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Predicting The Advective Flow Velocity In A Confined Aquifer Using A Single Well Tracer Test

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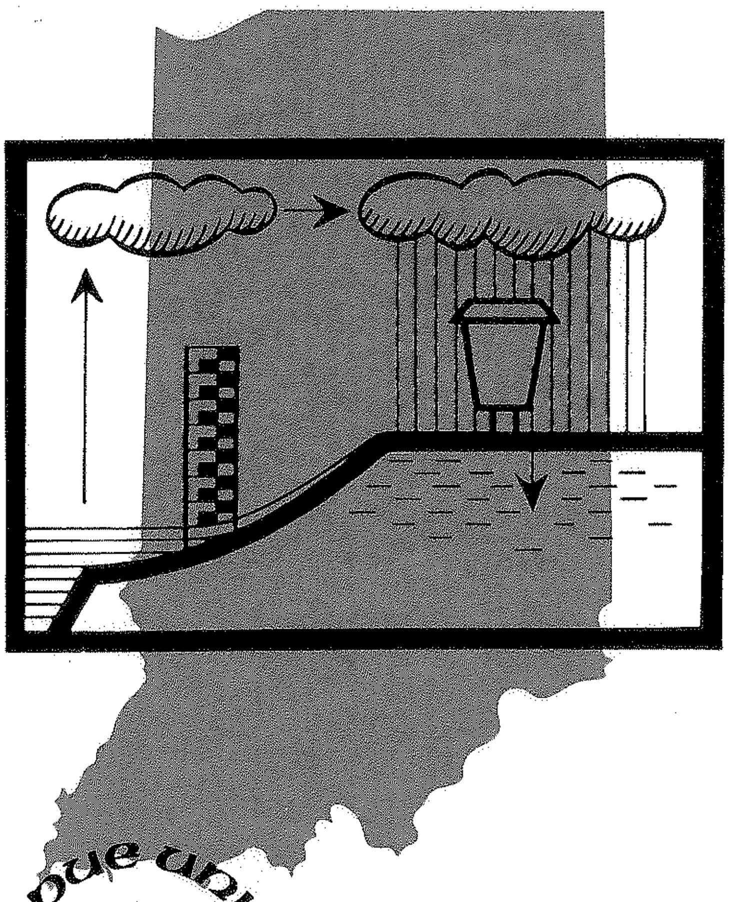

by

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and
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September 1985

PURDUE UNIVERSITY
WATER RESOURCES RESEARCH CENTER
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PREFACE

Two Pulse Single Well Method

A method described in Kaplan and Leap (1984) employed a single well in which two pulses would be injected and pumped back to the well sequentially. Although plans were made to test this method with the laboratory model, it was not possible due to equipment breakdowns which were not repairable before the deadline of this report.

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LIST OF SYMBOLS

a = minimum value in beta-probabilty density function

A = area

b = saturated thickness; maximum value in beta-probabilty density function

C = concentration; constant in beta-probability density function

C_o = original concentration

dh/ds = hydraulic gradient

E[x] = expected value

H = difference in head

K = hydraulic conductivity

L = distance between seepage faces

n = porosity

Q = discharge rate

r = distance along along a radial axis

S[x] = standard deviation

t = time

t_a = arrival time of tracer

t_i = injection time of tracer

t_p = time pumping commences

T = time

v = velocity

v_a = advective flow velocity

v^* = average bulk velocity

v' = calculated parameter

V_x = coefficient of variation

V = volume

V' = calculated parameter

W = volume

x = distance

α = shape parameter in beta-probabilty density function

β = shape parameter

λ = adjustment factor

π = the number pi

τ = finite difference of time

ABSTRACT

A study was conducted to determine the form of the solution required to solve for the advective flow velocity in a porous medium where observation is limited to a single penetrating well. An analytical equation expressing advective flow velocity as a function of well discharge and arrival time of an injected tracer was derived for a confined aquifer. The modelling process used to derive the velocity equation is described.

An experiment was conducted to test the accuracy of the velocity equation at the laboratory scale. A sodium chloride tracer was injected into a sand tank model of a confined aquifer, allowed to drift, and then recovered through a miniature well. The same well that was used for injection.

Initial results suggest that the equation is accurate using the methods described to obtain data. Field testing is suggested.

The stochastic properties of velocity are discussed as an aid to solving contaminant transport problems. A method for fitting a probability density function to the expected value of velocity is illustrated.

INTRODUCTION

The discipline of groundwater hydrogeology has traditionally concerned the questions of water supply and management of water resources. As such, mathematical and field methods address the quantity of water available for human consumption, its location, the capacity of the system, and quality of naturally occuring groundwaters. It now is recognized that man has the ability to degrade the quality of his groundwater resources. The issue of groundwater contamination has generated a new set of questions, of which two are of fundamental importance: how do we mitigate the damage already done, and what preventive measures can be taken to prevent further contamination.

These are not easy questions to answer. Within their broad framework an entire subset of questions can be addressed, for example: what are the buffering capacities of the natural system, what is the dilution capability of the groundwater cycle, and what engineering solutions are available. The list obviously goes on. Of the questions being asked, those regarding contaminant transport appear to draw the most of attention.

Let us define contaminant transport as the process by which something we don't want occurs somewhere other than where it was placed. In the context of groundwater hydrogeology, this is the ability of a contaminant to migrate into and degrade an economically valuable source of water or to threaten public health and safety as a direct consequence of that mobility. The rate of movement and the mechanisms by which that rate is determined are the fundamental parameters needed to manage our groundwater contamination problems. This suggests that all that needs to be done is either measure the rate at which a contaminant is moving or, by understanding the process, measure the physical properties of the flow domain and the contaminant itself in order to predict the behavior of the contaminant within the system. Because very minor amounts of some contaminants are known to be hazardous, a very high degree of accuracy in either measurement or prediction is implied. It is increasingly apparent that certain models for predicting contaminant transport produce questionable results in field situations (Gillham and Cherry, 1982).

The premise of this study assumes one fundamental parameter that must be described in order to make any accurate prediction about the behavior of a contaminant is a description of groundwater velocities.

PURPOSE AND OBJECTIVE OF STUDY

This study was conducted to determine the form of the solution required to solve for the advective flow velocity in a porous medium where observation is limited to a single penetrating well. An analytical solution was derived and tested against the behavior of a laboratory model to verify the accuracy.

A number of constraints were imposed by the utilitarian nature of the research. The solution should be representative of the environment on a scale compatable with the environmental and management concerns that necessitated the test. The assumptions made in the derivation of the solution should be simple and unambiguous. The calculations required for the solution should be simple. And, although not a part of this study, the methodology required to implement this test in the field should be inexpensive.

The constraints argue the point that this is a problem in applied hydrogeology as opposed to a theoretical investigation. By implication, given a number of possible ways of solving this problem, we are looking for the solution that most benefits the end user. The end user is perceived as a state or local agency in an initial stage of site characterization. Funds are assumed to be limited and training of personnel minimal.

Aside from the necessary assumptions that will be made in the process of deriving an equation the following point is emphasized: no hydrogeologic investigation is of any value until the basic geology of the site has been characterized.

REVIEW OF SINGLE WELL TRACER METHODS

The incentive behind the development of single well methods is economic. Groundwater investigations are often limited by lack of funds, equipment, and trained personnel. The assumption is that single well methods utilize limited resources more effectively than multiple well techniques. This is true only if the quality of the information obtained from a single well test is comparable to other available methods or if less information is sufficient than could otherwise be obtained.

A major effort in the implementation of single well techniques has developed around the use of artificial tracers. A substance with a distinct chemical or radioactive signature is introduced into the porous environment and the subsequent behavior is assumed to be a function of the physical state of the system.

The following is a review of two single well tracer techniques currently employed to obtain an advective flow velocity.

Point-Dilution Method

The point-dilution method, also called the borehole dilution method, is used to determine the average horizontal groundwater flow velocity at the well site by measuring the rate at which the concentration of an artificial tracer decreases with time. To conduct the test a portion of the well is isolated and the tracer introduced.

