213998)(76.125) = 20.50145$

$\hat{Y}_{FN} = 12.25683 - 0.106156\ X_{FN}$

Therefore the regression equation for the FN series for values of S between 66 and 84 percent is

$$\widehat{\log_{10} N} = 12.257 - 0.106\ S$$

$\hat{Y}_{FA} = 20.50145 - 0.213998\ X_{FA}$

Therefore the regression equation for the FA series for values of S between 66 and 84 percent is

$$\widehat{\log_{10} N} = 20.501 - 0.214\ S$$

SIGNIFICANCE TEST FOR DIFFERENCE IN CORRELATION COEFFICIENTS

$r_{FN} = -0.618625$

$r_{FA} = -0.936319$

H_o: $\rho_{FA} = \rho_{FN}$

H_1: $\rho_{FA} > \rho_{FN}$

$\alpha = 5\%$

$$z = (1.1513)\left[\log_{10}(1+r) - \log_{10}(1+r)\right]$$

$$\chi^2 = \Sigma(n-3)z^2 - \frac{\left[\Sigma(n-3)z\right]^2}{\Sigma(n-3)} = 44.686 - 38.386 = 6.30$$

$\chi^2_{1,.05} = 3.84$ $\therefore$ reject H_o, accept H_1

SIGNIFICANCE TEST FOR DIFFERENCE IN SLOPES β_{FN} and β_{FA}

$\hat{Y}_{FN} = 12.25683 - 0.106156\, X_{FN}$

$\hat{Y}_{FA} = 20.50145 - 0.213998\, X_{FA}$

H_o: $\beta_{FA} = \beta_{FN}$ where b_{FN} and b_{FA} are estimates of β_{FN}

H_1: $\beta_{FA} > \beta_{FN}$ and β_{FA} respectively

$\alpha = 5\%$ assumption: the two populations have a common variance

test for common variance

H_o $\sigma^2_{FN} = \sigma^2_{FA}$

H_1 $\sigma^2_{FN} > \sigma^2_{FA}$

$\alpha = 5\%$ $N_{FA} = 16$ $N_{FN} = 16$

$$F = \frac{\dfrac{\Sigma(Y_{FNi} - \hat{Y}_{FNi})^2}{N-2}}{\dfrac{\Sigma(Y_{FAj} - \hat{Y}_{FAj})^2}{N-2}} = \frac{\Sigma y_i(1 - r^2_{FN})}{\Sigma y_j(1 - r^2_{FA})} = \frac{255.4}{52.0} = 4.91$$

$F_{.95}(14,14) = 2.48$ $\therefore$ reject H_o, accept H_1

test for difference in slopes $t = \dfrac{b_{FA} - b_{FN}}{s_{b_{FA}-b_{FN}}}$

$$\text{where } s^2_{b_{FA}-b_{FN}} = s^2_{\Sigma}\left[\frac{1}{\Sigma(X_{FNi}-\bar{X}_{FN})^2} + \frac{1}{\Sigma(X_{FAj}-\bar{X}_{FA})^2}\right]$$

$$\text{and } s^2_{\Sigma} = \frac{\Sigma(Y_{FNi}-\hat{Y}_{FNi})^2 + \Sigma(Y_{FAj}-\hat{Y}_{FAj})^2}{N_{FN} + N_{FA} - 4}$$

(continued)

SIGNIFICANCE TEST FOR DIFFERENCE

IN SLOPES β_{FN} AND β_{FA} (continued)

$$s_E^2 = \frac{255.407875 + 52.001775}{32 - 4} = 10.9789$$

$$s^2_{b_{FA}-b_{FN}} = 10.9789 \left[\frac{1}{413.750} + \frac{1}{421.750}\right] = 0.052503$$

$$s_{b_{FA}-b_{FN}} = 0.229135$$

$$t = \frac{-0.213998 + 0.106156}{0.229135} = -0.470$$

$t_{28,.10} = -1.70$ $\qquad\qquad$ $\therefore$ accept H_o

CALCULATIONS FOR PREDICTION INTERVALS

Equation for prediction interval

$$L = a + b X_o \mp t_{\alpha(n-2)} s_E \sqrt{1 + \frac{1}{n} + \frac{(X_o - \bar{X})^2}{\Sigma(X_i - \bar{X})^2}}$$

FN Series

$X_o = 70, 75, \& 80$

$t_{14,.05} = 2.145$ $\qquad s_E = \sqrt{\frac{\Sigma(Y_i - \hat{Y}_i)^2}{n - 2}} = \sqrt{\frac{255.407875}{14}}$

