Towards extraction of vibration-based damage indicators

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Abstract
After a brief review of vibration based damage identification methods, three different algorithms for damage identification are applied to the case of the benchmark Z24 bridge in this paper. Data-driven as well as model-based methods are discussed, including input-output algorithms for taking into account the effect of environmental and/or operational sources on the variability of damage features. A further class of data-driven methods that use finite element information is finally introduced as a possible future development.

1 INTRODUCTION

The structural health monitoring process involves the definition of proper 'damage indicators' that can be estimated based on data recorded by a network of sensors. Periodically repeating the estimation of features extracted from the data allows to detect their variation in time and to obtain indications of possible anomalies linked to damage. Most of the different vibration based damage identification methods proposed in literature can be classified as data-driven or model-based. Data-driven methods extract damage features relying solely on the recorded response and are hence attractive for adoption within real-time damage identification but the absence of a numerical model of the system often hampers the estimation of damage severity. Model-based methods rely on the updating of significant parameters of a finite element (FE) model on the basis of vibration measurements. They are usually less attractive for real-time damage identification, due to the computational cost of the updating process but the availability of a FE model allows for an estimation of damage severity and may provide useful information regarding the remaining service life of the structure. In recent years, a third group of damage identification methods has begun to emerge. It includes data-driven methods that use FE information to define the damage indicators without model updating thus combining the possibility of automated damage identification with model-based features. While having appealing theoretical properties, many of the combined methods are still in an early phase of development hence herein they will be just presented: their application to a real case such as the Z24 bridge is subject of future investigations. Changes in operational conditions related to loading (e.g. traffic intensity or wind speed), environmental conditions (e.g. temperature or humidity) or to the conditions of acquisition of responses (e.g. finite number of samples, measurement noise) may induce statistical variability of the damage features, which is often mistakenly attributed to damage. A number of approaches have been proposed for incorporating or discarding this variability, mainly classified in two categories: output-only methods relying solely on response measurements and input-output methods aiming at modeling the relationship between the response and the extracted features with respect to measured operational conditions. In this paper, a data-driven damage localization method, a model-based damage identification algorithm and an input-output method able to take into account the statistical variability of the damage features will be applied to the real case of the bridge Z24.

2 METHODS FOR VIBRATION BASED DAMAGE IDENTIFICATION

Data-driven method are inverse methods that use models based on experimental response data recorded on the structure instead of physical models. Damage-sensitive features are extracted from data and their changes used to identify damage in the structures so the damage parameter *D* in these methods is the variation of the damage feature *d* between the inspection I and the reference R configurations: $D = d_I - d_R$. With respect to model-based methods the main advantages of data-driven methods are that they do not require a finite element model and may be applied with a limited number of available signals (both responses and excitations). Depending on the signal-processing tool used to extract the damage features, data-driven methods can be classified in Fourier-based methods, Time series methods and Time-variant methods. In Time-invariant Fourier-based methods, Fourier analysis is used as the primary signal-processing tool and time-invariant models are defined to follow the structural behavior. In these methods, damage features are usually defined in terms of modal parameters (mainly frequencies, modal shapes and/or their derivatives or combinations of both frequencies and modal shapes, e.g. flexibilities) or in terms of Operational Shapes retrieved from Frequency Response Functions. An extensive review is reported in [1]. Time-series methods use statistical tools for developing mathematical models describing one or more measured random signals and analyzing their observed and future behavior. A more detailed description of this family of methods can be found in [2]. Time-variant methods develop time-variant models that allow to identify sudden changes in the system characteristics. They can be classified into three major groups: time-dependent models using models with time-dependent coefficients (Kalman filter), time–frequency methods that analyse time variations of the spectral quantities using, for example, the Wigher-Ville distribution and time–scale methods that decompose the signal based on a priori chosen functions, e.g. wavelets. A review of these methods can be found in reference [3].

