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In an electromagnetic cloak based on a transformation approach, reduced sets of material properties are generally favored due to their easier implementation in reality, although a seemingly inevitable drawback of undesired scattering exists in such cloaks. Here, the authors suggest the use of high-order transformations to create smooth moduli at the outer boundary of the cloak, therefore completely eliminating the detrimental scattering within the limit of geometric optics. The authors apply this scheme to a nonmagnetic cylindrical cloak and demonstrate that the scattered field is reduced substantially in a cloak with optimal quadratic transformation as compared to its linear counterpart. © *2007 American Institute of Physics*. [DOI: 10.1063/1.2783266]

Recently, increasing attention has been focused on creating an electromagnetic cloak of invisibility based on various schemes, including anomalous localized resonance,[1–3] dipolar scattering cancellation,[4,5] tunneling light transmittance,[6] sensors and active sources,[7] and coordinate transformation.[8,9] The transformation approach, which generalized a similar idea on cloaking of conductivity,[10,11] has triggered enormous interest because the proposed device is supposed to render a macroscopic object invisible, and the design is not sensitive to the object that is being masked. Further theoretical treatments of the problem were recently presented.[12–14] The proof-of-concept demonstration of such a cloak operating at microwave frequency was experimentally reported,[15] which showed that the detour of waves and the reduced scattering from the cloaked object were indeed possible.

As pointed out in our recent paper,[16] the design for the microwave cloak[15] is not directly adaptable to optical frequencies due to the saturation effect of the scaling of split ring resonators (SRRs).[17] Replacing the SRRs with other high-frequency magnetic elements such as coupled nanorods,[18] nanoplates,[19] or nanostrips[20,21] is also questionable due to the lossy nature of such plasmonic magnetic structures. To realize an optical cloak that is inherently compatible with the natural inertness of magnetic responses in the visible range, we proposed the physical principle and implementation recipe of a nonmagnetic optical cloak, as detailed in Ref. 16, where a cylindrical region $r' \leqslant b$ is compressed into a concentric cylindrical shell $a \leqslant r \leqslant b$, and an anisotropic material with a set of reduced parameters without any magnetism requirement is utilized to form the shell. The idea of using reduced parameters to achieve an approximate cloak originated in Ref. 22. The major detrimental effect of using the nonmagnetic reduced set as compared to the perfect set is the nonzero scattering due to impedance mismatch at the outer surface of the cloaking system.

In this letter, we use a high-order coordinate transformation to eliminate the undesired scattering from the cylindrical cloak while sustaining the important nonmagnetic feature for optical frequencies. Mathematically, there are countless ways to perform the spatial transformation. Up to now, in all the reported literature on this topic,[8,15,16,22] a linear transformation function is used for this purpose,

$$r = (1 - a/b)r' + a. \tag{1}$$

This linear transformation, although straightforward and intuitive, prohibits any flexible control of the associated moduli. As a result, impedance mismatching and undesired scattering are inevitable for cloaks using any form of reduced parameters, including the demonstrated microwave cloak in Ref. 15 for TE incidence and the designed optical cloak in Ref. 16 for TM polarization.

This scenario, however, can be dramatically changed when using a high-order transformation instead of the linear one in Eq. (1). We may conceive any possible transformation function $r = g(r')$ from (r', θ', z') to (r, θ, z) in order to compress the cylindrical region $r' \leqslant b$ into a concentric shell. We allow several flexible variables in the expression of $g(r')$ for further adjustments, as will be detailed later. With the given form of the transformation, we calculate the Jacobian matrix for this coordinate change, and based on the techniques described in Refs. 8 and 23, the permittivity and permeability tensors can be determined as well,

$$\begin{aligned} &\varepsilon_r = \mu_r = (r'/r)\partial g(r')/\partial r', \quad \varepsilon_\theta = \mu_\theta = 1/\varepsilon_r, \\ &\varepsilon_z = \mu_z = (r'/r)[\partial g(r')/\partial r']^{-1}. \end{aligned} \tag{2}$$

For the linear transformation, the material parameters in Eq. (2) reduce to the simple forms given in Refs. 15 and 22. Note that for a closed-form expression of the parameters in the (r, θ, z) space, all r' in Eq. (2) should be replaced by $r' = g^{-1}(r)$. From the expressions above, we can see that the impedance at the outer boundary is perfectly matched; that is, $\sqrt{\mu_\theta/\varepsilon_z}|_{r=b} = \sqrt{\mu_z/\varepsilon_\theta}|_{r=b} = 1$, which is an important feature of an ideal cloak.

