On the robustness of cable configurations of suspended cable-driven parallel robots

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On the robustness of cable configurations of suspended cable-driven parallel robots

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Abstract—*Cable-driven parallel robot (CDPR) are parallel robots that use coilable cables as legs. We are interested here in* **suspended CDPR** *for which there is no cable that exert a downward force on the platform. If we assume that the cables are mass-less and not elastic it has been shown that at a given pose whatever is the number $m > 6$ of cables there will always be at most 6 cables under tension simultaneously. A* **cable configuration** *(CC) at a given pose is the set of cables number that are under tension and usually there are several possible CC for the same pose. These CC are not equivalent in terms of cable tensions, sensitivity to measurement errors and therefore it make sense from a control viewpoint to enforce the "best" CC to obtain the optimal robot configuration, which can be done by controlling the length of the cables that are not members of the CC so that we are sure that they are slack. Hence we are interested in ranking the different CC in term of robustness. We propose several ranking indices for a CC and algorithms to calculate these indices at a pose, on a trajectory or when the robot moves on a surface and we show examples for a CDPR with 8 cables.*

Keywords: cable-driven parallel robots,cable configurations, kinematics,statics

I. Introduction

Cable-driven parallel robot (CDPR) are robots whose platform are connected to the ground by a set of cables that can be uncoiled or coiled. The study of CDPR has started about 30 years ago with the pioneering work of Albus [2] and Landsberger [16] but there has been recently a renewed interest in such a robot, both from a theoretical and application viewpoint. For example kinematics analysis of CDPR is much more complex than the one of parallel robot with rigid legs as static equilibrium has to be taken into account [5], [14], [32] and is still an open issue especially as not all cables of a robot with m cables may be under tension [1], [3], [8], [6], [25] and that only stable solutions have to be determined [7]. Numerous applications of CDPRs have been mentioned e.g. large scale maintenance studied in the European project Cablebot [26], rescue robot [31], [22] and transfer robot for elderly people [19] to name a few.

The output point of the coiling mechanism for cable i will be denoted A_i while this cable is attached at point B_i on the platform. We define an absolute frame $(O, \mathbf{x}, \mathbf{y}, \mathbf{z})$ and we assume that the coordinates of A_i in this frame are known. In the same manner we define a mobile frame $(C, \mathbf{x_r}, \mathbf{y_r}, \mathbf{z_r})$ that is attached to the platform (figure 1). Without lack of generality C will be assumed to be the center of mass of the platform with coordinates (x_c, y_c, z_c). We assume that the coordinates of B_i in the mobile frame are known.

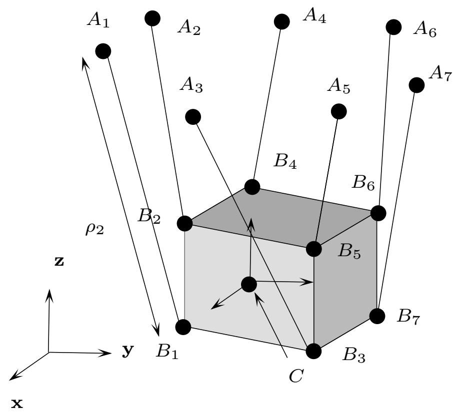

Fig. 1. A suspended CDPR

The proprioceptive measurement on such a robot is usually the cable lengths as other physical quantities such as orientation of the cables or their tensions are difficult to measure and are very noisy. The kinematic analysis of such robot is drastically influenced by the cable model that is used. We will assume here that the cables are mass-less and non deformable (which is a realistic assumption for some synthetic cables as soon as the size of the CDPR is not too large). We will also assume that the number of cables is larger than 6 in order, for example, to enlarge the size of the CDPR workspace. In this paper we consider a specific class of CDPR: *suspended CDPR* for which there is no cable that can exert a downward force, gravity being used as a virtual wire that is pushing the platform downward. In that case it has been shown that there will be at most 6 cables under tension at the same time [21]. The length of the cable will be denoted ρ_i and if it is not slack its positive tension will be τ_i. We will assume that the platform is submitted to gravity only so that the wrench $\mathcal{F}$ applied on the platform