In model-based methods, damage is identified through the updating of a finite element (FE) model ([4]-[6]). The basic premise is that structural damage results in a reduction of stiffness. The experimental data used for damage assessment through FE model updating most often consist of modal characteristics, which are extracted from measured response time histories using modal analysis techniques. Several types of modal data can be used. Most basically, model updating can be performed based only on natural frequencies or eigenvalues which are known to be affected by changes in structural stiffness, and can be measured fairly accurately. The model parameterization is usually limited to the following linear parameterization of the stiffness matrix:

$$K(\theta) = K_0 + \sum_j^N \theta_j K_j \quad (1)$$

where each K_j is the contribution of a single substructure to the global stiffness matrix and θ_j is a scaling representative of its effective stiffness. The measured data and computed data are confronted in a cost function $F(\theta)$. Many alternative formulations are possible, but most often it is expressed as a weighted least squares fit between predictions and data:

$$F(\theta) = \frac{1}{2}\eta(\theta)^T W\, \eta(\theta) \quad (2)$$

where η is the vector containing the residuals between predicted and measured data (natural frequencies, model shapes, etc) and W is a weighting matrix. In most practical applications, a diagonal weighting matrix is chosen, so that the cost function becomes a sum of squared residuals. The optimal value θ^* of the parameter vector is determined through the solution of a non-linear least-squares problem:

$$\theta^* = \arg \theta \min F(\theta) \quad (3)$$

Local gradient-based optimization methods are most often used to solve the optimization problem. For modal data such as natural frequencies and mode shapes, the gradient can be efficiently calculated analytically, avoiding the use of finite differences. Model updating is thus an inverse problem and is often prone to ill-posedness and ill-conditioning. Accounting for uncertainties due to measurement and modeling errors is therefore essential [6].

A critical issue relating to vibration-based SHM pertains to the susceptibility to the varying environmental conditions [7]. Available methodologies for incorporating or discarding the operational variability from models and indicators of structural response are primary classified into two classes: i) output-only methods (unsupervised learning) aiming to eliminate the influence of operational factors on the basis of vibration response measurements and/or extracted features [8] and ii) input-output methods (supervised learning) modeling the relationship between the measured vibration data and/or the extracted features with respect to measured operational conditions [9]. In the former category methods such as the Principal Component Analysis (PCA) [10] or its nonlinear ramifications (kernel PCA, Factor Analysis) and others have been employed for solving the problem by searching and discarding patterns, which reveal the influence of the unobserved input variables. The supervised learning alternative typically involves formulation of a regression problem. Information on the inputs may be extracted by deploying a small number of sensors tracking environmental agents or traffic/train crossing loads, along with the array of vibration sensors monitoring structural response.

3 DAMAGE IDENTIFICATION ON THE Z24 BRIDGE BENCHMARK

The aforementioned methods for damage identification are now illustrated for the benchmark case of the Z24 bridge [11] which was located in the canton Bern near Solothurn, Switzerland. It was part of the road connection between the villages of Koppigen and Utzenstorf, over-passing the A1 highway between Bern and Zürich. It was a classical post-tensioned concrete two-cell box-girder bridge with a main span of 30 m and two side spans of 14 m (Figure 1). The bridge was built as a freestanding frame with the approaches backfilled later. Both abutments consisted of triple concrete columns connected with concrete hinges to the girder. Both intermediate supports were concrete piers clamped into the girder. An extension of the bridge girder at the approaches provided a sliding slab.

