Sensitivity of the electrocardiography inverse solution to the torso conductivity uncertainties

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A simulation study

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Abstract

Electrocardiography imaging (ECGI) is a new non invasive technology used for heart diagnosis. It allows to construct the electrical potential on the heart surface only from measurement on the body surface and some geometrical informations of the torso. The purpose of this work is twofold: First, we propose a new formulation to calculate the distribution of the electric potential on the heart, from measurements on the torso surface. Second, we study the influence of the errors and uncertainties on the conductivity parameters, on the ECGI solution. We use an optimal control formulation for the mathematical formulation of the problem with a stochastic diffusion equation as a constraint. The descretization is done using stochastic Galerkin method allowing to separate random and deterministic variables. The optimal control problem is solved using a conjugate gradient method where the gradient of the cost function is computed with an adjoint technique . The efficiency of this approach to solve the inverse problem and the usability to quantify the effect of conductivity uncertainties in the torso are demonstrated through a number of numerical simulations on a 2D geometrical model.

Main Objectives

1. Propose a new method for solving the ECGI problem.
2. Introduce the uncertainty of the conductivity in the ECGI problem
3. Evaluate the effect of uncertainties on the forward and inverse solutions.

Methods

Stochstic forward problem of electrocardiography

We denote by D the space domain and Ω the probability space.

$$\begin{cases} \nabla.(\sigma(x,\xi)\nabla u(x,\xi)) = 0 \text{ in } D\times\Omega, \\ u(x,\xi) = u_0 \text{ on } \Gamma_{\text{int}}\times\Omega, \\ \sigma(x,\xi)\frac{\partial u(x,\xi)}{\partial n} = 0 \text{ on } \Gamma_{\text{ext}}\times\Omega, \end{cases} \tag{1}$$

where, Γ_{int} and Γ_{ext} are the epicardial and torso boundaries respectively, $\xi\in\Omega$ is the stochastic variable (it could also be a vector) and u_0 is the potential at the epicardial boundary.

Numerical descretization of the stochastic forward problem

We use the stochastic Galerkin method to solve equation (1). The stochastic conductivity and solution are projected on the probability density functions $\{\Psi_k(\xi)\}_{k=1}^{p}$

$$\sigma(x,\xi) = \sum_{i=0}^{p}\hat{\sigma}_i(x)\Psi_i(\xi, \qquad u(x,\xi) = \sum_{j=0}^{p}\hat{u}_j(x)\Psi_i(\xi)$$

The elliptic equation (1) projected in the stochastic basis could be solved

$$\begin{aligned} \sum_{i=0}^{p}\sum_{j=0}^{p}T_{ijk}\nabla.(\hat{\sigma}_i(x)\nabla)\hat{u}_j(x)) &= 0 \text{ in } D, \\ \hat{u}_0(x) &= u_0(x) \text{ on } \Gamma_{\text{int}}, \\ \hat{u}_j(x) &= 0 \text{ on } \Gamma_{\text{int}} \quad \forall j = 1,...p, \\ \hat{\sigma}_i(x)\frac{\partial\hat{u}_j(x)}{\partial n} &= 0 \text{ on } \Gamma_{\text{ext}}\,\forall\, i,j = 0,...p, \end{aligned} \tag{2}$$

where $T_{ijk} = E[\Psi_i(\xi),\Psi_j(\xi),\Psi_k(\xi)]$.