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is $\{0, 0, -mg, 0, 0, 0\}$ where m is the mass of the platform. Let $\mathcal{S}$ be a sextuplet of cables that may support the platform at a given pose $\mathbf{X}$. The inverse jacobian matrix of the robot for this sextuplet will be denoted $\mathbf{J^{-1}}$ while $\boldsymbol{\tau}$ will be the vector of the cable tensions. The sextuplet will support the platform if

$$\rho_i = ||\mathbf{A_iB_i}|| \ \forall\, i \in \mathcal{S} \quad \rho_i > ||\mathbf{A_iB_i}|| \ \forall\, i \notin \mathcal{S} \qquad (1)$$

which indicates that the length of the cables under tension shall be exactly the distance between A_i, B_i while for the slack cables this length should be larger than this distance. At the same time the sextuplet should satisfy the mechanical equilibrium condition

$$\mathcal{F} = \mathbf{J^{-T}}\tau \qquad (2)$$

with $\boldsymbol{\tau}$ having only positive components. In this equation $\mathbf{J^{-T}}$ is the transpose of the inverse jacobian matrix whose i-th column is

$$((\frac{\mathbf{A_iB_i}}{\rho_i} \quad \frac{\mathbf{CB_i} \times \mathbf{A_iB_i}}{\rho_i}))$$

where C is the center of mass of the platform.

For a given pose there will be usually several sextuplets of cables that satisfy equations (1,2), that will be called *valid cable configurations (CC)*. Each of the valid cable configuration exhibits a different set of tensions in the cables together with different kinematics capabilities. Note that even a perfect knowledge of the cable lengths it is impossible to determine what is the current cable configuration as it depends on the history of the system [21] and that a cable configuration may have from 1 to 6 cables.

From a control viewpoint the concept of cable configuration is usually ignored although it has a high influence on the behavior of the robot. An alternative for a trajectory has been proposed in [27]: instead of being passive with respect to the cable configuration the trajectory is split in several parts for each of which a specific cable configuration is chosen and is guaranteed to be reached by imposing on the cables that are not part of the cable configuration to be slack, this being obtained by choosing as desired length value $||\mathbf{A_iB_i}|| + \Delta\rho$ where $\Delta\rho$ is large enough to ensure that in spite of the control and model errors the cable length ρ will be larger than $||\mathbf{A_iB_i}||$. Clearly whenever possible the selected CC should have 6 cables under tension to ensure full controlability of the robot.

The purpose of this paper is to propose criterion to determine the best CC with 6 cables under tension. The purpose of these criterion will be basically to state how much we may disturb the robot (in different meanings) while keeping it in the same CC. However these criterion do neither take into account the mechanical stability of the platform which is another topic [4], [7] or its dynamics [9], [33].

II. Ranking criteria for cable configuration

A. Determining the valid cable configurations

For a given pose it is easy to determine all valid CC with 6 cables: we consider each possible CC, solve the linear system (2) and store as valid CC only the one with positive tensions for all cables. For a time-dependent trajectory which is described by time functions we have exhibited an algorithm that allow to determine time intervals (and therefore part of the trajectory) for which a given CC is valid [17]. As an example we consider a circular trajectory for the robot that will be presented in section IV-A and figure 2 shows the circular arcs for which the various CC are valid.

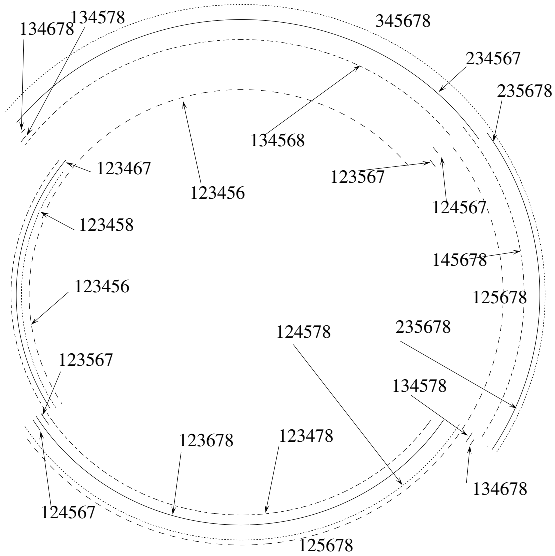

Fig. 2. The possible CC on a circular trajectory. The arc have the same radius but have been enlarged in order to show the different CC

B. First criteria for selecting the CC

Possible criterion for selecting the best CC may be based on kinematic properties. For example we may consider both for a pose, a trajectory or a surface:

- the minimum of the maximum of the cable tensions
- the minimum of the maximum positioning error. Assuming bounded errors $\boldsymbol{\Delta\rho}$ on the measurements of the $\boldsymbol{\rho}$ we may evaluate their effects on the positioning errors $\Delta\mathbf{X}$ by using the relation $\Delta\rho = \mathbf{J^{-1}}\Delta\mathbf{X}$

However we may adopt another point of view which is more related to the size of the disturbance that we may apply on the robot at a nominal pose with the CC being still valid.