The bridge, which dated from 1963, was demolished at the end of 1998, because a new railway adjacent to the highway required a new bridge with a larger side span. To monitor the bridge dynamics over a one-year timespan, 16 accelerations were measured on the bridge at different points and in different directions. Every hour, a sequence of 65536 acceleration samples, taken at the 16 sensors, was collected. Some accelerometers showed a considerable drift and a few of them failed during operation. Since the aim of the long-term monitoring test was quantifying the environmental variability of the bridge dynamics, all quantities considered of possible importance for the bridge dynamics were monitored. Since temperature was known to have a key influence on the dynamics of civil engineering structures, the bridge's thermal state was monitored in particular detail. In order for the subsequent progressive damage tests to be significant, it was made sure that they were relevant for the safety of the bridge and that the simulated damage occurred frequently. Figure 2 provides an overview of all progressive damage tests and illustrates two of them in detail. Before and after each applied damage scenario, the bridge was subjected to a forced and an ambient operational vibration test. With a measurement grid consisting of a regular 3 × 45 grid on top of the bridge deck and a 2 × 8 grid on each of the two pillars, 291 degrees of freedom have been measured: all displacements on the pillars, and mainly vertical and lateral displacements on the bridge deck.

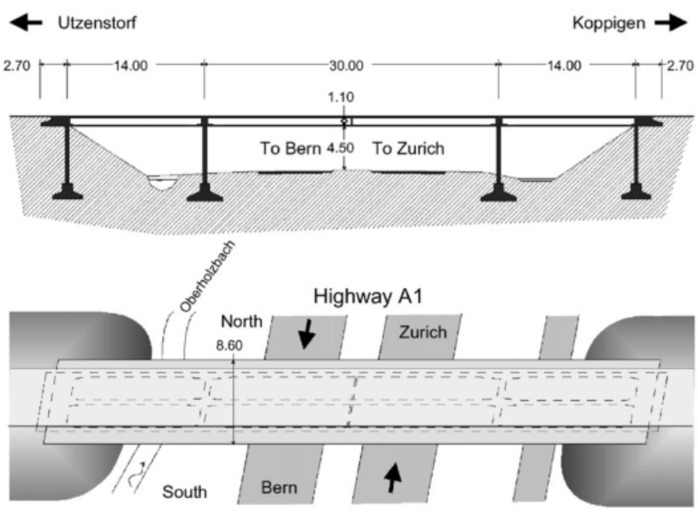

Figure 1: Front (top) and top view (bottom) of the Z24 bridge [11]

Date (1998)	Scenario
4 August	Undamaged condition
9 August	Installation of pier settlement system
10 August	Lowering of pier, 20 mm
12 August	Lowering of pier, 40 mm
17 August	Lowering of pier, 80 mm
18 August	Lowering of pier, 95 mm
19 August	Lifting of pier, tilt of foundation
20 August	New reference condition
25 August	Spalling of concrete at soffit, 12 m^2
26 August	Spalling of concrete at soffit, 24 m^2
27 August	Landslide of 1 m at abutment
31 August	Failure of concrete hinge
2 September	Failure of 2 anchor heads
3 September	Failure of 4 anchor heads
7 September	Rupture of 2 out of 16 tendons
8 September	Rupture of 4 out of 16 tendons
9 September	Rupture of 6 out of 16 tendons

Figure 2. Chronological overview of applied damage scenarios (left), settlement system used for damage scenarios 3 - 6 (right top) and cutting of tendons for scenarios 15 - 17 (right bottom). [12].

3.1 Damage identification on the Z24 by the data driven Interpolation Method

The Interpolation Damage Detection Method is a method of damage localization based on the use of a smooth function to interpolate the deformed shape of the structure. Specifically a damage feature is defined in terms of the variation, between a reference and an inspection configuration, of the error related to the use of an interpolating cubic spline function. The method does not need a physical model of the structure but merely response data recorded on the structure. In reference [13] a more detailed description of the IDDM is reported that, in its original version, was applied to Operational Deformed Shapes. Herein the extension of the method for the use with modal shapes is applied [14]. The damage feature is defined as the interpolation error $E(z)$ computed as the sum, over the n identified modes, of the 'modal interpolation errors':

$$E(z)=\sum_{i=1}^{n}\sqrt{\left[\phi^{(i)}{}_R(z)-\phi^{(i)}{}_S(z)\right]^2} \tag{4}$$