From the material properties of an exact cloak, the corresponding reduced parameters can be obtained.[15,16] As stated in Ref. 16, a nonmagnetic cloak is of particular interest at optical frequencies due to the absence of optical magnetism in nature. We focus on TM incidence with the mag-

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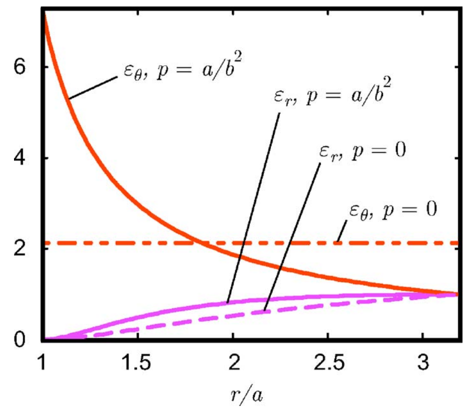

FIG. 1. (Color online) Anisotropic material parameters ε_r and ε_θ of two nonmagnetic cloaks with $p=a/b^2$ (optimal quadratic transformation, solid lines) and $p=0$ (linear transformation, dashed lines). μ_z equals unity in both cases. The shape factor (a/b) in this example is 0.31 and the diameter $(2b)$ is 4 μm.

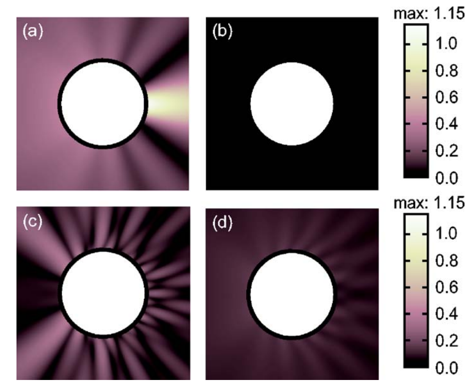

FIG. 2. (Color online) Full-wave field-mapping simulations of the magnitudes of normalized scattered field for a metal cylinder inside (a) a vacuum without any cloak, (b) an ideal linear cloak, (c) a linear nonmagnetic cloak with $p=0$, and (d) an optimal quadratic cloak with $p=a/b^2$.

netic field polarized along the z axis. In this case only μ_z, ε_r, and ε_θ enter into Maxwell's equations. To simplify the ε and μ tensors while maintaining the wave trajectory inside the cylindrical shell, in Eq. (2) we multiply ε_r and ε_θ by μ_z and obtain the following reduced set of nonmagnetic cloak parameters:

$$\varepsilon_r = (r'/r)^2, \quad \varepsilon_\theta = [\partial g(r')/\partial r']^{-2}, \quad \mu_z = 1. \tag{3}$$

From Eq. (3), we calculate the impedance at the outer boundary,

$$Z|_{r=b} = \sqrt{\mu_z/\varepsilon_\theta}|_{r=b} = \partial g(r')/\partial r'. \tag{4}$$

By setting it equal to unity, we can fix the function $g(r')$ together with all the material properties, and a nonmagnetic cloak with minimized scattering is achieved. The perfectly matched impedance also indicates that $(\partial r/\partial r')|_{r=b}=1$. This smooth modulus at the outer surface removes the discontinuity and minimizes the scattering after the high-order transformation.

Following the implementation guidance described above, we consider a quadratic transformation function with one flexible parameter p,

$$r = g(r') = [1 - a/b + p(r' - b)]r' + a. \tag{5}$$

We see that the boundary confinements $g(0)=a$ and $g(b)=b$ are fulfilled. By setting $Z|_{r=b}=1$, we fix the flexible variable $p=a/b^2$ and obtain the following optimal transformation:

$$r = g(r') = [(a/b)(r'/b - 2) + 1]r' + a. \tag{6}$$

All nonmagnetic material properties can be determined consequently using Eq. (3). To make sure that the transformation is monotonic, we require a shape factor of $a/b < 0.5$.