B.1 For a pose

For a pose we propose as ranking criteria for the robustness of a CC the minimal radius of the singularity-free sphere centered at the pose $\mathbf{X_0}$ for which the CC is valid for any point included in the sphere under the assumption that the platform orientation is constant. This ranking criteria clearly characterize how much it is possible to move away from the current pose while keeping the same CC.

We define a pose on the sphere as

$$\mathbf{X} = \mathbf{X_0} + r\mathbf{u}$$

where r is the sphere radius and $\mathbf{u}$ an arbitrary unit vector. The tension of cable i is established with Cramer's theorem as

$$\tau_i = \frac{|\mathbf{J_i^{-T}}|}{|\mathbf{J^{-T}}|}$$

where $\mathbf{J_i^{-T}}$ is the matrix obtained by substituting column i of $\mathbf{J^{-T}}$ by the wrench $\mathcal{F}$. Consequently a pose will lie on the sphere with maximal radius if one of the two following condition hold:
1. $|\mathbf{J_i^{-T}}| = 0$
2. $|\mathbf{J^{-T}}| = 0$
These conditions may be simplified. Indeed consider the matrix $\mathbf{M}$ that is obtained by removing the ρ in the denominator of the elements of $\mathbf{J^{-T}}$. It is easy to show that $|\mathbf{J^{-T}}| = |\mathbf{M}|/\prod_{j=1}^{j=6} \rho_j$. Therefore the determinant of $\mathbf{J^{-T}}$ cancels at the same place than the determinant of $\mathbf{M}$ and both determinants have the same sign as the ρ are positive. Hence this matrix can be used in place of $\mathbf{J^{-T}}$. A similar reasoning may be used for $|\mathbf{J_i^{-T}}|$. With this simplification and under our constant orientation assumption an analytic form may be obtained both for $|\mathbf{J_i^{-T}}|$ and $|\mathbf{J^{-T}}|$. We will consider each condition separately. For the first condition we have to determine in turn for each cable the maximum of r under the constraints $|\mathbf{J_i^{-T}}| = 0$ and $||\mathbf{u}|| = 1$. To solve this optimization problem we define the Lagrangian function H

$$H = r + \alpha|\mathbf{J_i^{-T}}| + \beta(||\mathbf{u}|| - 1) \quad (3)$$

If the components of $\mathbf{u}$ are (X_1, X_2, X_3) the maximum of r will be obtained if

$$\frac{\partial H}{\partial r} = 0 \;\; \frac{\partial H}{\partial \alpha} = 0 \;\; \frac{\partial H}{\partial \beta} = 0 \;\; \frac{\partial H}{\partial X_i} = 0 \; \forall i \in [1,3] \quad (4)$$

which is a square system of 6 equations. We have

$$\frac{\partial H}{\partial r} = 1 + \alpha \frac{\partial |\mathbf{J_i^{-T}}|}{\partial r} = 0$$

hence α cannot be 0 as usually $\partial|\mathbf{J_i^{-T}}|/\partial r$ will is not 0. However as this quantity has usually have a relatively large value any α will be small. We have also

$$\frac{\partial H}{\partial X_i} = \alpha \frac{\partial |\mathbf{J_i^{-T}}|}{\partial X_i} + 2\beta X_i = 0$$

As α is small it may be assumed that β will also be small. The other unknown of the system are the X_i's that have to lie in the range [-1,1] and r that can also be bounded (e.g. r cannot be such that all B_i points are outside the vertically lifted convex hull of the A). Hence all the unknowns of the problem may be bounded and the system (4) may be solved in a guaranteed manner with interval analysis.

We may proceed in the same way if we are looking at the singularity free condition by substituting $\mathbf{J_i^{-T}}$ by $\mathbf{J^{-T}}$ (this has to be done only once as the denominator of τ_i is identical for all cables).

Note that it is possible to remove the constraint $||\mathbf{u}|| - 1 = 0$ by choosing

$$\mathbf{u} = (\sin(\beta_1)\cos(\beta_2), \sin(\beta_1)\sin(\beta_2), \cos(\beta_1))$$

with $\beta_1 \in [0, \pi]$ and $\beta_2 \in [0, 2\pi]$ that will lead to a system of 5 equations. However our trials have shown that this system is more difficult to solve than the one we have chosen.

B.2 For a trajectory with a constant orientation

We assume that the translation parameters of the pose (i.e. the coordinates of C) are analytical functions of the bounded time t i.e. $x_c = f_1(t)$, $y_c = f_2(t)$, $z_c = f_3(t)$. The coordinates of B_i in the absolute frame are

$$\mathbf{OB_i} = \mathbf{OC} + \mathbf{CB_i}$$

as the orientation is assumed to be constant $\mathbf{CB_i}$ is a constant vector and consequently the coordinates of B_i can be expressed as time functions. Our ranking criteria will be the minimal radius of a sphere centered at any pose on the trajectory so that the given CC is valid.

