A 4DVar PIV-data assimilation for flow spatio-temporal reconstruction

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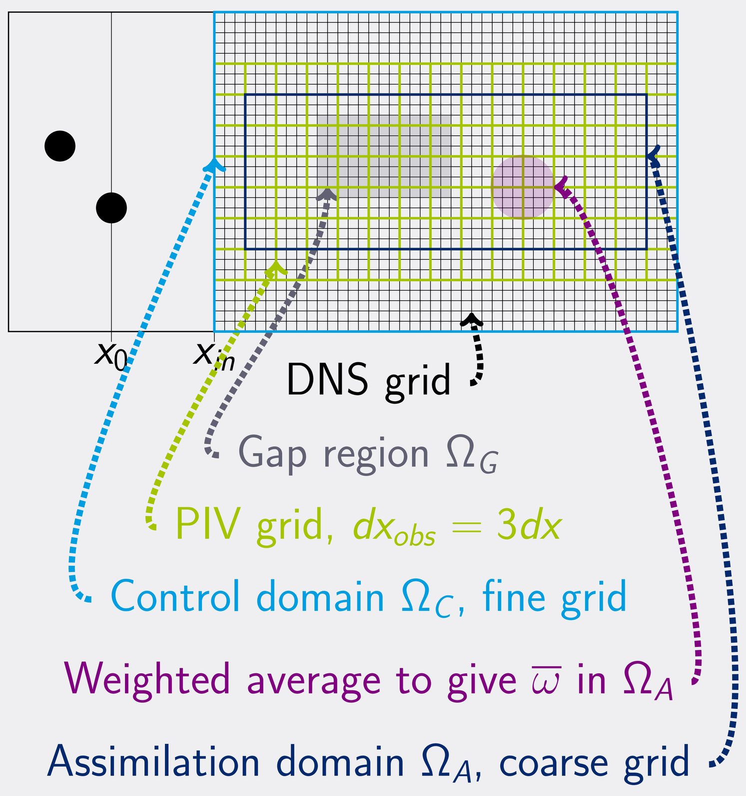
A variational data assimilation technique (4DVar) was used to reconstruct turbulent flows (Gronskis *et al.*, 2013). The problem consists in recovering a flow by modifying the initial and inflow conditions of a system composed of a DNS code coupled with some noisy and possibly incomplete PIV measurements. In the present study the ability of the technique to reconstruct flow in gappy PIV data was investigated.

Dynamical model

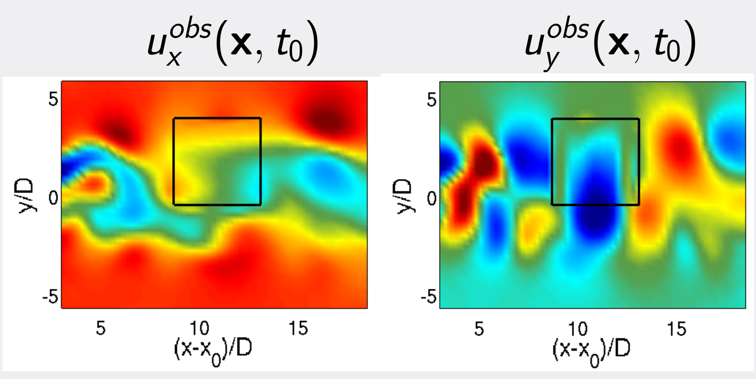
`Incompact3d` A DNS/LES code to solve incompressible flows with quasi-spectral accuracy (see Laizet & Lamballais, JCP 2009).

