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# Trading Horizons and the Value of Money<sup>1</sup>

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## Abstract

We develop an anonymous trading framework where specialization and trade are beneficial to society and trading arrangements are endogenous. Its key features are that individual actions can have long-lasting aggregate consequences and the current allocation of consumption can affect the future availability of productive resources. We study equilibrium patterns of exchange when the trading sequences are finite and deterministic, demonstrating that fiat money does not necessarily lose its beneficial allocative role. The reason is that individual actions that reduce the agent's current payoff—such as selling for money on its final trading date—can be incentive-compatible when this prevents others from making choices carrying adverse long-lasting aggregate consequences.

JEL: C70, E40.

Keywords: Monetary Equilibrium, Anonymous Trade, Finite Horizons, Repeated Games, Specialization.

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## 1 Introduction

A basic idea in economics is that allocations can be improved by specialization and trade and can be further expanded by innovations in the trading technology. Fiat money is such an innovation: several observers have noted how the use of ‘barren’ tokens facilitates beneficial trades in markets that are not well-functioning, especially if informational frictions exist (e.g. Ostroy, 1973, Townsend, 1989, Ostroy and Starr, 1990). Indeed, Kocherlakota (1998) suggests that money is simply a record-keeping technology; he demonstrates that a monetary allocation dominates non-monetary outcomes in several settings where it is impossible to commit to future actions or to freely access each others’ trade records.

Interestingly, virtually all fiat monetary models consider infinite trading sequences. The reason is that for intrinsically useless tokens to have value in equilibrium, agents must expect that someone will want to trade consumption for money at some future date (Cass and Shell, 1980). One may thus conclude that money *necessarily* loses its technological edge in environments where the monetary trade sequence is finite and deterministic (e.g. see Kocherlakota, p. 244). In this study we explain why this would be a rushed conclusion. In so doing, we make two contributions.

First, we add to the monetary theory literature by demonstrating that fiat-money can have a fundamental allocative role in economies characterized by finite trade sequences. Prior work has studied existence of monetary equilibria in finite-horizon economies where money has either an implicit role or the final trading date is uncertain (Faust, 1989, Kultti, 1995, Kovenock and De Vries, 2002).<sup>2</sup> We complement this work by studying economies where money has an *explicit* role and the trading horizon is publicly known.

More generally, we contribute to a literature concerned with allocative efficiency in environments with limited commitment and enforcement (for a monetary economy, see for example Corbae and Ritter, 2004). We provide a simple but intuitive example that demonstrates why incentive-compatibility need neither hinge on the agents’ ability to obtain current or future rewards, nor on the threat of future punishment. Precisely, in environments where individual actions can significantly affect some aggregate state variable, an agent may voluntarily take an action that reduces his current payoff if this prevents others from behaving in ways that carry adverse aggregate consequences.

To articulate these ideas we construct an economy with finitely many dynastic (altru-

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<sup>2</sup>The continuous-time model of Faust has a terminal date but an infinite number of trade rounds. The models of Kultti and Kovenock and De Vries share one or more of these features: (i) agents are asymmetrically informed on either the last trading date or their position at that date (buyer or seller) or (ii) money is a unit of account.

istic) agents from two overlapping generations. The initial old have one token each, the young are productive and society can benefit from specialization and trade. This simply means that the young should produce a high-value good for the old, avoiding other alternatives. A key aspect is that the current allocation of consumption can affect the future availability of productive resources. Indeed, we add a reason to worry about consumption of the old—relative to the usual (e.g. Antinolfi et al., 2001)—by letting a dynasty’s survival depend on the old’s consumption. Finally, we make explicit a set of information frictions that obstacle history-dependent credit trades and give money a central role. To do this, we assume pairwise anonymous matches with limited communication/commitment, i.e., trades are unobservable to third parties and agents can choose autarky at any point in time. Indeed, the dynastic formulation naturally motivates the existence of difficulties in accessing trade histories—much as ‘lack-of-recall’ does in Temzelides and Yu (2004)—and in making privately observed actions, public.

We relax the standard assumption of infinite trading sequence in two progressive steps. First, we let tokens have a finite deterministic life-span and then we do the same for the economy’s duration. We prove that multiple Pareto-ranked equilibria are possible, in some of which trade takes place only if supported by the exchange of tokens. Especially, equilibria with valued tokens exist both when they are perishable and the economy is infinite or when the economy has a finite life-span. In the latter case monetary trade occurs until a date sufficiently ahead of the economy’s last period. Either way, the incentive to swap consumption for money, on money’s last trading date, hinges on the desire to avoid an inferior equilibrium in the remaining periods.

What is the intuition? The fundamental feature is that individual actions can be strategically non-negligible and have permanent aggregate consequences. In the model, lack of consumption by an old permanently reduces the productive resources available to society and can spawn significant consumption risk in a small economy. Unfortunately, the allocation of goods cannot be based on individual histories or identities. It is the need to bridge this information gap that creates a role for money even if the trading horizon is finite. To see why, suppose tokens are valued up to a date  $t$  ahead of the economy’s last period. Making a unilateral transfer to someone who has a token in  $t$  is optimal if it is the old who own money and if, by refusing to make this ‘gift,’ the agent would severely impair his and his dynasty’s consumption prospects in the remaining periods.

## 2 Environment

We propose a conceptually simple model where the available productive resources depend

on past allocations of consumption, in order to bring to light the workings of an anonymous trading framework where specialization and trade are beneficial to society. Along the way, we will motivate its central features and its simplifying assumptions.

Time is discrete,  $t = 0, 1, \dots, J \leq \infty$ . In  $t = 0$  there are  $2N \in \mathbb{N}_+$  spatially separated dynastic agents divided in equal proportion between young and old members of two-period-lived overlapping generations. The agents are altruistic in that their objective is the maximization of their dynasty's lifetime utility from consumption of either of two types of non-storable goods, denoted 'market' and 'home'. The key difference from similar models of dynastic agents (e.g. Fuster et al., 2003) is that consumption of some good is a necessary input for the preservation of one's dynasty. Specifically, an old agent generates an offspring only if he consumes; otherwise, the dynasty dies out.<sup>3</sup> As in other overlapping generations models, we make productive resources available only to the young so the size of the new generation is a key aggregate state variable. These features capture—simply and intuitively—the notion that current productive resources depend on past allocations.

