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1 Introduction

Routing a fleet of robots in a known surface is a complex problem. It consists in the determination of the exact trajectory each robot has to follow to collect information. The objective is to maximize the exploration of the given surface. To ensure the robots can execute the mission in a collaborative manner, connectivity constraints are considered. These constraints guarantee that robots can communicate among each other and share the collected information. Moreover, the trajectories of the robots need to respect autonomy constraints.

Applications of this problem can be found in space scattered sensor visit problems for data collection and/or power up purposes. Again, robots can be used to restock particular resources to precise locations as, for instance, water in emergency areas or informations to soldiers in war zones ([1, 2]).

2 Problem definition

A set $\mathcal{K}$ of K robots needs to visit, over a discreet horizon $\mathcal{T}$, points on a triangular (isometric) grid $\mathcal{S}$ characterized by the length l of the equilateral triangles' edges that form it. Each robot is characterized by three parameters : its initial position $y_0 \in \mathcal{S}$, its autonomy A that limits the length of the robots' trajectory and the covering radius r that determines the area monitored by the robot.

At each time step, a robot moves from its current point $y \in \mathcal{S}$ to a point in its neighbourhood $\mathcal{N}_y$. $\mathcal{N}_y$ contains all the points that can be reached from y throughout an edge of $\mathcal{S}$. We suppose $y \in \mathcal{N}_y$, i.e., the robot is not obliged to move. When a robot stands on a point of the grid $\mathcal{S}$ we say the point is *visited*. When two robots stands on two vertices of $\mathcal{S}$ connected by the same edge, they are said *connected*.

The problem calls for the determination, over $\mathcal{T}$, of a trajectory for each robot in $\mathcal{K}$ in order to maximize the number of visited points respecting autonomy and connectivity constraints. While autonomy constraints are intuitive, connectivity constraints can be written as follows : for each $\mathcal{K}' \subset \mathcal{K}$ it exists a pair $(\bar{k}, k') \in (\mathcal{K} \setminus \mathcal{K}', \mathcal{K}')$ such that $y_{k'} \in \mathcal{N}_{y_{\bar{k}}}$, where y_k is the position of robot k.

2.1 A mathematical model for the multi-robot routing problem

Let us consider variables y_i^{kt} that equal 1 if the robot k is at position i at t, 0 otherwise. Variables x_{ij}^{kt} that equal 1 if robot k goes from i to j from period t to period $t+1$, 0 otherwise. Variables z_i equal 1 if point i is visited, 0 otherwise.

$$
\begin{aligned}
(RBT)\ \max \sum_{i\in\mathcal{S}} z_i & \qquad (1)\\
s.t.\ y_i^{k0} = y_0^k \quad \forall k \in \mathcal{K} & \qquad (2)\\
\sum_{i\in\mathcal{S}} y_i^{kt} = 1 \quad \forall k \in \mathcal{K},\ \forall t \in \mathcal{T} & \qquad (3)\\
\sum_{i\in\mathcal{S}} \sum_{k\in\mathcal{K}} y_i^{kt} = K \quad \forall t \in \mathcal{T} & \qquad (4)\\
y_i^{k(t-1)} \leq \sum_{j\in\mathcal{N}_i} y_j^{kt} \quad \forall j \in \mathcal{S},\ \forall t \in \mathcal{T}\setminus\{0\},\ \forall k \in \mathcal{K} & \qquad (5)\\
\sum_{j\in\mathcal{N}_i} x_{ij}^{kt} = y_i^{kt} \quad \forall i \in \mathcal{S},\ \forall t \in \mathcal{T},\ \forall k \in \mathcal{K} & \qquad (6)\\
\sum_{i\in\bar{\mathcal{N}}_j} x_{ij}^{kt} = y_j^{k(t+1)} \quad \forall j \in \mathcal{S},\ \forall t \in \mathcal{T},\ \forall k \in \mathcal{K} & \qquad (7)\\
\sum_{t\in\mathcal{T}} \sum_{i\in\mathcal{S}} \sum_{j\in\mathcal{N}_i} x_{ij}^{kt} \leq A \quad \forall k \in \mathcal{K} & \qquad (8)\\
\sum_{i\in S} \sum_{k\in\mathcal{H}} y_i^{kt} \leq K \sum_{i\in S} \sum_{k\in\bar{\mathcal{H}}} \sum_{j\in\mathcal{N}_i} y_j^{kt} \quad \forall \mathcal{H} \subset \mathcal{K}, \forall S \subset \mathcal{S}, |S| = |\mathcal{H}| & \qquad (9)\\
z_i \geq \sum_{t\in\mathcal{T}} \sum_{k\in\mathcal{K}} y_i^{kt} \quad \forall i \in \mathcal{S} & \qquad (10)\\
z_i, y_i^{kt}, x_{ij}^{kt} \in \{0,1\} \quad \forall i,j \in \mathcal{S},\ \forall t \in \mathcal{T},\ \forall k \in \mathcal{K} & \qquad (11)
\end{aligned}
$$

The objective function (1) maximizes the visited points of $\mathcal{S}$. Constraints (2) consider initial position of each robot. Constraints (3) and Constraints (4) impose that each robot stands in exactly one position at each time step and exactly K robots are used. Constraints (5)–(7) manage robots movements along the grid. Constraints (8) are autonomy constraints while Constraints (9) are the connectivity constraints. Constraints (10) force variables z_i to be zero if point i is not visited. Constraints (11) define the variables.

2.2 A two-phase solution method for the multi-robot routing problem

We propose a two-phase matheuristic in order to determine the trajectory of each robot. Let us suppose that exists $1 < \gamma \in \mathbb{N}$ such that $K = \gamma H$. First, we determine a triangular sub-grid $\mathcal{S}'$ of $\mathcal{S}$ where the length of the edge of $\mathcal{S}'$ is γl. We then consider H robots, called *parents* with a covering radius equal to γr. We then determine the trajectories on $\mathcal{S}'$ of these H robots.

Second, for each point visited by a parent robot h, we determine the sub grid $\mathcal{S}^h$ of $\mathcal{S}$ covered by h, i.e., all the points in $\mathcal{S}$ not further than γr from the position of h. We then calculate the trajectories for γ robots on $\mathcal{S}^h$. Composing these trajectories we obtain the final trajectory of each robot.

Preliminary computational tests have provided promising results and validation tests are currently being performed on a real platform.

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