Tutorial on arbitrary and state-dependent sampling

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Abstract— **This tutorial, presents basic concepts and recent research directions about sampled-data systems. We focus mainly on the stability of systems with time-varying sampling intervals. Without being exhaustive, which would be neither possible nor useful, we try to give a structural survey of what we think to be the main results and issues in this domain.**

Index Terms— **networked control systems, time-varying sampling, dynamic sampling, self-triggering, event-triggering.**

I. INTRODUCTION

In the literature, the analysis and design of sampled-data systems with periodic sampling is a well established domain. In the monographs [16], [26], [71], advanced topics such as optimal control, robust controller design, identification, etc. can be found. In practice however, it is difficult to maintain a constant sampling rate during the real-time control of physical systems. Embedded and networked systems are often required to share a limited amount of computational and transmission resources between different applications. This may lead to fluctuations of the sampling interval, due to the interaction between real-time control algorithms and task / communication scheduling protocols [15], [36], [73], [84]. From the control theory point of view, these variations in the sampling interval need to be treated in a robust manner since they may have a destabilizing effect if they are not properly taken into account [83], [46]. Furthermore, aperiodic controller implementations may actually have interesting properties in distributed control applications, when explicitly evaluating energy, computation and communication costs [11], [14], [14], [33]. The new trend in control is to intentionally modify the sampling interval as an additional control parameter using event- and self-triggering control schemes.

This tutorial will present basic concepts and research directions for systems with time-varying sampling intervals. After a short presentation of sampled-data systems (Section II), qualitative properties of sampled-data system are indicated in Section III. The main results concerning the analysis of systems with time-varying sampling intervals will be presented in Section IV. Finally, emerging research directions concerning the reduction of sampling events will be introduced in Section V.

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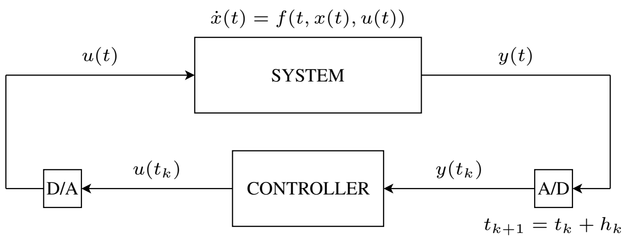

Fig. 1: Sampled-data system

II. SAMPLED-DATA SYSTEMS

The systems under consideration consist of a plant, a sampled-data control, and appropriate interface elements, such as represented in Figure 1, in which the blocks A/D and D/A correspond to an analog-to-digital converter (a sampler) and a digital-to-analog converter (a zero-order hold) respectively. There are various ways in which the controller synthesis for sampled-data systems can be done [7], [16]. One possible way is to design a continuous-time controller and then to approximate it using a sample-and-hold device. This approach is usually called *emulation*. Another approach is the *discrete-time* controller design where an exact or approximate discrete-time model of the plant is used to design a discrete-time controller, which will then be implemented to control the continuous-time plant using a zero-order-hold.

In this paper, the different concepts and results will mainly illustrated through the use of Linear Time-Invariant (LTI) sampled-data systems with linear state-feedback:

$$\dot{x}(t) = Ax(t) + Bu(t), \ \forall t \geq 0, \tag{1a}$$
$$u(t) = Kx(t_k), \ \forall t \in [t_k, t_{k+1}), \ k \in \mathbb{N}, \tag{1b}$$

with x the system state and u the control signal, although some of the presented results have been originally given in more general control configurations or in the nonlinear case.

The following associated discrete-time model at instants t_k is

$$x_{k+1} = \Lambda(h_k)x_k, \ \forall k \in \mathbb{N}, \tag{2}$$

with $\Lambda(h) := \mathrm{e}^{Ah} + \int_0^h \mathrm{e}^{As} ds BK$, $x_k := x(t_k)$. It is well known that for a constant sampling interval $h_k = T, \forall k \in \mathbb{N}$, the discrete-time system (2) is asymptotically stable if and only if the matrix $\Lambda(T)$ is Schur, i.e. all its eigenvalues are strictly within the unit circle. However, in the case of time-varying sampling intervals, the analysis of sampled-data systems is quite complex, even in the LTI case.