As flow gradually removes the tracer from the well a concentration versus time curve is generated.

The relationship between the change in concentration with time and the velocity through the well is given by

$$dC/dt = -Av^*C/W \qquad (3.1)$$

where

C = Tracer concentration.

t = Residence time of tracer in the well.

A = Vertical cross-sectional area through the center of the isolated portion of the well.

v^* = Average bulk velocity across the center of the well.

W = Volume of the isolated portion of the well.

A solution for v^* is obtained by integration and the use of the following initial condition

$$C = C_0 \text{ at } t = 0$$

where

C_0 = initial tracer concentration,

to give

$$v^* = (W/At)\ln(C/C_0). \qquad (3.2)$$

The natural flow velocity velocity v is related to v^* by

$$v = v^*/n\lambda \qquad (3.3)$$

where

n = porosity.

λ = an adjustment factor dependent on environmental conditions surrounding the well.

A detailed description of the method is given by Drost and Neumaier (1974) and Klotz et al (1978) and includes descriptions of the equipment used to monitor the wells. The application of the method to a shallow gravel aquifer using a fluoride tracer is described by Grisak et al (1977). A general discussion of the theory is well presented by Freeze and Cherry (1979).

Single Injected-Pulse Technique

The single-well, injected-pulse technique is used to determine the average flow velocity at the well site by relating the change in tracer concentration to pumped volume. An artificial tracer is introduced into the formation during an injection stage. Once the tracer has penetrated the formation the tracer is allowed to drift under natural flow conditions. After some time has elapsed, the well is pumped at a constant rate to recover the tracer. Monitoring the tracer generates a concentration versus volume curve.

The relationship between the change in concentration with pumped volume and the flow velocity v is derived by assuming cylindrical symmetry in pumping:

$$x = (Qt/\pi bn)^{1/2} \qquad (3.4)$$

where

x = the distance the tracer has travelled during the recovery phase.

Q = pumping rate during recovery, assumed constant.

t = pumping time required to recover 50% of the recovered tracer.

b = saturated thickness of the formation, assumed constant.

n = effective porosity of the formation.

The preceding equation is solved for x and the natural flow velocity is given as

$$v = x/t \tag{3.5}$$

A description of the method and the equations for a multi-layered aquifer are given in Gaspar and Oncescu (1972). Application of the method in a clastic aquifer is described by Borowczyk _et al_ (1966). Applications for the determination of dispersion coefficients are given by Fried (1975).

CONCEPTION AND DEVELOPMENT OF AN ANALYTICAL SOLUTION

The end result of the modelling process is an abstraction. It can be an equation, a graph, or an actual physical model. In all cases it is a symbolic representation of something perceived as real. The process of modelling a groundwater problem leads from a conceptual understanding of the physical system to a device that describes the process (Mercer and Faust, 1980). This device is the model itself. In this case the device sought is a mathematical equation.

Developing a Model

In developing a model an outline is required, a framework that will lead to the form of the solution sought. The following points are considered first steps in conceptualizing the problem.

Model Domain

This is synonomous with the concept of flow domain. Here we hope to define the hydrogeological environment. This involves describing the area of interest in terms of the geometry and boundary conditions. The fact that a three dimensional space in real co-ordinates is assumed does not restrict looking for a solution in one or two dimensions nor would it restrict mapping the problem into the complex plane.

Intensive Properties of the Flow Domain

There are certain fundamental physical parameters that determine the flow of fluid through a porous medium. There are properties that belong only to the fluid, properties that belong only to the matrix, and properties that result from direct interaction of the two. These properties may vary spatially and with time. The variation may be random or functionally related to something else. The fact that these properties must exist does not guarantee the ability to recognize them. The recognition of some fundamental property does not guarantee the ability to measure or describe it.

Extensive Properties

This addresses the problem of how the system responds to some external influence. We can also inquire as to how some external demand responds in contact with the system. An example is pumping. Drawdown would be a system response to an external influence.

Solution

Where do we want the solution? Changes in the boundary values of our flow domain may provide an answer sufficient to the problem at hand; or it could be that a solution is required at every point within the flow domain. What type of solution is either appropriate or desirable? A physical model is a solution. So is an analytical equation or a numerical model.

Adapting Model Paradigm to the Specific Problem

Certain conditions are predetermined by the statement of purpose. The dependent variable is labelled advective flow velocity. The model domain is porous and the field of observation is limited to a single well. To facilitate an analytical solution the model paradigm is used to help define both the independent and dependent variables. It is also used to help formalize the assumption required.

Problem Domain

The flow domain for which a solution is sought is a horizontal aquifer of some finite thickness bounded at top and bottom by confining beds assumed to be impervious. Boundaries normal to the confining beds extended to infinity.

Intensive Properties of the Problem Domain

The matrix properties are assumed to be isotropic. An assumption of homogeneity is made subject to the following condition: the matrix properties are homogeneous if the scale of observation is greater than the scale of individual inhomogeneities within the matrix (Bear, 1979).

Solution

The solution is the dependent variable called advective flow velocity. Within the context of the derivation that follows advective flow velocity is defined as the average rate at which a particle of water changes its position with time in response to the natural gradient.

Derivation of Velocity Equation

The velocity equation is derived in the following manner. Assume that the volume of water recovered from or injected into the flow domain is equal to an equivalent volume of the flow domain itself. The volume of water pumped or injected by the well can be expressed as,

$$V = \int Q(t)dt \tag{4.1}$$

where,

V = volume of water pumped or injected.

Q(t) = volume discharge rate expressed as a function of time.

For a constant volume discharge rate equation 4.1 can be integrated to give

$$V = Qt \tag{4.2}$$

where,

Q = constant volume discharge rate.

t = elapsed pumping or injection time.

In a homogeneous, isotropic, confined aquifer in which there is no prior hydraulic gradient then, can be assumed that the volume of aquifer affected by injection or withdrawal of water describes a cylinder whose axis is coincident with the well. This volume can be described as,

$$V = \pi r^2 nb \tag{4.3}$$

where,

r = the radius of the cylinder.

n = effective matrix porosity, assumed constant

b = aquifer thickness, assumed constant.

Equating the two volumes gives,

$$Qt = \pi r^2 nb \tag{4.4}$$

Rearranging equation 4.4 to express r as a function of t gives,

$$r = (Q/\pi nb)^{\frac{1}{2}} t^{\frac{1}{2}} \tag{4.5}$$

Equation 4.5 expresses the radius of an enlarging cylinder of water as a function of the injection or pumping time. This is true only under the assumptions previously stated.

The next step is to consider the behavior of a particle of water in the flow domain whose motion is influenced only by the natural hydraulic gradient. If we assume there is a steady-state, linear decrease in head across the flow domain, and that the motion of the particle describes a straight line parallel to the confining boundaries then the velocity of the particle is constant and can be described in one dimension. the advective flow velocity can be expressed as:

$$v_a = x/\tau \tag{4.6}$$

where,

x = the distance between two arbitrary points in the direction of the natural gradient.

τ = the travel time between those two points.

Consider now a reference system in which the position of a particle inserted into the well defines the origin and the direction of the natural gradient defines the axis of motion. Assume that a particle located at some arbitrary point along this axis would return along this same axis under the influence of a pumping well. In addition assume that velocity potentials created by a pumping well are radially symmetric. Assume further that the motion of the particle can be described as a linear function of the velocity potential due to pumping and the velocity potential of the natural system. Under these assumptions the position of the particle can be described by rearranging equation 4.6 to solve for x and superposing it with equation 4.5.