Endowments are as follows. Each initial old has an indivisible unit of fiat money, known to last until date  $1 \leq H \leq \infty$  included. Each young has a production opportunity that can be used in one of two ways. He can use it to produce alone one indivisible market good by supplying effort that generates disutility  $e \in (0, \beta u)$ . Here,  $u > 0$  is the period utility from consumption of a market good and  $\beta \in (0, 1)$  is the discount factor. To motivate the need for trade, we assume that the young do not derive utility from own market production (e.g. Diamond, 1982). The alternative use of the production opportunity is as follows. The young can team-up with someone else—young or old—in order to costlessly produce and then consume one home good. Each partner is assumed to derive  $\alpha u$  period utility from such team activity, with  $\alpha \in (0, \frac{1}{2})$ . Thus, although home goods are least preferred, team activity allows agents to improve over autarky in the stage game. Indeed, we set the agents free to choose autarky at every point in time.

At each date agents select independently and simultaneously either autarky—which generates zero current utility—or anonymous trade. By this we mean that the agent elects to match to someone else—whose identity and history cannot be observed—for the purpose of trading. We assume that, although the initial population is known, the number of traders cannot be directly observed. Traders' interaction occurs according to a directed

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<sup>3</sup>Hence, the model can be interpreted as a simpler version of one in which either survival probabilities or the discount factor depend explicitly on the allocation of consumption, approaches seen in the literature of life-cycle consumption choices (e.g. Ray and Struefert, 1993, or Shi and Epstein, 1993). The inherent simplicity of our formalization enhances the transparency of the analysis and of the model's workings.

matching process whose operation is detailed in the next section. Here, we simply note that this process is essentially an assignment rule that exhausts all mutually desirable pairings selecting traders at random. As is standard in many ‘deep’ models of money, we assume that traders cannot communicate across matches, are anonymous and are unable to enforce or commit to an action. Finally, we assume that paired agents can only observe their respective actions and money holdings and that their exchange is based on a direct trading mechanism that is taken as given.

In short, in this model trading arrangements are endogenous and the current allocation of consumption affects the future availability of productive resources. Money may have a fundamental role since the owners of productive resources cannot access trade histories. Indeed, the dynastic agents formulation naturally motivates such difficulties as well as the existence of obstacles in making public privately observed actions. These features allow us to make explicit a set of information frictions that obstacle credit-type trades.

### 3 Decentralized Symmetric Equilibria

Agents play a game of imperfect information that, at every date, has two stages. At the beginning of each period, the agent must choose whether to trade and who to trade with. Then, pairwise matches are formed among those who have selected to trade. In the second stage, each matched agent proposes a trading plan that is implemented only if the proposals are consistent. At each stage agents can select autarky.

Specifically, denote the agent’s state at the beginning of  $t$  by  $z(t) = (m, a)$  where  $m = 0, 1$  denotes money holdings and  $a = 0, 1$  is the age, young or old. The agent must choose whether and with whom to trade, taking as given the choices of others. He can choose either autarky, denoted 0, or a trading position denoted  $\tau = b, s$ , i.e., trade as a buyer or seller ( $b$  or  $s$ ). He also chooses whom to match with by selecting the partner’s desired characteristics: money holdings  $m'$  and trading position  $\tau'$ . We denote by  $\omega_z(t)$  these beginning-of-period  $t$  choices of a representative agent, so

$$\omega_z(t) = \begin{cases} 0 & \text{if autarky is selected} \\ (\tau, \tau', m') & \text{if trade is selected.} \end{cases}$$

Say, if  $\omega_1(t) = (b, s, 0)$  then we have an agent with money ( $z = 1$ ) who has chosen to match as a buyer ( $\tau = b$ ) to a seller without money ( $\tau' = s$  and  $m' = 0$ ).

Given these choices, pairings take place in a manner that reflects the traders’ selections. This can be thought of as a directed matching process that—without going into unnecessary details—has the following key characteristics. First, two traders can match

only if their choices  $\omega$  are mutually consistent. Second, matching must be feasible, a notion formalized by assuming that the probability of the desired matching depends on the number of buyers  $B$  and sellers  $S$ . In particular, if sellers want to match with buyers—and viceversa—every trader is matched as desired only if  $B = S$ . If  $B \neq S$ , then the representative trader on the longest side of the market matches with someone with probability  $\frac{\min\{S,B\}}{\max\{S,B\}}$  and remains unmatched otherwise. Matched traders are anonymous.<sup>4</sup>

Unmatched traders do nothing while paired agents interact via a direct trading mechanism. Each trader can take a single action that may depend only on  $z$ ,  $t$  and the partner's money holdings  $m'$ . Assuming the action induces a single outcome, we can think of traders as playing a coordination game simultaneously proposing a feasible transfer of goods (home or market) and tokens, denoted  $x_z(t, m') = (h, g, d)$ . Here  $d, g \in \{-1, 0, 1\}$  and  $h \in \{-1/2, 0, 1/2\}$  denote the proposed transfer ( $-$ ) or request ( $+$ ) of  $d$  money,  $g$  market and  $h$  home goods. Consistent proposals are implemented, else autarky results. Actions and outcomes are unobserved by others and the match breaks at the period's end.

We model limited commitment by requiring that the trading mechanism satisfies sequential rationality (see Kocherlakota, 1998). Without introducing additional notation, this simply means that the agent's strategy must be a mapping from all its possible information sets into actions and it specifies a weakly optimal action at each information set, given that all others follow their strategies in the current and all future information sets. In particular, we note that since an agent can always choose autarky in (or outside) a match, for any action taken by anyone else, then it must be the case that equilibrium actions be compatible with individual incentives.

In constructing an equilibrium we focus on symmetric strategies, checking only unilateral one-period deviations (the unimprovability principle allows us to do so), restricting attention to environments where multiple deviations are not allowed (e.g. matched agents cannot both deviate). To sum up, we adopt the following equilibrium concept

**Definition.** *An equilibrium is a list of sequentially rational strategies  $\{\omega_z(t), x_z(t, m')\}_{t=0}^J$ , that are identical for agents in an identical state.*

In what follows, we will say that at date  $t$  there is monetary trade if sellers of market goods require a token in exchange for their production, i.e. if  $d = 1$ . We start by studying time-invariant trading regimes when  $H = J = \infty$ , providing a condition insuring that

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<sup>4</sup>Thus, it may be helpful to think of this as a directed search process with frictions as in, say, Burdett, Shi and Wright (2001). For a rigorous discussion of anonymity in environments where agents are bilaterally matched see Aliprantis, Camera and Puzzello (2004).

money has a fundamental role in sustaining the efficient allocation. Then, we study the case in which money ceases to exist after date  $H < J = \infty$ . Finally, we study a finite horizon economy,  $J < \infty$ . In these two last cases we cannot consider time-invariant allocations (since money has no intrinsic value) and so we focus on equilibria with regime-switching; there is some date  $T < J$  that separates two different, time-invariant, trading regimes.