$= 4.2712$

$X_o = 70$ $\quad L_1 = 4.8259 - 9.6707 = -4.8448$
$\quad L_2 = 4.8259 + 9.6707 = +14.4966$

$X_o = 75$ $\quad L_1 = 4.2951 - 9.4452 = -5.1501$
$\quad L_2 = 4.2951 + 9.4452 = +13.7403$

$X_o = 80$ $\quad L_1 = 3.7643 - 9.7491 = -5.9848$
$\quad L_2 = 3.7643 + 9.7491 = +13.5134$

FA Series

$X_o = 70, 75, \& 80$

$t_{14,.05} = 2.145$ $\qquad s_E = \sqrt{\frac{52.001775}{14}} = 1.9273$

$X_o = 70$ $\quad L_1 = 5.5216 - 4.4360 = +1.0856$
$\quad L_2 = 5.5216 + 4.4360 = +9.9576$

$X_o = 75$ $\quad L_1 = 4.4516 - 4.2672 = +0.1844$
$\quad L_2 = 4.4516 + 4.2672 = +8.7188$

$X_o = 80$ $\quad L_1 = 3.3816 - 4.3320 = +0.9504$
$\quad L_2 = 3.3816 + 4.3320 = +7.7136$

APPENDIX D

DEFORMATION TEST DATA

TABLE 37

DEFORMATION TEST RESULTS FOR NON-AIR-ENTRAINED CONCRETE
SPECIMEN NO. 1

Specimen size: 12 inches high by 6 inches in diameter.
Type of loading: Compressive at a rate of approximately 10 psi./sec.
Deflection yoke: 10-inch gauge length.
Two diametrically opposed Ames dials.

Load (lbs.)	Dial Readings in Ten Thousands of an Inch							
	1st Cycle		2nd Cycle		3rd Cycle		4th Cycle	
	Dial #1	Dial #2	Dial #1	Dial #2	Dial #1	Dial #2	Dial #1	Dial #2
500	0	0	7	4	7	5	7	5
5,000	5	2	13	6	12	8	13	7
10,000	12	5	22	8	22	9	22	9
15,000	20	8	30	11	29	12	30	12
20,000	28	11	36	15	36	16	37	17
25,000	35	16	42	19	42	21	43	21
30,000	42	20	48	25	47	26	48	26
35,000	49	24	53	29	52	31	53	31
40,000	55	29	57	34	57	36	57	36
45,000	62	35	62	39	62	41	62	41
50,000	70	41	67	45	67	46	67	47
45,000	67	38	64	42	63	43	64	44
40,000	62	35	59	38	58	39	58	40
35,000	56	31	54	35	53	36	53	36
30,000	51	27	48	30	48	32	48	32
25,000	45	24	43	27	42	27	42	28
20,000	38	20	37	23	36	24	36	24
15,000	31	16	29	19	29	20	29	20
10,000	23	13	22	15	21	16	21	16
5,000	14	10	13	11	12	12	13	12
500	7	4	7	5	7	5	7	5

Ultimate static compressive strength of specimen after having endured the four load cycles . . . . 92,540 lbs.

TABLE 38

DEFORMATION TEST RESULTS FOR NON-AIR-ENTRAINED CONCRETE
SPECIMEN NO. 2

Specimen size: 12 inches high by 6 inches in diameter.
Type of loading: Compressive at a rate of approximately 10 psi./sec.
Deflection yoke: 10-inch gauge length.
Two diametrically opposed Ames dials.

Load (lbs.)	Dial Readings in Ten Thousands of an Inch							
	1st Cycle		2nd Cycle		3rd Cycle		4th Cycle	
	Dial #1	Dial #2	Dial #1	Dial #2	Dial #1	Dial #2	Dial #1	Dial #2
500	0	0	9	8	10	9	11	9
5,000	6	2	14	12	15	12	16	12
10,000	12	6	20	17	21	17	22	18
15,000	18	11	25	23	26	23	27	24
20,000	23	17	30	28	32	29	32	29
25,000	28	23	35	33	36	34	37	35
30,000	33	29	41	39	42	40	42	41
35,000	39	35	45	44	47	45	47	46
40,000	44	41	51	50	52	51	53	52
45,000	51	47	56	55	57	56	58	57
50,000	56	55	62	61	63	62	64	63
55,000	64	62	67	66	69	67	69	67
60,000	72	69	73	71	74	72	75	73
55,000	68	67	69	67	70	68	70	69
50,000	63	62	64	63	65	64	65	65
45,000	58	57	59	58	59	59	60	60
40,000	52	53	53	54	54	55	54	55
35,000	47	48	48	49	49	50	49	50
30,000	42	42	42	44	43	45	44	45
25,000	37	38	37	39	38	40	38	40
20,000	32	33	32	34	32	34	33	35
15,000	26	27	26	28	27	29	27	29
10,000	21	21	21	23	21	23	22	24
5,000	15	15	15	16	16	16	16	16
500	9	8	10	9	11	9	11	9

