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# Numerical modification of atmospheric models to include the feedback of oceanic currents on air-sea fluxes in ocean-atmosphere coupled models

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**Abstract:** In this technical report we present the modifications to bring to atmospheric models to account for relative winds (i.e. the difference between the near-surface winds and the oceanic currents) instead of absolute winds in the computation of air-sea fluxes. Because of the implicit treatment of the bottom boundary condition in most atmospheric models the use of relative winds involves a modification of both the surface layer parameterization and the tridiagonal matrix for vertical turbulent diffusion.

**Key-words:** ocean-atmosphere coupled models, relative winds, tridiagonal problem

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# Modification numérique des modèles d'atmosphère afin de prendre en compte l'effet des courants océaniques sur les flux air-mer dans les modèles couplés océan-atmosphère

**Résumé :** Dans ce rapport technique nous présentons les modifications à apporter aux modèles d'atmosphère pour prendre en compte les vents relatifs (i.e. la différence entre les vents de surface et les courants océaniques) au lieu des vents absolus dans le calcul des flux air-mer. Du fait du traitement implicite en temps de la condition d'interface dans la plupart des modèles d'atmosphère, l'utilisation des vents relatifs implique une modification non seulement de la paramétrisation des flux de surface mais également du problème tridiagonal pour la diffusion verticale turbulente.

**Mots-clés :** modèles couplés océan-atmosphère, vents relatifs, problème tridiagonal

# 1 Introduction

This document provides a detailed description of the changes required to include the effect of oceanic currents on air-sea fluxes. Let us consider the two following equations modeling the vertical turbulent mixing in the atmospheric air-column and in the oceanic water column

$$\partial_t \mathbf{u}_a = \partial_z \left( K_v^a(z) \partial_z \mathbf{u}_a \right) \qquad -K_v^a \partial_z \mathbf{u}_a = \overline{\mathbf{u}_a' w'}(z) \tag{1}$$

and respectively

$$\partial_t \mathbf{u}_o = \partial_z \left( K_v^o(z) \partial_z \mathbf{u}_o \right) \qquad -K_v^o \partial_z \mathbf{u}_o = \overline{\mathbf{u}_o' w'}(z) \tag{2}$$

where $\mathbf{u}_o = (u_o, v_o)$ represents the oceanic velocities and $\mathbf{u}_a = (u_a, v_a)$ the wind vector. $K_v^a$ and $K_v^o$ are two turbulent viscosity coefficients, $\overline{\mathbf{u}_a' w'}$ and $\overline{\mathbf{u}_o' w'}$ are the vertical turbulent fluxes, and $z$ is the vertical direction. Assuming that $\mathbf{u}_o$ and $\mathbf{u}_a$ are given in the same coordinate system, the bottom boundary condition $\left.\overline{\mathbf{u}_a' w'}\right|_b$ for the atmosphere and the surface boundary condition $\left.\overline{\mathbf{u}_o' w'}\right|_s$ for the ocean are given by

$$\rho_a \left.\overline{\mathbf{u}_a' w'}\right|_b = \rho_o \left.\overline{\mathbf{u}_o' w'}\right|_s = \boldsymbol{\tau} = (\tau_x, \tau_y) \tag{3}$$

where $\boldsymbol{\tau}$ is the surface wind stress vector expressed in N m$^{-2}$, and $\rho_a$, $\rho_o$ are the densities near the air-sea interface. The wind stress is given by an appropriate parameterization of the atmospheric surface layer under the form

$$\boldsymbol{\tau} = \rho_a \frac{u_\star^2}{\| \left.\mathbf{u}_a\right|_b - \left.\mathbf{u}_o\right|_s \|} \left( \left.\mathbf{u}_a\right|_b - \left.\mathbf{u}_o\right|_s \right) \tag{4}$$

with $u_\star$ the friction velocity usually obtained through a given *surface layer parameterization*. We can easily see that $\|\boldsymbol{\tau}\| = \rho_a u_\star^2$. A first modification to bring to the atmospheric model is to replace $\left.\mathbf{u}_a\right|_b$ by $\left.\mathbf{u}_a\right|_b - \left.\mathbf{u}_o\right|_s$ as an input to the surface layer parameterization (a.k.a. bulk formulation). Doing so the value of $u_\star$ will account for the relative winds. However we emphasize it is not the only modification that is needed.