Throughout the discussion, we retain the following assumption

$$\frac{u - e}{2} > \alpha u \geq \frac{-e + \beta u}{1 + \beta}. \quad (1)$$

Simply put (see later) the first inequality implies that at each date market production and trade is Pareto efficient. The second inequality implies that a young agent would rather avoid production of market goods and engage in home production, so that something must be done to prevent him from doing so.

### 3.1 Allocations under infinite trading sequences

Suppose  $H = J = \infty$ . Here there can be a non-monetary stationary equilibrium and also a monetary equilibrium, i.e. an equilibrium in which, at every date, sellers of market goods require a token in exchange for their production.

If money is expected to be valueless, market goods are not produced and home production is individually optimal. We call this a non-monetary pattern of exchange. Formally, the equilibrium strategies are independent of money holdings. If, without loss in generality, we call the young a seller and the old a buyer, then for all  $m$  and  $m'$  we have:

$$\begin{aligned} \omega_z(t) = \omega_z^{**} &= \begin{cases} (b, s, m') & \text{if } z = (m, 1) \\ (s, b, m') & \text{if } z = (m, 0) \end{cases} \\ x_z(t, m') = x_z^{**} &= \begin{cases} (1/2, 0, 0) & \text{if } z = (m, 1) \\ (-1/2, 0, 0) & \text{if } z = (m, 0). \end{cases} \end{aligned} \quad (2)$$

In short, at every date we have  $N$  young-old matches where the home good is produced and consumed in equal amounts. Since agents care about their descendants, every agent has stationary lifetime utility defined recursively by

$$v_c = \alpha u + \beta v_c \Rightarrow v_c = \frac{\alpha u}{1 - \beta}. \quad (3)$$

Now suppose agents believe money has value and old agents choose to buy market goods in exchange for money. Suppose also that young agents choose to sell market goods



for money. Thus, in each period there are  $N$  buyer-seller pairs and each agent alternates market production as young, to market consumption as old. We refer to this pattern of exchange as monetary trade. Formally, in equilibrium we have

$$\omega_z(t) = \omega_z^* = \begin{cases} (b, s, 0) & \text{if } z = (1, 1) \\ (s, b, 1) & \text{if } z = (0, 0) \end{cases} \quad (4)$$

$$x_z(t, m') = x_z^*(m') = \begin{cases} (0, 1, -1) & \text{if } z = (1, 1) \text{ and } m' = 0 \\ (0, -1, 1) & \text{if } z = (0, 0) \text{ and } m' = 1. \end{cases}$$

In equilibrium, let  $v_b$  be the stationary expected lifetime utility of an old buyer and  $v_s$  for a young seller. Trade generates  $u + \beta v_s$  payoff to a buyer and  $-e + \beta v_b$  to a seller. Defining  $v_b$  and  $v_s$  recursively we have

$$v_b = u + \beta v_s \quad \text{and} \quad v_s = -e + \beta v_b$$

so that

$$v_b = \frac{u - \beta e}{1 - \beta^2} > v_c \geq v_s = \frac{\beta u - e}{1 - \beta^2}. \quad (5)$$

The inequalities follow from (1), which is seen to imply  $u - \beta e > \alpha u(1 + \beta)$ . In short, monetary exchange is a deterministic trading cycle that—relative to home production—generates higher lifetime utility for the buyer and greater surplus in the match.

**Lemma 1.** *Let  $H = J = \infty$ . The strategies defined in (2) and (4) can both be an equilibrium. However, only (4) sustains the Pareto efficient outcome.*

**Proof.** Conjecture that either (2) or (4) is an equilibrium strategy. In either case autarky is suboptimal, since trade generates non-negative surplus.

Suppose that (2) is an equilibrium strategy, i.e., money has no value. Clearly, the actions specified in (2) are individually optimal. The alternative is earning zero payoff, for an old agent, and less than  $\alpha u + \beta v_c$ , for a young agent.

Suppose that (4) is an equilibrium strategy, i.e., money has value. Here, an old agent strictly prefers offering his money to a seller of a market good. In equilibrium a young agent optimally chooses to sell a market good for money. Indeed, given the equilibrium matching strategies, a young can only match to a buyer with money and an old cannot consume if he is without money. Thus, the actions specified in (4) are individually optimal. Finally, monetary trade maximizes the trade surplus since  $u - e > 2\alpha u$  from (1) and so it supports the Pareto efficient allocation. ■

To sum up, monetary trade of market goods and non-monetary exchange of home production can both be an equilibrium. They sustain deterministic consumption of different types of goods. The trade patterns coexist because money has extrinsic value in equilibrium, which is a standard result (e.g. Kiyotaki and Wright, 1989).<sup>5</sup> Monetary trade is Pareto efficient, as it mirrors the planner's choice of mandating a transfer of a market good from each young to each old. This pattern of exchange is beneficial in two ways; it maximizes the number of matches and it maximizes the surplus in each match, since,  $u - e > 2\alpha u$ . Note that although a young agent would weakly prefer home production to monetary trade, since  $v_c \geq v_s$ , no young agent deviates along the monetary equilibrium path as he realizes he would otherwise be unable to consume when old.

In a nutshell, this is a decentralized trading environment where society can benefit from specialization and trade: the young should specialize in market production and the old should consume it. A crucial question is whether there are non-monetary patterns of exchange that can sustain the efficient allocation. We answer it in the following

**Lemma 2.** *If*

$$N \geq \underline{N} \equiv \frac{2\beta(u-\beta e)}{(u+e)(1-\beta^2)} + \frac{u-e}{u+e}$$

*then the efficient allocation cannot be sustained without money.*

**Proof.** Suppose there exists a 'gift-giving' equilibrium that replicates the efficient allocation. By this we mean an equilibrium where matching and transfers of market goods is not conditional on money holdings and money transfers. In this case the equilibrium strategies are as in (4) but they are independent of  $m$  and  $m'$ . That is, for any  $m$  and  $m'$  and all  $t$  we have  $\omega_z(t) = \omega_z$  and  $x_z(t, m') = x_z$  where

$$\omega_z = \begin{cases} (b, s, m) & \text{if } z = (m, 1) \\ (s, b, m) & \text{if } z = (m, 0) \end{cases}$$

$$x_z = \begin{cases} (0, 1, -1) & \text{if } z = (1, 1) \\ (0, -1, 1) & \text{if } z = (0, 0). \end{cases}$$

In short, a young transfers (sells) a market good to anyone who asks for it, while the old demands (buys) a market good from anyone who is willing to supply it.