Example Consider the LTI sampled-data system (1) with

$$A = \begin{bmatrix} 0 & 1 \\ -2 & 0.1 \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \; K = \begin{bmatrix} 1 & 0 \end{bmatrix}. \qquad (3)$$

It is stable for both constant sampling intervals $T_1 = 1.5$s and $T_2 = 3$s, as both matrices $\Lambda(T_1)$ and $\Lambda(T_2)$ are Schur. One may think that alternating the sampling interval between T_1 and T_2 will not affect the stability. However, the sampled-data system with periodically time-varying sampling intervals $T_1 \to T_2 \to T_1 \to \cdots$ is unstable (see Figure 2, left). This is due to the fact that the Schurness of transition matrices is not preserved under matrix product, i.e. the matrix $\Lambda(T_2)\Lambda(T_1)$ is not Schur.

On the other hand, there may also exist stabilizing sampling sequences which are composed solely of sampling intervals corresponding to non-Schur matrices. For instance, the sampled-data system (1), (3) is unstable for both constant sampling periods $T_3 = 2.1$s and $T_4 = 4$s, but it is stable under the periodically time-varying sampling $T_3 \to T_4 \to T_3 \to \cdots$ (see Figure 2, right). Indeed, the system transition matrix $\Lambda(T_4)\Lambda(T_3)$ is Schur.

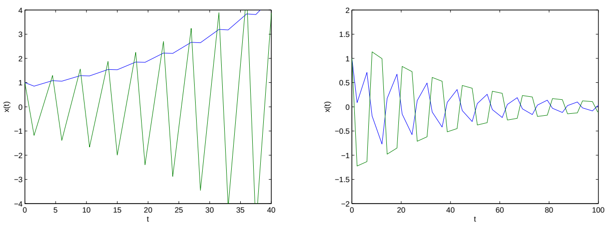

Fig. 2: Periodically time-varying sampling
Left: $T_1 = 1.5s \to T_2 = 3s \to T_1 \to \cdots$ - Unstable
Right: $T_3 = 2.1s \to T_4 = 4s \to T_3 \to \cdots$ - Stable

Thus, the following problems are raised:

- **PROBLEM A:** determine whether the sampled-data system is stable for any time-varying sampling interval h_k with values in a bounded subset $\Omega \subseteq \mathbb{R}_+$.

- **PROBLEM B:** design a sampling law $h_k = h(t, t_k, x(t_k), \cdots)$ that enlarges the sampling intervals while making the sampled-data system stable.

III. Qualitative properties of sampled-data systems

The choice of sampling intervals is a critical issue in sampled-data implementations of emulated controllers. Intuitively, when the original continuous-time controller guarantees stability, by choosing a sufficiently fast sampling frequency, the stability will be preserved under sampled-data implementation. This conjecture has been confirmed in [35], for the case of input-affine nonlinear systems. The main result proves the fact that the discretization of stabilizing continuous-time nonlinear control laws with Lipschitz properties preserves the stability for constant and sufficiently small sampling intervals. This has been generalized in [10] to the case of time-varying sampling intervals, with dynamical control laws discretized using Euler approximation. In [78] it is shown that when the periodic sampling is sufficiently fast, the Input-to-State Stability (ISS) property of a nonlinear system is semi-globally practically preserved.

In [6], [8], [43], [59], some results concerning the emulation approach were generalized and unified in a methodological framework, by considering the preservation of dissipation inequality under sampling. It is shown that if a continuous-time controller provides some dissipation properties, then the resulting sampled-data system satisfies similar properties in a semi-global practical sense for sufficiently small sampling intervals.

IV. Stability analysis under arbitrary time-varying sampling

This section concerns the estimation of the maximum allowable sampling intervals under arbitrary sampling variations.

