New Mathematical approaches in Electrocardiography Imaging inverse problem

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Context and objectives

Major objectives

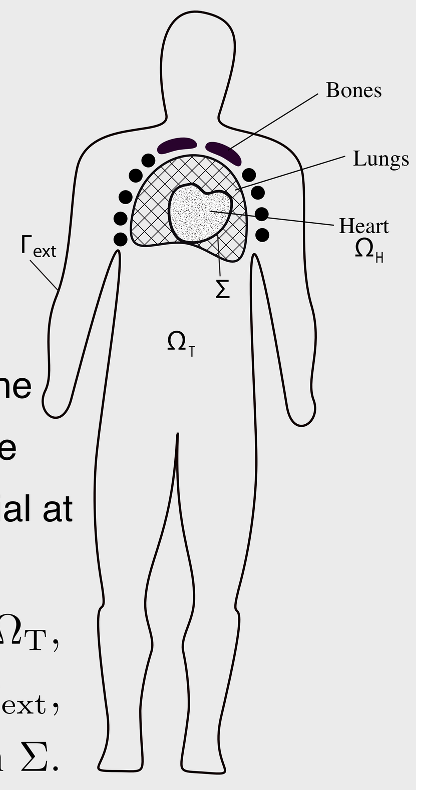
- ***Improve*** ECGI inverse problem reconstruction
- Introduce new mathematical approches to the field of the ECGI inverse problem
- Compare the performance of the new mathematical approaches to the state-of-the-art methods, mainly the MFS method used in commercial devices.
- In silico validation of the new approches.
- Assessment of some simplification hypothesis: Torso inhomogeneity
- Propose some uncertainty quantification aponches to deal with measurements errors

Mathematical model

Forward model

- If we know the heart potential we can compute the electrical potential

$$\begin{cases} \operatorname{div}(\boldsymbol{\sigma}_{\mathrm{T}}\nabla u_{\mathrm{T}}) = 0, \text{ in } \Omega_{\mathrm{T}}, \\ \boldsymbol{\sigma}_{\mathrm{T}}\nabla u_{\mathrm{T}}.\boldsymbol{n} = 0, \text{ on } \Gamma_{\mathrm{ext}}, \\ u_{\mathrm{T}} = u_{\mathrm{e}}, \text{ on } \Sigma. \end{cases}$$



Inverse problem

- If we know the electrical potential and the current density at the outer boundary of the torso and we look for the electrical potential at the heart surface

$$\begin{cases} \operatorname{div}(\boldsymbol{\sigma}_{\mathrm{T}}\nabla u_{\mathrm{T}}) = 0, \text{ in } \Omega_{\mathrm{T}}, \\ \boldsymbol{\sigma}_{\mathrm{T}}\nabla u_{\mathrm{T}}.\boldsymbol{n} = 0, \text{ and } u_{\mathrm{T}} = T, \text{ on } \Gamma_{\mathrm{ext}}, \\ u_{\mathrm{T}} = ?, \text{ on } \Sigma. \end{cases}$$

MFS approach

- Solve the linear system

$$\hat{A}\vec{a} = \vec{b}$$

$$\hat{A} = \begin{pmatrix} 1 & f(\|x_1 - y_1\|) & \cdots & f(\|x_1 - y_M\|) \\ \vdots & \vdots & \cdots & \vdots \\ 1 & f(\|x_N - y_1\|) & \vdots & f(\|x_N - y_M\|) \\ 0 & \frac{\partial f(\|x_1 - y_1\|)}{\partial n} & \cdots & \frac{\partial f(\|x_1 - y_M\|)}{\partial n} \\ \vdots & \vdots & \cdots & \vdots \\ 0 & \frac{\partial f(\|x_N - y_1\|)}{\partial n} & \cdots & \frac{\partial f(\|x_N - y_M\|)}{\partial n} \end{pmatrix} \quad \vec{a} = \begin{pmatrix} a_0 \\ a_1 \\ \vdots \\ a_M \end{pmatrix} \quad \vec{b} = \begin{pmatrix} u_T(x_1) \\ \vdots \\ u_T(x_N) \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

$$f(r) = \frac{1}{4\pi r}$$

$x_1, \ldots, x_N$: Torso points

$y_1, \ldots, y_M$: Heart points

- Regularization with CRESO

Optimal control approach

- Poincaré–Steklov variational formulation of the inverse problem.
- Minimize the following energy functional

$$J(\lambda) = \frac{1}{2}\int_{\Omega_{\mathrm{T}}} (\nabla u_{\mathrm{D}}(\lambda) - \nabla u_{\mathrm{N}}(\lambda))^2.$$