Due to anonymity, matching and transfers cannot be based on the partner's age, productivity or trade history. The key deviation, therefore, is for a young to go on the market asking for a transfer, instead of making one in  $t$ , reverting to equilibrium

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<sup>5</sup>There is always an equilibrium with autarky, where everyone chooses to not match with anyone else.

(consuming) when old, in  $t + 1$ . This deviation implies in  $t$  there are  $N + 1$  buyers and  $N - 1$  sellers, out of equilibrium. As a consequence, the deviator knows that in  $t$  he can either remain unmatched or match to a seller. Specifically:

1. With probability  $\frac{2}{N+1}$  the young deviator remains unmatched, in which case we denote the deviant's payoff by  $\beta\hat{v}_b$ , the discounted value of trading in  $t + 1$  as an old buyer, out-of-equilibrium. To determine  $\hat{v}_b$ , notice that due to the deviation some old agent does not consume in  $t$ . The deviation is undetected since this agent and his dynasty leave the economy and the number of traders is unobservable. Hence, in  $t + 1$  there are  $N - 1$  young sellers and  $N$  old buyers. Again, some old agent cannot consume. In particular, the old deviator consumes with probability  $\frac{N-1}{N}$ . Hence,  $t + 2$  starts with an equal number of old buyers and young sellers,  $N - 1$ . These agents follow equilibrium play as no deviation is made public. Thus:

$$\hat{v}_b = \frac{N-1}{N}(u + \beta v_s) = \frac{N-1}{N}v_b$$

2. With probability  $\frac{N-1}{N+1}$  the deviator matches to a seller. Here two old agents do not consume and their dynasties leave the economy. The deviator's payoff is  $u + \beta\check{v}_b$  where  $\check{v}_b$  denotes the expected payoff, as an old buyer, out-of-equilibrium in  $t + 1$ . The initial deviation remains undetected. In  $t + 1$  there are  $N - 2$  young who sell and  $N$  old who buy. Hence, two old agents do not consume in  $t + 1$  and two more dynasties exit the economy. Clearly, this can happen to the deviator in  $t + 1$ , since he is old and consumes only with probability  $\frac{N-2}{N}$ . It follows that in  $t + 2$  there are  $N - 2$  young sellers and  $N - 2$  old buyers, so

$$\check{v}_b = \frac{N-2}{N}(u + \beta v_s) = \frac{N-2}{N}v_b.$$

Thus, in  $t$  the young agent's expected payoff from deviating (buying instead of selling a market good) is:

$$\hat{v}_s = \frac{N-1}{N+1}(u + \beta\frac{N-2}{N}v_b) + \frac{2}{N+1}\beta\frac{N-1}{N}v_b.$$

The deviation is optimal if  $v_s < \hat{v}_s \Rightarrow v_s < \frac{N-1}{N+1}(u + \beta v_b)$ , or

$$N > \underline{N} \equiv \frac{2\beta(u-\beta e)}{(u+e)(1-\beta^2)} + \frac{u-e}{u+e}.$$

Notice  $\frac{\partial \underline{N}}{\partial e} < 0$ ,  $\frac{\partial \underline{N}}{\partial \beta} > 0$ ,  $\lim_{\beta \rightarrow 1} \underline{N} = \infty > \lim_{\beta \rightarrow 0} \underline{N} = \frac{u-e}{u+e}$ . ■

The intuition is simple. Suppose agents intend to alternate market production when young to consumption when old. The problem is that agents are anonymous so matching

and trade cannot depend on age, productivity, trade histories or other individual characteristics. A young agent may thus have incentives to misrepresent his needs, by claiming a market good instead of offering it. Whether or not this deviator gets to consume in  $t$ , his action deters consumption of at least one old agent, whose dynasty leaves the economy. This has an upside but also a downside for the deviator.

The upside is that the deviation cannot be made public, since the one who observes it is old and leaves the economy. Thus, since the number of traders is unobservable, no punishment can be triggered. The downside is represented by the permanent change in a crucial aggregate state variable—available production opportunities—as the deviation leads to a reduction in the size of new generations. Due to the matching process, this creates excess demand for market goods in the period following the deviation, which spawns consumption risk. The deviator’s assessment of this risk factor hinges on the size of the trading pool and on time-preferences. In particular, this risk is small if the initial population is large. Hence, there exists values  $N \geq \underline{N}$  such that a young prefers to demand a market good instead of supplying it. Naturally, greater production costs or a lower discount factor tend to raise the incentive to deviate, so  $\underline{N}$  falls.

Selling for money is an obvious remedy to these incentive problems. It helps allocating consumption to the initial old while giving the young a verifiable trade record that can be repeatedly passed on to future generations. Hence, we say that in a large market money has a fundamental role. The natural question is whether this is true when agents face a *finite* sequence of trading rounds, which is what we explore next.

### 3.2 Trading with Perishable Money

Let  $H = T - 1 < J = \infty$  an odd number, without loss in generality. That is, the money ‘vanishes’ at the end of  $T - 1$  although the economy continues on.

Conjecture an equilibrium where monetary trade takes place until  $T - 1$ , while home production occurs from  $T$  on. This is a combination of the two patterns of trade seen earlier; consumption is still deterministic, although in  $T$  agents switch to a less rewarding trade pattern. As market goods are only sold for money, we still call this a monetary equilibrium (slightly abusing the language). Formally, the strategies satisfy

$$\begin{aligned} \omega_z(t) &= \begin{cases} \omega_z^* & \text{if } t < T \\ \omega_z^{**} & \text{if } t \geq T \end{cases} \\ x_z(t, m') &= \begin{cases} x_z^*(m') & \text{if } t < T \\ x_z^{**} & \text{if } t \geq T. \end{cases} \end{aligned} \tag{6}$$

Let  $V_s(t)$  denote the equilibrium lifetime utility of a young agent (a seller) and  $V_b(t)$  for an old agent (a buyer), in  $t$ . Once again, due to altruistic motives, we define

$$V_s(t) = \begin{cases} -e + \beta V_b(t+1) & t < T \\ v_c & t \geq T \end{cases} \quad (7)$$

$$V_b(t) = \begin{cases} u + \beta V_s(t+1) & t < T \\ v_c & t \geq T. \end{cases}$$

Since  $T$  is an even number, a young seller in  $t = 0$  expects his dynasty to accomplish  $T/2$  monetary trade cycles, selling and buying a market good. Each cycle gives surplus  $-e + \beta u$ . In period  $T$  his descendant is an old buyer. Thus

$$V_s(0) = (-e + \beta u) + \beta^2(-e + \beta u) + \dots + \beta^{T-2}(-e + \beta u) + \beta^T v_c.$$