where $\phi^{(i)}{}_R$ and $\phi^{(i)}{}_S$ are the magnitudes of respectively the i-th identified modal shape and its spline interpolation. An increase of $E(z)$ in the inspection configurations with respect to the reference one at a certain location z, highlights a localized reduction of smoothness at z hence a loss of stiffness. Herein the method has been applied to detect the location of damage for the scenario inflicted to the Z24 bridge on August 18 (Figure 2) that is the settlement of the foundation of one of the supporting piers (at 95 mm). The modal shapes of the bridge deck identified from output-only (acceleration) data for the undamaged and for the damaged configuration by Reynders et al. [15] have been used. Figure 3 shows the results obtained considering in equation (4) respectively a total number of 1 and 5 modes. The threshold is calculated as $\Delta E_T(z)=\mu_{\Delta E}+1\cdot\sigma_{\Delta E}$ being $\mu_{\Delta E}$ and $\sigma_{\Delta E}$ respectively mean and standard deviation of the variation of the damage feature at all the instrumented locations. For the application reported herein only the components of the modal shapes along the central axis of the deck have been considered. The damage feature exhibits values higher than zero both at the damaged location and in other not damaged locations but the definition of the threshold allows discarding almost all the false alarms. At the increase of the number of modes the localization of damage becomes more refined (see Figure 3, right). Similar results have been obtained considering 3 and 4 modes. On the contrary it must be noted that if the contribution of mode 9 is added, the correct location of damage is still found but two false alarms appear at the ends of the girder.

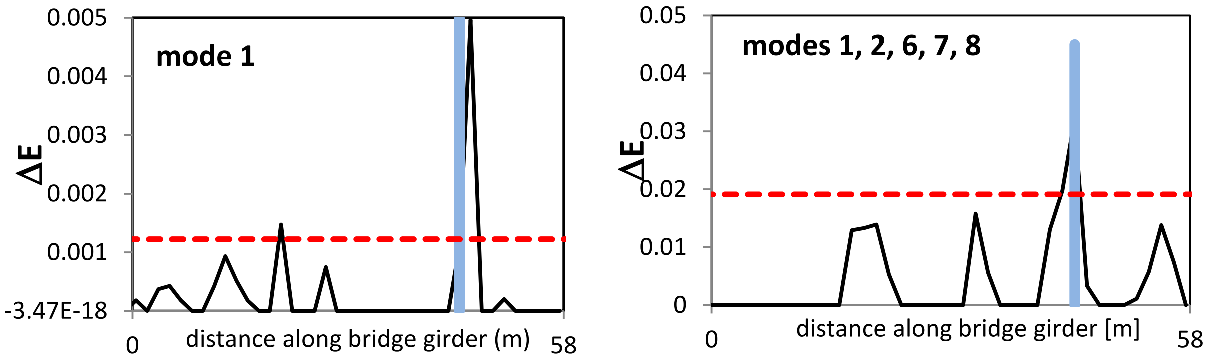

Figure 3. Values of the damage feature ΔE along the bridge girder considering 1 mode (left) and 5 modes (right). damage parameter threshold damage location