Ultimate static compressive strength of specimen after having endured the four load cycles . . . . 94,880 lbs.

TABLE 39

DEFORMATION TEST RESULTS FOR NON-AIR-ENTRAINED CONCRETE
SPECIMEN NO. 3

Specimen size: 12 inches high by 6 inches in diameter.
Type of loading: Compressive at a rate of approximately 10 psi./sec.
Deflection yoke: 10-inch gauge length.
Two diametrically opposed Ames dials.

Load (lbs.)	Dial Readings in Ten Thousands of an Inch							
	1st Cycle		2nd Cycle		3rd Cycle		4th Cycle	
	Dial #1	Dial #2	Dial #1	Dial #2	Dial #1	Dial #2	Dial #1	Dial #2
500	0	0	17	7	19	8	21	9
5,000	7	1	25	8	27	9	29	10
10,000	14	5	33	12	35	13	37	14
15,000	20	8	40	17	42	18	44	20
20,000	27	13	46	22	49	23	51	25
25,000	34	17	53	27	56	29	58	31
30,000	41	21	59	33	62	35	64	37
35,000	48	26	65	38	68	41	70	43
40,000	55	32	71	43	74	46	76	48
45,000	62	37	77	48	80	51	82	53
50,000	69	44	83	54	85	56	88	58
55,000	77	51	89	59	91	61	94	63
60,000	84	57	95	64	97	66	100	68
65,000	94	65	101	69	103	72	106	73
70,000	103	72	108	75	110	77	112	78
65,000	101	69	104	72	106	73	108	75
60,000	96	65	99	67	101	69	103	71
55,000	91	60	94	63	96	65	98	66
50,000	86	56	89	58	91	60	93	61
45,000	81	52	84	54	85	56	87	57
40,000	75	47	78	50	80	52	82	52
35,000	70	43	72	45	74	47	76	47
30,000	64	38	66	40	68	42	70	42
25,000	58	33	60	35	62	37	65	37
20,000	51	27	53	30	55	32	57	32
15,000	44	22	46	24	48	26	49	26
10,000	36	17	38	18	40	20	41	20
5,000	27	12	34	13	31	14	32	14
500	17	7	19	8	21	9	21	9

Ultimate static compressive strength of specimen after having endured the four load cycles . . . . 95,820 lbs.

TABLE 40

DEFORMATION TEST RESULTS FOR AIR-ENTRAINED CONCRETE
SPECIMEN NO. 1

Specimen size: 12 inches high by 6 inches in diameter.
Type of loading: Compressive at a rate of approximately 10 psi./sec.
Deflection yoke: 10-inch gauge length. Two diametrically opposed Ames dials.

Load (lbs.)	Dial Readings in Ten Thousands of an Inch							
	1st Cycle		2nd Cycle		3rd Cycle		4th Cycle	
	Dial #1	Dial #2	Dial #1	Dial #2	Dial #1	Dial #2	Dial #1	Dial #2
500	0	0	6	5	8	4	8	4
5,000	3	8	12	10	13	9	13	9
10,000	8	16	17	16	19	15	19	15
15,000	12	24	23	21	25	20	25	20
20,000	17	30	29	27	31	26	31	26
25,000	23	37	35	32	37	30	37	30
30,000	29	43	41	37	42	35	42	35
35,000	34	49	46	42	48	40	48	40
40,000	41	56	52	48	53	46	54	46
45,000	47	62	57	53	59	51	60	51
50,000	53	68	63	59	65	57	65	57
45,000	49	64	59	55	60	53	61	53
40,000	44	60	54	50	55	48	56	48
35,000	39	55	49	45	51	44	50	43
30,000	35	49	43	40	44	38	45	38
25,000	30	43	38	35	39	33	39	33
20,000	25	36	32	30	33	28	33	27
15,000	20	30	26	25	27	23	28	23
10,000	16	23	20	19	21	18	21	18
5,000	11	14	14	12	14	11	14	11
500	6	5	8	4	8	4	8	4

Ultimate static compressive strength of specimen after having endured the four load cycles . . . . 57,570 lbs.