A simple case is considered. At some time t_i a particle is inserted into the well and allowed to drift in a positive direction along an axis r at the advective flow velocity. After some period of time pumping of the well commences at time t_p. The particle eventually returns to the well at some time t_a. The position of the particle is given by,

$$r = v_a(t_p-t_i) + v_a(t_a-t_p) - (Q/\pi nb)^{1/2}(t_a-t_p)^{1/2} \tag{4.7}$$

If we let

$$T = t_a-t_i \tag{4.8}$$

and

$$t = t_a-t_p \tag{4.9}$$

then,

$$r = v_aT - (Q/\pi nb)^{\frac{1}{2}}t^{\frac{1}{2}} \tag{4.10}$$

at the well

$$r = 0 \tag{4.11}$$

therefore,

$$v_a = (Q/\pi nb)^{\frac{1}{2}}t^{\frac{1}{2}}/T \tag{4.12}$$

From the observation point, located at the well, we now have an equation for the advective flow velocity in terms of the drift and pump times at a constant discharge rate. A solution depends on the ability to either estimate or measure the porosity and thickness of the aquifer, and determination of the arrival time of the particle.

EXPERIMENT

The following experiment was conducted to test the accuracy of equation 4.12 derived in the previous chapter. A sodium chloride solution was injected into a fully penetrating well in a scale model of a confined aquifer. After a period of time, during which the tracer was allowed to drift, the well was pumped. The specific conductance of the recovered tracer was monitored during withdrawal. This process generted a specific conductance versus time curve for each trial. Points along the time axis corresponding to 50% of the recovered tracer were used to calculate the velocity. These values were compared against the pore velocities calculated at the seepage face of the model and velocities calculated in a one-way drift test.

Design and Construction of the Laboratory Model

The main body of the model, representing the aquifer, consists of a wood box. Interior dimensions are 4 feet wide by 8 feet long with a depth of 8 inches. The base of the box is a single sheet of 3/4 inch plywood. The sides are 3/4 inch thick Douglas fir. All interior wood surfaces were given three coats of polyurethane varnish and a finishing coat of polyester fiberglass resin.

Connected to each end of the main body are two constant head tanks (Figure 5.1). The difference in water elevation between the two tanks is controlled by a length of PVC tubing located at the center of each

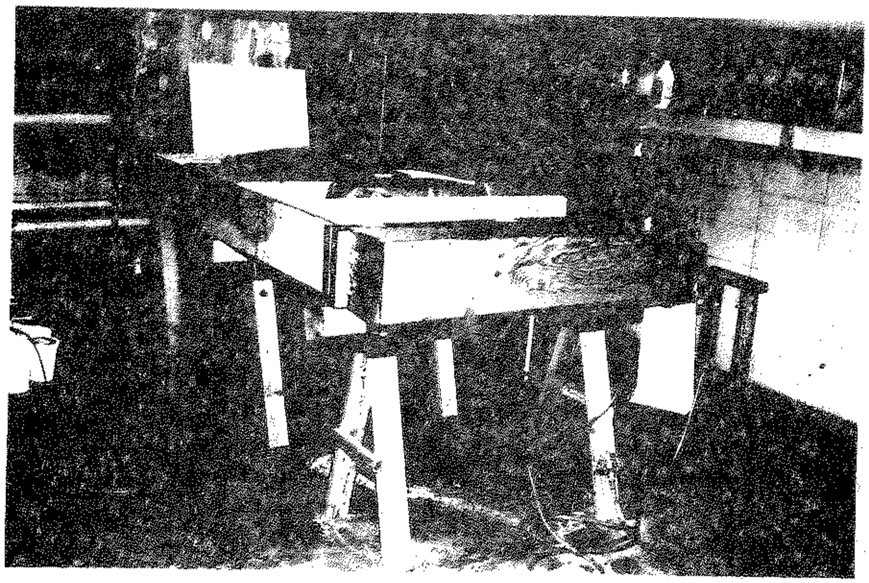

Figure 5.1 View of model. Flow is from the large tank in the rear towards small tank in foreground.

tank and penetrating the base (Figure 5.2). The height of each tube is adjustable. Difference in height determines the head gradient across the model.

The main body of the aquifer was filled with five centimeters of Ottawa sand; a clean, silica sand with a grain diameter of approximately one-half millimeter.

The upper confining bed is a 4 foot by 8 foot sheet of 1/2 inch thick, clear acrylic. Clear silicone sealant is used to bond the seams where the acrylic abuts the interior walls of the model. Small leaks, which occurred during the coarse of the experiment, were patched with plumber's putty. Two, 100 pound bags of sand, and 2 x 4 cross bracing holds the acrylic in contact with the upper surface of the sand.

A fully penetrating well is located in the center of the aquifer. The well is constructed of 5/8 inch outside diameter PVC tubing with an inside diameter of 7/16 inch. Perforations one millimeter in diameter were drilled into the portion of the tubing that penetrates the aquifer. This section was wrapped with three layers of nylon mosquito netting to prevent sand from entering the well. The well was sealed with a PVC cap at the top where it penetrates the acrylic (Figure 5.3). To allow injection of the tracer a single hole was drilled into the cap and sealed with clear silicone. The injection of tracer is accomplished by insertion (through the perforated well cap) of a 2 1/2 inch, stainless steel, hypodermic needle attached to a 50 milliliter syringe (Figure 5.4). Upon removal of the needle the clear silicone reseals itself.

The gross weight of the model when saturated is estimated as in excess of 1400 pounds. Three sawhorses support the unit at a height of

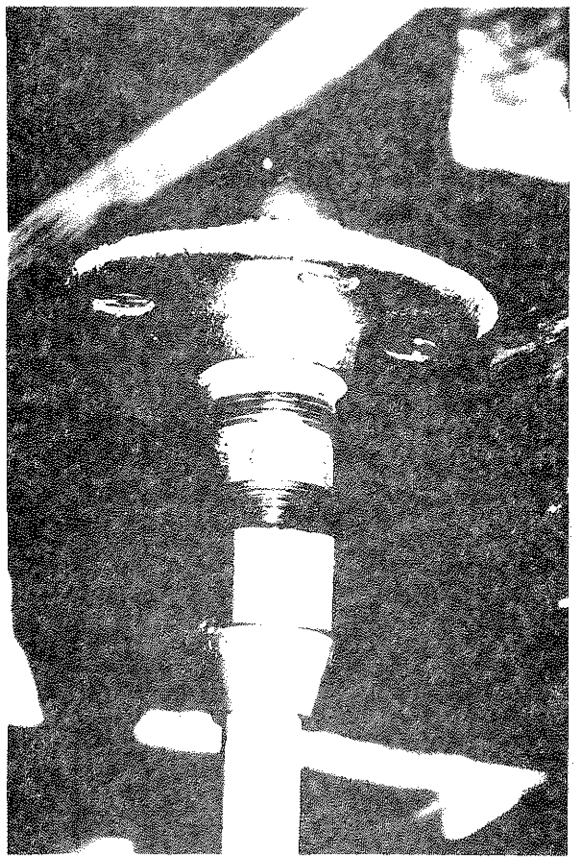

Figure 5.2 PVC tube penetrating base of constant head tank. Compression fitting allows tube to be raised or lowered in tank.

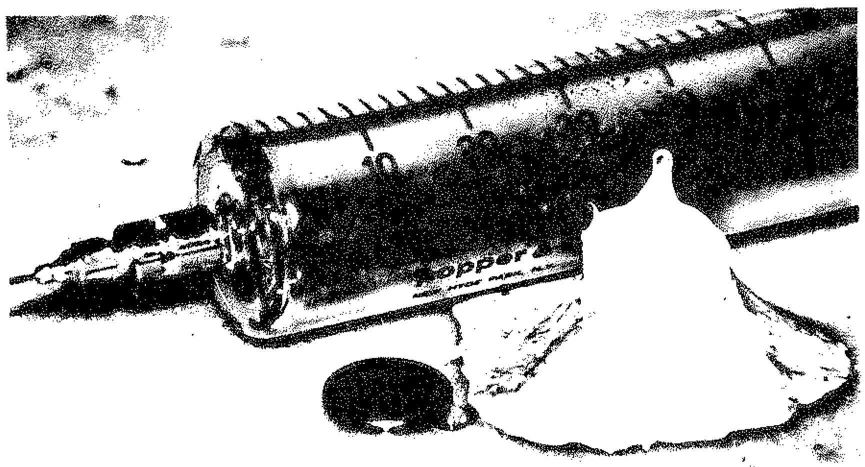

Figure 5.3 Top view of well. Mound of putty around base prevents seepage where well penetrates the acrylic.