3.2 Damage identification on the Z24 by model updating

For model-based damage identification, the bridge has been modeled with a beam model (6 degrees of freedom in each node) [11]. The girder was modeled with 82 beam elements; 44 beam elements were used to model the piers, columns and abutments. The concrete was considered to be homogenous; the initial values for the Young's modulus and shear modulus were taken as 37.5 GPa and 16 GPa, respectively. The principal axes of the piers were rotated to model the skewness of the bridge, and the width of the piers was taken into account by constraint equations. Mass elements were used for the cross girders and foundations.
Both concentrated translational mass and rotary inertial components were considered. In order to account for the influence of the soil, springs were included at the pier and column foundations, at the end abutments and around the columns.
Before and after each progressive damage test, a forced and ambient vibration test had been performed on the bridge in order to experimentally characterize the evolution of the natural frequencies and mode shapes. A total of 291 degrees of freedom were measured during each test. From the measured acceleration data, the modal characteristics were identified using reference-based stochastic [16] or combined [17] subspace identification.
In order to perform damage identification, the Young's modulus and shear modulus of the bridge deck were parameterized so as the make them vary along the length of the bridge in a piecewise linear way, with 7 unknown coefficients.
This makes a total of 14 updating variables. They have been tuned to minimize the difference between the identified modal characteristics and the modal characteristics computed with the finite element model as quantified by the least-squares cost function given by equation (3).
Modal updating has been performed both in the undamaged condition, and after the pier settlement of 95 mm (see Figure 4).
The updating has been performed both with the limited set of 6 modes obtained from the ambient vibration tests, and with a more elaborate set of 9 modes, obtained from the forced vibration test.
Comparing the identified stiffness in undamaged and damaged conditions (Figure 5, [15]) shows that a considerable loss of stiffness was identified after the pier settlement.

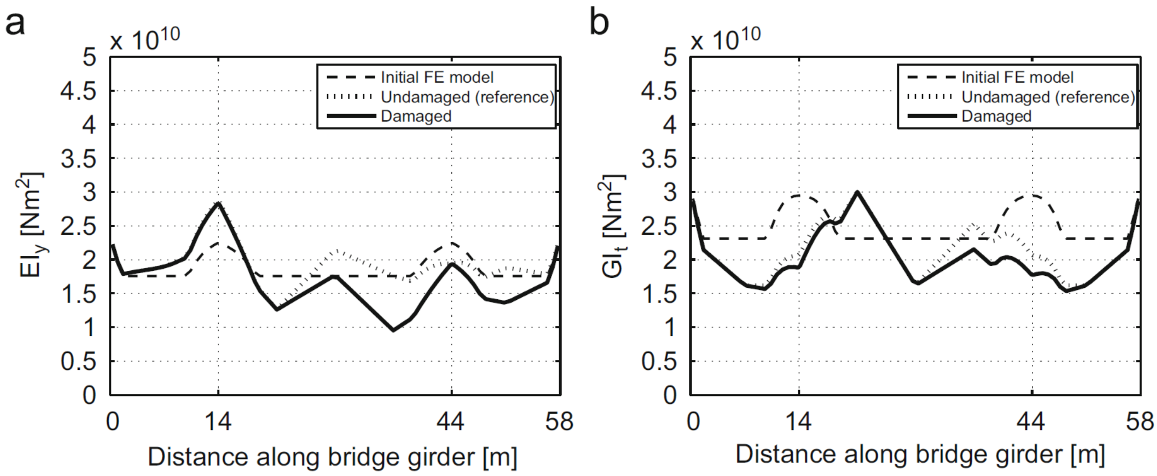

Figure 4. Identified bending stiffness (a) and torsional stiffness (b) of the bridge deck, as obtained from modal updating in the undamaged condition, and after a pier settlement of 95 mm., using forced vibration data [15].

3.3 A framework incorporating influence of environmental parameters

As an example of the supervised approach defined in section 2 for including the operational variability in the identification of damage features, Spiridonakos et al. [18], [19] propose the formation of a functional representation between inputs and outputs, tracking the behavior of the structure under study for a wide range of operational conditions. The algorithm relies on the expansion of appropriately selected structural features, such as modal characteristics of the healthy structure, onto a polynomial chaos basis (PCE) which conforms to the probability space of the measured influencing agents. Following the training phase, estimated statistical properties, such as the prediction error, may serve as condition indicators for the monitored system warning of irregular behavior or revealing evidence of a damaged/deteriorated system. For the Z24 case, the temperatures measured at six locations at the center of the middle span along with the air temperature serve as input variables, while the first four natural frequencies serve as outputs.