Anatomical data and computational mesh

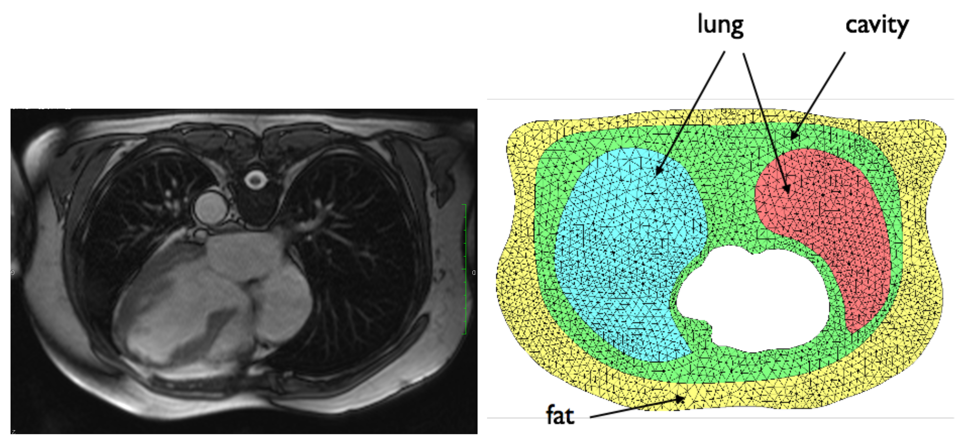

Figure 1: MRI 2D slice of the torso (left), 2D computational mesh of the torso geometry showing the different regions of the torso considered in this study: fat, lungs and torso cavity, (right).

Forward problem results

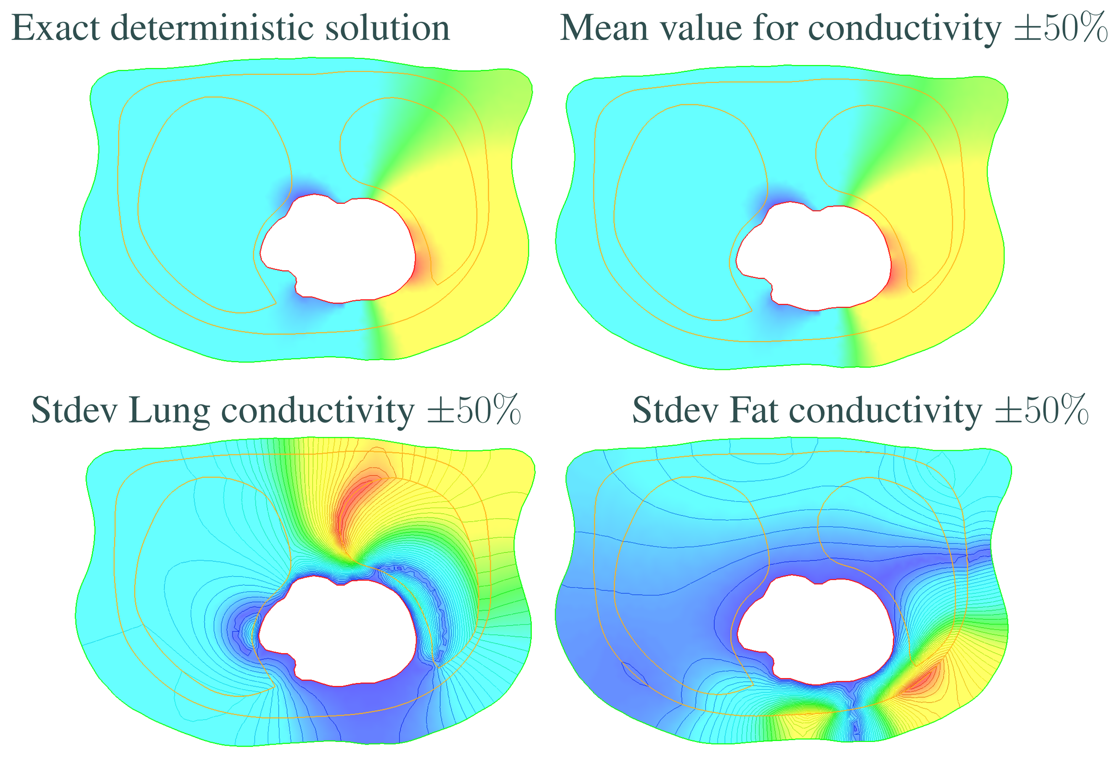

Figure 2: Stochastic solution of the forward problem: Exact solution (top, left), Mean value of the Stochastic solution for conductivity $\pm50\%$ (top, right). Standard deviation of the electrical potential for lung conductivity $\pm50\%$ (bottom, left) and fat conductivity $\pm50\%$ (bottom, right).