A preliminary check is to use the algorithm described in [17] for verifying if the trajectory may be completed with the same CC. To solve this optimization problem we use the same approach than for a pose except that we consider now time as an additional unknown. Hence we end up with a system of 7 equations in the 7 unknowns $r, X_1, X_2, X_3, \alpha, \beta, t$. However we have to consider that:
- the system may not have a solution and the minimal radius is obtained for one of the extreme value of t
- the system may provide a solution that lead to the maximal radius while the minimal one of is obtained at one of the end point of the trajectory

Hence to obtain the minimal radius it is necessary to use the algorithm described in the previous section for both end-point of the trajectory.

Note that as for the pose case all the unknowns may be bounded and interval analysis is an appropriate tool to find the roots of the system.

B.3 For a surface with a constant orientation

We assume that the translation parameters of the pose (i.e. the coordinates of C) are analytical functions of two bounded parameters t_1, t_2, i.e. $x_c = f_1(t_1, t_2)$, $y_c = f_2(t_1, t_2)$, $z_c = f_3(t_1, t_2)$ i.e. the robot moves on a surface. The coordinates of B_i in the absolute frame are

$$\mathbf{OB_i} = \mathbf{OC} + \mathbf{CB_i}$$

as the orientation is assumed to be constant $\mathbf{CB_i}$ is a constant vector and consequently the coordinates of B_i can be expressed as functions of the two parameters. Our ranking criteria will be the minimal radius of a sphere centered at any pose on the surface so that the given CC is valid for any pose within the sphere.

To solve this optimization problem we use the same approach than for a pose except that we consider now the parameters t_1, t_2 as additional unknowns. Hence we end up with a system of 8 equations in the 8 unknowns $r, X_1, X_2, X_3, \alpha, \beta, t_1, t_2$. As in the previous section the system may not have a solution or one that leads to the maximal radius. Hence it is necessary to use the algorithms described in the previous sections with t_1 and/or t_2 set to one of their extreme values.

Note that as for the pose case all the unknowns may be bounded and interval analysis is an appropriate tool to find the roots of the system.

C. *Second criteria for selecting the CC*

The previous criteria quantify the validity domain of a given cable configuration CC_1 with 6 cables. However it does not prove that for any trajectory included in the validity domain the robot will always have the cable configuration CC_1. Indeed the CC at a given time depends upon the control law, its discrete-time implementation and the time response of the coiling mechanism. For example we have shown that these elements may lead to CC changes when performing a trajectory that was theoretically feasible with a given CC both for non deformable cables [21] or for elastic cables [20] and such changes were experimentally observed. The previous criteria just allow to state that the robot may not change instantaneously from a CC with 6 cables to another one with also 6 cables because this situation is possible only at a singularity. But we have also to ensure that for given $\boldsymbol{\rho}$, a given pose $\mathbf{X}$ and a 6 cable configuration CC_1 there is no other CC CC_2 with less than 6 cables that is close enough from $\mathbf{X}$ so that a small disturbance may lead the robot from CC_1 to CC_2. Our ranking criteria will be a yes/no answer to the question: for a nominal pose $\mathbf{X}$ is there a solution to the forward kinematics of any cable configuration with 1 to 5 cables derived from the current CC in a ball centered at $\mathbf{X}$ with a fixed radius ?

For a given pose this amounts to show that all forward kinematics (FK) for any combination of less than 6 cables have no solution in a known neighborhood of $\mathbf{X}$. Although fully solving these FK problems is difficult and still not a settled issue we benefit here from the fact that we are looking only for pose within a bounded region and interval analysis will be appropriate.

D. *Ranking criteria for uncertainty management*

Real CDPR are submitted to uncertainties, namely in the the location of the A and B points and on the real cable lengths. We will assume in this section that these uncertainties are all bounded. To reach a given pose $\mathbf{X}$ the controller assume a knowledge of the location of the A, B and calculate the corresponding cable lengths $\boldsymbol{\rho}$ that are executed so that the cable reach the lengths $\boldsymbol{\rho^r}$ that differs from $\boldsymbol{\rho}$ by at most $\Delta\rho$. For the real robot the location of the i-th base attachment point is A_i^r which differs from A_i by at most ΔA. Similarly the real B_i will be denoted by B_i^r and differ from B_i by at most ΔB^r. We will also assume that a CC has been chosen and that the cable lengths of the cables not part of the CC have a length that ensure their slackness. When the cable control is executed the platform moves toward a pose $\mathbf{X^r}$ that is a FK solution of the robot whose parameters are $\boldsymbol{A^r}, \boldsymbol{B^r}, \boldsymbol{\rho^r}$. We will also assume that when moving toward $\mathbf{X}$ the robot starts with the CC and that there is no change of CC during the motion so that $\mathbf{X^r}$ is close to $\mathbf{X}$. The problem we address is to determine if CC is still valid at the pose $\mathbf{X^r}$ whatever are the $\boldsymbol{A^r}, \boldsymbol{B^r}, \boldsymbol{\rho^r}$ in their respective ranges.