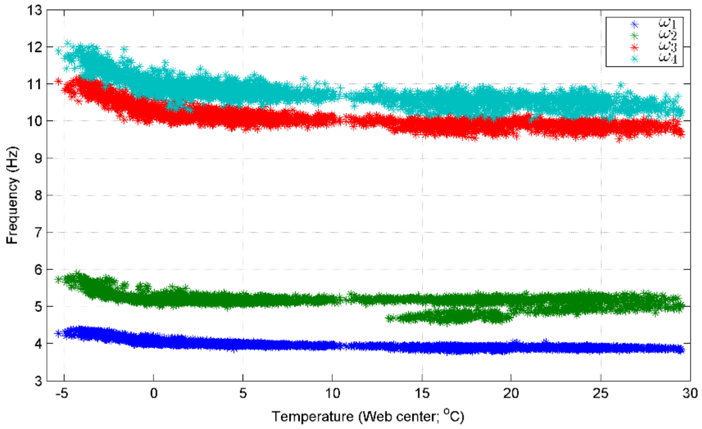

Figure 5. First four natural frequency estimates vs. temperature.

These are estimated through Stochastic Subspace Identification (SSI). The dependence of natural frequency estimates on the temperature is reported in Figure 5 indicating a bilinear temperature – natural frequency dependence, which is more influential than the effect of damage (discernable in the second mode). Independent Component Analysis (ICA) is implemented, for transforming the correlated input (temperature) measurements into independent latent variables. Additionally, in order to come up with a reduced order damage indicator the ICA method is again employed, inferring a reduced number of salient output quantities. The ICA-transformed input and output variable sets are fed into the PCE tool for identifying a model of their inter-relation. The estimation set comprises 1500 values from the first eight months of the monitoring period, while the remaining values are used for validation. A single input latent variable is enough for representing the temperature measurements, and the same applies for the natural frequency set. In Figure 6 the PCE-ICA prediction is plotted against the "true" value, derived from actual measurements. In the lower

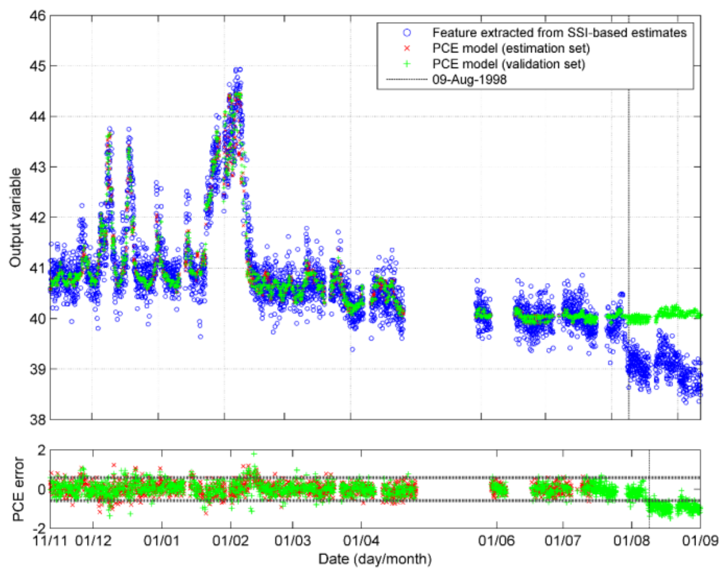

Figure 6. Extracted (measured) feature variable plotted against the ICA-PCE estimates (top) and the extracted damage index relying on the PCE prediction error - damage occurrence is noted via the black vertical line.

plot of Figure 6, a single damage index is delivered, defined as the error between measurement and prediction. Prior to the occurrence of damage (noted via a black vertical line), the error lies within prescribed thresholds, apart from isolated outliers. In the period following the point where the 1st damage occurs a persistent offset is manifested, serving as evidence of damage initiation and progression.