Subject to

$$\begin{cases} \operatorname{div}(\boldsymbol{\sigma}_{\mathrm{T}}\nabla u_{\mathrm{D}}(\lambda)) = 0, \text{ in } \Omega_{\mathrm{T}}, \\ u_{\mathrm{D}}(\lambda) = T, \text{ on } \Gamma_{\mathrm{ext}}, \\ u_{\mathrm{D}}(\lambda) = \lambda, \text{ on } \Sigma. \end{cases} \qquad \begin{cases} \operatorname{div}(\boldsymbol{\sigma}_{\mathrm{T}}\nabla u_{\mathrm{N}}(\lambda)) = 0, \text{ in } \Omega_{\mathrm{T}}, \\ \boldsymbol{\sigma}_{\mathrm{T}}\nabla u_{\mathrm{N}}(\lambda).\boldsymbol{n} = 0, \text{ on } \Gamma_{\mathrm{ext}}, \\ u_{\mathrm{N}}(\lambda) = \lambda, \text{ on } \Sigma. \end{cases}$$

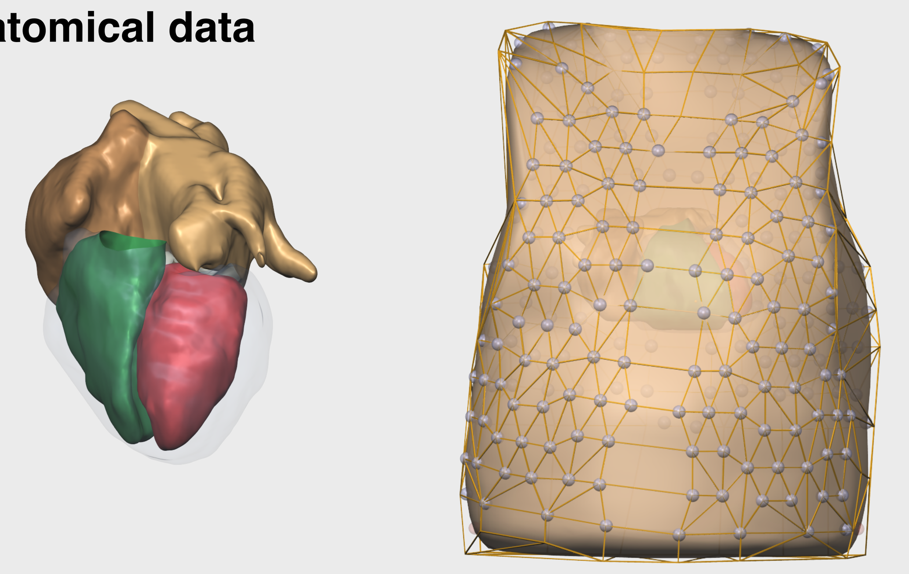
- Descent gradient methods

$$\nabla_\lambda J(\lambda) = \boldsymbol{\sigma}_{\mathrm{T}}(\nabla u_{\mathrm{D}}(\lambda) - \nabla u_{\mathrm{N}}(\lambda)).\boldsymbol{n}_{/\Sigma}$$

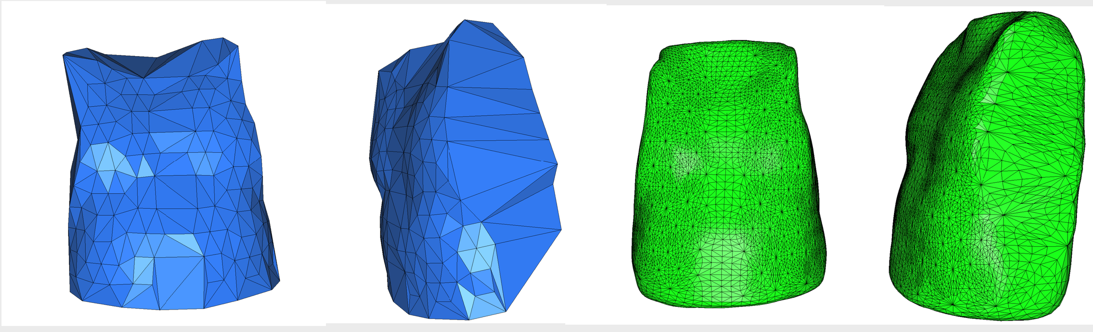
- Discretization with Finite elements method.