A first problem is that $\mathbf{X^r}$ is not known and must only be such that

$$\rho_j^r = ||\mathbf{A_j^r B_j^r}|| \tag{5}$$

where B_j^r is a function of $\mathbf{X^r}$. In the same manner the matrices $\mathbf{J_i^{-T}}$, $\mathbf{J^{-T}}$ are no more scalar matrices. Furthermore when expressed as functions of the unknowns their determinants are very large expressions that cannot be used. As we want to check if the tensions in the cables of the CC are always positive whatever are the values of $\boldsymbol{A^r}, \boldsymbol{B^r}, \boldsymbol{\rho^r}$ the problem amounts to verify if for any cable in the CC the following condition does not hold:

$$\begin{gathered}\exists A^r \in \hat{A}^r, B^r \in \hat{B}^r, \rho^r \in \hat{\rho}^r \\ \text{such that } |\mathbf{J_i^{-T}}||\mathbf{J^{-T}}| \leq 0\end{gathered}$$

where the hated quantities indicates intervals. Using the minimal parametrization of $\mathbf{X}$ this problem has 48 unknowns (6 for $\mathbf{X^r}$, 18 for the A_i^r, 18 for the B_i^r and 6 for the ρ_i^r) with only a single inequality to verify but we don't have to determine all the poses that may satisfy the inequality as finding a a single one will be sufficient. We will detail in section IV-D how interval analysis may be used to check the inequality.

Theoretically the proposed approach may be extended to trajectory or surface but has not be implemented.

III. Implementations

A. *Interval analysis*

Solving the system that result from the optimization problem is performed by using our interval analysis library `ALIAS`. This library allows to calculate exactly (i.e. with an arbitrary accuracy) all solutions of a system of equations that lie within a bounded region, called the *search space*. Without going into the details (that may be found in [12], [13], [23], [24]) the solving principle is first based on the *interval evaluation* of the equations: being given intervals for the unknowns $\mathbf{W}$ (which define a *box* in the unknowns space) and a function of these unknowns $f(\mathbf{W})$ the interval evaluation of f is an interval $[U, V]$ that is guaranteed to satisfy that for all vectors $\mathbf{W}$ whose components all lie in the corresponding intervals we have $U \leq f(\mathbf{W}) \leq V$. There are several methods for computing such an interval evaluation, all having the drawback that U may be underestimated (i.e. the minimum of f over the intervals is larger than U) and/or V may be overestimated (i.e. the maximum of f over the intervals is larger than V). However the differences between U, V and the minimum, maximum decrease with the size of the input intervals. Such an overestimation occurs when there are several occurrences of the same variable in f. A typical example of overestimation is to consider $f = x - x$ when $x \in [-1, 1]$ as $f([-1, 1]) = [-1, 1] - [-1, 1] = [-2, 2]$ that indeed include the solution 0 but with a large overestimation.

Clearly if $U > 0$ or $V < 0$, then f cannot cancel for any point in the box. The second component of an interval analysis algorithm is the *branch and bound* scheme. In this scheme we have a list $\mathcal{L}$ of box(es) which has, at the start of the algorithm, a single element, the search space and an index i initialized to 1. The algorithm look at the i-th box in the list and calculate the interval evaluation of each equation of the system for this box. If for each of these evaluations we have $U < 0$ and $V > 0$, then we bisect the box in two by selecting one of the unknowns and splitting its current interval at the mid-point. This process creates two new boxes that are stored at the end of $\mathcal{L}$ and the index i is incremented. If $U > 0$ or $V < 0$ then the index i is incremented. After each bisection the size of the box decreases so that we may use the third tool of interval analysis which is the Kantorovitch theorem. It states that if some conditions, that may be calculated with interval analysis, are fulfilled, then the box includes a single solution of the system and that this solution may be obtained by using the Newton-Raphson scheme with as initial guess the center of the box. If this theorem is fulfilled for a given box we have determined a solution of the system and the index i is incremented. The algorithm completes when the index i is larger than the number of elements in $\mathcal{L}$. Such an algorithm cannot miss a solution and will usually provide all the solutions in the search space unless the numerical accuracy is not high enough (in this case it is necessary to extend the floating point arithmetic and numerous packages allow to do it).