As an example, Fig. 1 shows the anisotropic material properties of two nonmagnetic cylindrical cloaks with $p=a/b^2$ (optimal quadratic transformation) and $p=0$ (linear transformation). The shape factor in this example is $a/b=0.31$. In the optimal quadratic case, all three material parameters ε_r, ε_θ, and μ_z are equal to unity at the outer boundary $r=b$, which perfectly matches the surrounding vacuum parameters.

To compare the performance of the nonmagnetic cloaks with different transformation methods, we conduct field-mapping simulations using the finite-element package COMSOL MULTIPHYSICS. The cloaking systems are examined at $\lambda=632.8$ nm with the same geometry as that in Fig. 1. The object hidden inside the cloaks is an ideal metallic cylinder with a radius the same as that of the inner surface. In Fig. 2, under four circumstances, we plot the magnitudes of the normalized scattered field. The scattered field from the object itself is shown in Fig. 2(a). The strong forward scattering at the right-hand side corresponds to the shadow cast behind the object. The scattered field outside an ideal cloak is illustrated in Fig. 2(b), which is essentially zero in magnitude. The results of nonmagnetic cloaks for both the linear transformation and the optimal quadratic case are illustrated in Figs. 2(c) and 2(d), respectively. The linear case exhibits an evident scattering pattern from the outer boundary of the system because of the impedance mismatch. On the other hand, the quadratic transformation function results in negligible scattering from the cloaking system. The figure of merit for cloaking (defined as the ratio of the scattering cross sections with and without the cloak) is about 10 for the considered quadratic cloak, and it increases toward infinity with the size of the cloaking system. The cloaking performance of an optimal quadratic nonmagnetic cloak, like that of the cloak of Leonhardt,[9] is exact as an ideal cloak within the realm of geometric optics.

To clearly illustrate the scattering and directivity properties of different cloaking systems, we plot in Fig. 3 the scattering radiation patterns corresponding to the four cases of Fig. 2. The curves in Fig. 3 show the energy flow in the radial direction normalized by the maximum value in the noncloaked case at a boundary outside the outer surface of

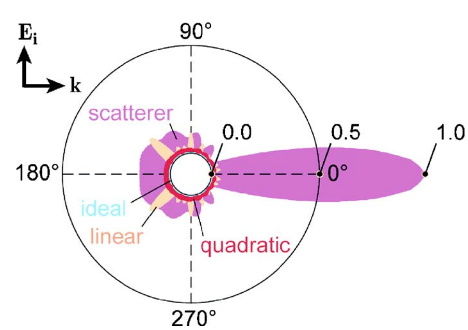

FIG. 3. (Color online) Scattering patterns from the four cases shown in Fig. 2. Purple: the metal cylinder (scatterer) with no cloak; blue: the cylinder with the ideal cloak; orange: the cylinder with a linear nonmagnetic cloak; red: the cylinder with an optimal quadratic nonmagnetic cloak.

the cloaks. In the ideal cloaking system, the scattering energy flow is zero, which is indicated by the solid inner circle in Fig. 3. The linear cloak with reduced parameters gives rise to a noticeable and strongly directional scattering pattern. In the nonmagnetic quadratic cloak, the overall scattering is much less significant. The peak value of the radial Poynting vector in the quadratic cloak is more than six times smaller than that of its linear counterpart. Moreover, the directivity in the scattering pattern is substantially suppressed, which is an important feature of a quasiideal cloak.

In conclusion, we proposed an electromagnetic cloak using high-order transformation to create smooth rather than discontinuous moduli at the outer interface. By this approach, the undesired scattering is completely eliminated within the limit of geometric optics, even for cloaks using nonmagnetic materials to simplify the implementation. We applied this scheme to the nonmagnetic cylindrical cloak and demonstrated that the scattered field is reduced by almost an order of magnitude in a cloak with optimal quadratic transformation compared to that with the usual linear compression. This nonmagnetic cloak with minimized scattering brings us one step closer to realizing an actual cloaking device at optical frequencies.

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