In silico gold standard

Anatomical data



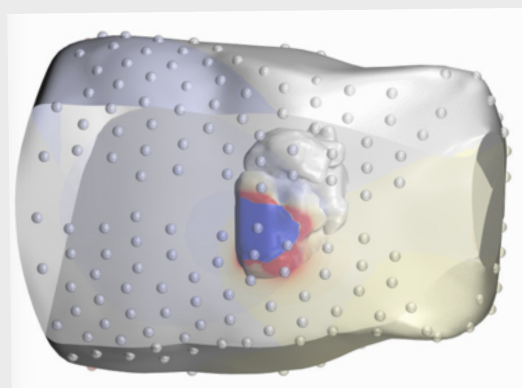
Computational heart and torso anatomical models + electrodes position

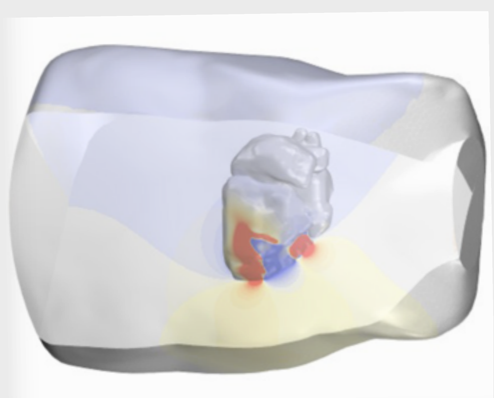


Computational torso meshes: 250 nodes mesh (blue). More accurate FE mesh with 6400 nodes (green)