# 2 Discrete formulation

Equations (1) and (2) are traditionally integrated in time using a backward Euler scheme to maintain good stability properties even if the vertical resolution is refined, as it is usually the case in the planetary boundary layers (PBLs). For a given variable $X$ ($X = u_a, v_a, u_o, v_o$), the space-time discretization reads

$$\frac{X_k^{n+1} - X_k^n}{\Delta t} = \frac{1}{\Delta z_k} \left[ K^x_{k+\frac{1}{2}} \frac{X_{k+1}^{n+1} - X_k^{n+1}}{\Delta z_{k+\frac{1}{2}}} - K^x_{k-\frac{1}{2}} \frac{X_k^{n+1} - X_{k-1}^{n+1}}{\Delta z_{k-\frac{1}{2}}} \right] \tag{5}$$

where the arrangement of variables on the computational grid is described in Fig. 1. The oceanic grid goes from $k = 1$ at the bottom to $k = N_o$ at the surface, while the atmospheric grid goes from $k = 1$ at the surface to $k = N_a$ at the top of the model, with $N_o$ and $N_a$ the number of points of the vertical discretization. At a discrete level, we thus have

$$\left.\mathbf{u}_a\right|_b = \left( \left.u_a\right|_{k=1}, \left.v_a\right|_{k=1} \right) \qquad \left.\mathbf{u}_o\right|_s = \left( \left.u_o\right|_{k=N_o}, \left.v_o\right|_{k=N_o} \right). \tag{6}$$

It is worth mentioning that $u_\star$ is generally cell-centered in the horizontal while $\left.u_a\right|_{k=1}$ and $\left.v_a\right|_{k=1}$ (and possibly $\left.u_o\right|_{k=N_o}$, $\left.v_o\right|_{k=N_o}$) are interfacial values. Equation (5) can be expressed in a matrix form $\mathbf{AX} = \mathbf{F}$ where

$$\mathbf{X} = (X_1^{n+1}, X_2^{n+1}, ..., X_N^{n+1})^t \tag{7}$$

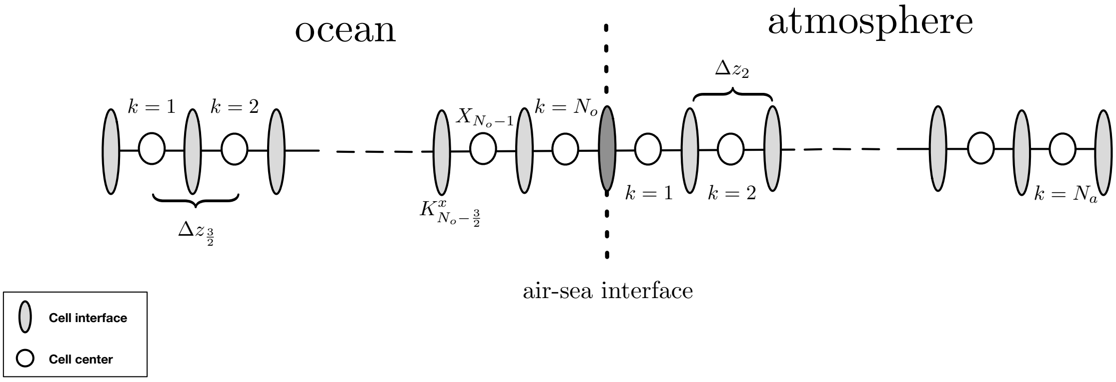


Figure 1: Arrangement of variables for the oceanic and atmospheric vertical grids from either side of the air-sea interface.

with $N = N_a$ or $N = N_o$ depending on the media under consideration. For $k = 2, N-1$, it is straightforward to see that the matrix $\mathbf{A}$ is a tridiagonal matrix by rearranging (5)

$$\mathrm{al}_k X^{n+1}_{k-1} + \mathrm{ad}_k X^{n+1}_{k} + \mathrm{au}_k X^{n+1}_{k+1} = X^n_k, \qquad k = 2, ..., N-1, \tag{8}$$

where

$$\mathrm{al}_k = -\frac{K^x_{k-\frac{1}{2}}\Delta t}{\Delta z_{k-\frac{1}{2}}\Delta z_k}, \qquad \mathrm{au}_k = -\frac{K^x_{k+\frac{1}{2}}\Delta t}{\Delta z_{k+\frac{1}{2}}\Delta z_k}, \qquad \mathrm{ad}_k = 1 - \mathrm{al}_k - \mathrm{au}_k. \tag{9}$$

$\mathrm{ad}_k$ is the term on the diagonal, $\mathrm{al}_k$ and $\mathrm{au}_k$ are respectively on the lower and upper diagonal of $\mathbf{A}$. Note that additional terms can appear depending on the PBL scheme under consideration (e.g. the nonlocal terms for K-profile parameterization like the Yonsei University scheme) but this is outside the scope here. To fully define the matrices $\mathbf{A}$ and $\mathbf{F}$, the boundary conditions must be considered. We assume here that the boundary conditions for $k = 1$ in the ocean and $k = N_a$ in the atmosphere are homogeneous Neumann conditions. For the boundary condition at the air-sea interface, the approach is different between the oceanic model and the atmospheric model.