This principle may be extended to deal with inequality. For example if the problem is to check if $f(\mathbf{W}) \leq 0$ a box will be deleted from the list $\mathcal{L}$ if $U > 0$ and a solution will be found if $V \leq 0$.

IV. Examples

A. The robot

As test example we consider a CDPR with 8 cables. The coordinates of the A, B points are provided in tables I,II and are derived from the robot presented in [10].

	A_1	A_2	A_3	A_4
x	-7.175120	-7.315910	-7.302850	-7.160980
y	-5.243980	-5.102960	5.235980	5.372810
z	5.462460	5.472220	5.476150	5.485390
	A_5	A_6	A_7	A_8
x	7.182060	7.323310	7.301560	7.161290
y	5.347600	5.205840	-5.132550	-5.269460
z	5.488300	5.499030	5.489000	5.497070

TABLE I. Coordinates of the A points (in meter)

	B_1	B_2	B_3	B_4
x	0.503210	-0.509740	-0.503210	0.496070
y	-0.492830	0.350900	-0.269900	0.355620
z	0.000000	0.997530	0.000000	0.999540
	B_5	B_6	B_7	B_8
x	-0.503210	0.499640	0.502090	-0.504540
y	0.492830	-0.340280	0.274900	-0.346290
z	0.000000	0.999180	-0.000620	0.997520

TABLE II. Coordinates of the B points in the mobile frame (in meter)

B. First and second criteria

In this section the orientation of the platform is assumed to be such that the axis vectors of the reference and mobile frame coincide.

B.1 For a pose

We consider the pose (0,1,2) and (1,0,2) for this robot. For (0,1,2) the following CC are valid: [1, 2, 3, 4, 5, 6], [1, 3, 4, 5, 6, 8], [2, 3, 4, 5, 6, 7], [3, 4, 5, 6, 7, 8]. For the first criteria we look at each cable of the CC in turn to determine the minimal distance between the current pose and a pose such that the tension in the cable become 0 ($|\mathbf{J_i^{-T}}| = 0$). We then look at the minimal distance between the current pose and and a singular pose ($|\mathbf{J^{-T}}| = 0$). The ranking for this first criteria will be the minimum of all these minima for all cables, The solving of the optimization problem provide the following solutions for the minimal r, r_{min} (the number in parenthesis indicates the cable number, d being the singular condition):

CC	[1, 2, 3, 4, 5, 6]	[1, 3, 4, 5, 6, 8]
r	4.4946264798 (1) 2.9411453193 (d) 4.050982548(2) 0.755452449 (3) 0.7600999421(5) 0.789817071 (4)	4.494269494(1) 2.9452771 (d) 0.759251662181 (3) 0.79751117 (6) 4.4942694944(7) 4.0509825480 (8)
r_{min}	0.755452449	0.75925166218

CC	[2, 3, 4, 5, 6, 7]	[3, 4, 5, 6, 7, 8]
r	4.0273984389 (2) 3.005642354 (d) 0.547094056 (4) 4.4946264798 (7)	0.759563273849 (5) 3.00951266946 (d) 0.780833856164(6) 4.0273984389697(8)
r_{min}	0.547094056081	0.759563273849

For (1,0,2) the following CC are valid: [1, 2, 5, 6, 7, 8], [1, 4, 5, 6, 7, 8], [2, 3, 5, 6, 7, 8], [3, 4, 5, 6, 7, 8]. For the first criteria the solving of the optimization problem provides the following solutions for the minimal r, r_{min}:

CC	[1, 2, 5, 6, 7, 8]	[1, 4, 5, 6, 7, 8]
r	2.940956688 (d) 4.977861655 (6) 0.63930665576 (7) 0.607434554380417 (8)	2.9452947689(d) 0.5191414561 (6) 0.6358634180(7)
r_{min}	0.60743455438	0.5191414561

CC	[2, 3, 5, 6, 7, 8]	[3, 4, 5, 6, 7, 8].
r	3.0087565393(d) 0.5343214029 (5) 4.686335251 (6) 0.5699690786(8)	3.01281464529 (d) 0.5324771755(5) 0.5606442268(6)
r_{min}	0.53432140	0.5324771755

The total computation time is 8mn and 26 seconds but note however that the implementation is not optimal. Indeed as soon as a possible value for r_{min} is found it may be used in the remaining computation to eliminate boxes that may contain solution (s) but will lead to a larger value than the current estimation of r_{min}. We have just tried this method for the configuration [1,2,5,6,7,8] where we have first the cables before considering the singularity condition: we have noted that after solving for cable 6 the calculation time for cable 7 decreases and even decreases much more significantly for cable 8. As for the singularity condition the algorithm exit almost immediatly. According to this experiment we believe that we may reduce the total computation by a factor of at least 10.

