Stochastic Reynolds theorem and generalized subgrid tensor

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Abstract We propose a representation that allows decomposing the flow velocity in terms of a smooth component and a highly oscillating random component. This decomposion leads through a stochastic representation of the Reynolds transport theorem to a large-scale expression of the Navier-Stokes equations. In this work we show the benefit of such a representation to construct low order dynamical systems that include naturally a dissipative term related to the action of the small-scale random component.

STOCHASTIC FLOW REPRESENTATION UNDER UNCERTAINTY

In this work we explore the potential of a stochastic representation of the Navier-Stokes equations [2]. Such a representation is built from the decomposition of the flow in terms of a smooth differentiable component and an uncertainty random component decorrelated in time and correlated in space. This decomposition consists in separating or filtering a rough velocity in terms of a smooth slow time scale component and a fast oscillating one. It is reminiscent to the Reynolds decomposition. However, the smooth large-scale dynamics is in that case built from a stochastic version of the Reynolds transport theorem in which emerges naturally an anisotropic subgrid tensor. The corresponding subgrid model, which depends on the uncertainty variance tensor, generalizes the Boussinesq eddy viscosity assumption. This principle ensues here rigorously from the incorporation of Brownian uncertainties and the application of a stochastic version of the Reynolds transport theorem[2]. The resulting Navier-Stokes representation is Gallilean invariant and keeps the scale invariance properties of the Euler equation when neglecting viscosity.

This representation under uncertainty paves the way to new approaches for large scale modeling. In this work, it has been applied to the constitution of a reduced order system based on proper othogonal decomposition and a Galerkin projection [1]. Compared to a usual reduced system, the solution proposed incorporates an additional dissipative term related to the truncated discard modes. These small-scale modes are modeled as random variables spanned by the neglected modes whereas the large scale modes are expressed on the reduced basis. Two different strategies have been proposed to estimate the small-scale variance tensor – and thus the subgrid tensor – from the residues. The first strategy is based on stationnary assumption, whereas the second one expresses at each point, the path of the subgrid tensor over the reduced temporal basis.

NUMERICAL RESULTS

The performances of the reduced system have been assessed on numerical data of a simulation at Reynolds 3.900 of a wake behind a cylinder. The sequence of data corresponds to the observation of three vortex sheddings. In the first figure we show a comparizon of the error evolution for a deterministic POD-Galerkin system, a POD Galerkin system with a modal eddy viscosity [3] and our model for a stationnary and a time varying variance tensor. The system proposed shows an improved accuracy and a better stability when compared to a modal eddies viscosity technique, which seems to be sensitive to the number of modes selected. Opposite to this latter method, no tuning of parameter are needed for the proposed system. This representation offers also a new inspection tool through the value of the variance incertainty tensor. The second figure plots the maps of the variance tensor magnitude (left) and its anisotropy (right). The streamlines represent the diffusion directions associated to the biggest (left) and smallest eigenvalues (right) of the one-point covariance-variance matrix respectively.

The approach proposed provides a physically relevant stochastic derivation of the large scale flow dynamics. Amoung other advantages, it enables the constitution of reduced models that outperform POD Galerkin reduced order systems with empirical eddy viscosity models.