## 2.1 Oceanic model

In most oceanic models, the surface boundary condition at $k = N_o$ is specified explicitly outside of the tridiagonal system. The matrix $\mathbf{A}$ is thus filled in by considering an homogeneous Neumann condition at $k = N_o$ and the actual boundary condition is simply applied as follows

$$u_o^{n+1}\big|_{k=N_o} = u_o^n|_{k=N_o} + \frac{\Delta t}{\Delta z_1}\frac{\tau_x}{\rho_o}, \qquad v_o^{n+1}\big|_{k=N_o} = v_o^n|_{k=N_o} + \frac{\Delta t}{\Delta z_1}\frac{\tau_y}{\rho_o}. \tag{10}$$

By doing so and because $\tau_x$ is a function of $u_o|_{k=N_o}$ (resp. $\tau_y$ is a function of $v_o|_{k=N_o}$) there is an additional CFL condition associated with (10). This stability condition does not seem problematic in practice. Again, one has to be careful about the discrete placement of $\tau_x$ and $\tau_y$ relative to $u_o$ and $v_o$.

*Adding the oceanic surface currents in the computation of the surface wind stress does not require any modifications in the oceanic model besides providing $u_o|_{k=N_o}$ and $v_o|_{k=N_o}$ to the coupling interface.*

## 2.2 Atmospheric model

In most atmospheric models, the bottom boundary condition is imposed in the tridiagonal system in an implicit way, we thus have for $u_a$

$$K^a_{\frac{1}{2}} \frac{u_a^{n+1}\big|_{k=1} - u_a^{n+1}\big|_{k=0}}{\Delta z_{\frac{1}{2}}} = \left[ \frac{u_\star^2}{\| \, u_a^n|_{k=0} - u_o^n|_{k=N_o} \, \|} \right] \left( u_a^{n+1}\big|_{k=1} - u_o^n|_{k=N_o} \right) \tag{11}$$

which amounts to write (5) for $k = 1$ under the form

$$u_a^{n+1}\big|_{k=1} - u_a^n|_{k=1} = \frac{\Delta t}{\Delta z_1} \left[ K^u_{\frac{3}{2}} \frac{u_a^{n+1}\big|_{k=2} - u_a^{n+1}\big|_{k=1}}{\Delta z_{\frac{3}{2}}} - \left[ \frac{u_\star^2}{\| \, u_a^n|_{k=0} - u_o^n|_{k=N_o} \, \|} \right] \left( u_a^{n+1}\big|_{k=1} - u_o^n|_{k=N_o} \right) \right]. \tag{12}$$

where

$$\mathrm{al}_1 = 0, \qquad \mathrm{au}_1 = -\frac{K^u_{\frac{3}{2}} \Delta t}{\Delta z_{\frac{3}{2}} \Delta z_1}, \qquad \mathrm{ad}_1 = 1 - \mathrm{au}_1 - \left[ \frac{\Delta t u_\star^2}{\Delta z_1 \| \, u_a^n|_{k=0} + u_o^n|_{k=N_o} \, \|} \right] \tag{13}$$

can be identified, on top of

$$f_1 = u_a^n|_{k=1} + \left[ \frac{\Delta t u_\star^2}{\Delta z_1 \| \, u_a^n|_{k=0} - u_o^n|_{k=N_o} \, \|} \right] u_o^n|_{k=N_o} \tag{14}$$

for the right hand side $\mathbf{F}$. Same remarks apply to $v_a$.

*The red term in (14) must be added to the atmospheric model when the oceanic currents are taken into account for the computation of the surface wind stress. By omitting this term we would suppress the rectification of the wind stress orientation by the oceanic currents.* In a model like WRF (Weather Research & Forecasting), this modification is somehow tedious because it must be done for each PBL parameterization since the building of the tridiagonal system is done locally in the parameterization and not through a common interface.

# 3 Concluding remarks

In order to properly account for the effect of oceanic currents on wind-stress magnitude and orientation in coupled models modifications are needed beyond the piece of code responsible for air-sea fluxes computation. We emphasize that the right-hand-side of the tridiagonal problem to solve for vertical diffusion has to be modified otherwise the influence of oceanic currents on wind-stress orientation is absent.

# Contents

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