For both poses we have then considered the second criteria and we have shown that there is no CC with less than 6 cables that may lead to a pose that belong to a ball of radius 0.03 centered at the pose.

C. Trajectories and surfaces

We consider now the circular arc of radius 1 described by $x_c = \cos(t)$, $y_c = \sin(t)$, $z_c = 2$ when t is restricted to lie in the range $[0, \pi/2]$. The start point and end point of this trajectory being (1,0,2) and (0,1,2) we deduce from the previous paragraph that only [3,4,5,6,7,8] is a valid CC for the whole trajectory. The solving algorithm regarding the first criteria provides the solution:

	cable 5	cable 6	cable 7	$\lvert\mathbf{J^{-T}}\rvert = 0$
r	0.94811	0.9722	4.4942	3.0085

For the start and end point the minimal r were respectively 0.759563273849347 and 0.532477175518869. Hence we deduce that the solving provide the maximal r and therefore the ranking criteria for this CC is 0.532477175518869.

Now if we look at the planar circle centered at (1,0,2) and radius 0.4 the solving algorithm provides:

	cable 5	cable 6	cable 8
r	0.1324778 0.932476	0.16064 0.960643	4.6332

and hence the index is 0.132477813874347.

D. Uncertainty criteria

The purpose of the criteria is to check if a given CC is valid for a nominal pose $\mathbf{X}$ for which the nominal cable lengths are ρ, assuming a given model for the A, B points. For the real robot the coordinates of the A, B points may have any value in the known intervals $\hat{A}^r, \hat{B}^r$ while the real cable lengths lie in the known intervals $\hat{\rho}^r$. For testing the validity of the CC we will check is there is any values of $\boldsymbol{A^r}, \boldsymbol{B^r}, \boldsymbol{\rho^r}$ in their respective ranges that verify

$$|\mathbf{J_i^{-T}}||\mathbf{J^{-T}}| \leq 0 \tag{6}$$

for all cables involved in the CC. If this inequality hold for at least one cable the mechanical equilibrium condition (2) hold only for a negative tension the cable and therefore CC is no more valid.

As unknown for this problem apart of A^r, B^r, ρ^r we have the coordinates $\mathbf{CB^m}$ of the B in the mobile frame. To parametrize the pose of the platform we use a redundant parametrization with the coordinates of all B in the reference frame together with the coordinates of C in this frame. This leads to a problem with 63 unknowns (18 for the A, 18 for $\mathbf{CB^m}$, 18 for the B, 3 for C, 6 for the $\boldsymbol{\rho}$).

The motivation of this parametrization is the assumption that the FK solution for any A^r, B^r, ρ^r will be close to $\mathbf{X}$ and this allow us to bound the values of these unknowns. Furthermore we have the following constraints:

$$||\mathbf{A_j^r B_j^r}||^2 = ||\mathbf{OB_j} + \mathbf{OA_j}||^2 = (\rho_j^r)^2 \tag{7}$$

$$||\mathbf{CB_j}||^2 = ||\mathbf{OC} + \mathbf{OB_j}||^2 = ||\mathbf{CB_j^m}||^2 \tag{8}$$

for all cable j in the CC. We have also for all pairs of cables (i, j):

$$\begin{aligned} ||\mathbf{B_i B_j}||^2 &= ||\mathbf{OB_j} - \mathbf{OB_i}||^2 = \\ & \quad ||\mathbf{CB_j^m} - \mathbf{CB_i^m}||^2 \end{aligned} \tag{9}$$

$$\mathbf{CB_i}.\mathbf{CB_j} = \mathbf{CB_i^m}.\mathbf{CB_j^m} \tag{10}$$

Hence we get 42 constraint equations together with inequality (6).

Another motivation to use this parametrization is that it allows one to use a classical technique of interval analysis whose purpose if to reduce the interval for the unknowns without resorting to a bisection. For example consider the relation $||\mathbf{A^r_j B^r_j}||^2 = (\rho^r_j)^2$. If x_a, y_a, z_a and x_b, y_b, z_b denote respectively the coordinates of A_j, B_j in the reference frame the constraint may be written as

$$(x_b - x_a)^2 + (y_b - y_a)^2 + (z_b - z_a)^2 = (\rho^r_j)^2$$

which may be written as

$$(x_b - x_a)^2 = (\rho^r_j)^2 - (y_b - y_a)^2 - (z_b - z_a)^2$$

We proceed to the interval evaluation of the right hand-side of this constraint to get the interval $[U, V]$. If V is negative the constraint cannot be satisfied. If $U < 0, V > 0$ then $(x_b - x_a)^2$ must lie in the range $[0, V]$. We deduce that

$$(x_b - x_a) \in [0, \sqrt{V}] \quad \text{or} \quad (x_b - x_a) \in [-\sqrt{V}, 0]$$

or

$$x_b \in x_a + [0, \sqrt{V}] \quad \text{or} \quad x_b \in x_a + [-\sqrt{V}, 0]$$