Simulated cases

- 6 single and double stimuli
- 14 reentry cases



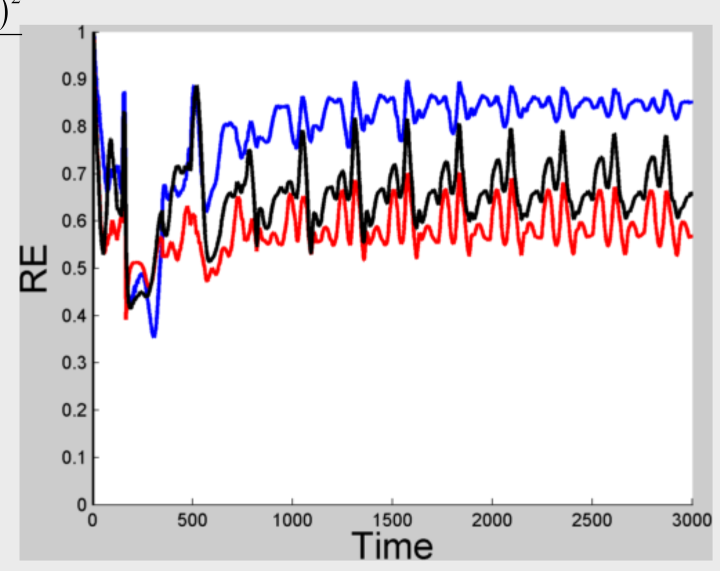


Results

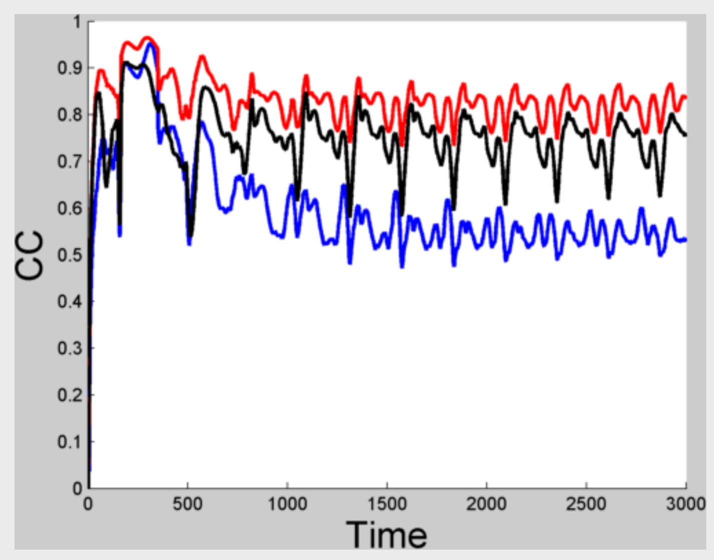
Relative error and correlation coefficient

Cases	metric	MFS + CRESO	O.C interpolated	O.C refined data
Single and double stimulus (6 cases)	RE	0.81±0.04	0.71±0.02	0.59±0.06
	CC	0.57±0.07	0.7±0.03	0.8±0.04
Re-entry (VT) (14 cases)	RE	0.78±0.06	0.67±0.04	0.59±0.05
	CC	0.6±0.08	0.73±0.04	0.83±0.04
All 20 cases	RE	0.79±0.06	0.69±0.04	0.59±0.05
	CC	0.59±0.07	0.72±0.04	0.82±0.04

- $RE = \sqrt{\frac{\sum_{i=1}^{L}(V_i^C - V_i^M)^2}{\sum_{i=1}^{L}(V_i^M)^2}}$
- MFS + CRESO (blue line)
- Optimal Control + Tikhonov data from vest interpolated on Finer mesh (black line)
- Optimal Control + Tikhonov more rich data from Finer mesh (red line)

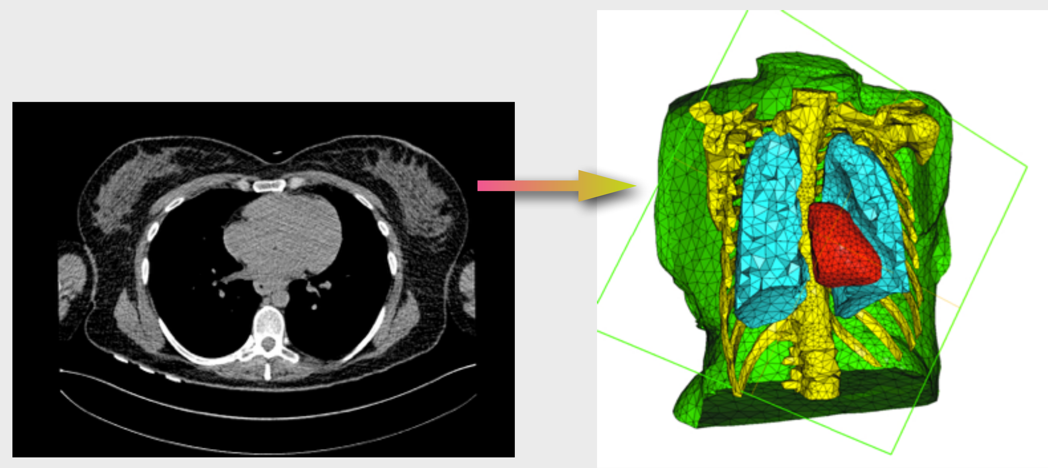


- $CC = \frac{\sum_{i=1}^{L}(V_i^M - \bar{V}^M)(V_i^C - \bar{V}^C)}{\sqrt{\sum_{i=1}^{L}(V_i^M - \bar{V}^M)^2}\sqrt{\sum_{i=1}^{L}(V_i^C - \bar{V}^C)^2}}$