A. *Time-delay approach*

This approach was initiated in [55] and further developed in [28] and in several other works [27], [41], [50], [75], [78], [85]. It consists in considering the discrete-time dynamics induced by the digital controller as a delay effect. For system (1), we may re-write $u(t) = Kx(t_k) = Kx(t-\tau(t))$, where the delay $\tau(t) = t - t_k$, $\forall t \in [t_k, t_{k+1})$, is piecewise-linear, and satisfies $\dot{\tau}(t) = 1$ for $t \neq t_k$, and $\tau(t_k) = 0$. The LTI system with sampled data (1) is then re-modeled with a time-varying delay

$$\dot{x}(t) = Ax(t) + BKx(t - \tau(t)), \; \forall t \geq 0, \qquad (4)$$

In this context of delay systems [72], stability is studied using Lyapunov-Krasovskii or Lyapunov-Razumikhin functionals [32] depending on the past system state values. LMI stability criteria are given in [27], [28], [74]. For the nonlinear case, we point to the works in [41], [50], [78], [85].

B. *Hybrid system approach*

In this approach, the LTI sampled-data system (1) is modelled as an impulsive system (i.e. a dynamical system with discontinuous state variables, representing the sampling effect), with the state $\xi(t) = [x^T(t), \, z^T(t)]^T$, where $z(t) = x(t_k)$, $\forall t \in [t_k, t_{k+1})$:

$$\begin{cases} \dot{\xi}(t) = \begin{bmatrix} A & BK \\ 0 & 0 \end{bmatrix} \xi(t), & t \neq t_k, \forall k \in \mathbb{N}, , \\ \xi(t_k) = \begin{bmatrix} x(t_k^-) \\ x(t_k^-) \end{bmatrix}, & t = t_k, \forall k \in \mathbb{N}. \end{cases}$$

Stability analysis in this context involves Lyapunov functions with discontinuities at the impulse times [9], [58]. For nonlinear systems, the $\mathcal{L}_p$-stability properties have been studied in the more general context of networked control systems (NCS) [61]. See also [12], [62], [67], for a particularization to the sampled-data case.

C. Discrete-time approach and convex-embeddings

For system (1) with time-varying sampling intervals in $[\underline{h}, \bar{h}]$, $\underline{h} > 0$, the convex-embedding approach [17], [29], [37], [31], uses the properties of the transition matrix $\Lambda(t - t_k) = e^{A(t-t_k)} + \int_0^{t-t_k} e^{As} ds BK$, describing the evolution of $x(t)$ over the sampling interval with respect to the initial value $x(t_k)$. The idea is to express the stability problem as a finite number of LMIs, by embedding the matrix $\Lambda(\theta)$, $\theta \in [\underline{h}, \overline{h}]$ in a larger polytope $\bar{\mathcal{W}} := \text{conv}\{\Lambda_i, i = 1, \cdots, N\}$ with finite number of vertices Λ_i. For quadratic Lyapunov functions $V(x) = x^T P x$, simple LMIs dependent on the polytope vertices may be obtained:

$$P \succ 0, \ \Lambda_i^T P \Lambda_i - P \prec 0, \ \forall i \in \{1, \cdots, N\}. \qquad (5)$$

A continuous-time stability analysis of sampled-data systems based on convexification arguments has been proposed in [22], [38]. The discrete-time approach has also been considered in the case of nonlinear sampled-data systems [6], [42], [57], [60], [63]. However the developments are complex even in the case of periodic sampling.

D. I/O approach

In this approach, the sampling effect is seen as a perturbation $w(t) = x(t_k) - x(t) = -\int_{t_k}^{t} \dot{x}(\tau) d\tau$, and tools from robust control theory are used to guarantee the system's stability. The main idea is to write the sampled-data system (1) on each interval $[t_k, t_{k+1})$ as the feedback interconnection of the operator $\Delta_{sh} : y \to w$ defined by: $w(t) = (\Delta_{sh}\, y)(t) = -\int_{t_k}^{t} y(\tau) d\tau$, $\forall t \in [t_k, t_{k+1})$, with an LTI system

$$\mathbf{G} := \begin{cases} \dot{x}(t) = A_{cl} x(t) + B_{cl} w(t), \\ y(t) = \dot{x}(t), \end{cases} \qquad (6)$$

where $A_{cl} = A + BK$ and $B_{cl} = BK$. The stability may be studied using classical robust control tools based on frequency domains analysis of the interconnection. See [40], [56], for a study based on the small gain theorem and [30] for a more general Integral Quadratic Constraints (IQCs) and Kalman-Yakubovich-Popov Lemma analysis. Extensions to nonlinear sampled-data systems have been provided in [64]–[66], [68] using dissipation theory.

