Estimation of probabilistic worst case execution time while accounting OS costs

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***Abstract*—The arrival of modern and more complex processors (e.g., use of caches, multi- and many-core processors) increases the timing variability of tasks, i.e., the worst case execution time (WCET) is becoming significantly larger, while the probability of appearance of a worst case execution time is extremely low. Approaches taking into account this probability have been the topic of last years research by defining the probabilistic worst case execution time of a task as a probabilistic bound on all execution time scenarios. Existing measurement-based approaches consider the execution of the tasks in isolation and our contribution provides hints for accounting OS costs.**

I. Introduction and related work

During the last twenty years different solutions have been proposed to time critical system designers through a pessimistic estimation of performances of the processors (thus increased costs) while using average time behavior processors. For instance DARPA estimates that in 2050 the construction of an airplane with current solutions will require the entire defense budget of USA [1].

The arrival of modern and more complex processors (e.g., use of caches, multi- and many-core processors) increases the timing variability of tasks, i.e., the absolute worst case execution time is becoming significantly larger, while the probability of appearance of a worst case execution time is extremely low [4]. Approaches taking into account this probability have been the topic of last years research either by measurement-based reasoning [8] [9] [13] [6] or static reasoning [2] [3]. The preemption costs are studied in presence of randomized caches [7] or simple processors [12]. Another stream of work considers that the worst case execution time of a task obtained in isolation is not becoming larger in the presence of the operating system (OSs) [10]. Such operating system is called compositional. In our work we are interested in studying the estimation of WCET of a task while the OS does not have the property of composability.

Our contribution Our paper concerns the probabilistic estimation of the WCET for non-compositional OSs. We consider the case of independent tasks scheduled using an off-line scheduler on one processor. We use measurement-based approaches for the estimation of the WCET to guarantee an approach that is scalable on more complex processors.

Organisation of the paper Section II introduces the definition of the probabilistic worst case execution time of a task. Section III presents the main steps of a probabilistic measurement-based approach for the estimation of the probabilistic worst case execution time of a task. Section IV summarizes the main steps of our approach. We conclude in Section V.

II. Probabilistic worst case execution time (pWCET)

Definition 1. *The probabilistic execution time (pET) of the job of a task describes the probability that the execution time of the job is equal to a given value.*

For instance the j^{th} job of a task τ_i may have a pET

$$\mathcal{C}_i^j = \begin{pmatrix} 2 & 3 & 5 & 6 & 105 \\ 0.7 & 0.2 & 0.05 & 0.04 & 0.01 \end{pmatrix} \tag{1}$$

If $f_{\mathcal{C}_i^j}(2) = 0.7$, then the execution time of the j^{th} job of τ_i has a probability of 0.7 to be equal to 2.

The definition of the probabilistic worst-case execution time (pWCET) of a task is based on the relation $\succeq$ between two probability distributions, provided in Definition 2.

Definition 2. *[11] Let $\mathcal{X}$ and $\mathcal{Y}$ be two random variables. We say that $\mathcal{X}$ is worse than $\mathcal{Y}$ if $F_{\mathcal{X}}(x) \leq F_{\mathcal{Y}}(x)$, $\forall x$, and denote it by $\mathcal{X} \succeq \mathcal{Y}$.*

For example, in Figure 1 $F_{\mathcal{X}_1}(x)$ never goes below $F_{\mathcal{X}_2}(x)$, meaning that $\mathcal{X}_2 \succeq \mathcal{X}_1$. Note that $\mathcal{X}_2$ and $\mathcal{X}_3$ are not comparable.