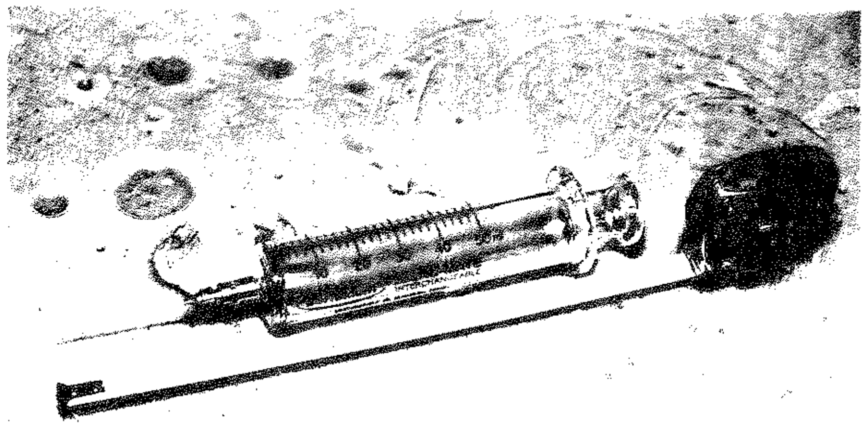

Figure 5.4 Syringe used for injection of tracer.

30 inches from ground level to the base of the model. Leveling is accomplished by placing a two-ton, hydraulic jack on a jackstand under the corners of the model and inserting wood shims between the sawhorses and the bracing at the base of the main tank (Figure 5.5).

Choice of Tracer

The properties required of a tracer for the proposed experiment are conservative, or ideal behavior and easily detectable signature. Conservative behavior implies that the tracer should describe fluid flow in a porous medium without modifying the transmission characteristics (Kaufman and Orlob, 1956). Signatures can be chemical, colormetric, or radioactive.

Sodium chloride was chosen as a tracer because of documentation of its ideal behavior (Kaufman and Orlob, 1956; Cahill, 1966). The chloride ion is easily detected by measuring the electrical conductivity of the fluid between two electrodes (Cahill, 1973).

Instrumentation

Instrumentation for the experiment consisted of the following equipment.

Flowmeters

Two flowmeters were used, one to monitor discharge from the well, the second to monitor discharge from the downstream constant head tank. Both were glass float Gilmont meters with a range of 10 to 850 mL per minute (Figure 5.6). Each meter carries a serial number and comes with a calibration chart that allows graphical conversion of the meter scale reading to flow rates.

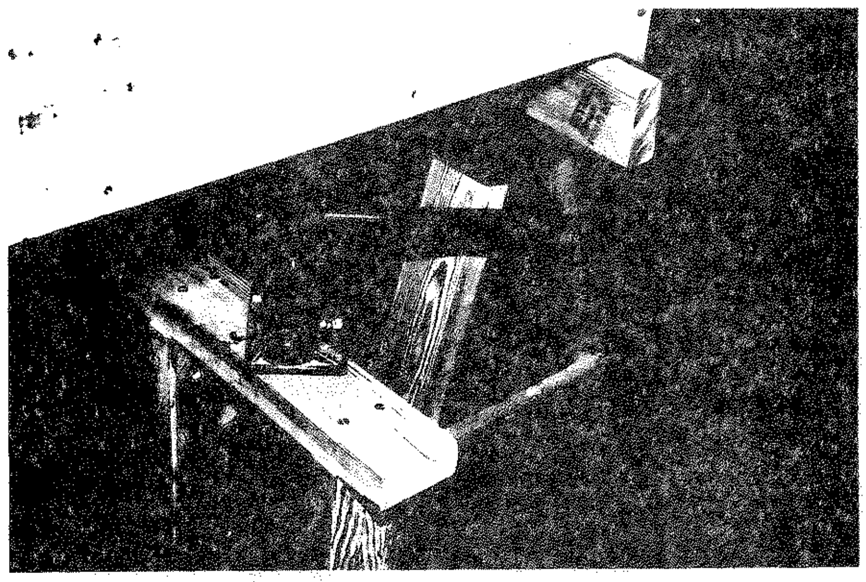

Figure 5.5 Hydraulic jack in corner location.

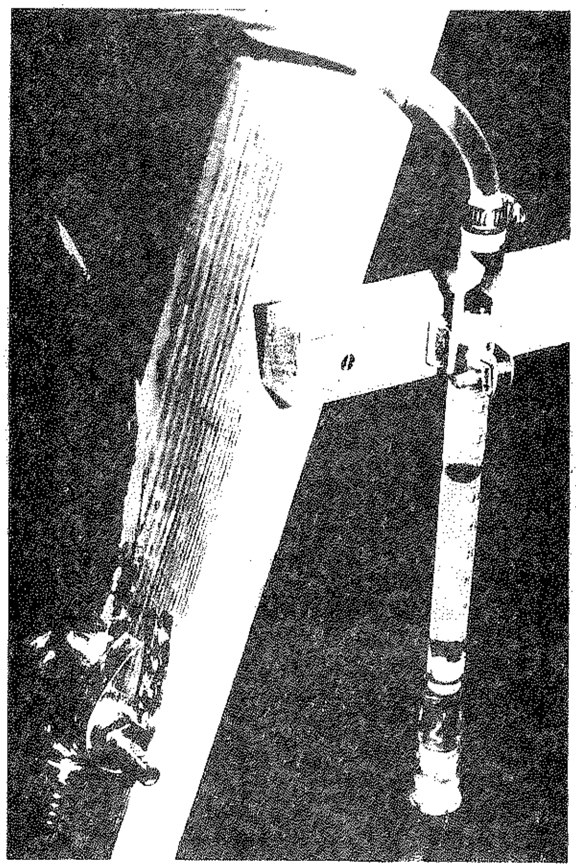

Figure 5.6 Gilmont flowmeter.

Electrodes

A flow type conductivity cell with a cell constant of 0.1 reciprocal centimeter and platinum electrodes was used to monitor the specific conductance of the recovered tracer (Figure 5.7). The cell was installed in line between the well and the flowmeter with lengths of flexible Tygon tubing (Figure 5.8). A U-shaped length of tubing between the conductivity cell and the well functions as a sediment trap.

To monitor tracer concentration downstream from the well an electrode pair consisting of two 3-1/2 inch, 22 gauge, stainless steel hypodermic needles were installed (Figure 5.9). Only the shape of the tracer concentration versus time curve is used to calculate an arrival time. Therefore, no attempt was made to determine the cell constant of this electrode pair. A long term test indicated that the behavior of the electrode pair was stable.

Conductance Meter

A Yellow Springs Instrument Company model 32 conductance meter was used to monitor tracer concentration (Figure 5.10). The instrument has a range that extends from 0.1 micromhos to 200 millimhos. Only non-temperature compensated signals are presented at the recorder terminals.

Chart Recorder

A Linear model 355 chart recorder was used to keep a continuous record of specific conductance versus time (Figure 5.11). The instrument has variable input voltages and chart speeds.