V. DYNAMIC CONTROL OF THE SAMPLING

In this section, the sampling interval h_k is considered as a control parameter. Increasing h_k means reducing the quantity of information sent between the sensors and the actuators. In the literature, three main approaches cover the definition of such a sampling law $h_k = h(t, t_k, x(t_k), \cdots)$. In the first approach, the *event-triggered control* [13], [34], [48], [54], [77], [1], [79], [80], the system state is continuously monitored and the control actuation is performed only when certain events occur. These events are usually generated when the system state crosses a frontier in the state space. A dedicated hardware is required in order to monitor continuously the plant and generate such events. The second approach, the *self-triggered control* [45], [81], [82], [51], [52], [3], [2], [4], [5], aims at emulating event-triggered control without dedicated hardware, by pre-computing at each sampling instant the next admissible sampling interval based on previously received data and the knowledge of the plant dynamics. The third approach, the *periodic event-triggered control* (PET Control) [19], [70] considers uniform monitoring of the system state and event-triggering conditions that are verified periodically. In the following, we present a brief classification of some research directions in event- and self-triggered control.

A. Deadband control approach [13], [69]

The main idea of this event-triggered controller is that it is not necessary to update the control of the system when its state is close enough to the equilibrium point. The control is updated only when the state leaves some neighbourhood, called deadband. In [13] for example, the authors aim at reducing the number of actuations, while guaranteeing that the system state stays bounded. The chosen deadband is $\|Cx\| \leq z_{max}$ for some $z_{max} > 0$. In order to ensure that the disturbances will not make the output drift away from zero, the control outside the deadband is designed as $u(t) = -\text{sgn}(Cx(t))$. Inside the deadband, the control is $u(t) = \text{sat}(K\hat{x}(t))$, and it is based on a simulation $\hat{x}(t)$ of the ideal evolution of the system, with initial conditions updated at triggering times ($\hat{x}(t_k) = x(t)$ when $\|Cx\| = z_{max}$).

B. Perturbation rejection approach [44], [48], [76]

This type of event-triggered control scheme tries to take into account exogenous perturbations in event-triggering control by estimating and rejecting them. The sensors need to include an observer which estimates the perturbation. The event-generator used for this kind of controller is similar to the one used for deadband control (*i.e.* information is sent from the sensors to the actuators only when the state leaves a neighbourhood of the origin), except that, here, the error generating the trigger is computed with respect to the estimated state, instead of the equilibrium point. The events are thus generated when the measured state $x(t)$ leaves the vicinity $\Omega(\hat{x}(t)) = \{x | \|x - \hat{x}(t)\| \leq \bar{e}\}$ of the estimated state $\hat{x}(t)$, for a given threshold $\bar{e}$.

C. Lyapunov function levels approach [79]

Another approach to event-triggered control consists in updating the control only when a chosen Lyapunov function crosses some predetermined energy levels. In [79], it is considered a nonlinear sampled-data system with an event generator defined by some levels of a Lyapunov function V: $V(x(t)) = \eta V(x(t_k))$, for some given scalar $0 < \eta < 1$. Note that the trigger occurs when entering the region $V(x(t)) \leq \eta V(x(t_k))$ therefore stability is not granted for all values of η. To ensure stability with such an event generator, it is necessary to guarantee that after each sampling instant t_k there will be a time $t > t_k$ for which the event-triggering condition $V(x(t)) = \eta V(x(t_k))$ will be satisfied. The method proposed in [79] consists in computing an upper-bound η^* of the minimal admissible η (i.e. such that the previously mentionned property is satisfied for any $\eta \geq \eta^* > 0$). A gridding of the state space is used to this aim.