Configuration

Data: PIV cylinder wakes at Re=112
Test: artificial gap without velocity



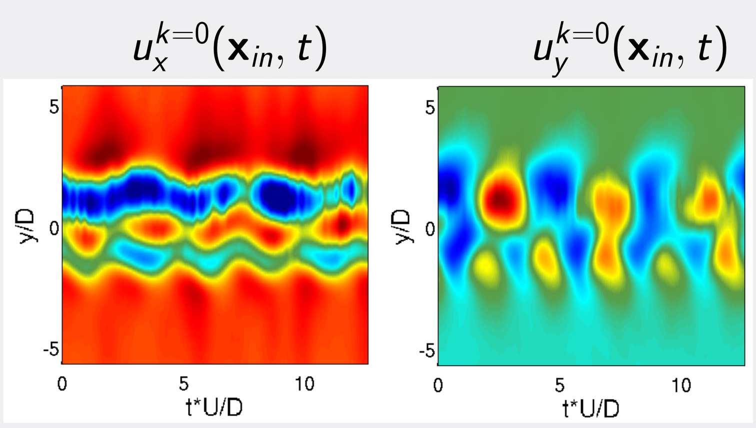
Initial condition



Initial condition in Ω_G (IC)

1. Uniform stagnant flow
2. Velocity interpolation

Inflow condition



- From PIV sequence with Taylor's hypothesis

Variational Data Assimilation

Problem: Recover a system's state X obeying a dynamical model $\mathbb{M}$, given observations at discrete times separated by Δt_{obs}. This is formalized by the state dynamics

$$\partial_t X(\mathbf{x}, t) + \mathbb{M}(X(\mathbf{x}, t), \eta(t)) = 0 \quad \text{and} \quad X(\mathbf{x}, t_0) = X_0(\mathbf{x}) + \epsilon(\mathbf{x}),$$

where $X_0(\mathbf{x})$ is the initial state, $\eta(t)$ and $\epsilon(\mathbf{x})$ are control variables.

Optimization problem

Minimization problem: dependence of the system's state variable X on the control variable γ,

$$X \equiv \mathbf{u}(\gamma), \quad \gamma = \{\epsilon(\mathbf{x}), \eta(t)\} \equiv \{\overline{\mathbf{u}}(\mathbf{x}, t_0) - \mathbf{u}_0(\mathbf{x}), \overline{\mathbf{u}}(\mathbf{x}_{in}, t) - \mathbf{u}_{in}(t)\}.$$

Cost functional: deviations of the DNS values from the PIV observations weighted by the accuracy of the observations, plus deviations of the control variable fields and their background fields weighted by the accuracy of the backgrounds.

$$J(\gamma) = \int_{t_1}^{t_N}\int_{\Omega_A} \| \overline{\omega}(\mathbf{x}, t) - \omega^{obs}(\mathbf{x}, t)\|_R^2 \, d\mathbf{x}\, \Delta_{t^*} dt + \int_{\Omega_{C_{obs}}} \|\overline{\mathbf{u}}(\mathbf{x}, t_0) - \mathbf{u}_0(\mathbf{x})\|_{Q_{obs}}^2 \, d\mathbf{x} + \int_{t_0}^{t_N}\int_{\Omega_{C_{in}}} \|\overline{\mathbf{u}}(\mathbf{x}_{in}, t) - \mathbf{u}_{in}(t)\|_{Q_{in}}^2 \, d\mathbf{x}\, dt$$