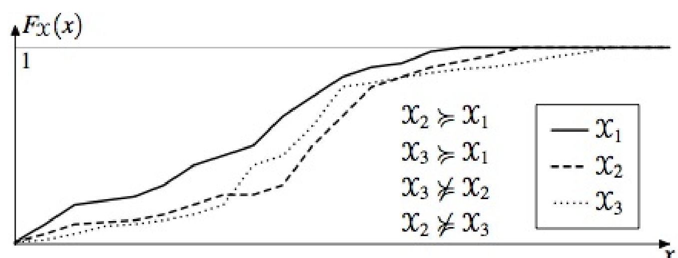

Fig. 1. Possible relations between the CDFs of various random variables

Definition 3. *The probabilistic worst-case execution time (pWCET) $\mathcal{C}_i$ of a task τ_i is an upper bound on the pETs $\mathcal{C}_i^j$ of all jobs of τ_i $\forall j$ and it may be described by the relation $\succeq$ as $\mathcal{C}_i \succeq \mathcal{C}_i^j$, $\forall j$.*

Graphically this means that the CDF of $\mathcal{C}_i$ stays under the CDF of $\mathcal{C}_i^j$, $\forall j$.

III. MEASUREMENT-BASED OF THE PWCET OF A TASK

A measurement-based approach for estimating the pWCET of a task has two main parts: (i) collecting the execution time traces of the task; and (ii) estimation the pWCET based on the set of execution time traces obtained during the first step.

The second step is mainly today done by using the Extreme Value Theory [8] [9] [13] [6]. This theory provides a pWCET estimation from a set of tasks by fitting the data to the closest Gumbel probability distribution. The Gumbel probability distribution is a particular case of the Generalised Extreme Value distribution which has the following Cumulative Distribution Function:

$$F_\xi(x) = \begin{cases} e^{-(1+\xi\frac{x-\mu}{\sigma})^{\frac{1}{\xi}}} & \xi \neq 0 \\ e^{-e^{-\frac{x-\mu}{\sigma}}} & \xi = 0 \end{cases}$$

This distribution is defined by three parameters: shape (ξ), scale (σ) and location (μ). If the shape parameter $\xi = 0$ then F_ξ is a Gumbel distribution.

Existing work considers the execution time traces obtained in isolation. Our contribution is obtaining the execution time traces while different instances of the task are executed in presence of an OS, whose preemption cost is accounted. For this purpose we propose a methodology for answering the first step (i) of a measurement-bases approach.

IV. ESTIMATION OF PWCET WHILE ACCOUNTING OS COSTS

This paper is an initial step forward solving the more general problem consisting in estimating the WCET of a task while OS costs are accounted. We consider here the same framework as the one defined in [12] (see page 720, Section VIII). The tasks are executed on the same processor. The scheduler policy is defined with respect to an off-line static table.

During the execution of the tasks, we consider two cases: (a) the preemption costs are contained within the WCET estimation as an exact absolute worst case value and (b) the preemption cost is not approximated within the WCET and we measure its possible different values.

For this precise case we know that the shortest feasibility interval with respect to the schedulability is defined by $(0, S_i + P)$, where

- $S_1 = O_1$;
- $S_i = \max\{O_i, O_i + \lceil \frac{S_{i-1} - O_i}{T_i} \rceil T_i\}, \forall i \in \{2, 3, \ldots, n\}$.

where P is the lcm of all periods [5].

The steps of our collection of execution times traces is done within the following steps:

STEP 1 The task is executed in isolation on the processor and a pWCET estimate is obtained using a method similar to [6].

STEP 2 The absolute worst case preemption cost is calculated as the maximum number of preemptions from all higher priority tasks.

STEP 3 The system is executed on the processor within the feasibility interval $(0, S_n + P)$, where n is the number of tasks.

We obtain two pWCET estimates: one solving problem (a) and a second solving a problem (b). The two estimates may be compared using a function like $CRPS = \sum_{i=0}^{+\infty}\{f_{pWCET_1}(i) - f_{pWCET_2}(i)\}^2$. This allows to calculate the pessimism of absolute worst case preemption costs with respect to the more finer approach that takes into account the variation of preemption costs during the execution.

V. CONCLUSION

We provide in this short paper a first solution for taking into account OS costs in WCET estimation of a task, without a good knowledge of the internal structure of the processor. Thus our method has the advantage of being scalable to more complex processor. The first extension of our work is the introduction of more complex processors like caches or pipelines and more complex schedulers.

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