Similar considerations can be made for a buyer, implying that in equilibrium

$$V_b(t) = \frac{1-\beta^{T-t}}{1-\beta^2}(u - \beta e) + \beta^{T-t} v_c \quad (8)$$

$$V_s(t) = \frac{1-\beta^{T-t}}{1-\beta^2}(-e + \beta u) + \beta^{T-t} v_c.$$

Clearly,  $V_b(t) > V_s(t)$  and, from (1), we have

$$V_b(t) - V_b(t-1) < 0 < V_s(t) - V_s(t-1) \text{ if } t < T.$$

Also, since  $v_c \geq v_s$  then

$$v_s \leq V_s(t) \leq v_c < V_b(t) < v_b.$$

In short, the allocation is inefficient because the exchange of market production is sustained on a subset of the economy's life-span. Indeed, a buyer has a lower continuation value—relative to the case  $H = \infty$ —because his descendants will eventually switch to less desirable home production. The opposite holds for a seller since  $\alpha u(1 + \beta) \geq -e + \beta u$ .

The crucial feature of this conjectured equilibrium is that a young who sells in  $T - 1$  knows that he will enter period  $T$  without money. From then on he and everyone else will engage in home production. Selling in  $T - 1$ , therefore, amounts to making a gift of market consumption to someone that will never be met again. Of course, this must be in the seller's best interest. Indeed, this course of action is individually optimal if it satisfies individual rationality and incentive compatibility requirements.

To start, participation in trade must be individually rational. That is, everyone must weakly prefer trade to autarky, at the beginning of a period. Clearly  $\omega_z(t) \neq 0$  for all  $t$  if

$V_b(t) \geq 0$  and  $V_s(t) \geq 0$ , which are satisfied by (8). The reason is that every trader expects to earn some surplus, in the conjectured equilibrium. In addition, a trader's actions must be incentive compatible. This amounts to verifying that the agent does not strictly prefer a different matching choice or a different trading outcome, than what prescribed by the equilibrium. We have the following

**Lemma 3.** *Conjecture an equilibrium based on the strategies (6). If*

$$N \leq \bar{N} \equiv \frac{\beta\alpha u}{e(1-\beta)},$$

*then the strategies in (6) are individually optimal.*

**Proof.** Conjecture an equilibrium with strategies given by (6). Lemma 1 insures that these strategies are individually optimal in  $t \geq T$ . Now consider  $t < T$ . Clearly, in equilibrium an old agent strictly prefers to buy a market good with his money since any deviation prevents consumption and so generates zero continuation payoff (the agent and his dynasty leave the economy). Thus, focus on a young in  $t < T$ .

This agent has no interest in deviating by trying to match to a seller as the deviator would end up unmatched. The reason is that in equilibrium sellers are young agents who desire to match only with someone who has money, i.e.,  $\omega_{(0,0)}(t) = (s, b, 1)$  in  $t < T$ . Choosing to sell to a buyer is at least weakly preferred, in equilibrium, since the agent can refuse to trade at any stage. Thus, focus on a seller-buyer equilibrium match (young-old match).

Since exchange must be voluntary, the key deviation here is the young agent refuses to sell for money. Denote by  $\hat{V}_s(t)$  the seller's expected payoff from doing so. Hence, trade is incentive compatible if  $V_s(t) \geq \hat{V}_s(t)$  for all  $t < T$ . Clearly, in equilibrium we have  $V_s(t) \geq \hat{V}_s(t) = 0$  for all  $t \leq T - 2$  because a young who does not sell cannot obtain money hence cannot consume when old. This deviation gives zero payoff and so selling is incentive compatible in  $t \leq T - 2$ , i.e. we have  $x_{(0,0)}(1) = (0, -1, -1)$  in  $t \leq T - 2$ .

Now focus on period  $T - 1$ . Here a seller has a greater incentive to not produce a market good, since money exhausts its record-keeping role, in this period. Home production, from  $T$  on, is independent of money holdings. Hence,  $\hat{V}_s(T - 1) = \beta\hat{v}_c$  where

$$\hat{v}_c = \frac{N-1}{N}(\alpha u + \beta v_c)$$

is the lifetime utility of the deviator who, as an old agent, attempts to engage in home production in  $T$ . Following the same logic of prior proofs, the deviation lowers by one the number of young in  $T$  (an old does not consume in  $T - 1$ ). This deviation is undetected so

there are  $N - 1$  young sellers and  $N$  old buyers in  $T$ . Once again, one old will not match and consume so in  $T + 1$  there are  $N - 1$  young sellers and  $N - 1$  old buyers.

Incentive compatibility in  $T - 1$ , therefore, requires  $x_{(0,0)}(T - 1, 1) = x_{(0,0)}^*$ , i.e.,

$$-e + \beta v_c \geq \beta \hat{v}_c \Rightarrow \beta(v_c - \hat{v}_c) \geq e.$$

This can be rearranged as

$$N \leq \bar{N} \equiv \frac{\beta \alpha u}{e(1-\beta)}.$$

Thus,  $\frac{\partial \bar{N}}{\partial e} < 0$ ,  $\frac{\partial \bar{N}}{\partial \beta} > 0$ ,  $\lim_{\beta \rightarrow 1} \bar{N} = \lim_{e \rightarrow 0} \bar{N} = \infty$  and  $\lim_{\beta \rightarrow 0} \bar{N} = 0$ . ■

The intuition is this. By Lemma 1, the strategies listed in (6) are individually optimal in all  $t \geq T$ . Hence, we need to check for deviations only in  $t < T$ . In those periods, old agents would not deviate because their continuation payoff is positive only if they consume. Due to the equilibrium matching scheme, this requires that the old follow the proposed strategy. Young agents want to sell in  $t < T - 1$  since in equilibrium they cannot engage in home production and they cannot buy (having no money). The key period, therefore, is  $T - 1$  when money exhausts its record-keeping role. Here, swapping a market good for money amounts to making a unilateral transfer since the seller receives no current or future reward. Note that due to anonymity and lack of communication there is no threat of individual or collective punishment either. So why should a young agent sell for money in  $T - 1$ ?

There are two contrasting elements that affect a seller's incentive to do so. The positive aspect is that by refusing to sell the young agent can sidestep current disutility  $-e$ . Note that the failure to acquire money is irrelevant here, since money effectively cannot be carried into the next period. The negative aspect is that the deviation reduces the future number of productive agents, spawning consumption risk. The relative strength of these contrasting effects hinges on market size, production disutility and impatience.