Main Remarks

1. *The mean value of the stochastic solution matches with the exact forward solution. This comes from the linearity of the forward problem.*
2. *For each organ, the uncertainty on the conductivity is reflected by a high uncertainty of the solution at its boundary*
3. *The direction of the standard deviation iso-values are are modified when they cross the the organ for which we introduce the uncertainty.*
4. *The magnitude of the uncertainty does not exceed $\pm2\%$ of the magnitude of the forward solution*

Stochastic ECGI Inverse Problem

Mathematical formulatin

We look for the current density and the value of the potential on the epicardial boundary $(\eta,\tau)\in L^{-\frac{1}{2}}(\Gamma_{\text{int}})\times L^{\frac{1}{2}}(\Gamma_{\text{int}})$ by minimizing the following cost function under a stochastic constraint on v

$$\begin{cases} J(\eta,\tau) = \frac{1}{2}E\Big(\|v(x,\xi)-f\|^2_{L^2(\Gamma_{\text{ext}})} + \frac{1}{2}\Big\|\sigma(x,\xi)\frac{\partial v(x,\xi)}{\partial n} - \eta\Big\|^2_{L^2(\Gamma_{\text{int}})}\Big) \\ \text{with } v(x,\xi) \text{ solution of :} \\ \nabla.(\sigma(x,\xi)\nabla v(x,\xi)) = 0 \;\; in\; D\times\Omega, \\ v(x,\xi) = \tau \;\; on\; \Gamma_{\text{int}}\times\Omega, \\ \sigma(x,\xi)\frac{\partial v(x,\xi)}{\partial n} = 0 \;\; on\; \Gamma_{\text{ext}}\times\Omega. \end{cases} \tag{3}$$

In order to solve this minimization problem, we use a conjugate gradient method as used in [1] where the components of the gradient of the cost function are computed using an adjoint method. The gradient of the functional J is given by:

$$\begin{cases} <\frac{\partial J(\eta,\tau)}{\partial\eta}.\phi> = -E[\int_{\Gamma_{\text{int}}}(\sigma\frac{\partial v}{\partial n}-\eta)\phi d\Gamma_{\text{int}}] \;\; \forall\,\phi\in L^2(\Gamma_{\text{int}}), \\ <\frac{\partial J(\eta,\tau)}{\partial\tau}.h> = E[\int_{\Gamma_{\text{int}}}\sigma\frac{\partial\lambda}{\partial n}hd\Gamma_{\text{int}}] \;\; \forall\,h\in L^2(\Gamma_{\text{int}}), \\ \text{with } \lambda \text{ solution of :} \\ \nabla.(\sigma(x,\xi)\nabla\lambda(x,\xi)) = 0 \;\; \text{on } D\times\Omega, \\ \lambda(x,\xi) = \sigma(x,\xi)\frac{\partial v(x,\xi)}{\partial n} - \eta \;\; \text{on } \Gamma_{\text{int}}\times\Omega, \\ \sigma(x,\xi)\frac{\partial\lambda(x,\xi)}{\partial n} = -(v-f) \;\; \text{on } \Gamma_{\text{ext}}\times\Omega. \end{cases} \tag{4}$$

We use the conjugate gradient method to minimize the energy function J.