References

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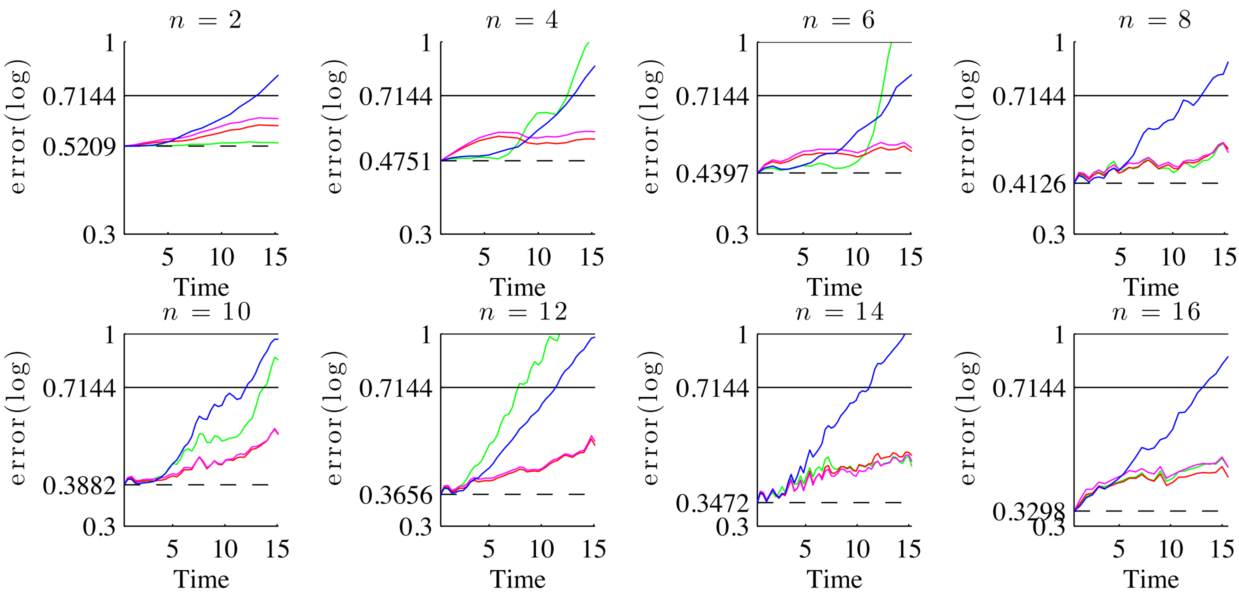

Figure 1. Normalized error for $n = 2$, 4, 6, 8, 10, 12, 14 and 16 modes. The error is normalized by the energy of the solution. The blue line is the POD-ROM deterministic model. The red one is our model with a stationnary variance tensor. The magenta one is our model with a variance tensor expressed as a linear function of the POD temporal modes. The green one is a constant modal Eddy Viscosity model. The modal Eddy Viscosity are estimated on the first quarter of the sequence. The doted line is the error due to the choice of the number of modes. The black solid line is the error considering only the temporal mean velocity.

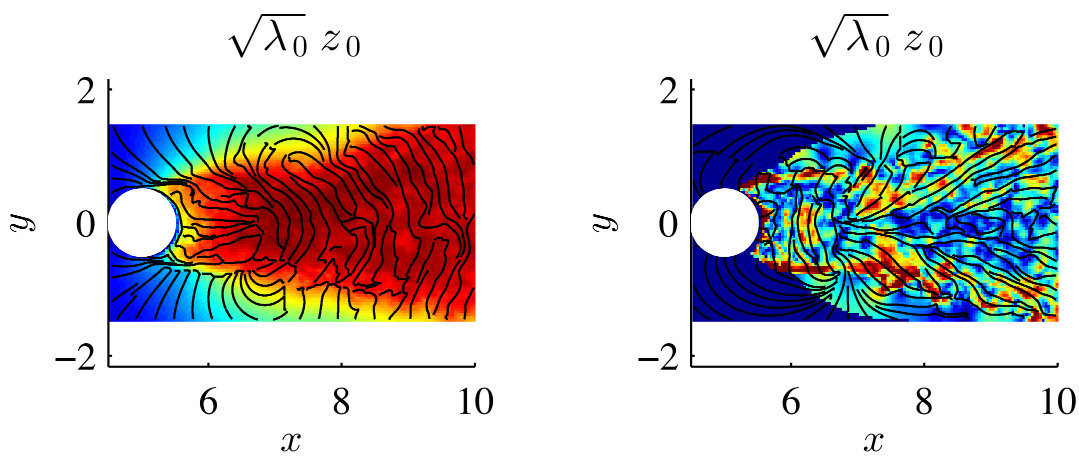

Figure 2. Local spectral representations of the uncertainty variance tensor, a, in the horizontal section at $z = 0$, for n = 2 modes. The colour represents, on the left, the magnitude and, on the right, the anisotropy of the variance. The streamlines represent, on the left, the eigenvectors corresponding to the biggest eigenvalues and, on the right, the eigenvectors corresponding to the smallest eigenvalues.