In a market populated by many impatient traders, a small reduction in the aggregate production capabilities generates small consumption risk and consequently a small expected loss, in present value. The opposite is true in a small market which is why the proposed actions are individually optimal only if  $N \leq \bar{N}$ . Naturally, this upper bound grows with greater patience and smaller production costs, as these factors strengthen the incentives to play equilibrium. This and the prior results lead to the following

**Proposition 4.** *Let  $H < J = \infty$ . If  $\beta > \bar{\beta} \in (0, 1)$  then there exist  $N \in (\underline{N}, \bar{N}) \subset (1, \infty)$  such that the monetary trade pattern supported by the strategies (6) is an equilibrium. The corresponding allocation improves on any other possible non-monetary allocation.*

**Proof.** We start by proving the second part of the proposition. By Lemma 2 we know that trade of market goods cannot be sustained without the exchange of money, if  $N > \underline{N}$ . The allocation associated to (6) is even harder to sustain without money as the continuation payoffs in  $T - 1$  are even lower than when  $J = \infty$ .

To prove the first part of the proposition recall Lemma 3, indicating the individual optimality of (6) when  $N \leq \bar{N}$ . Thus, we need only provide a condition guaranteeing that  $[\bar{N}, \underline{N}]$  is a non-empty set. To do so recall that  $\bar{N}$  and  $\underline{N}$  both increase in  $\beta$  and diverge to  $\infty$  as  $\beta \rightarrow 1$ . Also,  $\lim_{\beta \rightarrow 0} (\bar{N} - \underline{N}) < 0$  while  $\lim_{\beta \rightarrow 1} (\bar{N} - \underline{N}) > 0$  since

$$\bar{N} - \underline{N} = \frac{\beta}{1-\beta} \left( \frac{\alpha u}{e} - \frac{2(u-\beta e)}{(u+e)(1+\beta)} \right) - \frac{u-e}{u+e}$$

where  $\frac{\alpha u}{e} > \frac{2(u-\beta e)}{(u+e)(1+\beta)}$  from (1). Since  $\bar{N}$  and  $\underline{N}$  are continuous in  $\beta$  then, by the intermediate value theorem, there exists a  $\bar{\beta} \in (0, 1)$  satisfying  $\underline{N} = \bar{N}$ . This  $\bar{\beta}$  is unique since  $\frac{\partial(\bar{N}-\underline{N})}{\partial\beta} > 0$  always. Hence, there exists  $\beta > \bar{\beta}$  such that  $\bar{N} > \underline{N} \geq 1$  so that we can pick  $N \in (\underline{N}, \bar{N})$ . ■

The central lesson is that the incentives for monetary trade in the early life of the economy hinge on the value generated by switching to a different trade pattern, later on. A seller's deviation in  $T - 1$  lowers permanently society's productive capabilities, creating consumption risk that lowers the deviator's continuation payoff. In a market of moderate size with patient agents this aggregate effect is sufficiently noxious that it disciplines the seller's actions. This occurs despite the lack of commitment, enforcement or the possibility to rely on punishment triggered by publicly announced deviations.

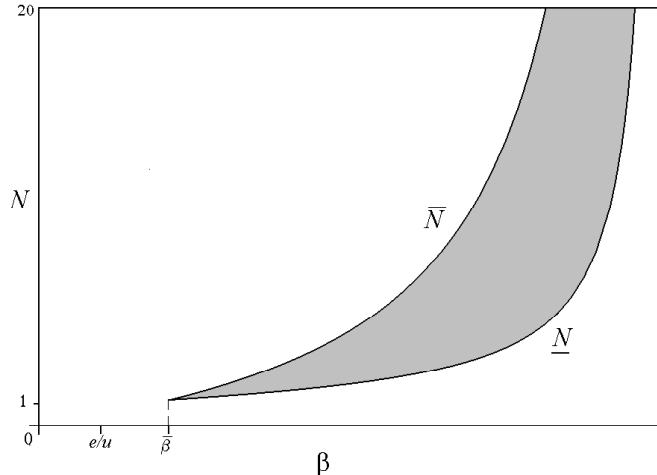
What about small or large economies? In a small economy, it is easy to sustain an equilibrium where the young make a transfer to the old. Any deviation would be very costly as it would severely disrupt the process of exchange, so money does not have a fundamental allocative role. Conversely, in a large economy a deviation would induce small consumption risk so every young would deviate in  $T - 1$ . A standard backward induction argument suggests money would lose value.

It is important to recognize that money need not change hands in  $T - 1$  (unlike prior dates) because it loses its record-keeping role after that date. However, money remains a fundamental element of the trading mechanism because it is crucial that the old consume. Due to anonymity, the young have an incentive to pass themselves as buyers (see Lemma 1) so that it is optimal to match and trade only with someone who has money.

Figure 1 reports the results of a numerical simulation of a sequence of infinitely-lived economies characterized by different initial populations and preferences for future



consumption.<sup>6</sup> The area above the curve  $\underline{N}$  sustains the monetary equilibrium defined by (4), when  $H = \infty$ . For economies where  $H < \infty$ , instead, (6) is an equilibrium only in the shaded area. As we see in this second case the parameter set supporting monetary trade is smaller due to the earlier recognized trade-off between market size and patience. Indeed, as  $\beta$  increases, monetary trade can be sustained for larger values of  $N$  since greater patience gives more weight to the harm resulting from a young agent's defection in  $T - 1$ .



### 3.3 Trading in a Finite Horizon

We now show that monetary trade can take place even if the economy has a finite life. Indeed, this amounts to showing that society can benefit from the adoption of a record-keeping technology. To do so, we set  $1 \leq J < \infty$ . Since no one would produce for a token in period  $J$ , we construct an equilibrium with a simple ‘regime switch’ occurring at some date  $H < J$  when tokens lose value. Here, the agents engage in monetary trade until  $H$  and engage in non-monetary home joint production, afterward.

We let  $H = T - 1$ , for continuity with earlier notation, where  $1 \leq T \leq J$ . In equilibrium, this pattern of transactions is characterized by strategies that are time-invariant in the sub-intervals  $[0, T - 1]$  and  $[T, J]$ . Formally, we have

$$\omega_z(t) = \begin{cases} \omega_z^* & \text{if } t < T \\ \omega_z^{**} & \text{if } T \leq t \leq J \end{cases} \quad (9)$$

$$x_z(t, m') = \begin{cases} x_z^*(m') & \text{if } t < T \\ x_z^{**} & \text{if } T \leq t \leq J. \end{cases}$$

<sup>6</sup>Here  $u = 10$ ,  $e = 0.99$ ,  $\alpha = 0.45$ . This ensures that (1) is satisfied.