Consequently if the interval for x_b is $\hat{x_b}$ the only valid part of this interval with respect to the constraint is

$$(\hat{x_b} \cap (\hat{x_a} + [0, \sqrt{V}])) \cup (\hat{x_b} \cap (\hat{x_a} + [-\sqrt{V}, 0]))$$

which may allow to reduce the size of $\hat{x_b}$. As for the determinant of the matrices we just proceed to the interval evaluation of their elements. Then classical methods of determinant expansion are used to calculate the interval evaluation of the determinant with the drawback that we have a large overestimation of the interval evaluation. But we use another approach based on the test for checking the regularity of an interval matrix (i.e. determining if there is at least one singular matrix among the all the matrices of the set) [28]. Basically this test amounts to calculate the sign of the determinant of a finite number of scalar matrices which are obtained by taking as elements the extreme values of the elements of the interval matrix: if the sign is the same for all determinants, then the interval matrix does not include singular matrices. If this is the case, then it is sufficient to select a scalar matrix in the set of interval matrix and to calculate the sign of its determinant that will be the sign of the determinant of any matrix in the set.

This algorithm has been used to check the robustness of the CC [3,4,5,6,7,8] at the pose (0,1,2) for a possible error on the location of the A of $\pm$ 5mm, an uncertainty on the ρ of $\pm$ 5 mm and an uncertainty of $\pm$ 1 mm for the location of the B. It was assumed that the B and C that may be solution of the FK where located in a ball centered on their nominal location with radius 5 cm. It was found that the CC was always valid in a computation time of 9mn58s.

Theoretically this algorithm may be extended to deal with a trajectory or a surface. For a trajectory the nominal pose will be a time function and we will add the time as additional unknown. For a given time interval we will be able to calculate ranges $\hat{\mathbf{X}}$ for the nominal pose and $\hat{B}$ for the nominal B that will allow us to determine ranges $\hat{\rho}$ for the nominal $\boldsymbol{\rho}$. For the other unknowns the ranges on $\mathbf{CB^m}$, A will remain the same while the ranges for ρ^r will become $\hat{\rho} + \Delta\rho$ and the ranges for B, C will become $\hat{B}, \hat{\mathbf{X}}$ that will be expanded by a value $W = K\text{Max}(w(\hat{\rho} + \Delta\rho), w(\hat{\mathbf{X}}))$ where w denote the width of an interval and K is a safety factor that can be conservatively chosen as 10. For example if $\hat{\mathbf{X}} = [p, q]$ the range for C will be chosen as $[p - W, q + W]$. A similar method may used for a surface.

V. Conclusions

Determining the cable configuration of a suspended CDPR is crucial for their command and safety as the CC influences drastically the cable tensions and positioning accuracy of the robot. A possible control strategy is to enforce a selected cable configuration (by setting the lengths of the cables not member of the CC so that they will be slack) to avoid the disturbances due to CC changes. But the choice of the CC shall take into account how robust is the CC. We have defined as robustness indices:

- how far away the robot may move from its nominal pose while keeping the same CC
- a binary index that is set to unsafe if there is in the vicinity of the nominal pose a pose that may be reached with less than 6 cables under tension
- a binary index that is set to unsafe if the CC is no more valid when uncertainties on the geometry and control of the robot are taken into account

We have then provided algorithms that allow one to calculate all these indices. As possible extension we should mention that the two first indices have been calculated by assuming that the orientation of the platform remains constant. This has allowed us to define the index as a length but is not appropriate if the orientation of the platform may change. For the robustness of the CC with respect to uncertainties we have only implemented the algorithm for a pose but shall investigate the case of a trajectory or a surface. A similar work may be done for elastic cables that may present as well slack cables. For large scale robot the sagging of the cable may have to be taken into account. The concept of cable configuration here shall be different has such a cable is never slack (in the sense that it is always submitted to a tension) but may contribute negatively to the support of the platform (i.e. pulling downward). For such a cable we may expect an even higher complexity as both the inverse and forward kinematics are not yet mastered [11], [15], [18], [30], [29]. Clearly the proposed indices should be completed by other one that address the mechanical stability of the CC.

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