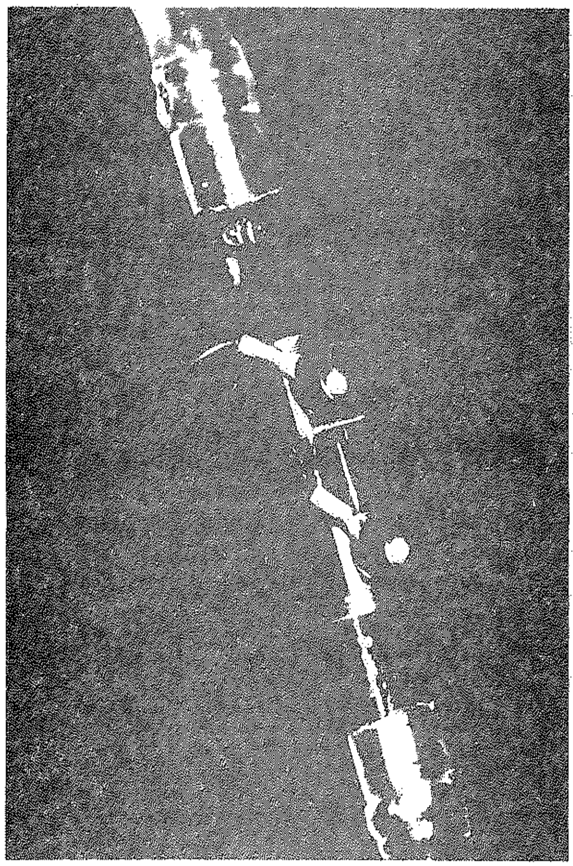

Figure 5.7 Conductivity cell.

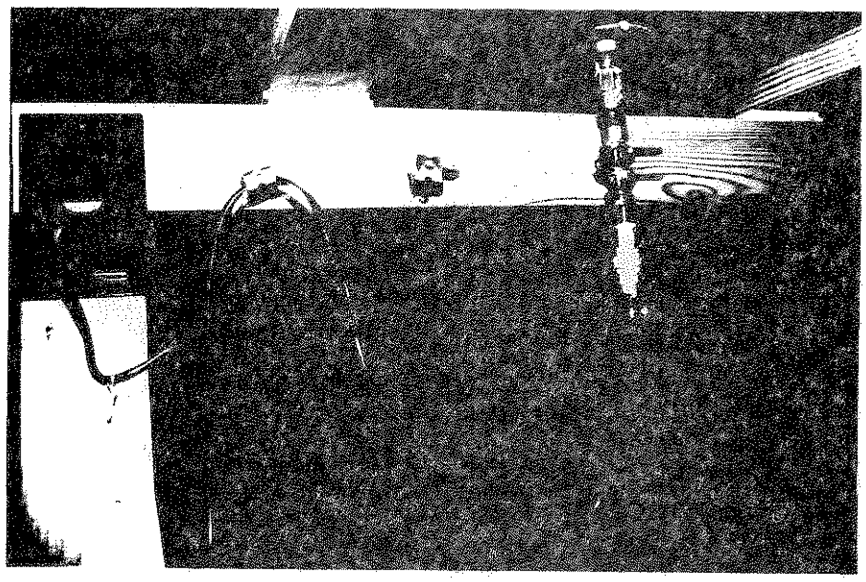

Figure 5.8 Relation of conductivity cell to well. U-shaped length of tubing functions as a sediment trap.

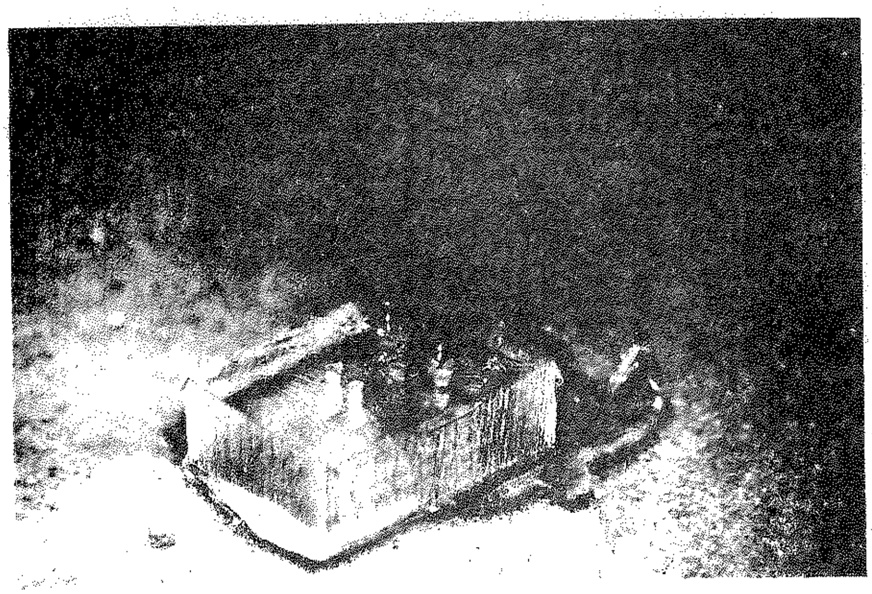

Figure 5.9 Stainless steel electrodes. Electrode pair penetrates the entire saturated thickness of the aquifer.

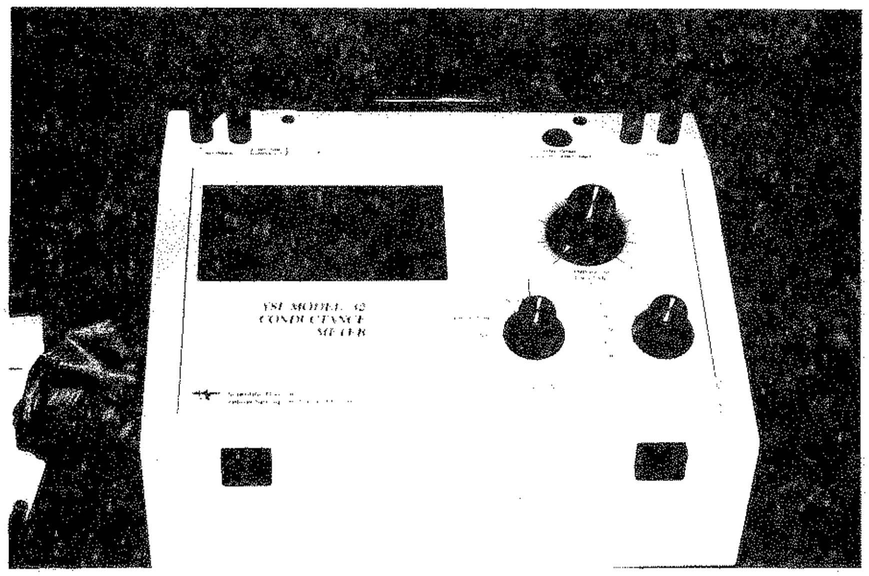

Figure 5.10 Conductance meter.

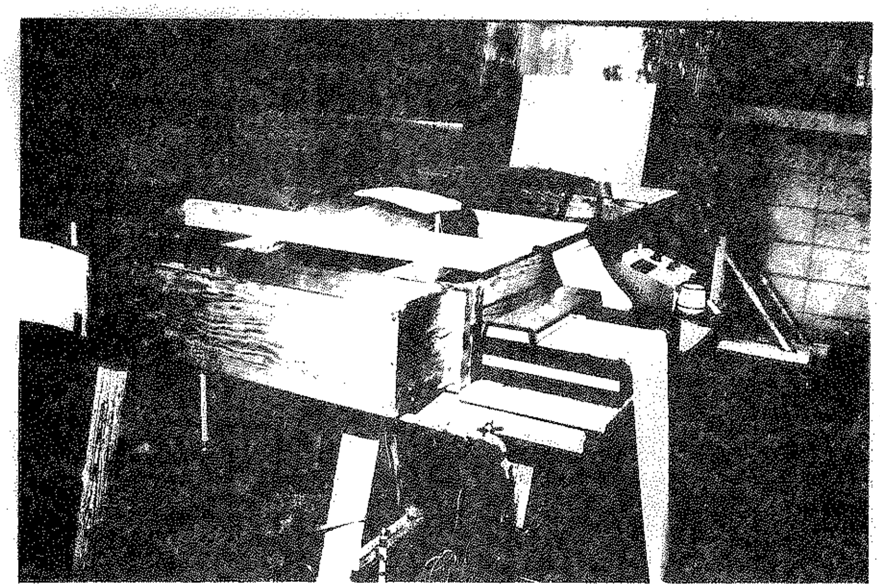

Figure 5.11 Chart recorder.