4 FURTHER DAMAGE LOCALIZATION METHODS: COMBINED DATA-DRIVEN AND MODEL-BASED

Besides the methods presented in this paper, a third group of methods for damage localization and quantification has started to develop in recent years that lies between the data-driven and model-based approaches. This class is based on data-driven features from measurements of the reference and damaged states, which are confronted to a FE model of the investigated structure to define damage indicators for the elements of the FE model, without updating its parameters. These methods try to unite advantages of both data-driven and model-based methods:

- Data-driven features are used as in data-driven approaches, together with some information from a FE model in addition. This leads to damage indicators that are strongly related to the measurements, but that are also built on the basis of physical information, lying beyond the geometry defined by the sensor positions. Furthermore, the application to arbitrary structural types is possible as with model-based methods, without limitations regarding type (e.g. as beam or plate-like).
- The requirements on the accuracy of the FE model are less strict than for model-based methods, since the parameters of the reference FE model are not updated. The analysis with respect to the FE model is less profound compared to an updating approach.
- Possible ill-posedness as in the updating problem is avoided by dividing the problems of damage localization and quantification into two separate steps.

A careful definition of the data-to-model distance measure and its statistical evaluation are required. Two methods are briefly sketched in the following as examples.

1) The *damage locating vector* (DLV) approach systematically interrogates changes in the structural flexibility in order to locate damages. In its generalization to output-only data [20], a vector is estimated in the null space of the transfer matrix difference between reference and damaged states, which is entirely obtained from the data. The theory shows that when applying this vector as a virtual load to the FE model of the structure, the resulting stress over the damaged elements is zero. From this property, a damage indicator is defined for each of the elements of a model, and damage is located where the indicator is zero. Since the indicator is a random variable computed from measurements, it is compared to zero in a statistical test [21]. Once damage is located, it can be quantified in the damaged elements.

2) *Statistical fault isolation methods* with a background in automatic control offer a theoretical framework for damage localization, where the problem is formulated as follows: *given measurements from a reference and a damaged state, together with a structural parametrization from a FE model, which of the parameters have changed?* The conceptual difference to FE model updating is that the information on the value of the parameter change is not asked for in the first step, only the information *if* a parameter has changed, which is carried out by statistical hypothesis testing. In particular, such a localization method is based on a residual function that denotes the distance of a data-driven reference model to newly collected data [22]. This data-based residual is then tested for a change along directions that are defined by the model-based sensitivities of structural parameters using a statistical χ^2

test–parameters associated with the damaged region result in large values of the test. Once damage is located, it can be quantified by estimating the absolute parameter change only in the damaged elements [23]. While having appealing theoretical properties, many of the combined methods are still in the beginning of their development. Numerical examples and applications on structures in the lab show promising results (see references above). The next step should involve application studies on structures in the field such as Z24 Bridge.

5 CONCLUSIONS

In this paper three vibration based approaches to damage assessment at different levels (detection, localization and quantification) have been applied to the case studyof the Z24 bridge. A model-based approach based on the updating of a FE model in the damaged state allowed the quantification of the extent of damage (in terms of reduction of the bending and torsional stiffness) beyond the detection of its presence and location. A data-driven approach enabled the correct localization of damage but not its quantification while a framework able to take into account environmental variability was employed for damage detection only based on modal frequencies. Generally speaking, data-driven methods are able to capture the loss of stiffness, not of strength, and are therefore inadequate for predicting the remaining service life of the structure. On the other side model-based methods are hardly compatible with real time on line damage identification algorithms hence for adoption within an automated monitoring system for which data-driven method are more attractive due to the lower computational effort related to the estimation of the damage parameters. In terms of the methods applicable in accounting for the influence of environmental and operational conditions, it should be noted that the majority of these methods are formulated in the context of detection (previously mentioned stage (1)). The incorporation of spatial information (such as mode shapes) as well as a link to model-based approaches would be beneficial for progressing to stages (2) and (3) of the damage assessment procedure.

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