This pattern of exchange mirrors (6) with the key difference that after  $T-1$  agents engage in home production for  $J-T+1 < \infty$  periods. Then the economy ends.

Conjecturing an equilibrium according to (9), the recursive formulation of expected lifetime utilities in (7) must be amended. Precisely, in what follows we work with

$$\begin{aligned} V_s(t) &= \begin{cases} -e + \beta V_b(t+1) & t < T \\ v_c(J-t+1) & T \leq t \leq J \end{cases} \\ V_b(t) &= \begin{cases} u + \beta V_s(t+1) & t < T \\ v_c(J-t+1) & T \leq t \leq J \end{cases} \end{aligned} \quad (10)$$

where for  $t = T, \dots, J$  we define

$$v_c(J-t+1) = \frac{(1 - \beta^{J-t+1}) \alpha u}{1 - \beta} = (1 - \beta^{J-t+1}) v_c. \quad (11)$$

Note that the continuation payoff at date  $t = T$ , i.e.,  $v_c(J-T+1)$ , now depends on the number  $J-T+1$  of trade rounds that separate the end of monetary trade from the economy's terminal date. A first implication of having a terminal date, therefore, is that continuation payoffs are uniformly smaller than when  $J = \infty$ . Precisely

$$\begin{aligned} V_b(t) &= \frac{1 - \beta^{T-t}}{1 - \beta^2} (u - \beta e) + \beta^{T-t} v_c(J-T+1) \\ V_s(t) &= \frac{1 - \beta^{T-t}}{1 - \beta^2} (-e + \beta u) + \beta^{T-t} v_c(J-T+1) \end{aligned}$$

where  $v_c(J-t+1) < v_c < v_s < v_b$ . As we move to an infinite horizon economy,  $J \rightarrow \infty$ , then  $v_c(J-t+1) \rightarrow v_c$ . Also, using l'Hospital rule  $\lim_{\beta \rightarrow 1} \frac{1 - \beta^{J-T+1}}{1 - \beta} = J - T + 1$ , thus  $\lim_{\beta \rightarrow 1} v_c(J-t+1) = (J-T+1)\alpha u$ . Clearly in all  $t$  we have

$$V_b(t)|_{J=\infty} > V_b(t)|_{J<\infty} > V_s(t)|_{J<\infty} \geq 0,$$

so participation in trade is individually rational at all dates, i.e.,  $\omega_z(t) \neq 0$  for all  $t$ . This also means that the value of money in this economy is smaller than in an infinite-life economy. Also for  $t < T$  we have

$$V_b(t) - V_b(t-1) < 0 \leq V_s(t) - V_s(t-1)$$

since (1) implies  $V_s(t) < v_s \leq v_c < v_b$ .

To find parameters under which the actions described in (9) are individually optimal we follow the same procedure as before, resulting in

**Lemma 5.** *Conjecture an equilibrium based on the strategies (9). There exist  $(J, T)$  pairs satisfying  $1 \leq T \leq J < \infty$  with a corresponding value*

$$\bar{N} = \frac{\beta \alpha u (1 - \beta^{J-T+1})}{e(1-\beta)},$$

*such that if  $N \leq \bar{N}$ , then the strategies in (9) are individually optimal.*

**Proof.** Suppose the equilibrium strategies are as in (9) for some  $1 \leq T \leq J < \infty$ . Clearly, the arguments concerning periods  $t < T - 1$  that we laid out in Lemma 3 apply also here. The only item we have to revisit is the deviation of the matched seller in period  $T - 1$ , since the continuation payoffs are different due to  $J$  being finite.

Thus, consider  $t = T - 1$ . Suppose the young agent refuses to sell to a buyer and denote the deviant's discounted continuation payoff by  $\beta \hat{v}_c(J - t + 1)$  where, following the same logic as before, we have

$$\hat{v}_c(J - T + 1) = \frac{N - 1}{N} [\alpha u + \beta v_c(J - T)].$$

Selling is incentive compatible if  $-e + \beta v_c(J - T + 1) \geq \beta \hat{v}_c(J - T + 1)$  rearranged as

$$\frac{\alpha u}{1-\beta} (1 - \frac{N-1}{N} \beta - \frac{1}{N} \beta^{J-T+1}) \geq \frac{N-1}{N} \alpha u + \frac{e}{\beta}. \quad (12)$$

Thus, (12) holds for

$$N \leq \bar{N} \equiv \frac{\beta \alpha u (1 - \beta^{J-T+1})}{e(1-\beta)} = \bar{N} (1 - \beta^{J-T+1}).$$

It is clear that  $\frac{\partial \bar{N}}{\partial \beta} > 0$ ,  $\frac{\partial \bar{N}}{\partial (J-T)} > 0$ ,  $\lim_{\beta \rightarrow 1} \bar{N} = \frac{\alpha u (J-T+1)}{e}$  and  $\lim_{J-T \rightarrow \infty} \bar{N} = \bar{N}$ . It follows that there exists  $(J, T)$  pairs and values  $N \leq \bar{N}$  such that (12) holds and hence (9) is individually optimal. ■

This result is virtually identical to Lemma 3. The main difference is the presence of the term  $J - T$ , which must be sufficiently large. Indeed, a young producer will choose to not misrepresent his consumption needs in  $T - 1$  only if he has a sufficiently large continuation payoff. This requires a sufficiently long string of home consumption, i.e.  $J - T$  must be sufficiently large. The key implication is that even when the trading horizon is finite and publicly known, tokens may have a beneficial role for society.

**Proposition 6.** *There are  $(J, T)$  pairs,  $1 \leq T \leq J < \infty$ , such that if  $\beta > \bar{\beta} \in (0, 1)$  then there exist values  $N \in (\bar{N}, \bar{N}) \subset (1, \infty)$  such that the monetary trade pattern supported by the strategies (9) is an equilibrium. The corresponding allocation improves on any other possible non-monetary allocation.*

**Proof.** Conjecture (9) is an equilibrium. Lemma 5 tells us that (9) is individually optimal when  $N \leq \bar{N}$ . To prove that the allocation improves over a non-monetary allocation we cannot use Lemma 2, since  $J < \infty$ . Thus, we simply have to prove that, in the absence of money, on date  $T - 1$  a young would prefer to misrepresent his needs by choosing to trade claiming consumption instead of offering it.