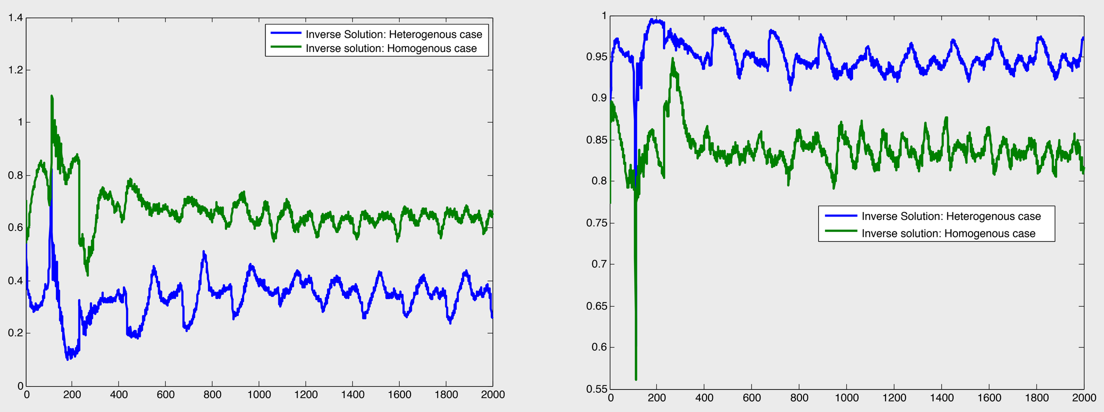


Torso Heterogeneity effect

Anatomical data

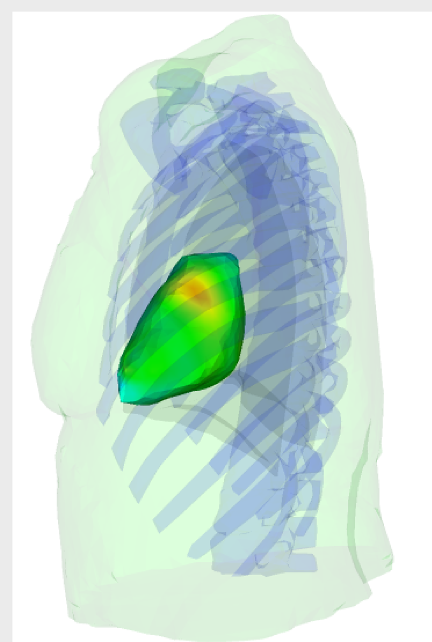


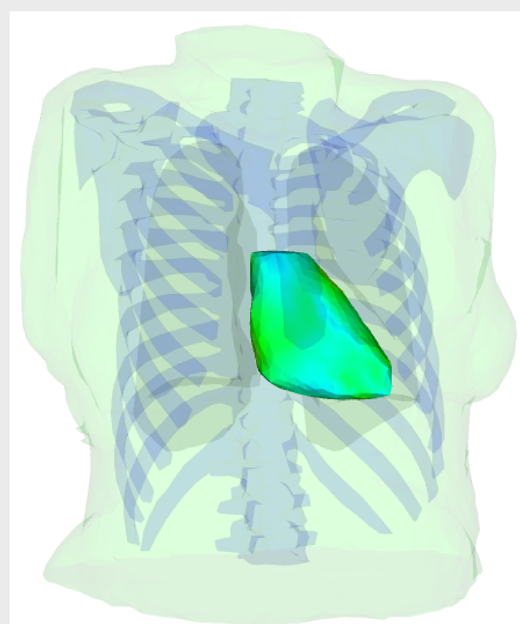
CT scan of a 43 years old women (left) and computational mesh obtained after segmentation (right)



Comparison of the optimal control solution for heterogeneous (bleu) and homogeneous (green) torso conductivities

Space distribution of the error





Space distribution of the RE over time: Left (left) and right (right) ventricles views

Remarks

- Introducing the torso heterogeneity is natural with FEM. also anisotropy could be introduced
- The error is more important in the left ventricle

Conclusions

Main results and perspectives

- New mathematical approches for solving the inverse problem in electrocardiography imaging based on optimal control
- Over all the 20 cases used in this study the optimal control method performs better than the MFS both in terms of relative error and correlation coefficient:
  - RE was improved from 0.79±0.06 to 0.59±0.05
  - CC was improved from 0.59±0.07 to 0.82±0.04
- Our results show that the heterogeneity in the torso has an impact on the accuracy of the solution both in terms of RE and CC.
- We are working on other new approches for solving ECGI problem and also quantifying the effect of the torso conductivity uncertainties on the ECGI solution

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