Operational Behavior of the Model

The hydraulic conductivity of the model was calculated prior to commencement of tracer tests. Discharge from the downstream constant head tank was monitored with a Gilmont flowmeter. Darcy's Law was used to obtain the following expression for hydraulic conductivity.

$$K = QL/AH \tag{5.1}$$

where,

K = hydraulic conductivity (cm/s)

Q = model discharge rate (cm^3/s)

L = distance between seepage faces in model, 250 cm

A = area of seepage face, 610 cm^2

H = difference in head between constant head tanks (cm).

Substituting in values for L and A, which are constant, gives

$$K = 0.41Q/H \tag{5.2}$$

All values for H are referenced to the base of the aquifer. Values of hydraulic conductivity in the range of 0.20 to 0.30 centimeters per second were expected. Actual values are given in Table 5.1.

Initial values were an order of magnitude larger than predicted. Seepage faces were compacted and the model pounded with a rubber mallet. This resulted in a dramatic reduction in hydraulic conductivity. Further reductions were obtained by the placement of 200 pounds of sand on the exterior of the upper confining sheet of acrylic.

Injection of red food coloring at the well and in the upstream head tank confirmed the suspicion of preferential flow paths within the model. These paths were located at the sharp corner where the acrylic

Table 5.1 Variations in Model Hydraulic Conductivity During Calibration.

Head (cm) upstream	Head (cm) downstream	H(cm)	Q(cm/s)	K(cm/s)
15.0	13.5	1.5	8.30	2.30
14.4	13.4	1.0	6.20	2.50
14.4	13.9	0.5	3.00	2.40
14.4	9.4	5.0	13.30	1.10
14.4	9.4	5.0	8.30	.66
14.4	9.4	5.0	7.30	.59
14.4	9.4	5.0	6.30	.50
14.4	9.4	5.0	7.70	.61
14.4	8.4	6.0	7.50	.50
14.4	7.9	6.5	8.30	.51
14.4	7.4	7.0	8.75	.50
14.4	6.9	7.5	9.50	.51
14.4	6.9	7.5	8.80	.47
15.4	6.9	8.5	10.00	.47
15.4	5.9	9.5	11.30	.48
13.0	6.0	7.0	6.83	.39
13.0	5.5	7.5	6.92	.37
10.5	5.5	5.0	4.30	.35
10.5	5.5	5.0	3.00	.24
11.5	5.5	6.0	3.58	.24
12.5	5.5	7.0	4.42	.25
13.5	5.5	8.0	5.58	.28
14.0	5.5	8.5	6.25	.29
14.0	7.0	7.0	5.75	.33
14.0	7.0	7.0	6.25	.36
14.0	7.5	6.5	5.50	.34
14.0	7.5	6.5	5.75	.35
14.0	6.5	7.5	6.67	.36
14.0	5.5	8.5	6.83	.32
10.0	5.5	4.5	2.50	.22
10.0	5.5	4.5	2.75	.24
10.0	5.5	4.5	3.08	.27
1ω.0	6.5	4.5	3.50	.3ω
12.0	7.5	4.5	3.50	.3ω
12.0	5.5	6.5	4.75	.29
13.5	5.5	8.0	6.25	.3ω
15.5	5.5	10.0	8.75	.35

was in contact with the sides of the main tank and randomly at the boundary between the acrylic and the sand aquifer. This accounts, in part, for the higher than expected values of hydraulic conductivity. There is also evidence that conductivity increases as a function of increasing pressure head in the model. This is attributed to the elastic behavior of the acrylic confining layer. Separation of this layer from the porous matrix was confirmed by the behavior of dye injected at the well which moved in streamers downgradient (Figure 5.12).

Entrapped air, also noticeable in Figure 5.12, was a problem for several days following saturation of the model. Continuous running of the model between tests reduced the problem substantially. Depressions in the upper surface of the sand remained after degassing.

Values of hydraulic conductivity during the tracer tests are given in Table 5.2. The model stabilized at a value slightly greater than twice the lowest value recorded during calibration.

Data Collection

Injection and recovery data are listed in Table 5.3. Instrument settings are given in Table 5.4. In addition to the data listed, a continuous record of specific conductance versus time was obtained for each trial. Adequate time was allowed between trials to flush any residual tracer from the system.

Only distilled water was used in the model. Two overnight trials were conducted to monitor fluctuations in background concentration. The first of these tests showed rapid fluctuations in specific conductance. The magnitude of the fluctuations was, at that time,

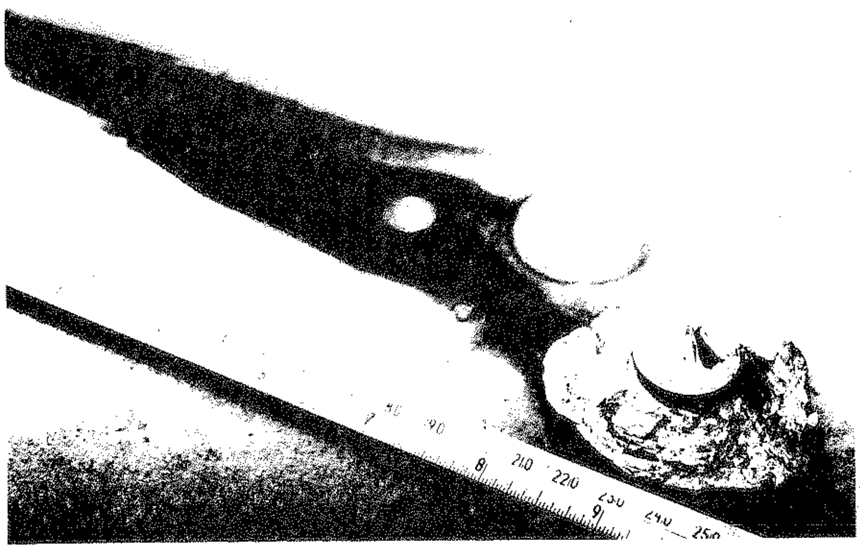

Figure 5.12 Dye tracer in upper flow zone. Note also the presence of entrapped air.

Table 5.2 Variations in Model Hydraulic Conductivity During Tracer Tests

Head(cm) upstream	Head(cm) downstream	H(cm)	Q(cm/s)	K(cm/s)	Trial No.
15.5	5.5	10.0	11.67	.48	1
15.5	5.5	10.0	11.67	.48	2
15.5	5.5	10.0	11.67	.48	3
15.5	5.5	10.0	11.83	.49	4
15.5	5.5	10.0	11.83	.49	5
15.5	5.5	10.0	12.67	.52	6
15.5	5.5	10.0	12.92	.53	7
10.5	5.5	5.0	5.58	.46	8
10.5	5.5	5.0	5.58	.46	9
10.5	5.5	5.0	5.58	.46	10
10.5	5.5	5.0	5.58	.46	11
10.5	5.5	5.0	5.58	.46	12
10.5	5.5	5.0	5.42	.44	13
10.5	5.5	5.0	5.58	.46	14
10.5	5.5	5.0	5.58	.46	15
10.5	5.5	5.0	5.58	.46	16
10.5	5.5	5.0	5.58	.46	17
10.5	5.5	5.0	5.58	.46	18
10.5	5.5	5.0	5.58	.46	19
13.0	5.5	7.5	9.17	.50	20
13.0	5.5	7.5	9.58	.52	21
13.0	5.5	7.5	9.17	.50	23
13.0	5.5	7.5	9.17	.50	24
13.0	5.5	7.5	9.50	.52	25
13.0	5.5	7.5	9.50	.52	26
13.0	5.5	7.5	9.33	.51	27
13.0	5.5	7.5	9.17	.50	28
13.0	5.5	7.5	9.42	.51	29
13.0	5.5	7.5	9.17	.50	30
10.5	5.5	5.0	5.58	.46	31
10.5	5.5	5.0	5.58	.46	32
10.5	5.5	5.0	5.58	.46	33
13.0	5.5	7.5	8.50	.46	34
13.0	5.5	7.5	8.33	.46	35
13.0	5.5	7.5	8.33	.46	36