By an argument identical to the one presented in the proof of Lemma 2, the expected payoff to a young deviator in  $T - 1$  is denoted

$$\hat{v}_s = \frac{N-1}{N+1}[u + \beta \check{v}_c(J-T+1)] + \frac{2}{N+1}\beta \hat{v}_c(J-T+1). \quad (13)$$

The deviator consumes with probability  $\frac{N-1}{N+1}$ , does nothing otherwise and

$$\begin{aligned} \check{v}_c(J-T+1) &= \frac{N-2}{N}[\alpha u + \beta v_c(J-T)] \\ \hat{v}_c(J-T+1) &= \frac{N-1}{N}[\alpha u + \beta v_c(J-T)] \end{aligned} \quad (14)$$

are the deviator's (undiscounted) continuation values from entering period  $T$  with or without consuming in  $T - 1$ . Using (11) in (14) we can rearrange (13) as

$$\hat{v}_s = \frac{N-1}{N+1}[u + \beta v_c(J-T+1)].$$

The deviation is optimal if  $-e + \beta v_c(J-T+1) < \hat{v}_s$ , rearranged as

$$N > \bar{N} \equiv \frac{2\beta\alpha u(1-\beta^{J-T+1})}{(u+e)(1-\beta)} + \frac{u-e}{u+e} < \bar{N}.$$

Notice  $\frac{\partial \bar{N}}{\partial e} < 0 < \frac{\partial \bar{N}}{\partial \beta}$ ,  $\lim_{\beta \rightarrow 0} \bar{N} = \frac{u-e}{u+e}$  and  $\lim_{\beta \rightarrow 1} \bar{N} = \frac{2(J-T+1)+u-e}{u+e}$ . Therefore,  $\lim_{\beta \rightarrow 0} (\bar{N} - \bar{N}) < 0$  while  $\lim_{\beta \rightarrow 1} (\bar{N} - \bar{N}) > 0$  since

$$\bar{N} - \bar{N} = \frac{\beta(1-\beta^{J-T+1})}{1-\beta} \left( \frac{\alpha u}{e} - \frac{2\alpha u}{u+e} \right) - \frac{u-e}{u+e}$$

and  $u+e > 2\alpha u$  (from (1)). Since  $\bar{N}$  and  $\bar{N}$  are continuous in  $\beta$  then, by the intermediate value theorem, there exists a  $\bar{\beta} \in (0, 1)$  satisfying  $\bar{N} = \bar{N}$ . This  $\bar{\beta}$  is unique since  $\frac{\partial(\bar{N} - \bar{N})}{\partial \beta} > 0$  always. Also,  $\lim_{J-T \rightarrow \infty} \bar{N} = \bar{N}$ . Thus, there exists  $\beta > \bar{\beta}$  and  $J-T$  sufficiently large such that  $\bar{N} > \bar{N} > 1$  so that we can pick  $N \in (\bar{N}, \bar{N})$ . ■

In short, even in a finite horizon economy, society might have reasons to exploit the availability of fiat money. This occurs early on in the life of the economy and only if there

are alternative non-monetary allocations that—although generating smaller returns from trade—can be supported in a finitely repeated game without commitment, enforcement or recall. In our model, this is possible because young-old pairs can costlessly (and jointly) produce low-value home goods. Of course, since monetary exchange can only take place on an initial set of periods, the unilateral transfers made in  $T - 1$  must be compensated by a sufficiently long spell of this less valuable consumption, in the remaining periods. Hence, as the life of the economy shrinks, so does the duration of initial monetary exchange. This explains why if  $\beta$  is too low, then money cannot be valued.

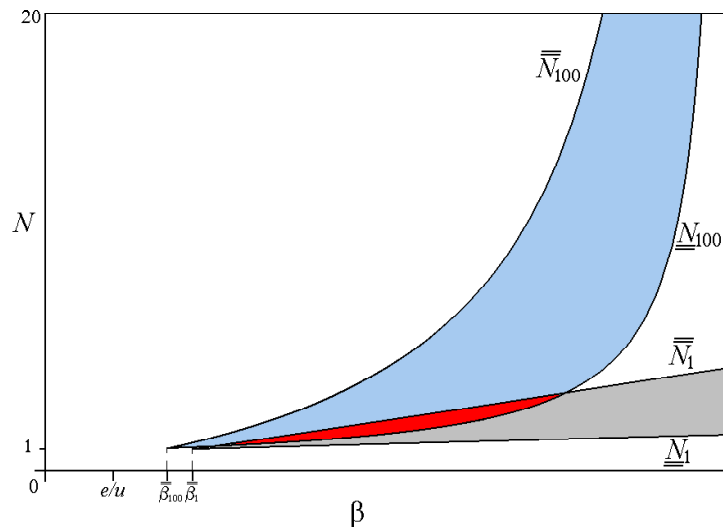


Figure 2 illustrates this result. It simulates two ‘types’ of economies, longer- and shorter-lived. In one we have  $J - T + 1 = 100$  and in the other we set it to 1. The index on the curves  $\overline{N}$  and  $\underline{N}$  reflects this distinction. As indicated earlier, we see that monetary exchange can be sustained in economies with larger markets and with lower discount factors the larger is the continuation value to a seller in date  $T - 1$ , i.e., the greater is  $J - T$ . The figure also indicates that we can find  $\beta$  and  $N$  values that sustain monetary exchange in both short- and longer-lived economies (the dark area created by the intersection of the curves  $\underline{N}_{100}$  and  $\overline{N}_1$ ).

#### 4 Concluding Remarks

We have proposed an anonymous trading environment where agents cannot commit to future actions, enforce transfers or base actions on histories of play. We have proved that, despite the presence of a publicly known finite trading horizon, society can expand the set of allocations through monetary exchange. First, we have presented an infinitely-lived

economy in which monetary exchange supports a better allocation on a subset of the economy's life. Then, we have illustrated how this can happen even if the economy has a finite and deterministic life-span.

These findings have been substantiated by means of simple, albeit rigorous, theoretical examples that suggest the following insights. First, the presence of a known finite trading horizon is generally insufficient to prevent society from exploiting some of the gains associated with specialization and monetary exchange. Second, in environments where individual actions can significantly affect some aggregate state variable, an individual will voluntarily take an action that reduces his current payoff if this prevents others from making choices that carry adverse long-lasting aggregate consequences. This is despite the lack of enforcement, commitment or the threat of punishment.

These basic insights should be robust to alternative specifications meant to enrich our model's more 'stylized' design features. For example, one can think of other reasons why an incorrect allocation of consumption can have noxious aggregate effects, an insufficient accumulation of human or physical capital being one such reason. Indeed, we surmise that the basic lessons of this paper should emerge in any framework where individual actions have a non-negligible strategic impact or can carry long-lasting aggregate consequences.

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