$dx_{obs} > dx$
DNS spatial average
initial condition
inflow condition

Gradient Evaluation on Discretized Model

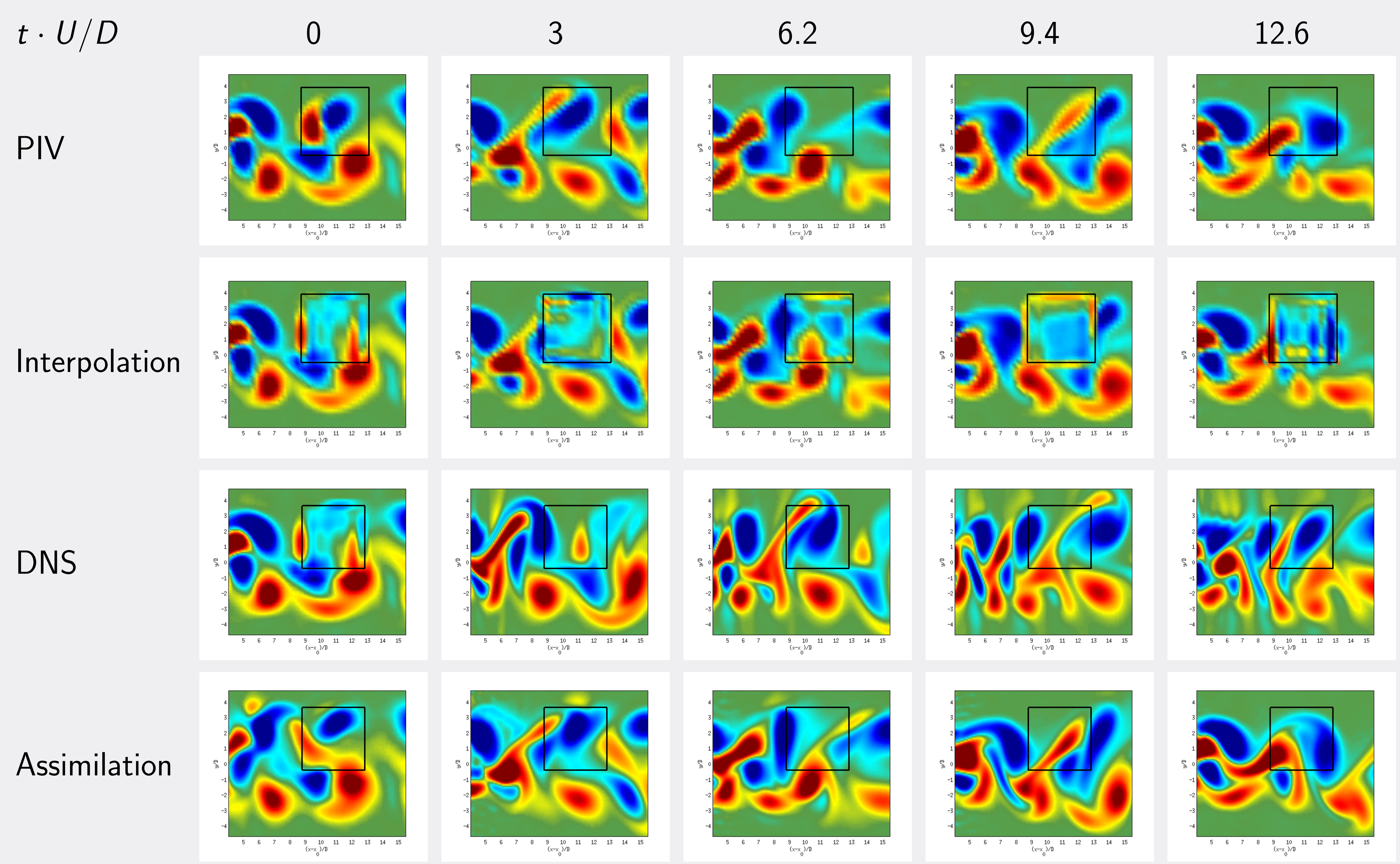
Adjoint formulation: $\nabla J(\gamma)$ obtained by

1. Forward integration of dynamical system
2. Backward integration of adjoint dynamical model

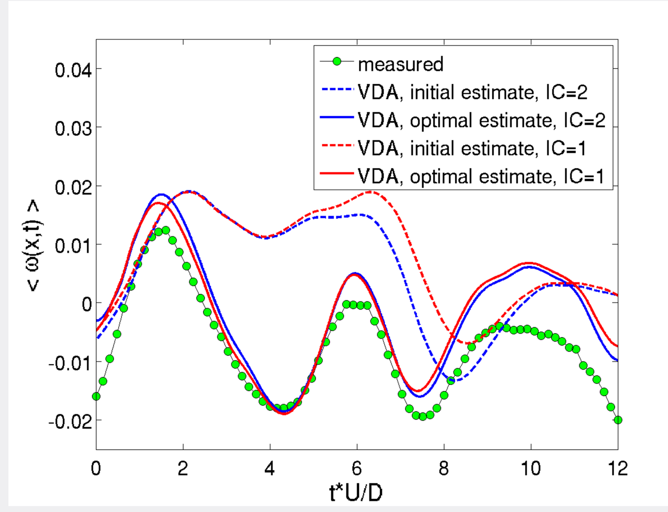
$$-\partial_t \lambda(\mathbf{x}, t) + (\partial_X \mathbb{M})^* \lambda(\mathbf{x}, t) = (\partial_X \mathbb{H})^* R^{-1}(\mathcal{Y} - \mathbb{H}(X(\mathbf{x}, t)))$$
$$\lambda(\mathbf{x}, t_f) = 0$$