Table 5.3 Injection and Recovery Data

Trial No.	INJECTION Amount (cm^3)	Concentration (ppm)	RECOVERY Pump Rate (cm^3/s)	Drift Time (min)
1	20	450	0.33	15.0
2	20	450	0.42	15.0
3	20	450	0.67	30.0
4	20	450	3.25	15.0
5	20	450	1.67	30.0
6	20	450	3.50	30.0
7	20	450	3.08	45.0
8	25	450	2.08	105.0
9	25	450	4.33	221.0
10	25	450	5.42	15.0
11	20	450	1.50	15.0
12	20	450	2.00	20.0
13	20	450	2.58	720.0
14	20	450	2.00	25.0
15	20	450	2.17	30.0
16	20	450	1.83	36.0
17	20	450	3.17	45.0
18	20	450	5.83	60.5
19	20	450	6.00	75.0
20	20	450	2.00	20.0
21	20	450	1.92	30.0
22			4.03	1175.0
23	20	900	2.08	20.0
24	20	900	3.42	20.0
25	20	900	6.08	25.0
26	20	900	5.83	30.0
27	20	900	6.08	35.0
28	20	900	6.25	40.0
29	20	900	6.00	40.0
30	20	900	6.42	45.0
31	20	900	1.08	20.0
32	20	900	1.17	20.0
33	20	900	1.92	50.0
				45.0
				40.0
34	20	900	4.67	40.0
				35.0
				30.0
35	20	900	2.50	55.0
				45.0
				35.0
36	20	900	8.08	55.0
				45.0
				35.0

Table 5.4 Recorder and Conductance Meter Settings During Tracer Tests

Trial No.	RECORDER Chart Speed (cm/min)	RECORDER Input Voltage (volts)	CONDUCTANCE METER Sensitivity
1	3.0	1.0	2 mmhos
2	6.0	2.0	2 mmhos
3	3.0	1.0	2 mmhos
4	6.0	2.0	2 mmhos
5	6.0	2.0	2 mmhos
6	6.0	2.0	2 mmhos
7	6.0	2.0	2 mmhos
8	6.0	2.0	2 mmhos
9	6.0	2.0	2 mmhos
10	6.0	2.0	2 mmhos
11	6.0	2.0	2 mmhos
12	6.0	2.0	2 mmhos
13	6.0	2.0	2 mmhos
14	6.0	2.0	2 mmhos
15	3.0	1.0	2 mmhos
16	3.0	1.0	2 mmhos
17	2.0	2.0	200 μmhos
18	2.0	2.0	200 μmhos
19	2.0	1.0	200 μmhos
20	6.0	1.0	2 mmhos
21	2.0	2.0	200 μmhos
23	3.0	1.0	2 mmhos
24	3.0	0.5	2 mmhos
25	2.0	0.5	2 mmhos
26	1.0	0.5	2 mmhos
27	1.0	1.0	200 μmhos
28	1.0	1.0	200 μmhos
29	1.0	1.0	200 μmhos
30	0.5	0.5	200 μmhos
31	6.0	2.0	2 mmhos
32	6.0	2.0	20 mmhos
33	2.0	2.0	200 μmhos
34	1.0	1.0	200 μmhos
35	1.0	1.0	200 μmhos
36	1.0	2.0	200 μmhos

greater than the tracer concentrations expected from the recovery of the salt solution. This problem was corrected and the second test showed no detectable variations in background concentration.

DATA INTERPRETATION

The purpose of the data is to solve equation 4.12 for the advective flow velocity. Three values are assumed constant, the number pi, the porosity of the sand matrix, and the thickness of the aquifer. The pumping rate Q, which varies from trial to trial, is measured directly in line by a flowmeter past the electrodes. The two required time parameters are both functions of the arrival time of what, to this point, has been called a particle. The major working hypothesis leading to a solution is that there is some identifiable and unique point in the distribution of recovered mass that behaves like a particle.

In mechanics there is one point on a moving body that exhibits particle behavior. That point is the center of mass (Resnick and Halliday, 1977). The analogy in our case would be the center of mass of the recovered tracer. The time sought is that time at which the center of mass returns to the well.

As stated, specific conductance versus time was recorded for each of the tests. Our interest is in the distribution of mass with time. Because specific conductance is one measure of mass concentration no conversion to traditional units of mass is required.

Another way of viewing the problem is by asking the following question. At what time did half the mass recovered reach the well? A compensating polar planimeter was used to measure the entire area under the specific conductance versus time curve. Then, by iteration with the planimeter, the area was divided in half. The point on the time axis corresponding to that division was used as the arrival time.

Velocities were calculated using different pumping rates and drift times at two different head gradients across the model.

Porosity of the Ottawa sand was calculated using three different volume displacement techniques. Values of 0.388, 0.380, and 0.376 were obtained. The value of 0.38 was used for all velocity calculations.

Table 6.1 presents the calculated data and velocities for the trials conducted at a head difference of 7.5 centimeters between constant head tanks.

The apparent increase in velocity with increasing drift time, and therefore with distance travelled from the well, is considered real. Rather than interpret the increase as random error, it is more likely that a velocity shadow exists downstream from the well.

Fully penetrating, stainless steel electrodes were installed at a distance of 23 centimeters from the exterior of the well in the downstream direction. Tracer was injected into the well and allowed to drift. The specific conductance versus time curve generated as the tracer passed the electrodes showed two peaks. The first peak occured after four minutes. The velocity calculated from this first peak is 0.096 centimeters per second. Arrival time of the second peak occured at 38.4 minutes giving a velocity of 0.010 centimeters per second.

The first peak is interpreted as a high velocity flow zone located at the boundary between the acrylic confining layer and the sand matrix.

Table 6.1 Velocities Calculated at a Head Difference of 7.5 Centimeters

Trial No.	Drift Time (min)	t (s)	T (s)	Q (cm^3/s)	n	b (cm)	v_a (cm/s)
23	20	360	1560	2.08	.38	5	0.007
24	20	192	1392	3.42	.38	5	0.008
25	25	202	1702	6.08	.38	5	0.008
26	30	339	2139	5.83	.38	5	0.009
27	35	606	2706	6.08	.38	5	0.009
28	40	849	3249	6.25	.38	5	0.009
29	40	1116	3516	6.00	.38	5	0.010
30	45	1476	4176	6.42	.38	5	0.010

This was qualitatively confirmed by observing the behavior of red food coloring injected at the well.

The second peak is interpreted as representative of flow velocities within the body of the aquifer. Comparison with the velocities calculated for the two 40 minute drift and pumpback trials, which would have travelled about 23 centimeters before recovery, shows a difference of 0.001 centimeters per second in the case of trial 28 and no difference with the velocity calculated for trial 29.

Discharge of water from the seepage face of the model was 9.17 and 9.42 cubic centimeters per second for trials 28 and 29 respectively. Seepage velocity (Harr, 1972) is given by

$$v = Q/nA \qquad (6.1)$$

Using an average value for Q of 9.3 cubic centimeters per second for the two trials a value for the seepage velocity of 0.040 centimeters per second is obtained using equation 6.1. This value is four times greater than the value of 0.010 centimeters per second obtained from the injection and pumpback technique. Because of the presence of high velocity zones at the boundaries of the model, the value of 0.040 centimeters per second is not believed representative of velocities within the main body of the aquifer.

Table 6.2 presents the calculated data and velocities for the trials conducted at a head difference of 5.0 centimeters between constant head tanks.