TABLE 41

DEFORMATION TEST RESULTS FOR AIR-ENTRAINED CONCRETE
SPECIMEN NO. 2

Specimen size: 12 inches high by 6 inches in diameter.
Type of loading: Compressive at a rate of approximately 10 psi./sec.
Deflection yoke: 10-inch gauge length.
Two diametrically opposed Ames dials.

Load (lbs.)	Dial Readings in Ten Thousands of an Inch							
	1st Cycle		2nd Cycle		3rd Cycle		4th Cycle	
	Dial #1	Dial #2	Dial #1	Dial #2	Dial #1	Dial #2	Dial #1	Dial #2
500	0	0	18	1	19	2	19	2
5,000	10	1	27	3	28	3	28	4
10,000	20	4	37	5	37	6	37	6
15,000	30	6	45	9	45	10	46	10
20,000	39	10	52	14	53	15	53	15
25,000	48	14	60	18	60	19	61	20
30,000	57	17	67	23	67	24	68	24
35,000	65	22	74	26	74	26	75	28
40,000	73	26	80	31	81	32	81	33
45,000	81	32	87	35	87	36	87	37
50,000	89	37	92	40	92	41	93	42
55,000	97	43	98	45	99	45	99	46
60,000	105	48	104	50	105	50	105	51
55,000	101	45	101	46	101	46	101	47
50,000	96	41	95	42	95	42	96	43
45,000	90	37	90	38	90	38	90	39
40,000	85	33	84	34	84	35	84	35
35,000	78	29	78	30	78	31	78	31
30,000	71	25	71	26	71	27	71	27
25,000	64	22	64	23	64	23	65	24
20,000	56	18	56	19	57	19	57	20
15,000	48	14	48	15	48	15	48	15
10,000	39	10	39	11	39	11	39	11
5,000	29	6	30	6	30	7	30	8
500	18	1	19	2	19	2	20	2

Ultimate static compressive strength of specimen after having endured the four load cycles . . . . 101,720 lbs.

TABLE 42

DEFORMATION TEST RESULTS FOR AIR-ENTRAINED CONCRETE
SPECIMEN NO. 3

Specimen size: 12 inches high by 6 inches in diameter.
Type of loading: Compressive at a rate of approximately 10 psi./sec.
Deflection yoke: 10-inch gauge length.
Two diametrically opposed Ames dials.

Load (lbs.)	1st Cycle Dial #1	1st Cycle Dial #2	2nd Cycle Dial #1	2nd Cycle Dial #2	3rd Cycle Dial #1	3rd Cycle Dial #2	4th Cycle Dial #1	4th Cycle Dial #2
	Dial Readings in Ten Thousands of an Inch							
500	0	0	-1	10	-1	11	-1	12
5,000	5	5	3	16	3	17	3	18
10,000	11	12	8	24	8	25	8	25
15,000	16	19	14	31	13	33	13	34
20,000	21	27	19	39	19	40	19	41
25,000	26	35	24	46	24	47	24	49
30,000	30	42	29	53	29	55	29	55
35,000	35	49	33	60	33	62	33	62
40,000	39	56	38	66	38	67	38	68
45,000	43	62	42	73	42	74	42	75
50,000	48	68	47	78	47	79	47	81
55,000	51	75	51	84	51	86	51	86
60,000	56	82	56	89	55	91	55	92
65,000	61	90	60	95	60	96	59	97
70,000	67	98	64	102	63	103	63	104
65,000	64	95	61	98	61	100	60	100
60,000	60	90	58	93	57	94	57	95
55,000	56	86	53	88	53	89	52	90
50,000	51	81	51	83	49	84	49	85
45,000	47	76	45	78	44	79	44	80
40,000	42	70	40	73	40	74	40	74
35,000	37	65	35	68	35	68	35	69
30,000	32	60	30	62	30	63	30	64
25,000	27	54	25	56	25	57	24	57
20,000	21	47	19	50	19	51	19	51
15,000	15	40	13	42	13	43	13	44
10,000	9	32	8	34	8	35	8	35
5,000	3	22	3	23	3	24	3	25
500	-1	10	-1	11	-1	12	-1	12

Ultimate static compressive strength of specimen after having endured the four load cycles . . . . 112,510 lbs.