Build Adjoint Code through Automatic Differentiation (AD)

Results: Vorticity fields in the assimilation window

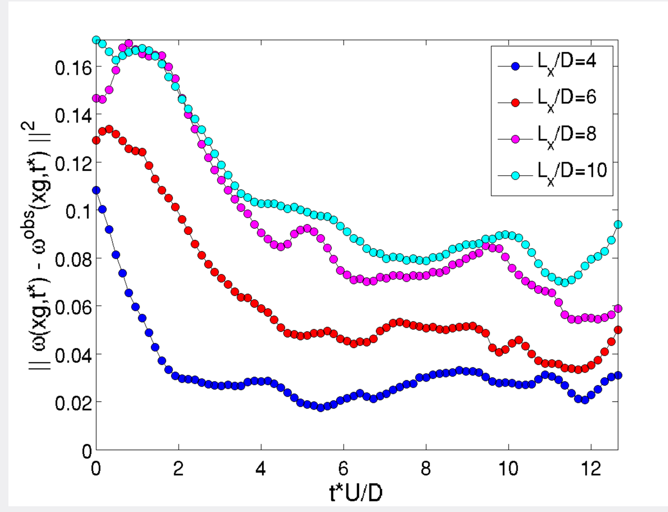


Influence of Initial Cond.



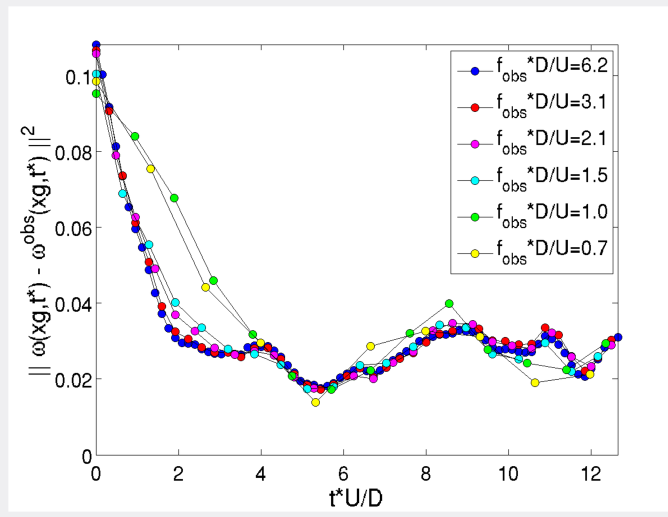
L_x/D=4, $f_{obs}D/U$=6.2
Method's accuracy did not depend on the initial condition in the gap.

Influence of gap size



IC=1, $f_{obs}D/U$=6.2
Method's accuracy was strongly related to the size of the gap.

Influence of obs. frequency



L_x/D=4, IC=1
Error decreased with increasing observations frequency $St_{obs} = f_{obs}D/U$.

Conclusions

- A new method to reconstruct turbulent flows in gappy PIV data was introduced.
- Method's accuracy did not depend on the initial condition in the gap.
- Method's accuracy was strongly related to the size of the gap.
- Error decreased with increasing observations frequency St_{obs}. From St_{obs}=1.5 an asymptotic state was reached, above St_{obs}=3 additional observations had less effects.

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Gronskis et al. *J. Comput. Phys.* 242 (2013) 480-497
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