D. Upper-bound on the system decay rate [52], [53]

Unlike the Lyapunov function levels approach, here the sampling occurs when the state moves away from the equilibrium point, that is when the system does not satisfy a specified decay rate for a chosen Lyapunov function. Considering V, a Lyapunov function with exponential decay-rate λ_0 for the closed-loop system with continuous feedback, and the map $\delta_c(x(t_k),t) := V(x(t)) - V(x(t_k))\mathrm{e}^{-\lambda(t-t_k)}$, for some $0 < \lambda < \lambda_0$, the methods uses as a trigger the condition $\delta_c(x(t_k),t) = 0$ for $t > t_k$. Note that implicitly, by construction, stability is granted in this approach. A self-triggering policy may be derived by computing online a Veronese embedding.

E. ISS - approach [3], [4], [52], [77]

ISS Lyapunov functions constitute another popular dynamic sampling control approach used to perform both event-triggered and self-triggered control. It was initiated by [77], and further developed in [52], [3], [2] and [4]. The general approach in [77] considers the the reset system

$$\left\{ \begin{array}{l} \dot{x}(t) = Ax(t) + BK(x(t) - e(t)),\ \forall t \geq 0, \\ e(t_k) = 0, \end{array} \right. \tag{7}$$

where $e : \mathbb{R}_+ \to \mathbb{R}^n$ is the measurement error between the current state and the last sampled state ($e(t) = x(t) - x(t_k)$). The considered system is supposed to be ISS-stable with respect to the measurement error e, i.e. there exists a Lyapunov function V such that $\frac{\partial V}{\partial x}(Ax + BK(x+e)) \leq -a\|x\|^2 + b\|e\|\|x\|$ for some scalars a and b. To enlarge the sampling intervals while ensuring stability, the control is updated when $b\|e(t)\| = a\|x(t)\|$. An extension to homogeneous systems, state-dependent homogeneous systems, and polynomial systems has been proposed in [3]. Further developments are proposed in [2] and [4], where the notion of isochronous manifolds is used to design the sampling function.

F. $\mathcal{L}_2$-stability approach [81], [82]

This approach is based on the notion of $\mathcal{L}_2$-stability and involves algebraic Riccati equations:

$$0 = PA + A^TP - PBB^TP + I + \frac{1}{\gamma^2}PEE^TP, \tag{8}$$

where γ represents the $\mathcal{L}_2$ gain. Considering the sampling error $e(t) = x(t) - x(t_k)$, and matrices M and N defined as: $M = (1-\beta^2)I + PBB^TP$, $N = \frac{1}{2}(1-\beta^2)I + PBB^TP$, it is shown that the $\mathcal{L}_2$-stability of the system is ensured with the event generator $e^T(t)Me(t) = x^T(t_k)Nx(t_k)$. Furthermore, by analyzing the evolution of the term $e^T(t)Me(t)$ for $t \geq t_k$, it is possible to compute at each sampling instant a lower-bound estimation of the next allowable sampling interval, and thus perform a self-triggered control scheme. Differently from previous Lyapunov-based approaches, this approach allows the use of non-monotonously decreasing Lyapunov functions.

G. Discrete-time and switched systems [18], [49]

This approach considers the exact system discretization at sampling instants, given by (2). When the sampling interval h_k is restricted to a finite number of values, the design of stabilizing sampling sequence h_k can be related to the problem of designing a stabilizing switching law for a switched linear system [47]. This idea has been exploited in several publications using periodic [46] or state-dependent sampling laws [18], [49]. Alternatively, one may also find in the literature some approaches based on PET Control [19]. In this case, the discrete-time implementation of several triggering conditions (based on state error, $\mathcal{L}_2$ stability, decay of Lyapunov functions, etc.) leads to Piecewise Linear (PWL) discrete-time models. This allows the use of classical LMI-based approaches [20], [39] for analysing the system's stability and performance.