Inverse problem results

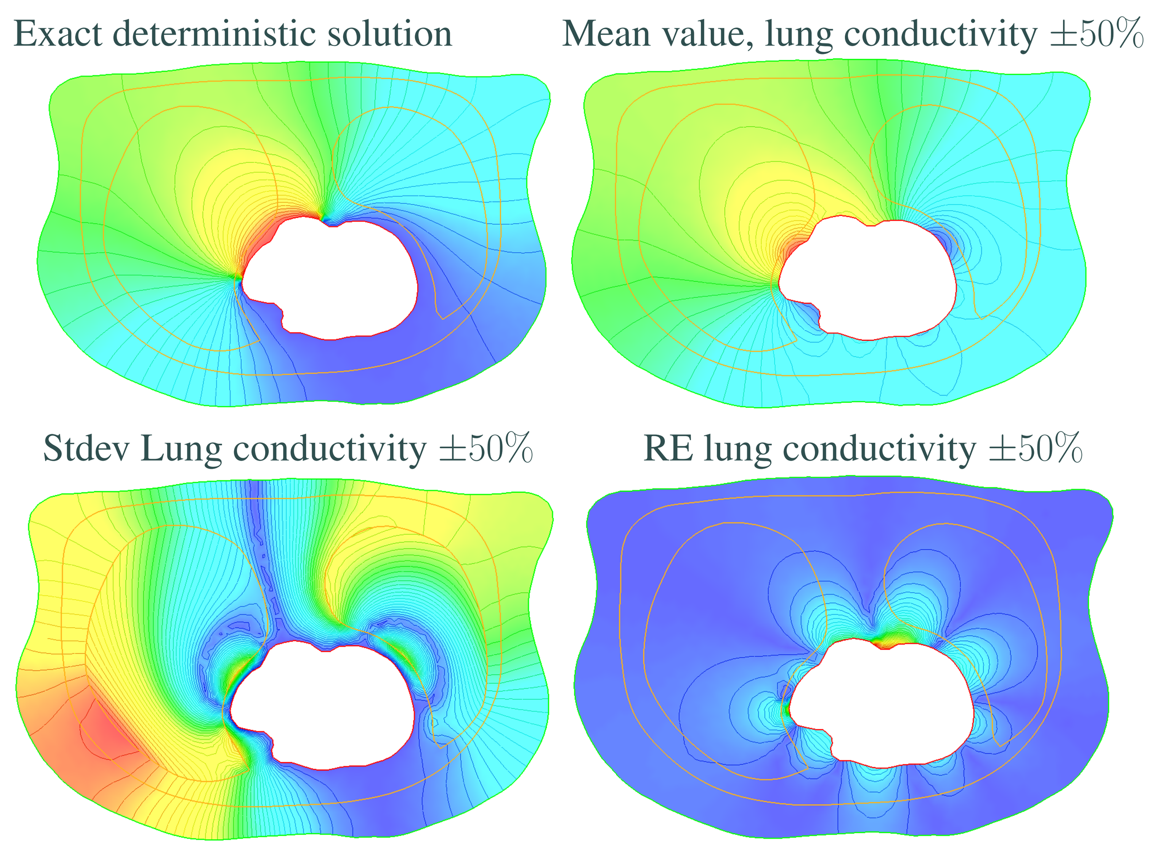

Figure 3: Stochastic solution of the inverse problem: Exact solution (top, left), Mean value of the Stochastic inverse solution for lung conductivity $\pm50\%$ (top, right). Standard deviation of the electrical potential for lung conductivity $\pm50\%$ (bottom, left) and relative error (RE) between the mean value and the exact solution (bottom, right).

Organ	% uncertainties	0%	±10%	±20%	±30%	±50%
Lungs	relative error	0.1245	0.1439	0.2208	0.3333	0.485
	Corr coeff	0.9930	0.9899	0.9767	0.9660	0.885
Fat	relative error	0.1245	0.1248	0.1248	0.1251	0.127
	Corr coeff	0.9930	0.9945	0.9943	0.9980	0.991

Table 1: Relative error and correlation coefficient of the stochastic inverse solution for different levels of uncertainty on the fat and lungs conductivities

Main Remarks

1. *The relative error between the mean value of the stochastic solution and exact forward solution reaches 50%.*
2. *Like for the forward problem, the direction of the standard deviation iso-values are modified when they cross the the organ for which we introduce the uncertainty.*
3. *The magnitude of the uncertainty reaches its maximum at the edge of the considered orrgan*

Conclusions

- The main contribution of this work was to introduce a new method for solving the ECGI inverse problem. This method is based on stochastic Galerkin approche. And the optimal control problem that we proposed allowed us to incorporate the uncertainties on the conductivity values as a constraint. The conjugate gradient method allow to take into account the conductivity uncertainties during the optimization procedure.
- The results show a low effect of conductivity uncertainties on the forward problem. On the contrary, their effect on the inverse solution is very important.
- For both inverse and forward solution the standard deviation of the stochastic solution achieves its maximum at the boundary of the organ for which the uncertainty was considered.

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Acknowledgements

We would like to thank the LIRIMA Laboratory which financially supported the teams ANO and EPICARD to perform this work. This work was partially supported by an ANR grant part of "Investissements d'Avenir" program with reference ANR-10-IAHU-04