The same increase in velocity with drift time is seen and attributed to the same cause. Discharge of water at the seepage face of the model was 5.58 cubic centimeters per second during all the

Table 6.2 Velocities Calculated at a Head Difference of 5.0 Centimeters

Trial No.	Drift Time (min)	t (s)	T (s)	Q (cm^3/s)	n	b (cm)	v_a (cm/s)
14	25	108	1608	2	.38	5	0.004
15	30	142	1942	2.17	.38	5	0.004
17	45	270	2970	3.17	.38	5	0.004
18	60.5	487	4027	5.83	.38	5	0.005
19	75	877	5377	6.00	.38	5	0.006

trials listed. Using equation 6.1 a seepage velocity of 0.024 centimeters per second is obtained. This value is considered excessive for the same reasons as previously stated.

As of this time a one-way drift test past the stainless steel electrodes has not been conducted.

Stochastic Properties of Velocity

Up to this point the dependent variable, independent variables, and constants have been treated as single-valued numbers. This is pure delusion. Advective flow velocity has already been defined as an average value. Implied in this definition is the concept of a distribution of values about the mean. Assuming this is the case, it would be advantageous to have some idea of the properties of that distribution.

Except for the number pi there is uncertainty in every value used to calculate a flow velocity. Some error comes from the act of observation and reading of the instrumentation, some from the instrumentation itself. Usually ignored are the very real uncertainties inherent in the process being described and the parameters chosen to describe it.

Velocity is a parameter that describes the interaction of a fluid with its environment. It can be defined one dimensionally in terms of Darcy's Law as

$$v = K(dh/ds)/n \tag{6.2}$$

This is an equivalent statement to equation 6.1 for seepage velocity, that is

$$Q/nA = K(dh/ds)/n \tag{6.3}$$

If we accept equation 6.2 as a deterministic statement of velocity then given single values of K, dh/ds, and n, velocity will always be a single-valued, predictable number. In other words, equation 6.2 is an empirical description of cause and effect. It can also be said that, given single values for K, dh/ds, and n, the probability exists that velocity is greater than or less than the value calculated and is defined by a probability density function. This case admits uncertainty in the values of the independent variables, and the velocity must be described in probabilistic rather than deterministic terms (Neuman, 1982). This uncertainty can be accepted as a failure to adequately describe or measure; or, the fact can be accepted that a deterministic model of a natural process is fundamentally ill-suited (Bronowski, 1953).

Redefining velocity in equation 6.2 as the expected value of a stochastic process, then a random sample of velocities within the flow domain described by the expected values for K, dh/ds, and n would produce any number of values whose distribution can be defined by a mean, coefficient of variation, and minimum and maximum values.

It would be reasonable to assume that the stochastic properties of the dependent variable are functionally related to the stochastic properties of the independent variables, including so-called constants.

Consider the properties of K, the hydraulic conductivity. This term is a physical descriptor of permiability that varies more than 13 order of magnitude (Freeze and Cherry, 1979). A conservative estimate of the coefficient of variation of K, defined as

$$V_K = (S[K]/E[K])100 \tag{6.4}$$

where

S[K] = standard deviation of K

E[K] = expected value of K

would be 60% (Harr, class notes). Published values for the coefficient of variation of porosity n for coarse-grained sand are on the order of 10% (Harr, 1977). Values for the coefficient of variation of the hydraulic gradient dh/ds are arbitrarily assigned a value of 10%.

With the values of the coefficients of variation, the problem of a stochastic description of the velocity v can be approached in the following manner. If equation 6.2 is written as a function of three uncorrelated variables, in the form

$$v = K^1 \cdot (dh/ds)^1 \cdot n^{-1} \tag{6.5}$$

then it can be argued (Harr, class notes) that the approximate solution for the expected value and coefficient of variation v can be given as

$$E[v] \sim E[K] \cdot E[dh/ds]/E[n] \tag{6.6}$$

$$(V_v)^2 \sim (V_K)^2 + (V_{dh/ds})^2 + (V_n)^2 \tag{6.7}$$

Substituting the values for the coefficients of variation into equation 6.7, and taking the square root

$$V_v \sim 61.6\% \tag{6.8}$$

This number defines the ratio of the standard deviation of velocity to the expected value. It was derived from Darcy's Law under the assumptions stated and estimates of the coefficients of variation.

Within the context of this experiment we are looking for a way to describe the probability distribution of the advective flow velocity expressed by equation 4.12. Accepting the values in Table 6.1 and 6.2

as expected values whose coefficient of variation is reasonably approximated by the value already derived, then standard deviations and minimum and maximum values are required to determine the distribution of velocities.

Taking trial number 30 as an example, the expected values is

$$E[v] = 0.010 \text{ cm/s} \tag{6.9}$$

and from the definition of the coefficient of variation given in equation 6.4

$$S[v] = (0.010\text{cm/s})(61.6)/100 = 0.0062 \text{ cm/s} \tag{6.10}$$

The end points are obtained by choosing zero as a realistic minimum velocity and defining the maximum velocity as

$$v(max) = 4S[v] + E[v] = 0.035 \text{ cm/s} \tag{6.11}$$

With the information now at hand it is possible to describe the probability density function of velocity. The function used is the beta-probability density function defined over the range [a,b] as

$$f(x) = (1/C)(x-a)^{\alpha}(b-x)^{\beta} \tag{6.12}$$

where

$$C = \alpha!\beta!(b-a)^{\alpha+\beta+1}/(\alpha+\beta+1) \tag{6.13}$$

A detailed description of the properties of this function and its application are given in Harr (1977).

Alpha and beta are approximated in the following manner.

$$v' = (E[v]-a)/(b - a) \tag{6.14}$$

$$V' = (S[v]^2/(b - a)^2 \tag{6.15}$$

$$\alpha = (v'^2/V')(1-v') - (1 + v') \tag{6.16}$$

$$\beta = (1/v')(+1) - (+2) \quad (6.17$$

Substituting values already calculated gives

$$v' = 0.286 \quad (6.18)$$

$$V' = 0.031 \quad (6.19)$$

$$\alpha = 0.60 \quad (6.20)$$

$$\beta = 3.0 \quad (6.21)$$

Because the value for alpha is not an integer the following expression is used for the beta-probability density function

$$f(v) = [(1/b-a)B(\alpha+1,\beta+1)][(v-a)/b-a)]^{\alpha}[b-v)/b- a)]^{\beta} \quad (6.22)$$

where

$$B(\alpha+1,\beta+1) = \Gamma(\alpha+1)\Gamma(\beta+1)/ \Gamma(\alpha+\beta+2) \quad (6.23)$$

Solving equation 6.23 for B(1.6,4) yields a value of 0.087. Substituting this and the other known values into equation 6.24 gives the beta-probability distribution function for trial 30.

$$f(v) = 5.7 \times 10^{7}\ v^{.6}(0.035 - v)^{3} \quad (6.24)$$

It is not the purpose of this section to explore the properties of the beta-probability density function. The point is velocity can be expressed in terms of a probability density function, using information available and some educated guesswork. Estimates of the distribution of velocity are fundamental to the solution of contaminant transport problems. Probability techniques are powerful tools in obtaining those estimates.

SUMMARY AND CONCLUSIONS

Data collected and interpreted indicate, to date, that the single well, injection and recovery technique and equation 4.12 provide a realistic estimate of the advective flow velocity in a confined aquifer at laboratory scale. Field scale testing is suggested subject to the following limitations.

Assumptions Required for Field Test

The aquifer is confined and reasonably horizontal. The well is fully penetrating. The formation is homogeneous in the sense that the scale of observation is greater than the scale of individual inhomogeneties within the aquifer. The formation behaves isotropically during pumping. The thickness of the aquifer is known. The porosity of the formation is known. Previous work with tracer techniques utilizing single well, injection and drawback assumes an average value based on the nature of material comprising the aquifer (Borowczyk *et al*, 1966). The groundwater divide induced by pumping is greater than the thickness of the aquifer. These assumptions are common to many popular methods for the determination of aquifer properties (Kruseman and de Ridder, 1979).

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