H. Convex embeddings and state-dependent sampling [21], [22], [24], [25]

This approach represents the counterpart of the convex embedding method [38] (used for analysing stability under arbitrary sampling) to the case of controlled sampling. It uses the properties of the transition matrix $\Lambda(t-t_k) = e^{A(t-t_k)} + \int_0^{t-t_k} e^{As}dsBK$ over the sampling interval $[t_k, t_{k+1})$ in order to derive a sampling map defined by conic partitions of the state space. For the case of quadratic Lyapunov functions $V(x) = x^TPx$, the stability condition $\dot{V}(x(t)) \leq -\beta V(x(t)), \forall t \in [t_k, t_k + h(x_k))$ may be re-expressed in the form $x_k^T\Phi(\theta)x_k \leq 0$, for all $\theta \in [0, h(x_k))$ where $x_k = x(t_k)$ and

$$\Phi(\theta) = \begin{bmatrix} \Lambda(\theta) \\ I \end{bmatrix}^T \begin{bmatrix} A^TP + PA + \beta P & PBK \\ * & 0 \end{bmatrix} \begin{bmatrix} \Lambda(\theta) \\ I \end{bmatrix}.$$

Stabilizing sampling maps may be designed using the fact that a sampling interval $h(x_k) = \tau$ ensures the decay of the Lyapunov function $V(x)$ in the region

$$\mathcal{Z}(\tau) = \{x \in \mathbb{R}^n :\ x^T\Phi(\theta)x \leq 0, \forall \theta \in [0,\tau]\}.$$

For practical implementations, considering a polytopic approximation with N vertices $\Phi_i(\tau)$, $i = 1, \cdots, N$, such that $\Phi(\theta) \in \mathrm{conv}\{\Phi_i(\tau),\ i = 1, \cdots, N\}$ for all $\theta \in [0,\tau]$, $\mathcal{Z}(\tau)$ may be over-approximated by a finite intersection of conic regions

$$\hat{\mathcal{Z}}(\tau) = \{x \in \mathbb{R}^n :\ x^T\Phi_i x \leq 0, \forall\ i = 1, \ldots, N\} \supseteq \mathcal{Z}(\tau).$$

A stabilizing sampling map $h(x)$ may be defined as

$$h(x_k) = \max\{\tau \in \mathbb{R}^+ : x_k \in \hat{\mathcal{Z}}(\tau)\}.$$

The approach allows to provide some optimisation in the design of sampling maps by optimising the choice of Lyapunov functions using LMI based tools. It is possible, for example, to optimise the choice the the Lyapunov function $V(x) = x^TPx$ which enlarges the lower bound of the sampling map $h(x_k)$, by solving the optimization problem

$$\begin{gathered}\sup \tau \text{ such that} \\ \exists P \succ 0 \\ \Phi(\theta) = \begin{bmatrix} \Lambda(\theta) \\ I \end{bmatrix}^T \begin{bmatrix} A^T P + PA + \beta P & PBK \\ * & 0 \end{bmatrix} \begin{bmatrix} \Lambda(\theta) \\ I \end{bmatrix} \prec 0 \\ \forall \theta \in [0, \tau] \end{gathered}$$

which may also be transformed in a finite set of LMIs using polytopic approximations of $\Phi(\tau)$.

This optimization procedure may also be used to analyse the system's stability with given conic regions $\mathcal{R}_i$ of the state-space and associated bounds on the sampling intervals τ_i. This problem setting is called *state-dependent sampling*. In this context, convex embeddings have been used in [22], [24], and Lyapunov-Krasovskii functionals in [23].

VI. Conclusion

This tutorial has presented some of the basic concepts and recent research directions in sampled-data systems: time-delay, hybrid, discrete-time and input-output models for sampled-data systems; stability for systems with arbitrary sampling intervals; design of stabilizing state-dependent sampling laws. It is to be emphasized that this brief overview is far from being an exhaustive survey on the stability of sampled-data systems with time-varying sampling intervals. Such a research topic is still wide open and continuously growing. Unavoidably, not all possible results are mentioned here.

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